

# *Collective* light-matter interactions

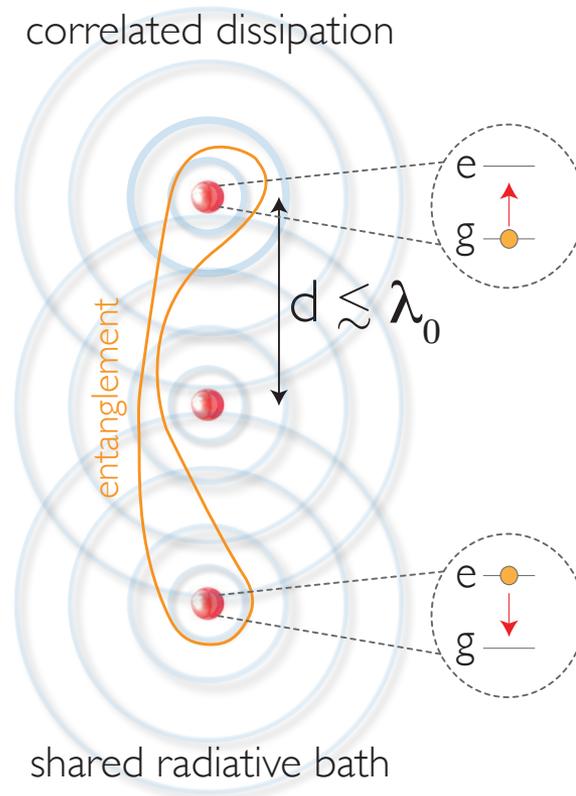
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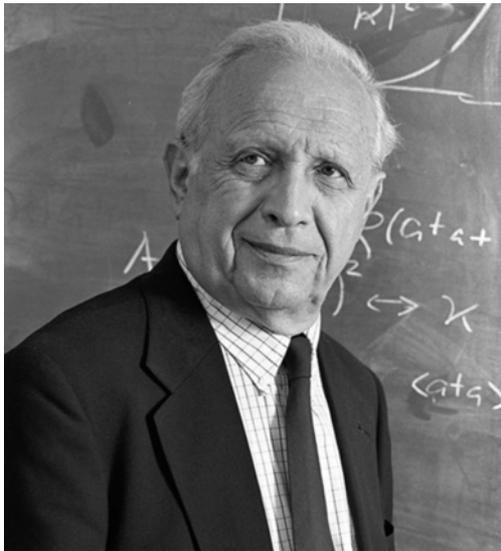


# Collective light-matter interaction: the physics of correlated dissipation



main idea: dissipation as a resource

# A remarkable insight...

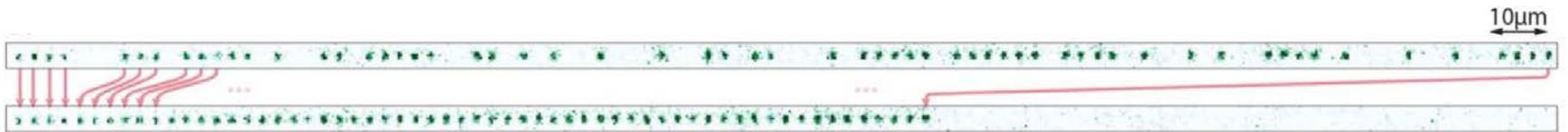


Glauber, Les Houches  
summer school, 1964

“[...] all of the delicate and ingenious techniques of optics are exercises in the constructive use of noise”

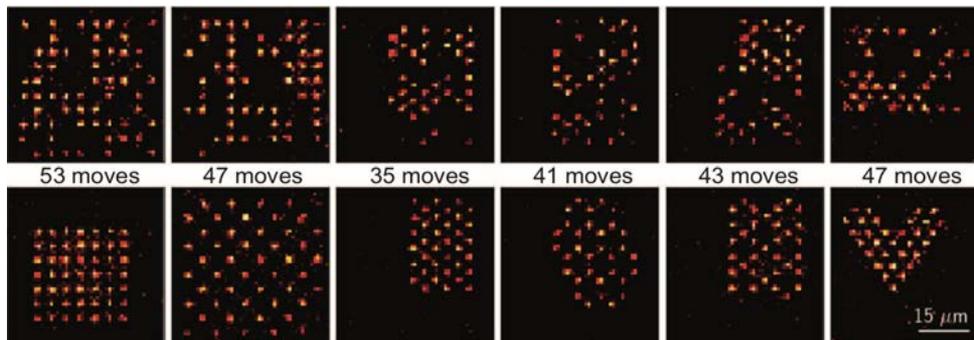
~dissipation

# Recent experimental developments



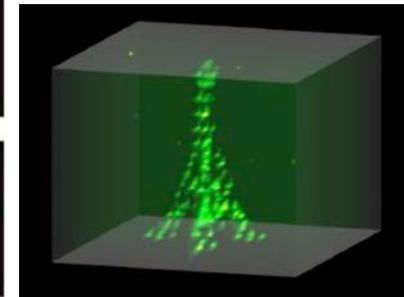
1D

Endres et al., Science **354**, 1024 (2016)



2D

Barredo et al., Science **354**, 1021 (2016)



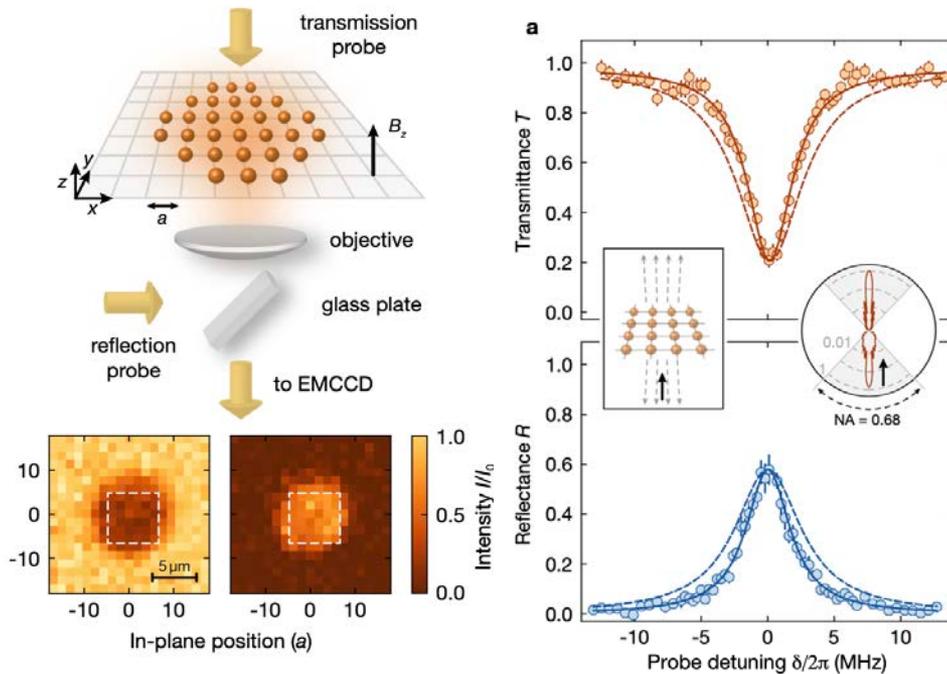
3D

Barredo et al.,  
Nature **561**, 79 (2018)

and many others....

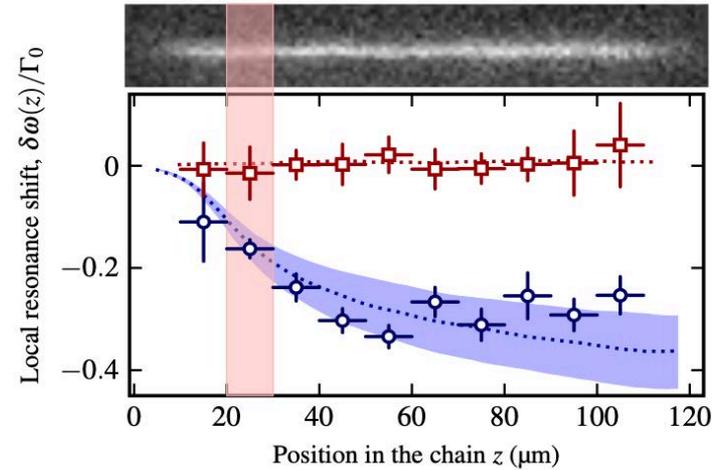
# Recent optical experiments in ordered arrays

## 2D atomic array as a perfect mirror



Rui et al., Nature 583, 369 (2020)

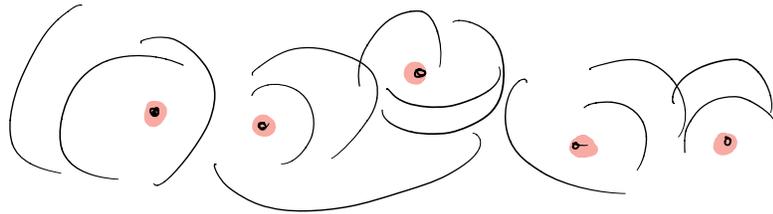
## Collective frequency shift in 1D arrays



Glicenstein et al., PRL 124, 253602 (2020)

## Collective atom-atom interactions mediated by light

Today's lecture is about a specific driven-dissipative system, ensembles of atoms interacting (collectively) with light



Dissipation in the form of photon emission can be a resource - why is this physics interesting  $\Rightarrow$  physics of interference.

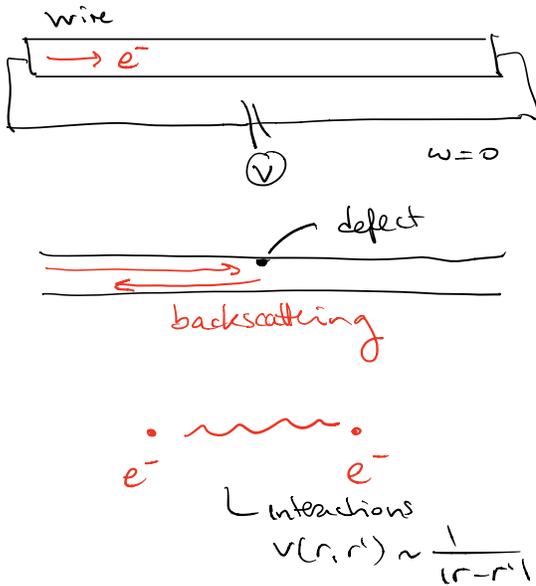
- Fundamental science: many-body physics in open systems  
Emergence of macroscopic quantum coherence via dissipation
  - \* Superradiance
  - \* Subradiance
- Applications: ensembles as efficient light-matter interfaces
  - Quantum communication, computing: memories, photon-photon gates
  - Metrology
  - Generation of quantum states of light
  - Characterization of many-body dynamics via the light that comes out of the system.

Outline:

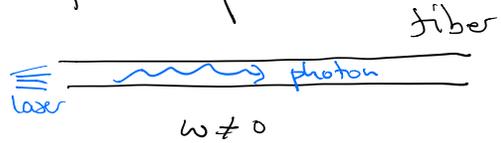
- How are optical (open) systems different from condensed matter (closed) systems?
- Basic formalism
- Collective atomic states = super and subradiance
- Many-body decay

# 1. Optical (open) vs condensed-matter (closed) systems

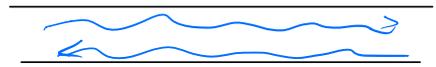
Condensed matter system



Optical system



optical systems are inherently open!



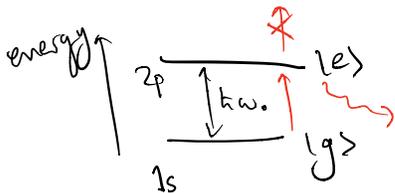
photons do not interact with each other: they need matter to do so.

Atomic ensembles: light-matter interfaces that provide non-linearities and where dissipation can be used as a resource.

## 2. Basic formalism

### 2.1. Atoms as "quantum" dipoles

Atomic spectrum is very anharmonic  $\Rightarrow$  atoms are highly non-linear "2-level systems"



decay rate (inverse time to emit a photon)

atom decays bc it is embedded in a continuum of ~~all~~ modes

(in a cavity, dynamics is reversible)  
"good"

Typical energy scales

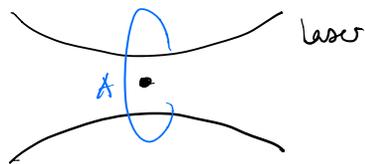
frequency:  $\omega_0 \sim 300 \text{ THz}$   
 linewidth:  $\Gamma_0 \sim 10 \text{ MHz}$

$$Q = \frac{\omega_0}{\Gamma_0} \sim 10^7 \text{ very high } Q \text{ resonators}$$

- These numbers depend on specific atom (typical are alkali, Cs, Rb)
- Both can be tuned by environment (Purcell enhancement, Lamb shift)
- atoms, if not illuminated, are in the ground state
- Artificial atoms: quantum dots, color centers, SC qubits
- Atoms interact with light via an induced dipole moment

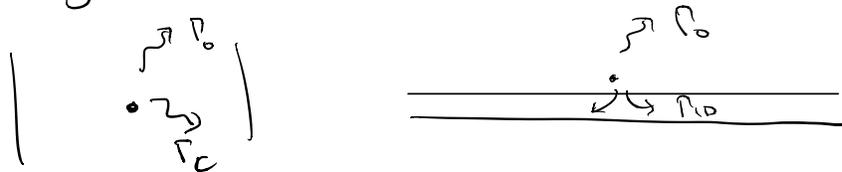
$$\begin{array}{c} \sim \overline{g} \begin{array}{c} e \\ \updownarrow \\ g \end{array} \sim \vec{p} \propto \vec{p} \sigma_{ge} \\ \text{transition} \quad \swarrow \quad \searrow \\ \text{matrix element} \quad \quad \text{atomic coherence operator } |g\rangle\langle e| \end{array}$$

- Atoms interact weakly with light



probability of photon absorption  
 $\sim \lambda^2 / A \ll 1$   
 due to diffraction limit

That is why people interface them with cavities or waveguides



Decay depends on the environment (Purcell)

- If atoms are close to each other (in free space) they radiate collectively (Dicke)

$\leftarrow d \ll \lambda \rightarrow$  Superradiance / subradiance  


- Question of today:

what happens in extended, ordered atomic arrays?

o o o o o o o

"low symmetry" scenario

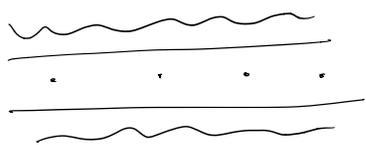
(different from cavity)

Complicated question because:

- Hilbert space atoms  $2^N$
- Infinite # of photon modes

Not easy  $\Rightarrow$  but we can integrate out photons and reduce complexity

## 2.2. "Spin" model for atom-atom interactions



Let's imagine we have atoms in some given dielectric medium such as vacuum, cavity, or something more complicated.

Traditional approach: # that has both atomic and photonic degrees of freedom. Find normal mode decomposition of field and associate bosonic  $\hat{a}^\dagger, \hat{a}$  to each mode.

- This is fine if only few modes are relevant (like high Q cavity)

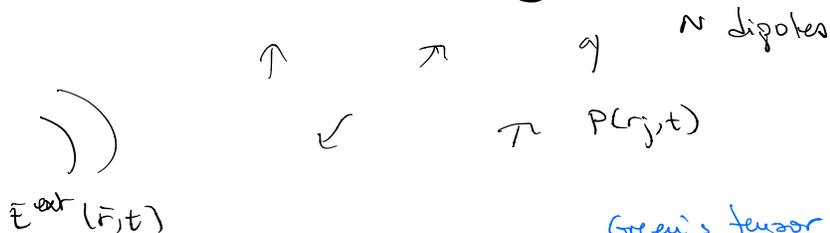
- However, it is complicated if there are many modes or if the expression of modes is difficult (complex dielectric structures)

- We want a more general approach  $\Rightarrow$  quantization using Green's functions

$\Rightarrow$  solve EM problem first (integrate out photonic degrees of freedom)

$\Rightarrow$  then do Quantum mechanics.

• Handway approach = "quantizing classical dipoles"



Generally, for a collection of dipoles:  $\checkmark$  Green's tensor: propagator of EM field. depends on dielectric environment

$$\vec{E}(\vec{r}, \omega) = \underbrace{\vec{E}^{\text{ext}}(\vec{r}, \omega)}_{\text{drive}} + \mu_0 \omega^2 \sum_{j=1}^N \underbrace{G(\vec{r}, \vec{r}_j, \omega)}_{\text{scattered field}} \cdot \vec{p}(\vec{r}_j, \omega)$$

Handway Hamiltonian

$$\mathcal{H} \sim \sum_j \vec{p}(\vec{r}_j, t) \cdot \vec{E}(\vec{r}_j, t)$$

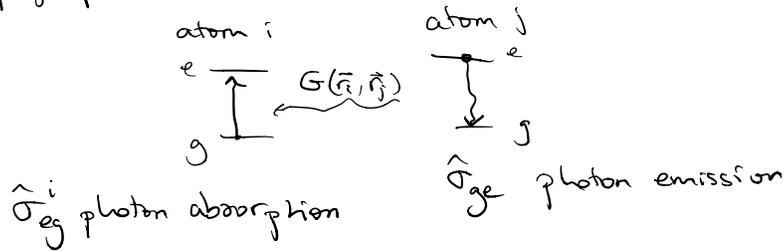
Since, for an atom  $\vec{p} \rightarrow \hat{d} \propto \hat{p} \sigma_{ge} + \text{h.c.}$

$$\mathcal{H} \sim \sum_j \Omega_j (\sigma_+^j + \sigma_-^j) \leftarrow \text{Rabi term}$$

$$+ \mu_0 \omega^2 \sum_{ij} \vec{p}^* \cdot G(\vec{r}_i, \vec{r}_j, \omega) \cdot \vec{p} \hat{\sigma}_{eg}^i \hat{\sigma}_{ge}^j$$

atom-atom interactions

We find a non-Hermitian Hamiltonian that describes "flip-flop" interactions



How do we get here formally?

### • Formal approach

Following Bohmann "Dispersion forces" book

Also: Gruner, Welsch, PRA 53, 1818 (1996)

Dung, Knoll, Welsch, PRA 66, 063816 (2002)

Bohmann, Welsch, Prog. Quantum Electron. 31, S1 (2007)

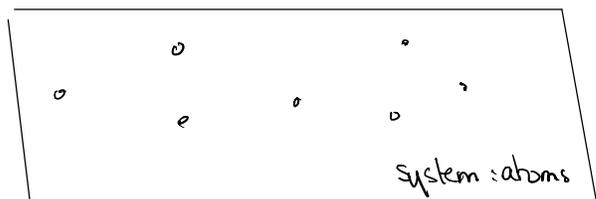
We consider multipolar coupling and just focus on dipole term.

$$\mathcal{H}_{af} = - \sum_{j=1}^N \hat{p}(r_j) \cdot \hat{E}(\vec{r}_j)$$

$\hat{p} \hat{\sigma}_{ge}^j + \hat{p}^* \hat{\sigma}_{eg}^j$       "  $\hat{E}^+ + \hat{E}^-$       retaining non-rotating terms

We want to integrate out photonic degrees of freedom (photons become bath). Once you trace  $\Rightarrow$  irreversible dynamics (needs density matrix).

$\Rightarrow$  Master equation for dipole-dipole interacting atoms



environment: photons

$$\rho = \text{Tr}_{\text{field}} \{ \rho_{\text{as}} \}$$

After some algebra, we find the spin model:

$$\dot{\rho} = \underbrace{-\frac{i}{\hbar} [\mathcal{H}, \rho]}_{\text{coherent}} + \mathcal{L}[\rho] \quad \leftarrow \text{dissipative (Lindbladian)}$$

$$\mathcal{H} = \hbar \omega_0 \sum_j \sigma_{zj} + \hbar \sum_{ij} J_{ij} \sigma_{xj} \sigma_{xi}$$

$$\mathcal{L}[\rho] = \sum_{ij} \frac{\Gamma_{ij}}{2} (2\sigma_{zj}^i \rho \sigma_{zj}^i - \sigma_{zj}^i \sigma_{zj}^i \rho - \rho \sigma_{zj}^i \sigma_{zj}^i)$$

where:

$$\Gamma_{ij} = -\frac{\mu \omega_0^2}{\hbar} \beta^* \cdot \text{Re} G(r_i, r_j, \omega_0) \cdot \beta$$

$$\Gamma_{ij} = \frac{2\mu \omega_0^2}{\hbar} \beta^* \cdot \text{Im} G(r_i, r_j, \omega_0) \cdot \beta$$

XY model with long-range interactions, open

To avoid dealing with Lindblad operators and density matrix

define:

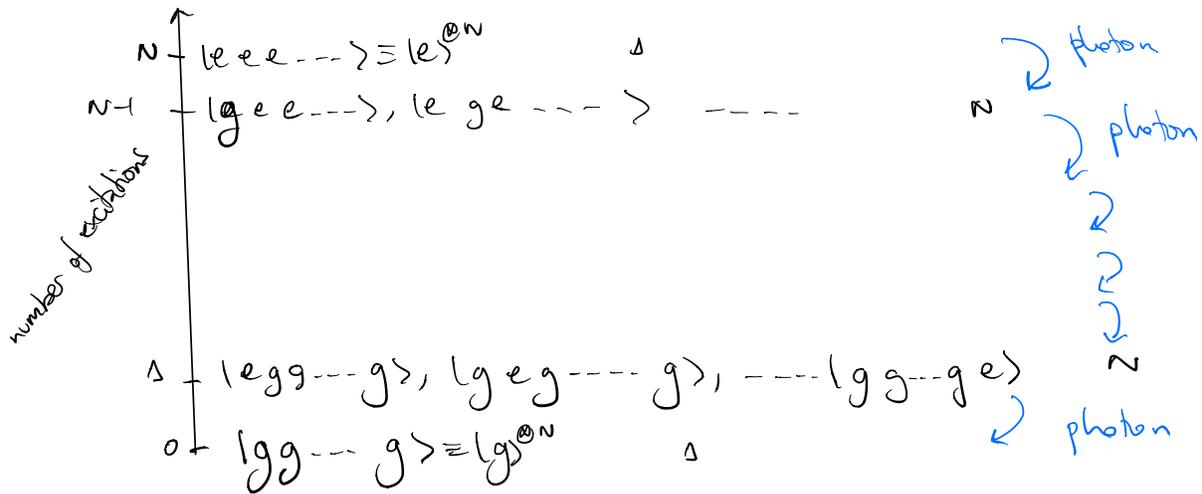
$$\mathcal{H}_{\text{eff}} = -\mu \omega_0^2 \sum_{ij} \beta^* \cdot G(r_i, r_j, \omega_0) \cdot \beta \sigma_{zj}^i \sigma_{zj}^i$$

- non-Hermitian, but conserves number of excit.  $\rightarrow$  non-unitary dynamics
- to evolve apply jumps at random times (stochastic wavefunction approach)

Still we have to deal with  $2^N$  degrees of freedom. We do not have to deal with EM modes but the price is that dynamics is open.

### 3. Collective atomic states = super and subradiance

$$\chi_{\text{eff}} = -\mu_0 \omega_0^2 \sum_{ij} \beta_i^* \cdot G(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \beta_j \alpha_{eg}^i \alpha_{ge}^j$$



We can understand collective states within a manifold of a given number of excitations.

Let's look at single excitation (easier!) for 1D chain

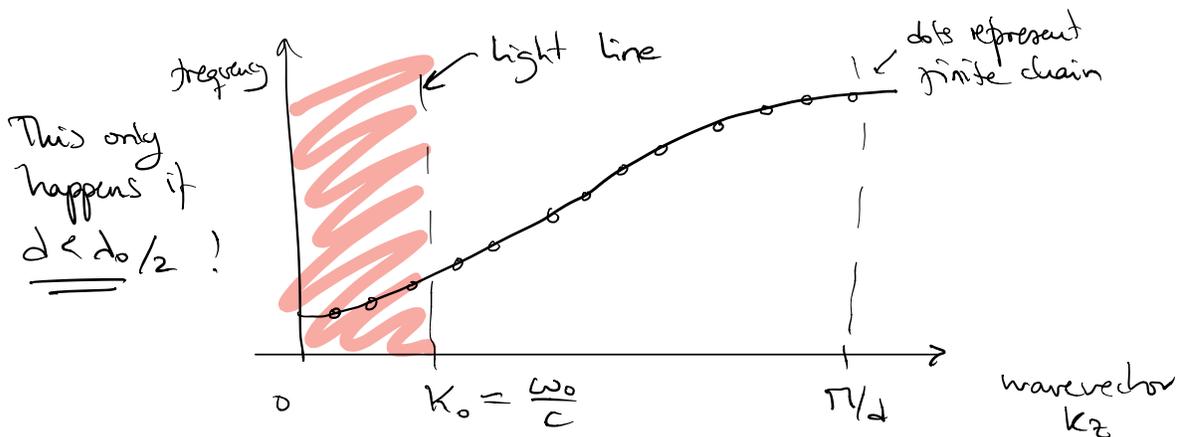


Free space (see Jackson's textbook for instance):

$$G(\mathbf{r}_i, \mathbf{r}_j) = \frac{e^{i\mathbf{k}_0 \cdot \mathbf{r}}}{4\pi k_0 r} \left[ (k_0^2 r^2 + i k_0 r - 1) \mathbb{1} + (-k_0^2 r^2 - 3i k_0 r + 3) \frac{\mathbf{r}_i \otimes \mathbf{r}_j}{r^2} \right]$$

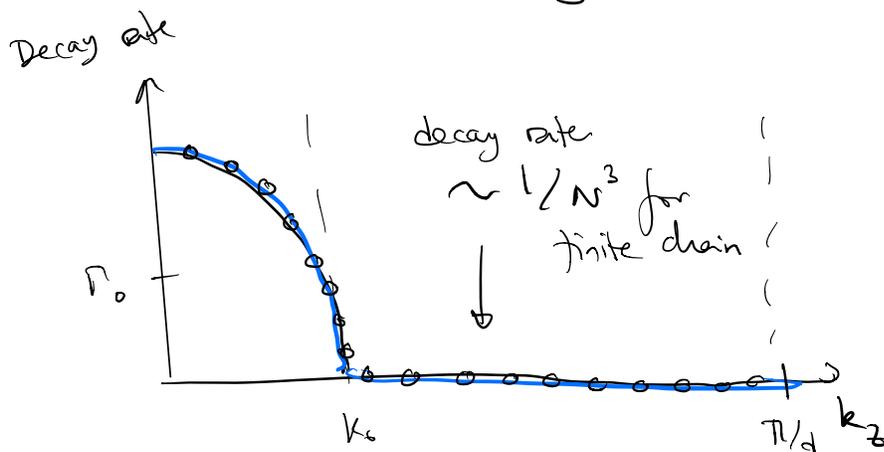
For an infinite chain, we can diagonalize using Bloch waves:

$$|\psi_{\mathbf{k}_z}\rangle \sim \underbrace{\sum_j e^{i\mathbf{k}_z \cdot \mathbf{r}_j} |e_j\rangle}_{\text{superposition}} |g\rangle^{\otimes N-1}$$



If  $|k_z| < k_0$ : superradiant modes - constructive interference (decay faster than single atom)

If  $|k_z| > k_0$ : subradiant modes - destructive interference (decay much slower than a single atom).



Dark states emerge that are protected from decay and dissipation! Emergence of coherence in an open system.

These can be used for improving fidelities of quantum memories, or realize better quantum gates...

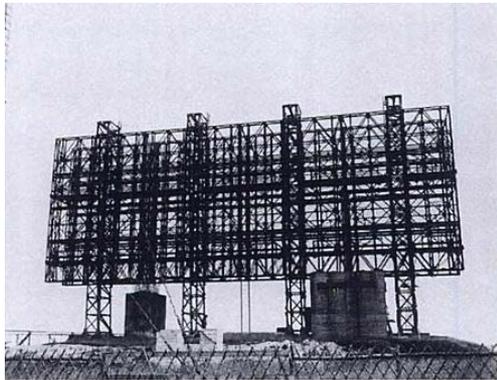
But... what happens for more excitations ??

The problem becomes "true quantum"

# Atomic chains: miniature antennas/waveguides (at the single-excitation manifold)

Size

~30 m x 16 m

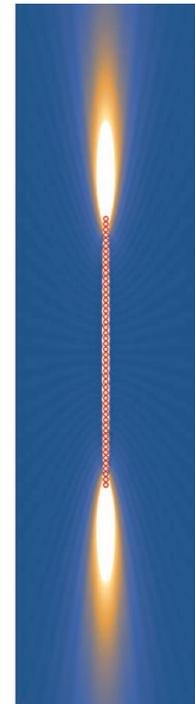


“Mammut” Radar, Germany 1944

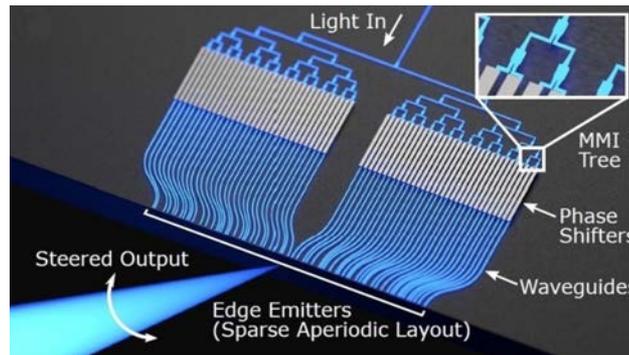
~1 cm x 1mm



probably the tiniest possible

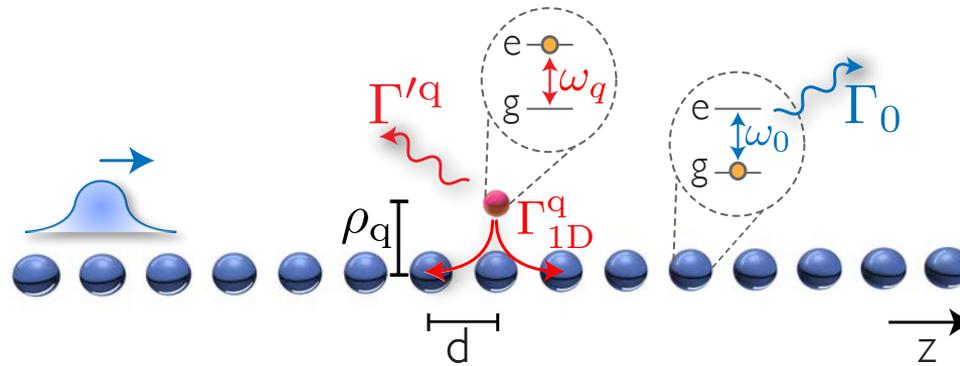


but quantum for multiple excitations!



Courtesy of the Lipson group @ Columbia

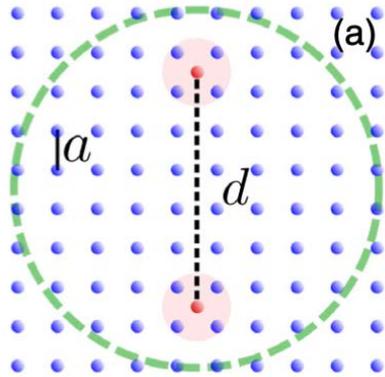
# 1D chains as (quantum) waveguides



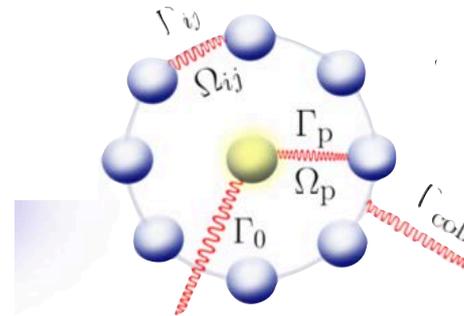
- Purcell: decay rate of an emitter depends on its environment
- An atomic chain can behave as a bath for a “qubit” impurity (we integrate-out the atoms under Born-Markov approximation)
- It can also mediate interactions between “qubit” impurities (“atomic-waveguide QED”)

# Recent suggestions in other geometries

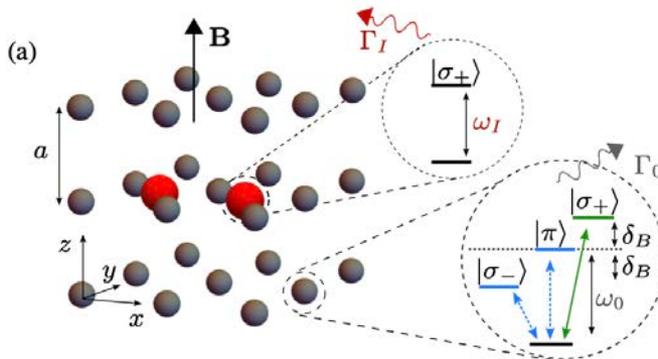
**2D:** Patti et al.,  
PRL 126, 223602 (2021)



**Ring:** Holzinger et al.,  
PRL 124, 253603 (2020)

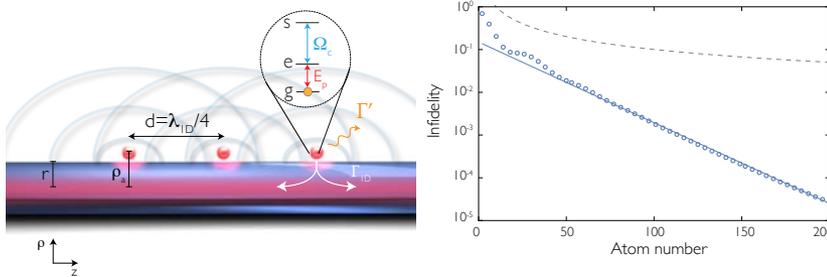


**3D:** Brechtelsbauer and Malz,  
arXiv: 2012.1277(2020)



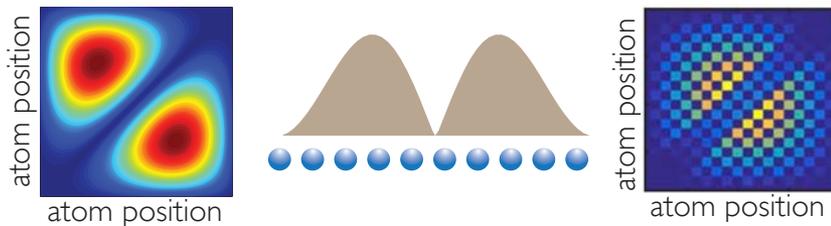
# Atom arrays as light-matter interfaces

Exponential improvement in photon retrieval fidelity when coupled to 1D waveguides

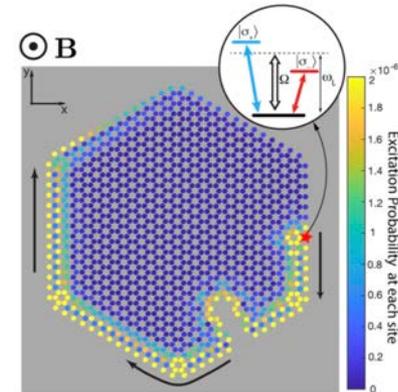
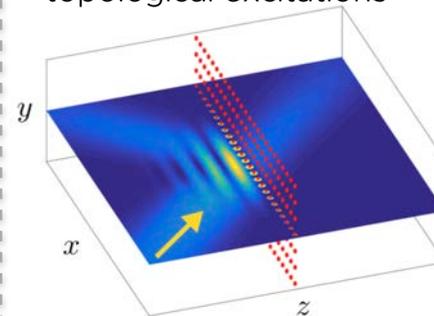


AAG et al., PRX **7**, 031024 (2017)

Many-body physics: emergence of fermionization in the dark states.



Perfect mirrors and topological excitations



Bettles et al., PRL **116**, 103602 (2016)

Shahmoon et al., PRL **118**, 113601 (2016)

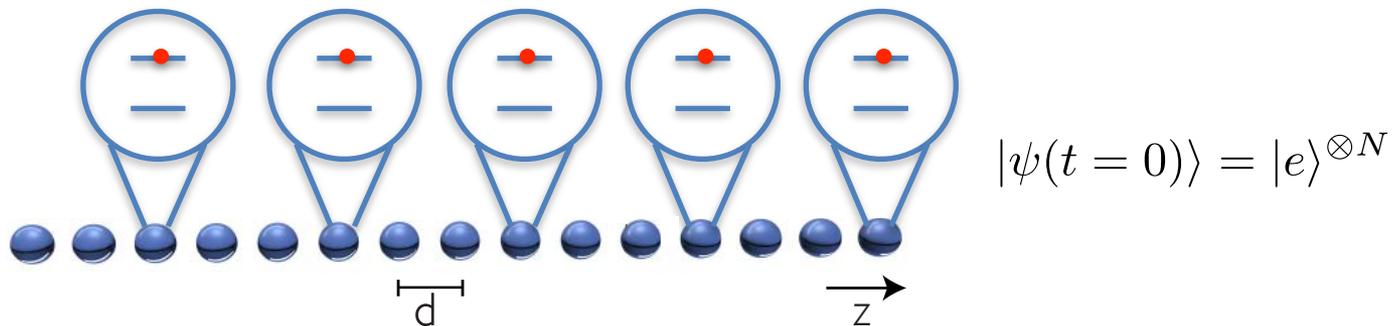
Perczel et al., PRL **119**, 023603 (2017)

# Many-body dissipative physics: what happens with many photons in the chain?

Large excitation densities do not support dark states

When photons are packed together, radiation is unavoidable

In the most extreme case, all atoms are inverted



What are the many-body signatures of collective decay?

# Dicke SR: many atoms radiate *differently*, not just more

PHYSICAL REVIEW

VOLUME 93, NUMBER 1

JANUARY 1, 1954

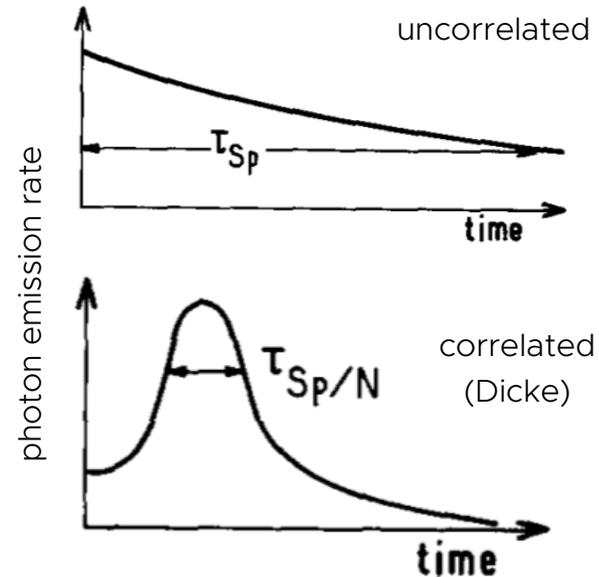
## Coherence in Spontaneous Radiation Processes

R. H. DICKE

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received August 25, 1953)

**I**N the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other. To justify this assumption it might be argued that, as a result of the large distance between molecules and subsequent weak interactions, the probability of a given molecule emitting a photon should be independent of the states of other molecules. It is clear that this model is incapable of describing a coherent spontaneous radiation process since the radiation rate is proportional to the molecular concentration rather than to the square of the concentration. This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent. The model is wrong in principle and many of the results obtained from it are incorrect.



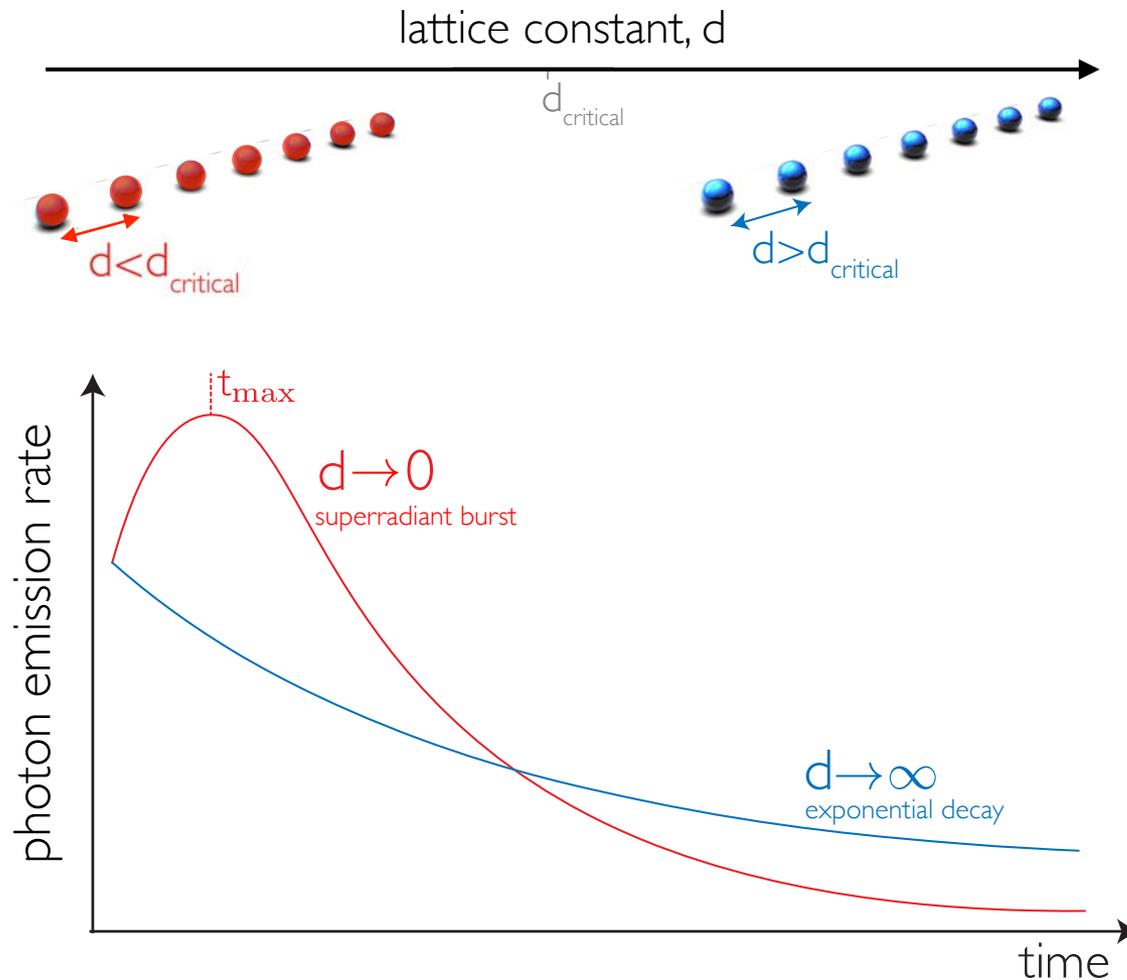
Gross, Haroche, Physics Reports 93, 301 (1982)

Example of emergence of macroscopic coherence through dissipation

Dicke solved the problem in a cavity (high-symmetry)

**Question:** what happens for extended arrays?

In extended lattices, there has to be a crossover between Dicke SR and exponential decay



# Why is this a hard problem?

Initial state: all emitters inverted

Initially there is no coherence, they start decaying due to vacuum fluctuations

Decay rates

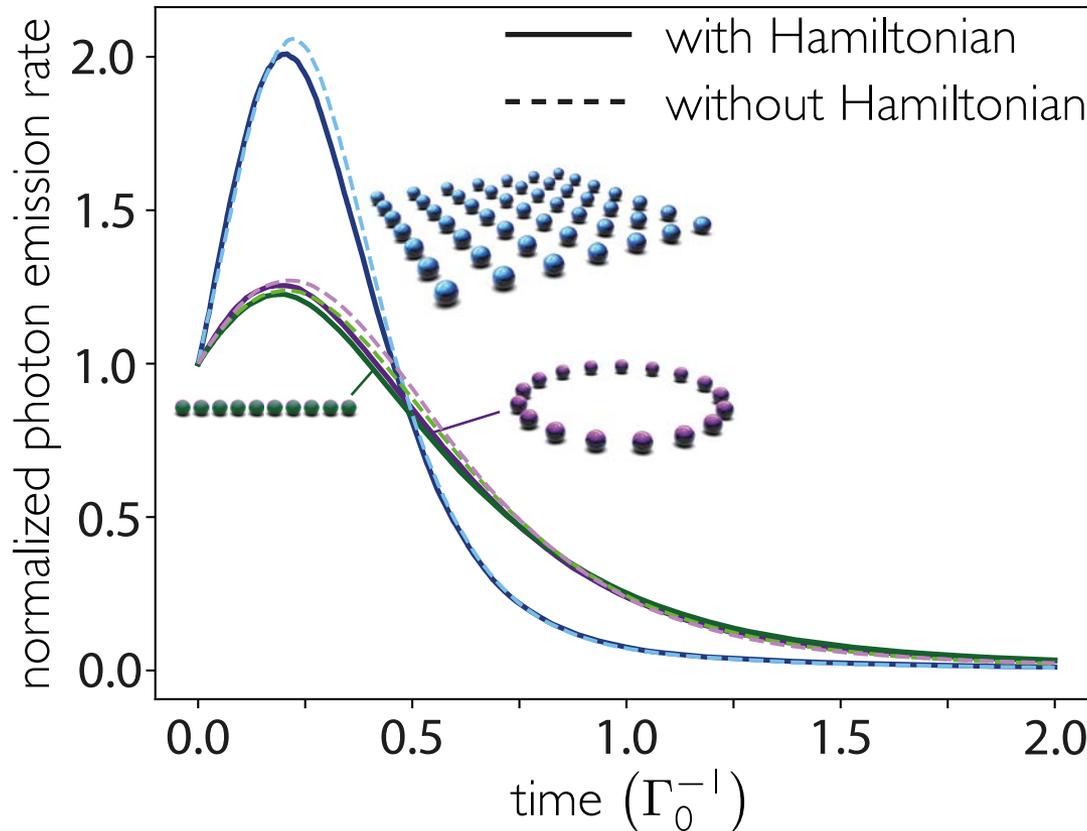
Collective jump operators: photon emission

$$\dot{\rho} = -\frac{i}{\hbar} [\mathcal{H}, \rho] + \sum_{\nu=1}^N \frac{\Gamma_{\nu}}{2} \left( 2\hat{\sigma}_{\nu} \rho \hat{\sigma}_{\nu}^{\dagger} - \rho \hat{\sigma}_{\nu}^{\dagger} \hat{\sigma}_{\nu} - \hat{\sigma}_{\nu}^{\dagger} \hat{\sigma}_{\nu} \rho \right)$$

in Dicke's case,  
does not contribute

Lattice constant	States	Decay
Zero	$\sim N$	Burst
Finite	$\sim 2^N$	?
Infinite	$\sim N$	Exponential

We can only do calculations for few emitters (16!)



Is this all we can do? No! understanding the physics gives us a hint...

We can exponentially reduce the complexity:  
let's just look at early dynamics!

Emitters synchronize at the beginning or not at all

“Minimum burst”: the first photon enhances the emission of the second

$$g^{(2)}(\tau = 0) = \frac{P(2 \text{ photons})}{(P(1 \text{ photon}))^2} > 1$$

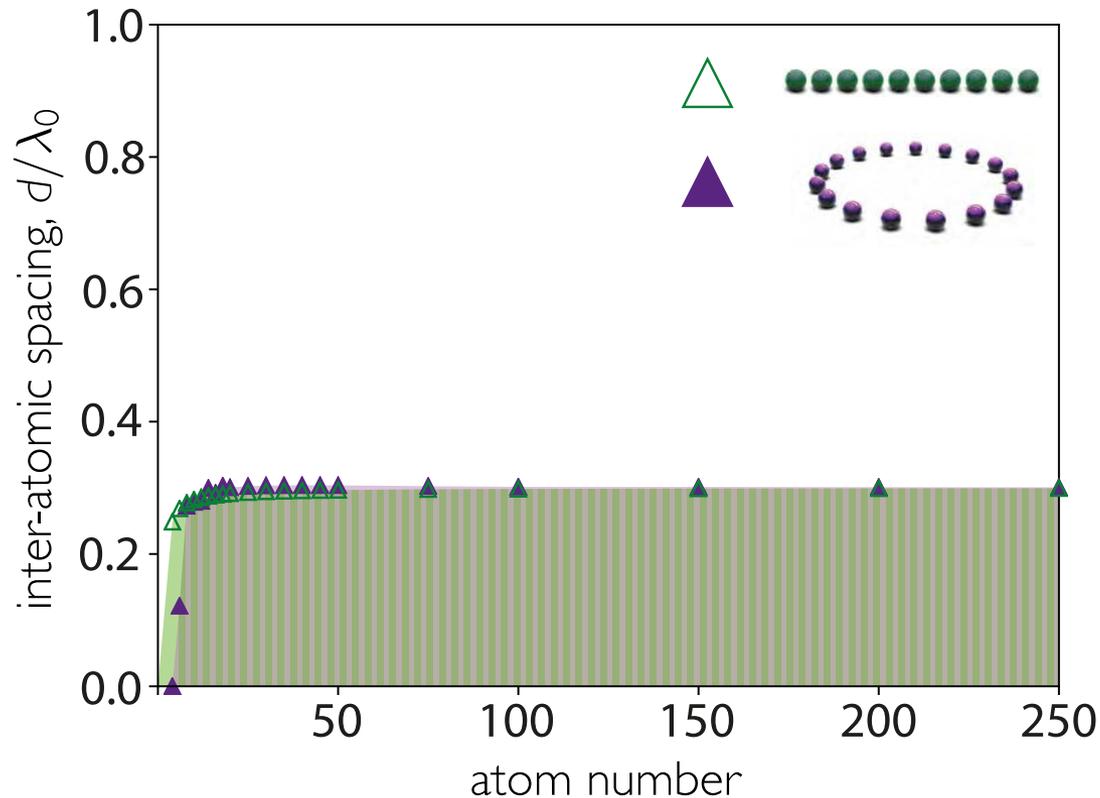
$$\text{Var.} \left( \frac{\{\Gamma_\nu\}}{\Gamma_0} \right) > 1$$

eigenvalues of dissipative  
interaction matrix  $\text{Im}\{G_0(\mathbf{r}_i, \mathbf{r}_j)\}$

From solving a differential equation in an exponentially large space to diagonalizing a NxN matrix

This equation is universal: valid for all geometries

Dicke SR is universal... occurs for any lattice as long as lattice spacing is small enough



Expression works for all geometries, potentially also in other baths.

Complexity reduced from  $2^N$  to  $N$

# Outlook

Physics of correlated quantum dipoles with  
dissipation and long range interactions

New playground for multidisciplinary physics

Ideas to explore further:

- Higher dimensions: richer dispersion relations
- Coherent control of single-photon states:  
(dynamical) dispersion engineering
- Beyond one excitation: photon-photon interactions  
and gates, many-body physics

# Acknowledgements



**Stuart Masson**



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for details:

S. J. Masson, AAG, Phys. Rev. Research 2, 043213 (2020)

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Phys. Rev. Lett. 125, 263601 (2020)

S. J. Masson, AAG, arXiv:2106.02042 (2021)



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