THEORY OF THE MAGNETIC DAMPING CONSTANT

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Abstract—The aim of this paper is to express the effects of basic dissipative mechanisms involved in the dynamics of the magnetization field in terms of the one most commonly observed quantity: the spatial average of that field. The mechanisms may be roughly divided into direct relaxation to the lattice, and indirect relaxation via excitation of many magnetic modes. Two illustrative examples of these categories are treated; direct relaxation via magnetostriction into a lattice of known elastic constant, and relaxation into synchronous spin waves brought about by imperfections. Finally, a somewhat speculative account is presented of time constants to be expected in magnetization reversal.

Index terms — dissipation mechanisms, micromagnetics, non-linear effects

I. INTRODUCTION

Damping of the motion of the magnetization vector, particularly the damping of large motion, plays a significant role in the performance of magnetic recording devices.

At the phenomenological level, a great deal is known about the behavior of small motions, such as occur in ferromagnetic resonance experiments, and modest advances into the realm of not quite so small motions have been made for the case of resonance at high signal powers. However, the problem of dissipation in truly large motions remains, at least in structures whose size exceeds typical dimensions of a domain wall.

To be of use in practice, the formulation of loss processes should be as far as possible in terms of the one quantity that appears in the ultimate applications: the spatial average $\vec{m}(t)=\langle\vec{m}(\vec{x},t)\rangle$ of the magnetization vector. The vast number of microscopic (or even mesoscopic) degrees of freedom responsible for the losses should be encapsulated as constants, or at least as simple functional structures, in the equations of motion of that spatial average alone. This presentation emphasizes that viewpoint. A price must be paid for this: eliminating unwanted degrees of freedom in favor of the wanted ones makes the equations of motion of the latter non-local in time. But in the appropriate limiting cases, local equations can usually be recovered.

II. DIRECT AND INDIRECT DAMPING MECHANISMS

A. *Direct Damping*

First, we note that there are two main pathways for degradation of the uniform mode. 1. Direct flow of energy from the uniform mode into lattice motions (and possibly conduction electrons, in the case of metals) and 2. Energy flow from that mode into non-uniform magnetic modes which we shall call magnons or spin waves (even when these are not small). Of course, these modes must eventually decay to the lattice, but this has no direct effect on the uniform mode (Fig. 1).

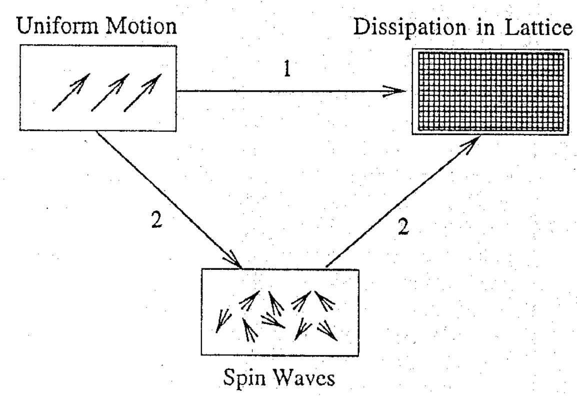

Figure 1. Two paths for degradation of uniform motion: 1) Direct relaxation to the lattice; 2) Decay into non-uniform motions, which in turn decay to the lattice.

For samples smaller than a domain wall thickness, mechanism 2 will be inhibited, because spatial variation of the spin orientations would entail too much exchange energy. Then we have the relatively simple case 1, and we begin by analyzing the case in which the coupling to the lattice is furnished by the magnetoelastic energy, and the sound velocity is considered infinite (small samples). All strains and stresses are then uniform. For simplicity, we shall consider shear motions of the lattice only. Reference [1] gives the shear part of the total elastic energy: $F=\mu e_{ij}^2+Be_{ij}m_im_j$ (repeated subscripts being summed over), where μ is the shear modulus, B a measure of the magnetoelastic energy, m_i the components of the magnetization vector, and $e_{ij}=(\frac{1}{2})(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$ is the strain tensor in terms of the displacement components u_i of the lattice point at $\vec{x}$. The equations of motion for the strains: Viscous stress tensor=minus elastic stress tensor $\partial F/\partial e_{ij}$ are [2]

$$\eta\dot{e}_{ij}+\mu e_{ij}+\frac{1}{2}Bm_im_j=0 \quad (1)$$

where η is the shear viscosity of the lattice. Equation (1) is solved for the e's in terms of the m's, and substituted in the equation of motion $\dot{\vec{m}}=\gamma\vec{m}\times(\partial F/\partial\vec{m})$ for $\vec{m}$. The result is

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$$\dot{\vec{m}}(t) = \gamma\vec{m}(t)\times\left[H - \frac{B^2}{2\eta}\int_{-\infty}^{t} dt'\vec{m}(t')(\vec{m}(t)\cdot\dot{\vec{m}}(t'))e^{-\lambda(t-t')}\right] \quad (2)$$

where $\lambda = \mu/\eta = E/2\eta(1+\sigma)$, ($E$ = Young's modulus, σ=Poisson's ratio). Note that the second term in the parentheses vanishes in a singular manner as the shear viscosity goes to zero. But, because for small viscosity the memory is short, an asymptotic expansion can be constructed by a change of variable to $\tau = t-t'$, and making a moment expansion in powers of τ. To third order the resulting damping terms have the form

$$\alpha_1\vec{m}\times\dot{\vec{m}} + \alpha_2\vec{m}\times\ddot{\vec{m}} + \alpha_3\vec{m}\times[\dddot{\vec{m}} + 3\dot{\vec{m}}(\dot{\vec{m}})^2] \quad (3)$$

the first of which has Gilbert's form. The constants are

$$\alpha_1 = 2\eta\gamma B^2(1+\sigma)^2/E^2;$$
$$\alpha_2 = -8\eta^2\gamma B^2(1+\sigma)^3/E^3;$$
$$\alpha_3 = 8\eta^3\gamma B^2(1+\sigma)^4/E^4$$

The expansion (3), being only asymptotic, fails at high frequencies. To estimate its limit of validity, the small signal case, for which (2) is easily solved exactly, should suffice. It gives a damping constant $2B^2\gamma\omega/(\eta(\lambda^2+\omega^2))$, which attains its maximum at $\omega = \lambda$. (There is also a small in-phase frequency dependent shift of amount equal to (ω/λ) times the damping rate which slightly alters the resonance condition in an applied field.) When shear wave propagation effects are included, equation (1) becomes (repeated subscripts being summed over)

$$\rho\ddot{u}_i - \mu\partial e_{ij}/\partial x_j - \eta\partial\dot{e}_{ij}/\partial x_j = B\partial(m_i m_j)/\partial x_j \quad (4)$$

where ρ is the mass density of the lattice. At one GHtz, the sound wavelength should be of order of one micrometer, just about of order of the length at which wall formation becomes energetically feasible. For samples in excess of this size, both sound propagation and non-uniform magnetization fields must be considered. By the same procedure as before, we find an equation of the form (2), except that t, t' must be replaced by the four-vectors $(t,\vec{x}), (t',\vec{x}')$, and

$$e^{-\lambda(t-t')} \Rightarrow e^{-\lambda(t-t')}R(|\vec{x}-\vec{x}'|, t-t')$$

where

$$R(|\vec{x}|, t) = e^{-\rho|\vec{x}|^2 t/\left(\eta(t^2+|\vec{x}|^2/c^2)\right)}$$
$$\cos\left\{\phi + \rho|\vec{x}|^2 t/\left(\eta(t^2+|\vec{x}|^2/c^2)\right)\right\},$$
$$\phi = \tfrac{1}{2}\arctan(|x|/ct) \quad (5)$$

and c is the shear wave velocity. The integration limits on t' are, of course, still $-\infty$ and t, while the x' integrations extend throughout space. Writing $t' = t-\tau$, $\vec{x}' = \vec{x}-\vec{\chi}$, averages of the form $\langle\tau^n\vec{\chi}^{2m}\rangle$ can be constructed from the kernel, and the first few terms of the asymptotic series can be found. The first few terms correspond to (n =1, m=0), (n =1, m=1) and (n =0, m=1). The last of these merely gives a small shift in the exchange field and is loss free. The other two imply loss, with a torque of the form

$$\gamma\vec{m}\times\left\{\langle\tau\chi^0\rangle\ddot{\vec{m}} + \tfrac{1}{6}\langle\tau\chi^2\rangle\left[\nabla^2\dot{\vec{m}} + 2\frac{\partial\vec{m}}{\partial x_i}\left(\frac{\partial\vec{m}}{\partial x_i}\cdot\dot{\vec{m}}\right)\right]\right\}$$

(summation convention). The last two terms show that loss can occur as the result of twisting of the magnetization vector, as in domain wall motion. That terms of this form can occur in principle is known [3, 4]. Here we have a recipe for evaluating them in the case of magnetoelastic coupling.

We have also derived the form of the memory kernel for the case of eddy currents by eliminating the electromagnetic field in favor of the magnetization vector. This will be discussed in a forthcoming submission.

B. *Indirect Damping*

We next turn to mechanism 2: degrading of the uniform motion by coupling to other modes of the spin system for sufficiently large samples. Embryonic forms of the special effects to be expected here are already apparent in the case of small motions in ferromagnetic resonance experiments with driving fields in the microwave range; hence we begin with a brief review of this subject. The magnetization vector in these experiments deviates only very slightly from complete lineup in the direction of the dc field. In lowest approximation the spatial Fourier coefficients of the small components transverse to the lineup behave like uncoupled two-dimensional harmonic oscillators, called magnons, or spin waves. In higher approximations, the spin waves are coupled, the coupling terms being classified according to the number of spin wave amplitudes occurring in them. Neglecting crystalline anisotropy and disregarding boundary conditions except those affecting the uniform mode, the excitation frequencies of these spin waves are given by

$$\omega_k = \sqrt{\left(\omega_H - N_z\omega_M + \omega_{ex}\ell^2k^2\right)\times\left(\omega_H - N_z\omega_M + \omega_{ex}\ell^2k^2 + \omega_M\sin^2\theta\right)} \quad (6)$$

where $\omega_H = \gamma H$, $\omega_M = 4\pi\gamma M$, and ω_{ex} the exchange energy in frequency units. ℓ is a length of order of the lattice spacing. θ is the angle between the directions of H and k. This formula breaks down for excitations of wavelength comparable with, or larger than, sample dimensions. So-called Walker modes [5] must then be used. The most important of these is the uniform mode, whose excitation frequency is

$$\omega_0 = \omega_H - (N_z - N_T)\omega_M$$

where a spheroidal sample has been assumed, with demagnetizing factors N_z, N_T respectively along and transverse to, the field applied along the axis (0z) of the spheroid.

Motion in this uniform mode can be degraded by splitting into spin waves synchronous (or 'degenerate') with it, both in regard to excitation frequency (energy) and momentum. Thus, for suitable ranges of the field, we can have $\omega_0 = \omega_k + \omega_{-k} = 2\omega_k$ and this can hold for a whole manifold of momenta. Similarly, for a different manifold of k - values, we can have two uniform precessian quanta turning into two spin wave quanta: $2\,\omega_0 = \omega_k + \omega_{-k} = 2\omega_k$ and so on for higher splittings, but only these two have been explored experimentally and theoretically. These processes can occur spontaneously as well as by thermal agitation; the latter process contributes a small temperature dependent line width noticeable only in extremely pure samples. The former process can result in an unstable growth of the spin waves in the entire degenerate manifold to large non-thermal values, provided the uniform mode exceeds a certain critical value determined by the damping of the spin wave modes. These run-away modes, in turn provide extra damping of the uniform mode, until a compromise is reached in which the uniform mode cannot be excited further, no matter how large the microwave field.

In a sample with imperfections, the dominant loss mechanism is usually scattering from the uniform mode by the process $\omega_0 = \omega_k$ + momentum $-\vec{k}$ absorbed by an imperfection. A very simple calculation shows that the resulting decay constant for the uniform mode must have the form:

$$\sqrt{\sum_{\omega_k=\omega_0} |p_k|^2}$$

where p_k is the coupling strength (in frequency units) of the uniform mode to spin wave k. (for realistic imperfections, the calculation of the p's is non-trivial [6]). All these processes have one thing in common: they do not preserve the magnitude of the uniform mode. Therefore, in the desired equation of motion for the uniform mode alone, they cannot be described by a damping term of either Gilbert or Landau-Lifshitz form. Clearly this feature must carry over to the case of large motions also. It follows that this kind of damping, leaving aside the above mentioned instabilities for the moment, must in general give an equation of motion of the form (m now refers to the uniform component only)

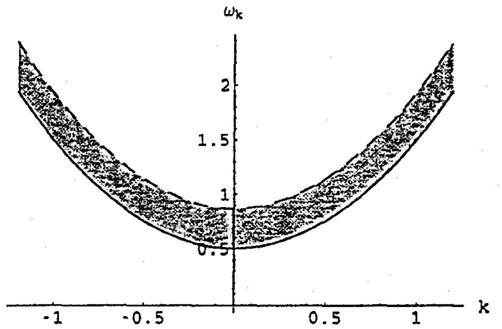
$$\delta\dot{m}_i = \left(\vec{m}\times\vec{H}\right)_i - \sum_{j=1}^{3} \frac{1}{T_{ij}} \delta m_j \qquad i \;=1,2,3 \qquad (7)$$

reminiscent of the equations used in paramagnetic and nuclear resonance. δm_j is the deviation of m_j from its equilibrium value. Of course, the relaxation times T_{ij} are constants only in the small motion limit. In the general case, they will also depend on scalars like $\vec{m}^2$ and/or $\vec{m}\cdot\vec{H}$. (A formal expression for the relaxation times will be presented in a forthcoming submission.)

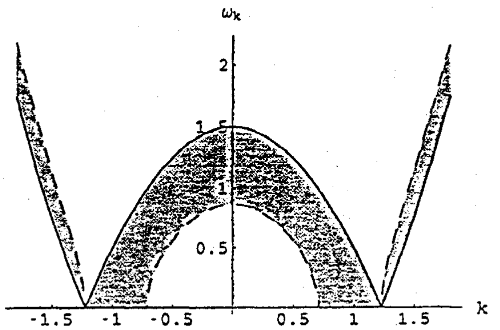
III. VERY LARGE MOTIONS

Next, we discuss some insights furnished by spin wave theory into the very large motions of the magnetization that occur in the process of switching by sudden reversal of the applied field. Disregarding thermal activation, switching is considered to begin for a value of the applied field at which the minimum in magnetic energy turns into a saddle point. At that value, one of the curvatures of the energy surface at the stationary point changes sign. Neglecting precession effects and retaining only the Landau-Lifshitz term, this results in an initial growth rate (which may be identified with the initial switching rate) that is proportional to the loss parameter in the Landau-Lifshitz equation: zero loss parameter-no switching. This procedure fails to take account of the losses due to the second mechanism, which cannot very well be forced into Landau-Lifshitz or Gilbert form.

To illustrate this point, assume that the Landau-Lifshitz loss parameter is, in fact, zero. Suppose that the applied magnetic field is suddenly decreased to zero from an initial value in excess of $N_z\omega_M$. This causes the portion of the portion of the spin wave spectrum with wave numbers $|\vec{k}| < |\vec{k}_{crit}| = \sqrt{N_z\omega_M / \ell^2\omega_{ex}}$ to turn negative, indicating instability. We know that the magnetization will break up into a more or less complex domain structure. However, initially, for a brief moment, the magnetization vector that was uniformly aligned along H stays put in its original direction until some thermal agitation causes it to precess slightly in what is now a negative internal field. Weakly excited spin waves with less than the critical wave number are in a similar position. So, for a brief instant we may once again assign positive excitation energies to disturbances with $k < k_{\text{crit}}$, effectively flipping over the portion of the spectrum that has sunk below the axis, as shown in Figure 2. Now any small thermal agitation of the uniform motion will cause unstable excitation of synchronous pairs of spin waves, conserving energy and momentum. These, in turn, can likewise yield unstable growth of other spin waves. Even though one may be inclined to dismiss this entire approach as fanciful, it seems to have features close to the actual state of affairs. The magnetization breaks up into a domain structure that minimizes the total energy, including that in the fringing fields. Simple considerations show that in a sample without imperfections, the resulting domain structure will favor a certain periodicity characterized by a wave number, K, say. In this state, the excitation spectrum will have minima at $|\vec{k}| = K$. The value of K, representing a balance between exchange forces, that would like to keep it small, and demagnetizing forces, that would like to keep it large, must, in fact, be of order of k_{crit}. It follows that Figure 2b is a reasonable facsimile of the true excitation spectrum of the collapsed state, and presumably is almost unaffected by the cascade of unstable spin waves effecting the collapse. Before turning to the second part of the process, in which H goes from zero to $-H$, we calculate the order of magnitude of the initial switching rate. To begin with, the appropriate spin waves feed on the thermal deviation of the uniform mode from complete alignment with the (now defunct) applied field. A prominent process is the spontaneous split $\omega_0 \Rightarrow \omega_k + \omega_{-k}$, for which there is evidently plenty of phasespace (see reference [7] for details). Although the analysis is done most easily in terms of spin wave amplitudes, a more picturesque description has been championed by the Russian school [8], using the language (though of course not the axiomatic structure) of quantum mechanics. Let $n_k = a_k^* a_k$ be the number of spin waves (complex amplitudes a_k) with wave number k in the manifold satisfying $\omega_0 = \omega_k + \omega_{-k}$, and $b_k = \langle a_k a_{-k} \rangle$ the 'anomalous' average of the product of two spin wave amplitudes with opposite momenta, pulled into coherence by coupling to the uniform mode. Then



(a)



(b)

Figure 2. Spin wave spectra. In filled regions between the graphs waves propagate inclined at angles θ between 0 and π/2 to the z-axis. Solid lines: θ=0, dashed lines: θ=π/2. Wave number k in units of reciprocal exchange length. (a) Normal spectrum of the saturated sample $(H/4\pi M_z(0))-N_z=0.5$; (b) Initial spectrum after sudden reversal of the field to a value such that $|H|/(4\pi M_z(0))+N_z=1.5$.

$$\dot{n}_k = \left(g_k b_k^* + g_k^* b_k\right)\sqrt{n_0}, \quad \dot{b}_k = 2 g_k n_k \sqrt{n_0}, \qquad (8)$$
$$\dot{n}_0 + \sum_{2\omega_k=\omega_0} \dot{n}_k = 0$$

where g_k, of order ω_M, is a measure of the coupling energy $\sum g_k a_k^* a_{-k}^* a_0 +$ comp. conj., and the phase of the uniform mode a_0 has been chosen real. Initially, before n_0 has a chance to change, the first two equations show that n_k increases exponentially at the rate $\omega_M n_0$. This is also the rate at which the spatially uniform part of m_z will decline. (Even though that quantity appears to be conserved according to the last equation of (8), it will decline if it is expanded beyond second order in the amplitudes). Also, at reasonable temperatures, $n_0 \cong kT/\omega_0$. Thus the initial switching rate is temperature dependent, equal to $\omega_M(kT/\omega_0) \cong kT/N_z$, (kT in frequency units). (We suspect that this rate gets larger as more and more spin waves split into others.)

Next we consider the process of reassembling the uniform magnetization when the applied field goes from zero to $-H$. Evidently the magnetization reassembles partly by sweeping domain walls out of the sample, and partly by coalescence of the favorably aligned domains at the expense of the others. Within the present formalism we cannot say much about the former process, but the latter can be mimicked in the spin wave picture as long as the reversed field is not too much larger than $N_z\omega_M$. With H reversed and greater in magnitude than $N_z\omega_M$, the spin wave spectrum is normal again, and the domain structure persisting for an instant longer may be regarded as a highly excited spin wave state with wave number k_{crit}. If $|H|$ is not too large, so that

$$\omega_0 < \tfrac{1}{6}\omega_M\left[8N_z + 1 - \sqrt{16N_z^2 + 16N_z + 1}\right] \qquad (9)$$

then pairs of spin waves with wave number $k_{crit}/2$ can grow unstable. This implies a spatial period doubling tending to bring the system closer to the uniform state, with magnetization reversed. For sufficiently small fields, further period doubling by unstable growth of pairs with wave numbers $k_{crit}/4$ can take place, and so on. The mathematics is essentially the same as for the unstable growth of ($\pm$ k) spin waves described earlier. Eventually a uniform, reversed state may be reached, provided the applied field is small enough.

For larger reversed fields, the phasespace available to these processes disappears. The route to the reversed magnetization should then be governed by direct losses to the lattice, for which an example was given above. In that case two distinct damping rates might be observed during the switching process. In a high quality sample, the first half of the switching process should be rapid, governed by spin wave excitations, while the second half should proceed much more slowly, with conventional damping by direct coupling to the lattice degrees of freedom. Although some experiments apparently are consistent with this conclusion [9], it is too early to claim it as a universal feature of magnetization reversal.

Finally, one more loss mechanism should be mentioned. During the second phase of the reversal, domain wall motion takes place, under certain circumstances resulting in losses by radiation of spin waves. This occurs when the wall velocity equals the group velocity $d\omega_k/dk$ of some spin waves. Details of this mechanism are found in ref. [10].

The discussion of damping mechanisms presented here is not sufficiently exhaustive to cover all cases of interest. For example, in yttrium iron garnet with certain rare earth impurities, the energy levels of these impurity may be a sensitive function of the orientation of the magnetization of the host [11], which leads to giant anisotropy peaks [12]. Kramers-Kronig-like relations must exist between crystal anisotropy and damping, (in fact, strong damping is observed in these cases [6]), but anisotropy and its implications have been ignored in the above discussion.

CONCLUSION

This investigation has shown that, for sufficiently small particles, the magnetization coupled to shear distortions of the lattice with very small shear viscosity leads to the Gilbert form of the damping term (equivalent tot Landau-Lifshitz form in that limit). The damping constant is expressed in terms of the elastic constants and is proportional to the shear viscosity. Also, the first few higher terms in an asymptotic expansion in powers of the viscosity are derived. These, of course, no longer have Gilbert-Landau-Lifshitz (GLL) form, but still preserve magnitude of the uniform part of the magnetization. For samples larger than a wavelength of sound, propagation effects lend to a damping contribution that depends on spatial variation of the magnetization field. However, for samples of this size, degradation of the uniform motion by spin wave excitations needs to be taken into account. Then the damping of the uniform motion no longer conserves its length, and the GLL damping term no longer applies. Instead, damping terms take forms similar to those found in paramagnetic resonance. Finally, an estimate is made of the initial switching rate of magnetization reversal caused by spin wave excitations. The possibility emerges that the switching rate may pass through two regimes: a fast rate prevailing until the uniform part of the magnetization vector is fully destroyed, and a slow rate prevailing while it reassembles to its new orientation.

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