

# “Dynamic-thermal” reversal in a fine micromagnetic grain: Time dependence of coercivity

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An analytic model for reversal field versus pulse time in a fine magnetic grain has been developed for the case when the external magnetic field  $\mathbf{H}_0$  is parallel to the axis of uniaxial anisotropy. The expressions include both thermal fluctuations and magnetization dynamics. Therefore application is for fields both less than and greater than the intrinsic uniaxial anisotropy field  $H_K$ . For the case of  $-H_0/H_K > 1$  the mean first passage time from the initial saturated state to the thermodynamically equilibrium reversed state is calculated. For the case of  $-H_0/H_K < 1$  we calculate the mean first passage time from the thermodynamically equilibrium state of the metastable well to the thermodynamically equilibrium stable reversed state. It is shown that simple Néel–Arrhenius analysis is applicable for times greater than at least  $100/\alpha\gamma H_K$ . © 2000 American Institute of Physics. [S0021-8979(00)83508-2]

## I. INTRODUCTION

Advanced media for ultrahigh density recording is comprised of such small grains that thermal fluctuations are important. Three time regimes are of interest: the short pulse time corresponding to the recording process ( $\sim 1$  ns), intermediate times corresponding to hysteresis loop measurements ( $\sim 1$  s), and long times corresponding to storage ( $\sim 10$  years). For medium to long times Néel–Arrhenius analysis has been utilized assuming a phenomenological attempt frequency  $f_0$ .<sup>1</sup> Under a variety of approximations the Fokker–Planck equation for the effect of thermal fluctuations on a single grain has been derived and solved.<sup>2–6</sup>

Recently, a new theoretical formulation has been proposed in which a particular simple form of the Langevin equation is utilized.<sup>7,8</sup> This approach is based on a particularly physical form of the phenomenological damping and is applicable to all energy barriers. In this article we use this formulation to derive the reversal time for any applied magnetic field.<sup>9</sup>

## II. MODEL

We consider the dynamics of a single-domain magnetic grain as a coherent rotation of magnetization in the effective magnetic field  $\mathbf{H}_{\text{eff}} = -(\partial\mathcal{E}/V)/\partial\mathbf{M}$ , where

$$\mathcal{E} = K_u V \sin^2 \theta - M_s V H_0 \cos \theta \quad (1)$$

is the energy,  $V$  is the grain volume,  $K_u$  is the uniaxial anisotropy constant. In Eq. (1)  $\theta$  is the angle between the vector magnetization  $\mathbf{M}$  ( $|\mathbf{M}| = M_s$ ) and the anisotropy axis.  $H_0$  is the external magnetic field oriented along the anisotropy axis.

The coherent rotation of the magnetization including thermal agitation can be described as a “random walk” motion of nonlinear oscillators in two separated energy wells. In Fig. 1 the energy (1) is plotted versus magnetization compo-

nent along the anisotropy axis direction ( $M_h = M_s \cos \theta$ ) for a reverse applied field. We describe the magnetization component  $M_h$  in terms of classical occupation numbers:  $N_1$  and  $N_2$  correspond to the magnetization variation left and right of the energy maximum in Fig. 1, respectively. Thus,  $M_h^{(\text{left})} = M_s(1 - N_1/S)$  and  $M_h^{(\text{right})} = -M_s(1 - N_2/S)$ , where  $S = M_s V / \hbar \gamma$  is the net spin corresponding to the magnetization  $M_s$ . Even though we formulate this problem in terms of the spin involving the gyromagnetic precession constant  $\gamma$  and Planck’s constant  $\hbar$ , this is a purely classical analysis and  $\hbar$  does not enter any final result.

In the first well the energy (1) has the form

$$\mathcal{E}_1(N_1) = -\hbar \gamma S H_0 + \hbar \omega_1 (1 - N_1/2N_{\text{top},1}) N_1,$$

$$0 \leq N_1 < N_{\text{top},1}, \quad N_{\text{top},1} = S(1 + h). \quad (2)$$

Here  $\omega_1 = \gamma(H_K + H_0)$  and  $h \equiv H_0/H_K$ . At the top of the energy barrier ( $N_{\text{top},1}$ ) the magnetization changes its direction of rotation.

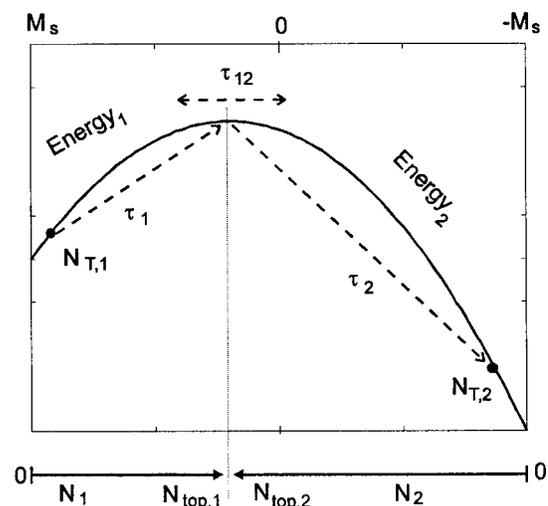


FIG. 1. The energy barrier (in relative units) and coordinates of nonlinear oscillators.  $H_0/H_K = -0.2$ .

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In the second well the energy (1) is

$$\mathcal{E}_2(N_2) = \hbar \gamma S H_0 + \hbar \omega_2 (1 - N_2/2N_{\text{top},2}) N_2,$$

$$0 \leq N_2 < N_{\text{top},2}, \quad N_{\text{top},2} = S(1-h). \quad (3)$$

Here  $\omega_2 = \gamma(H_K - H_0)$ .

In Ref. 8 it has been shown that the stochastic differential (Langevin) equations for  $N_j$  ( $j=1,2$ ) can be written as

$$dN_j = 2(-\eta_j N_j + D_j)dt + 2\sqrt{N_j D_j} dW_j, \quad (4)$$

where  $W_j$  describe independent random processes,  $\eta_j = (\alpha_j/\hbar)d\mathcal{E}_j/dN_j$  are the thermodynamically consistent nonlinear relaxation rates and  $D_j = \alpha_j k_B T/\hbar$  are the diffusion coefficients. In principal, the dimensionless damping parameters  $\alpha_1$  and  $\alpha_2$  can differ from each other. These parameters can be obtained from ferromagnetic resonance experiments.

We derive the mean first passage time  $\tau$  from one well to another. It is natural to choose the starting point of random motion as  $N_{T,1}$  and the finishing point as  $N_{T,2}$ , where

$$N_{T,j} = \frac{\int_0^{N_{\text{top},j}} dN \exp[-\mathcal{E}_j(N)/k_B T]}{\int_0^{N_{\text{top},j}} dN \exp[-\mathcal{E}_j(N)/k_B T]} \quad (5)$$

are the Boltzmann equilibrium occupation numbers (or, equilibrium magnetizations) in the first and second wells, respectively.

The mean first passage time is comprised of three contributions:

$$\tau = \tau_1 + \tau_2 + \tau_{12}. \quad (6)$$

As illustrated in Fig. 1,  $\tau_1$  is the passage time from initial state  $N_{T,1}$ , to the top of the energy barrier.  $\tau_2$  is the passage time from the top of the barrier to the final state  $N_{T,2}$ .  $\tau_{12}$  describes an additional delay because once a magnetization is at the energy maximum, thermal fluctuations can drive the system to either well.

### A. Thermal reversal

Consider the case  $-1 < H_0/H_K \leq 0$  when the energy barrier separates two wells. The mean first passage time Eq. (6) from the thermodynamically equilibrium state of the metastable well to the thermodynamically equilibrium stable reversed state can be written as

$$\alpha_1 \gamma H_K \tau_1 = \xi(1+h)(1-\bar{n}_1) \int_0^1 dv \int_0^1 du \exp\{\xi(1+h)^2 \times \zeta_1(v)(1-u)[2-\zeta_1(v)(1+u)]\}, \quad (7)$$

$$\alpha_2 \gamma H_K \tau_2 = \xi(1-h)(1-\bar{n}_2)^2 \int_0^1 dv \frac{1-v}{\zeta_2(v)} \times \int_0^1 du \exp\{-\xi(1-h)^2(1-\bar{n}_2) \times u(1-v)(2-u)[1-\zeta_2(v)]\}, \quad (8)$$

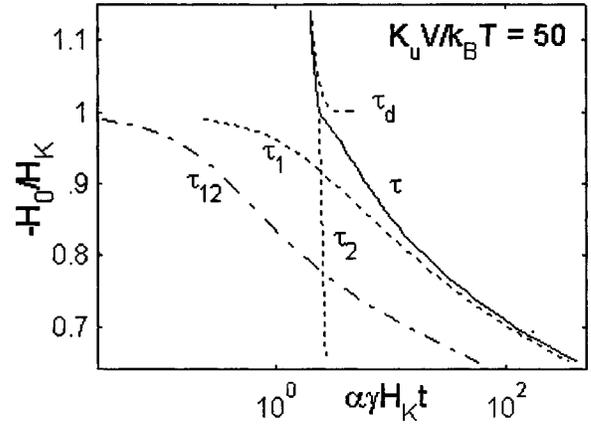


FIG. 2. Time dependence of coercivity vs exit time (solid line) and its components.

$$\alpha_1 \gamma H_K \tau_{12} = \xi(1+h)(1-\bar{n}_2) \exp(4h\xi) \int_0^1 \frac{dv}{\zeta_2(v)} \times \int_0^1 du \exp\{-\xi(1+h)^2 u(2-u)\} \times \exp\{\xi(1-h)^2 \zeta_2(v)[2-\zeta_2(v)]\}. \quad (9)$$

Here  $\zeta_j(v) = (1-\bar{n}_j)v + \bar{n}_j$ ,  $\bar{n}_j = N_{T,j}/N_{\text{top},j}$  and  $\xi = K_u V/k_B T$ .

### B. Dynamic reversal

For the case  $H_0/H_K \leq -1$  there is only one well and the mean first passage time from the initial saturated state ( $N_2 = 2S$ ) to the thermodynamically equilibrium reversed state  $N_2 = N_{T,2}$  is given by  $\tau_2$  only ( $\tau_1 = \tau_{12} = 0$ ):

$$\alpha_2 \gamma H_K \tau_2 = 2\xi(1-\bar{n}_d)^2 \int_0^1 dv \frac{1-v}{\zeta_d(v)} \int_0^1 du \times \exp\{-4\xi(1-\bar{n}_d)u(1-v)\} \times [1-h-u-(2-u)\zeta_d(v)], \quad (10)$$

where  $\zeta_d(v) = (1-\bar{n}_d)v + \bar{n}_d$  and  $\bar{n}_d = N_{T,2}/2S$ .

For comparison we can derive the corresponding ‘‘dynamic’’ time  $\tau_d$  which is calculated without thermal agitation:

$$\alpha_2 \gamma H_K \tau_d = \frac{1}{2(1-h)} \ln \left[ \frac{h-1+2\bar{n}_d}{(h+1)\bar{n}_d} \right]. \quad (11)$$

### III. DISCUSSION

Simple scaling relations apply for this case of applied field parallel to the anisotropy axis. The only parameters that occur in Eqs. (7)–(9) are the scaled variables  $h = H_0/H_K$ ,  $\xi = K_u V/k_B T$ ,  $\alpha_1 \gamma H_K$ , and  $\alpha_2 \gamma H_K$ . Here we will illustrate these results by plotting field versus reversal time. For simplicity, we consider the case  $\alpha_1 = \alpha_2 = \alpha$ .

In Fig. 2 normalized reversal field ( $-H_0/H_K$ ) is plotted versus normalized time  $\alpha \gamma H_K t$  for the case  $K_u V/k_B T = 50$ . The solid curve utilizes the net passage time Eq. (6) obtained from Eqs. (7)–(9). For comparison the contributions  $\tau_1$ ,  $\tau_2$ ,  $\tau_{12}$ , and  $\tau_d$  from Eqs. (7)–(10) are shown, respectively. For

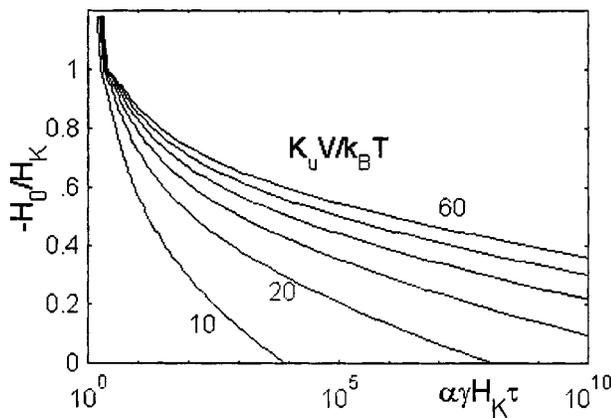


FIG. 3. Time dependence of coercivity for various  $K_u V/k_B T$ .

very long times and fields not too small ( $-H_0 > 0.2H_K$ )  $\tau_1$  is the major contributor. For short times where  $-H_0$  approaches  $H_K$  both  $\tau_1$  and  $\tau_2$  are important. For extremely short times where reversal fields exceeding  $H_K$  are required, the  $\tau_2$  approaches the purely dynamic limit  $\tau_d$ . One can see that for  $K_u V/k_B T = 50$  the reversal field equals  $H_K$  for pulse times about  $2.5/\alpha\gamma H_K$ . The “dynamic time”  $\tau_d$ , which does not take into account a thermal agitation, asymptotically goes to infinity when  $h = H_0/H_K$  increases to  $-1$ . The diffusion time  $\tau_{12}$ , relatively small in the vicinity of  $h = -1$ , increases with increasing  $h$  and equals  $\tau_1$  at  $h = 0$ . This fact has a simple physical interpretation: for a symmetric barrier the chance to move to a new well and the chance to return back are equal.

The effect of relative energy barrier  $K_u V/k_B T$  on the time dependence of coercivity for factors  $10 \leq K_u V/k_B T \leq 60$  is shown in Fig. 3. For all but the shortest times the reversal field depends strongly on the energy barrier. At very short times the reversal field depends only on  $\alpha\gamma H_K t$  independent of  $K_u V/k_B T$ . For every energy barrier there is a time for which the reversal field vanishes. For measurement times greater than these critical values the particles are “superparamagnetic.”

It is interesting to express the time dependence of coercivity  $-H_0/H_K$  in terms of the simple Néel–Arrhenius result  $(1 - \sqrt{(k_B T/K_u V)\ln(f_0\tau/\ln 2)})$ .<sup>1</sup> Figure 4 demonstrates that this dependence is close to linear for  $K_u V/k_B T > 50$  and  $-H_0/H_K < 0.8$ . In agreement with Fig. 2 this simple Néel–Arrhenius result may be used to estimate coercivity for times

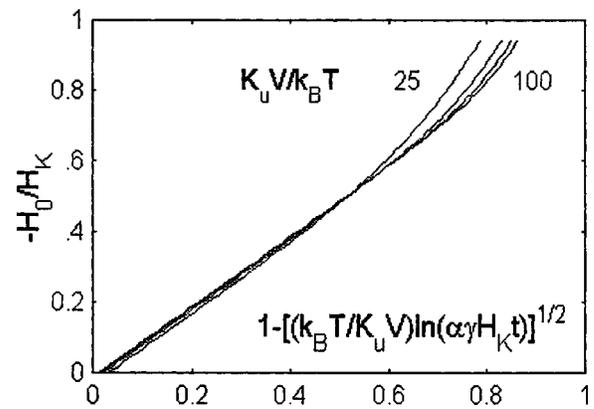


FIG. 4. Coercivity vs time expressed in terms of Sharrock’s expression for various  $K_u V/k_B T$ .

greater than  $\alpha\gamma H_K t \sim 100$ . For example, for  $K_u V/k_B T > 50$  at  $\alpha\gamma H_K t \sim 100$  the difference between our result and Néel–Arrhenius is about 10% and vanishes asymptotically at much greater times.

The scaling laws give  $f_0 \approx 2\alpha\gamma H_K$  for the time regime where the Néel–Arrhenius result is applicable. For example, for  $H_K = 2000$  Oe,  $\alpha \sim 0.1$ , and  $\gamma = 1.76 \times 10^7$  Oe<sup>-1</sup> s<sup>-1</sup>,  $f_0 \approx 0.7 \times 10^{10}$  s<sup>-1</sup>, which is typical of experimental measurements.<sup>10</sup>

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