LECTURE I: METASTABLE STATES OF THE JOSEPHSON JUNCTION

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OUTLINE

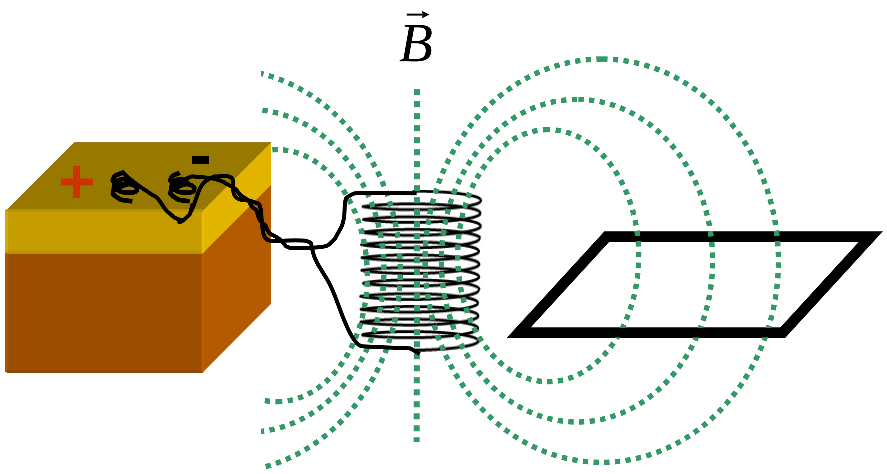
Lecture I: Metastable States of the Josephson Junction

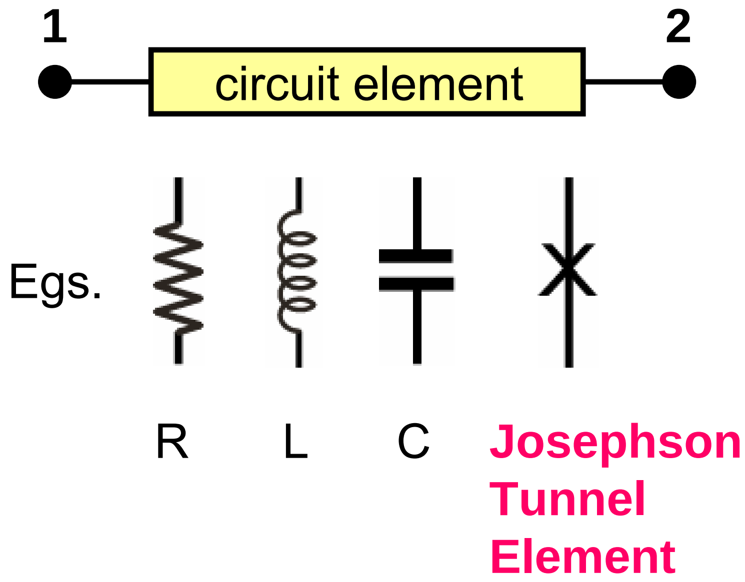
- Josephson junction dynamics
- Non-linear Josephson inductance
- DC / RF current biased junction
    - metastable states
    - escape dynamics
- Bifurcation amplification

Lecture II: Bifurcation Readout for Superconducting Qubits

- Quantum information and superconducting qubits
- Quantronium qubit
    - DC switching readout
    - RF bifurcation readout
- Coherence measurements
- Information flow
- Stark shift spectroscopy and relaxation

GENERALIZED FLUX





$$\Phi_{mag}(t) = \int_{surface} \vec{B}(t) \cdot d\vec{A}$$

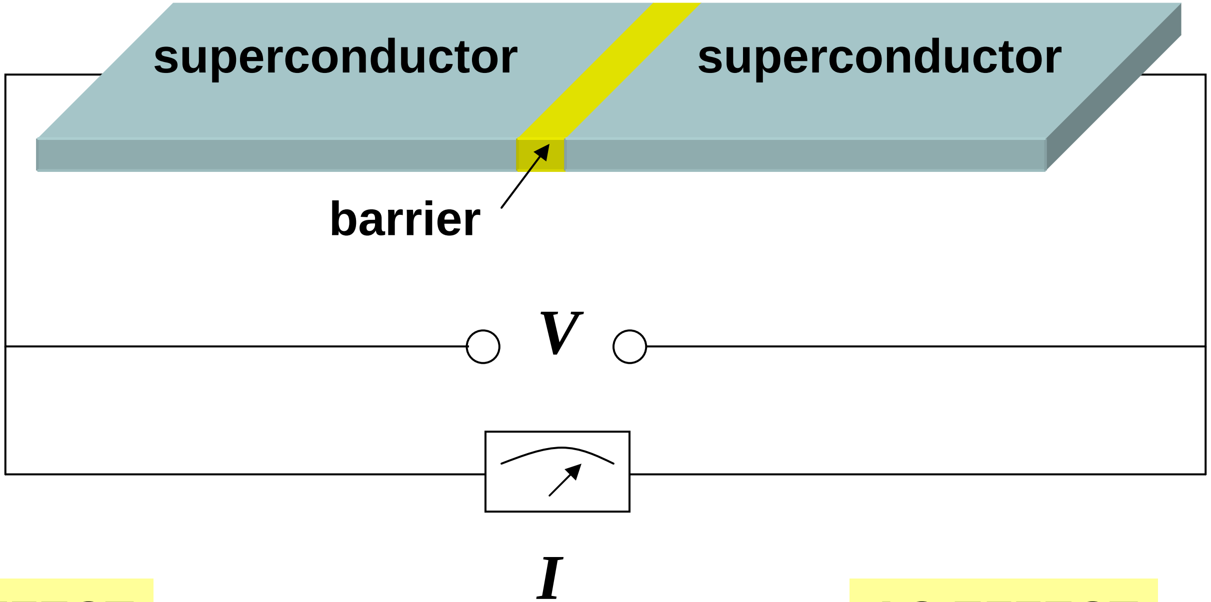
$$\int_{perimeter} \vec{E}(t) \cdot d\vec{l} = -\frac{d\Phi_{mag}(t)}{dt}$$

$$\Phi(t) = -\int_{-\infty}^{t} \int_{1}^{2} \vec{E}(t') \cdot d\vec{l}\, dt'$$

$$= \int_{-\infty}^{t} V_{12}(t') dt'$$

JOSEPHSON TUNNEL JUNCTION

$$\Psi_{left} = \psi_l \, e^{i\varphi_l} \qquad \Psi_{right} = \psi_r \, e^{i\varphi_r}$$



DC EFFECT

$$I = I_0 \sin(\Delta\varphi = \varphi_l - \varphi_r)$$

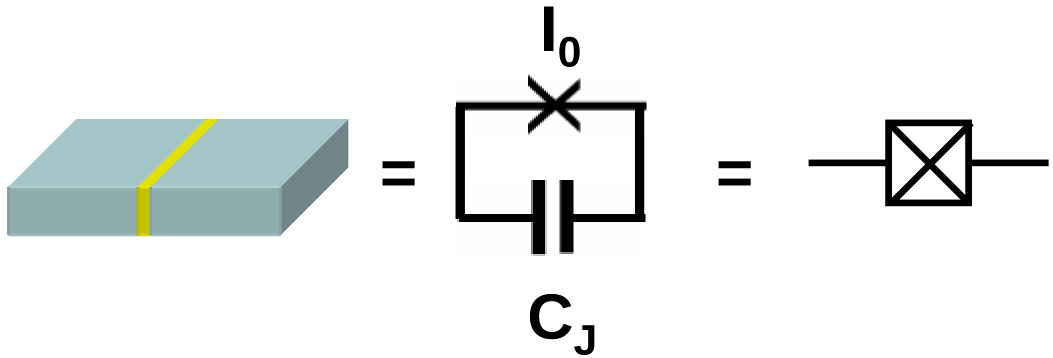
- Supercurrent with V=0
- I_0 is material dependent (nA-mA)

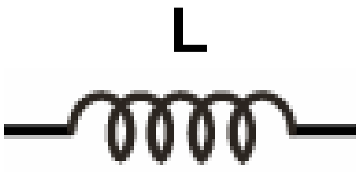
AC EFFECT

$$V = \frac{\hbar}{2e}\frac{d(\Delta\varphi)}{dt} = \varphi_0 \frac{d(\Delta\varphi)}{dt}$$

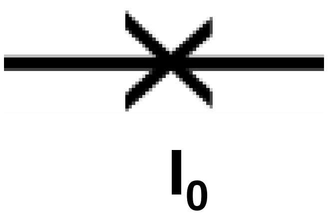
- Fixed V, I oscillates
- Magnitude I_0 , ν=2eV/*h (483 MHz/μV)*

THE NON-LINEAR JOSEPHSON INDUCTOR



- 

$$I(t) = \frac{1}{L}\Phi(t)$$

- 

$$I(t) = I_0 \sin[\Phi(t)/\varphi_0]$$

$$= \frac{I_0}{\varphi_0}\Phi(t) - \frac{I_0}{6}\left(\frac{\Phi(t)}{\varphi_0}\right)^3 + \ldots$$

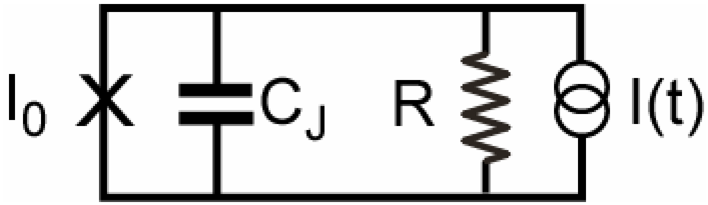
Josephson Inductance

$$L_J \equiv \frac{\varphi_0}{I_0} = \frac{\hbar R_T}{\pi \Delta_{BCS}}(BCS)$$

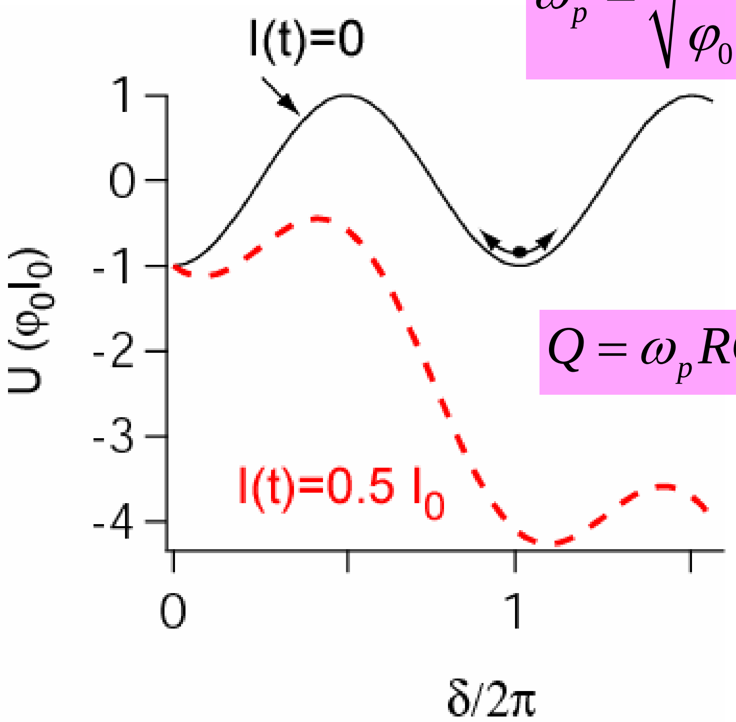
Gauge Invariant Phase Difference

$$\delta \equiv \frac{\Phi}{\varphi_0} \qquad \Delta\varphi \sim \delta \bmod 2\pi$$

THE EQUATION OF MOTION



$$\omega_p = \sqrt{\frac{I_0}{\varphi_0 C_J}}$$



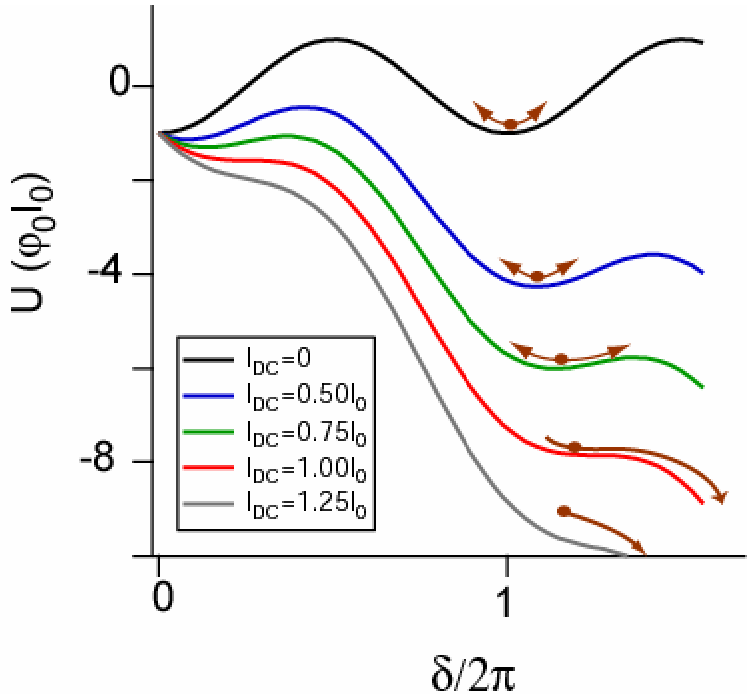
$$Q = \omega_p R C_J$$

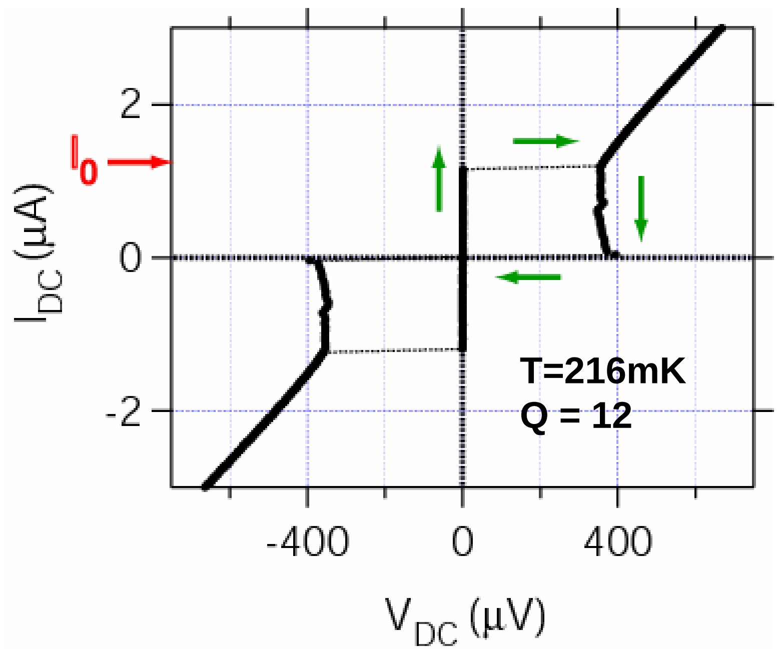
$$I(t) = \frac{V}{R} + C_J \frac{dV}{dt} + I_0 \sin\delta$$

$$\underbrace{\varphi_0^2 C_J}_{mass} \frac{d^2\delta}{dt^2} + \frac{\varphi_0^2}{\underbrace{R}_{drag}} \frac{d\delta}{dt} + \underbrace{\varphi_0 I_0 \sin\delta - \varphi_0 I(t)}_{\frac{\partial U(\delta)}{\partial \delta}} = 0$$

$$U(\delta) = -\varphi_0 I_0 \cos\delta - \varphi_0 I(t)\delta$$

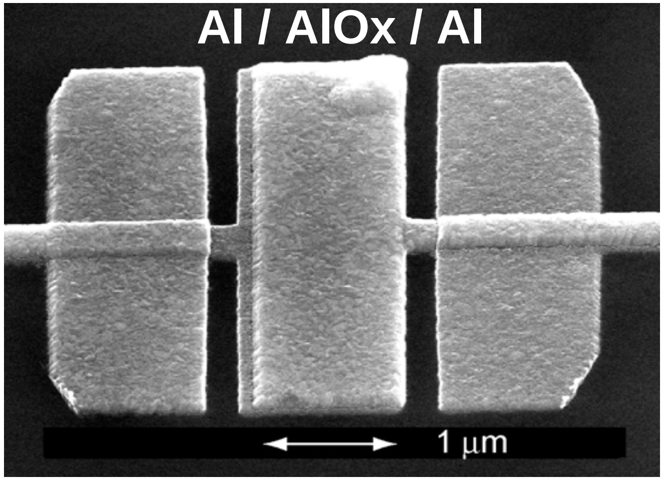
DC CURRENT BIAS I: I-V Curve



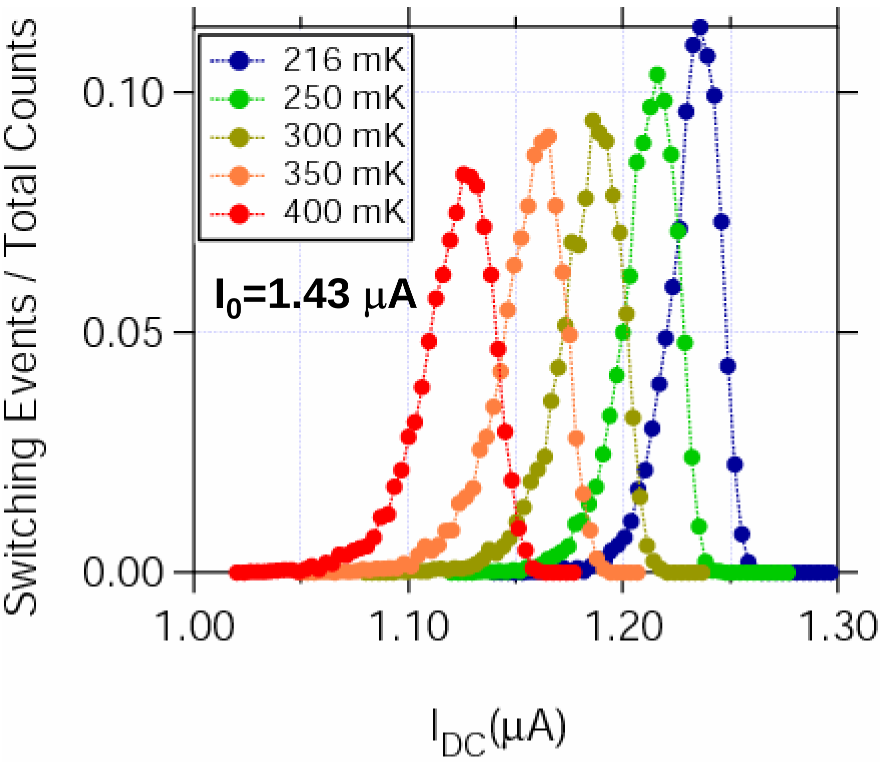


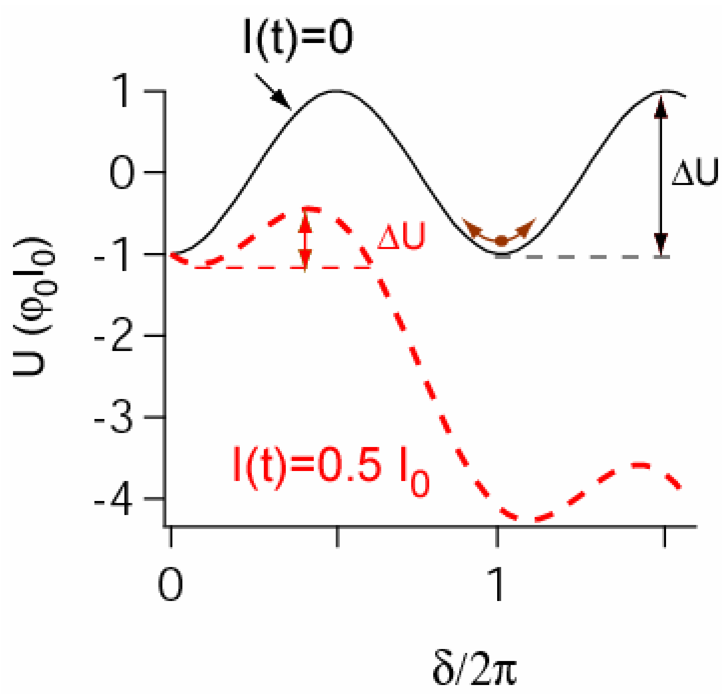
- $V_{DC} = \varphi_0 \langle \delta \rangle$
- $I_{DC} < I_0 : \langle \delta \rangle = 0 \longrightarrow$ superconducting

  $I_{DC} > I_0 : \langle \delta \rangle \neq 0 \longrightarrow$ dissipative



DC CURRENT BIAS II: Metastability & Switching





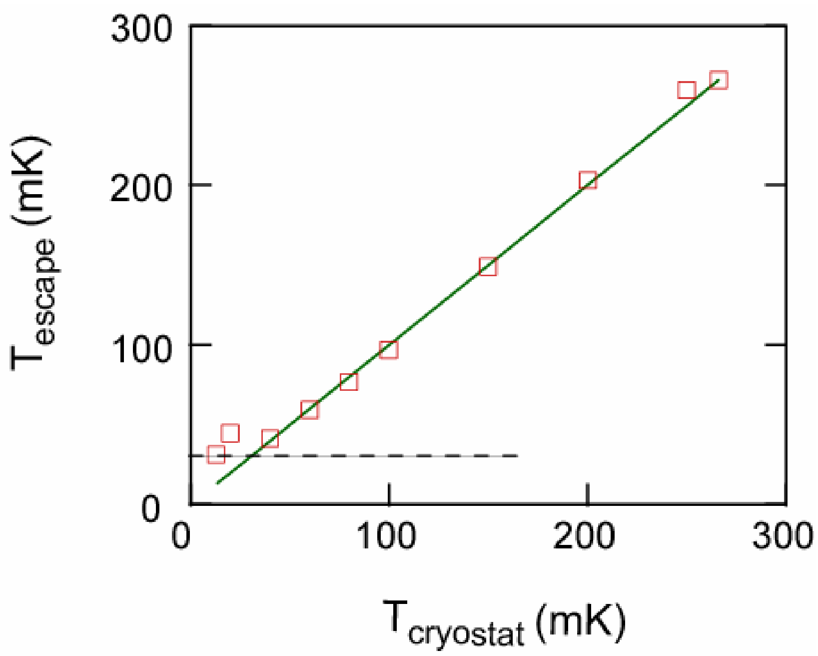
$$\Gamma_{0\to 1}(I_0, I_{DC}, T) = \frac{\omega_p}{2\pi}\exp\left(-\frac{\Delta U}{kT}\right)$$

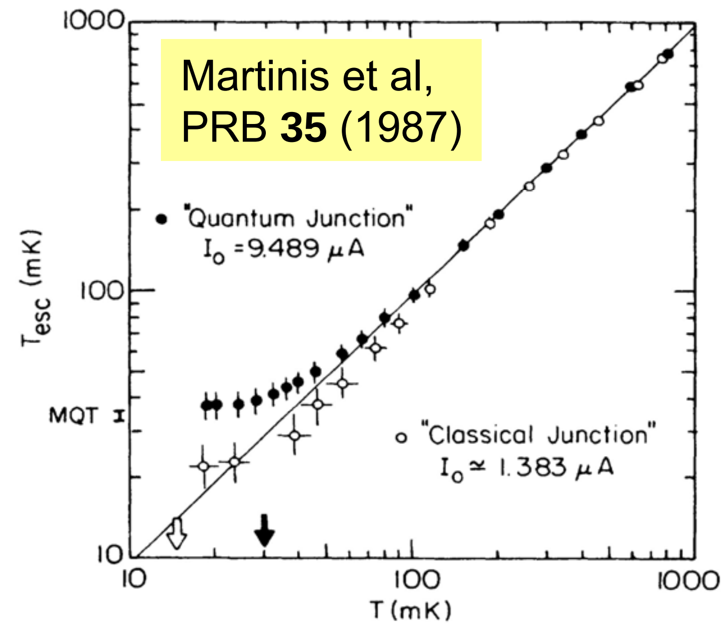
$(kT >> \hbar\omega)$

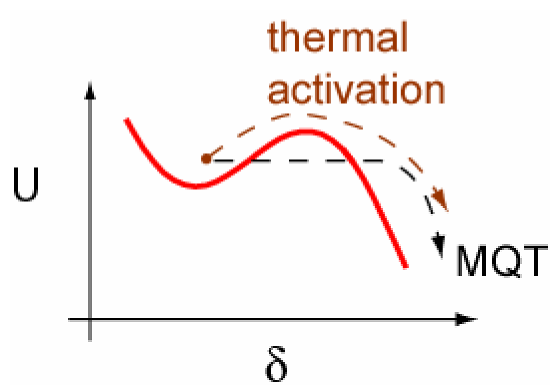
$$\Delta U(I_0, I_{DC}) = \underbrace{\left[\frac{2\sqrt{2}}{3}\frac{\hbar}{e}I_0\right]}_{u_0 \approx 50\,\mathrm{K}} \cdot \left(1 - \frac{I_{DC}}{I_0}\right)^{3/2}$$

$I_0 \downarrow$ 1μA

DC CURRENT BIAS III: Macroscopic Quantum Tunneling (MQT)







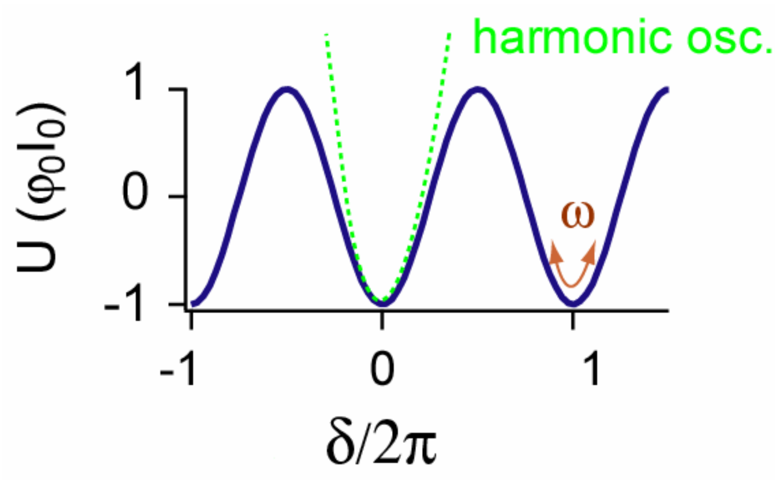
$$T > T^* = \frac{\hbar\omega_p}{7.2k} \quad : \; \Gamma \propto e^{\frac{-\Delta U}{kT}}$$

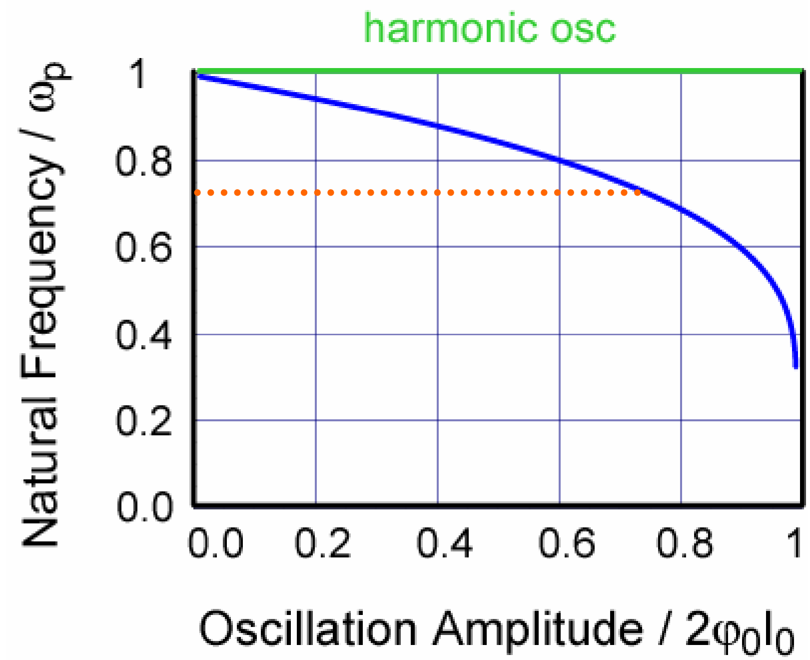
thermal activation

$$T < T^* = \frac{\hbar\omega_p}{7.2k} \quad : \; \Gamma = \text{constant}$$

MQT

RF CURRENT BIAS I: SOFTENING POTENTIAL





$$I(t) = I_{RF}\sin(\omega t)$$

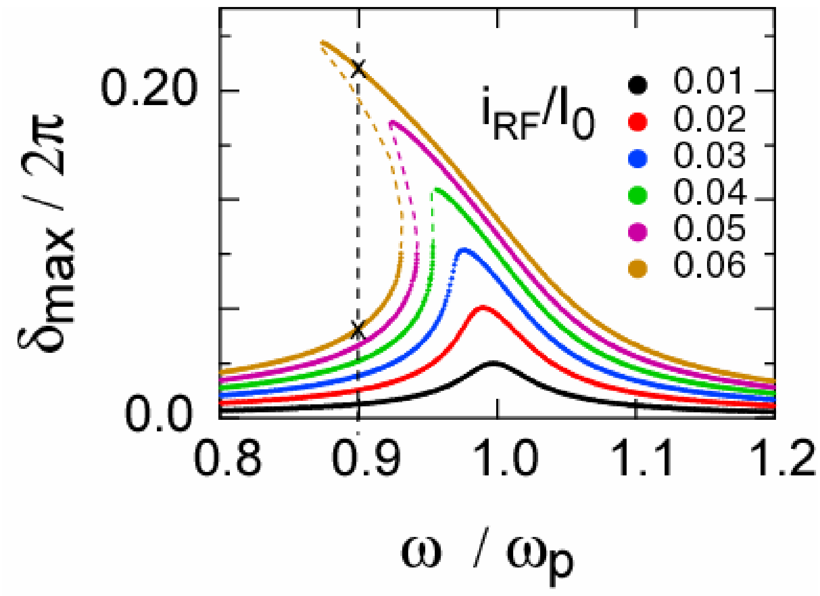
$$\delta(t) = \delta_{\max}\sin(\omega t + \gamma)$$

$$V(t) = \varphi_0 \dot{\delta}$$

- Frequency decreases w. drive amplitude

- For $\omega<\omega_p$, weak drive → off resonance
  strong drive → on resonance

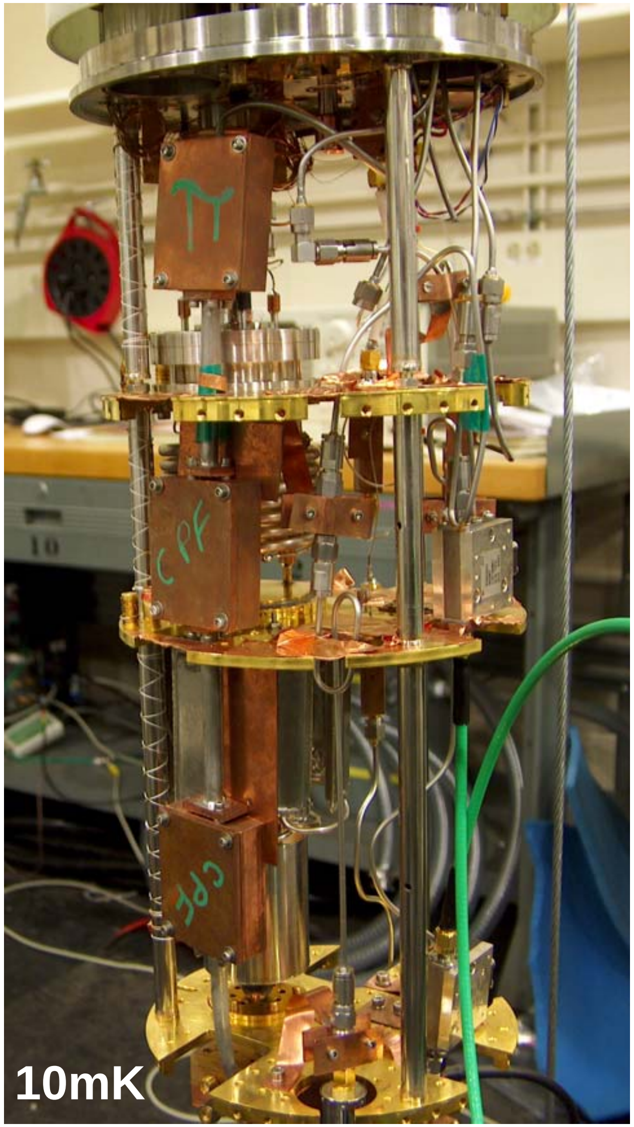
RF CURRENT BIAS II: TWO DYNAMICAL STATES



- If $\Delta\omega > \frac{\omega_p}{Q}\frac{\sqrt{3}}{2}$, bistability
- Dynamical states differ in oscillation amplitude & phase

$$\varphi_0^2 C_J \frac{d^2\delta}{dt^2} + \frac{\varphi_0^2}{R}\frac{d\delta}{dt} + \varphi_0 I_0\left(\delta - \frac{\delta^3}{6}\right) - \varphi_0 I_{RF}\sin(\omega t) = 0$$

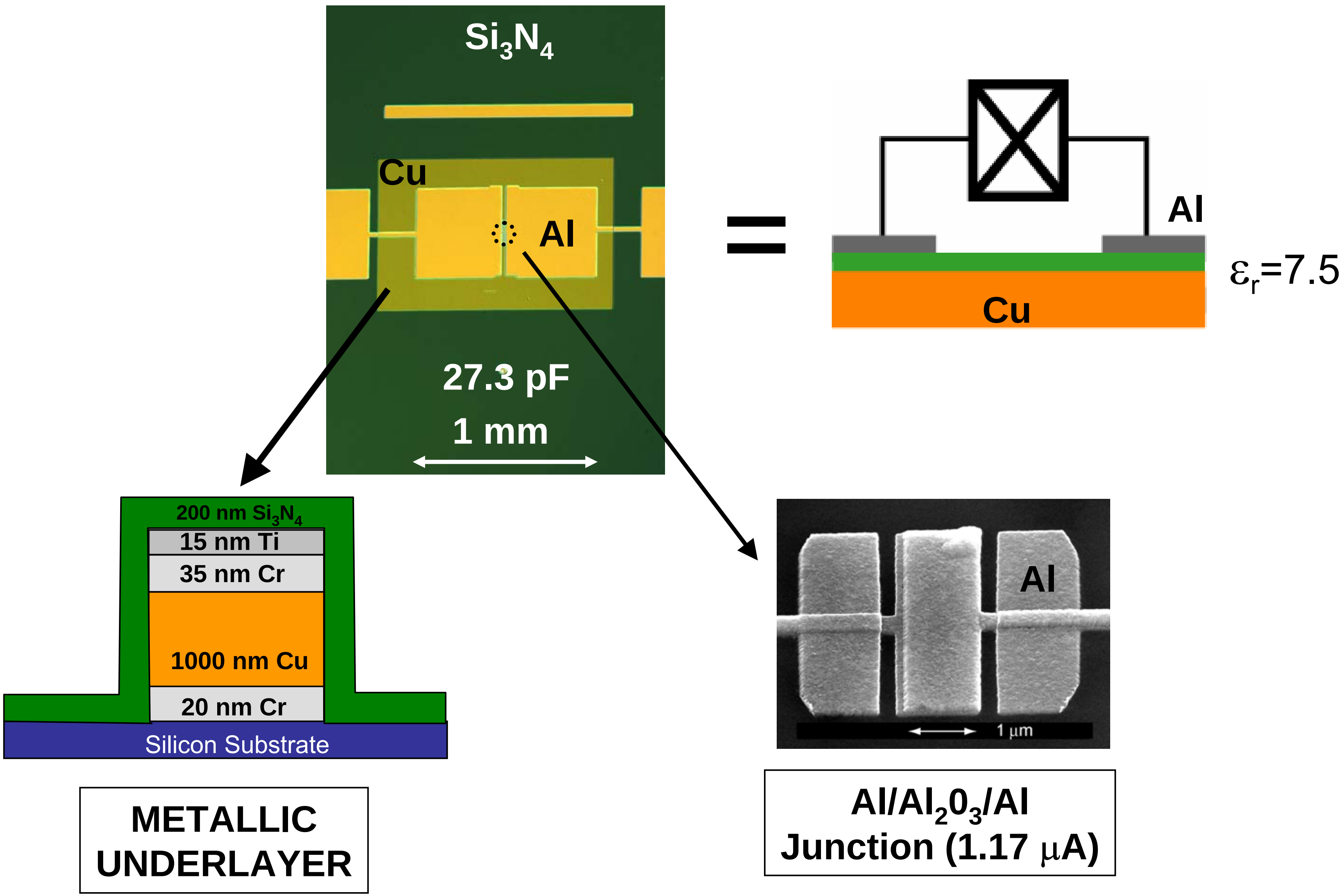
EXPERIMENTAL SETUP



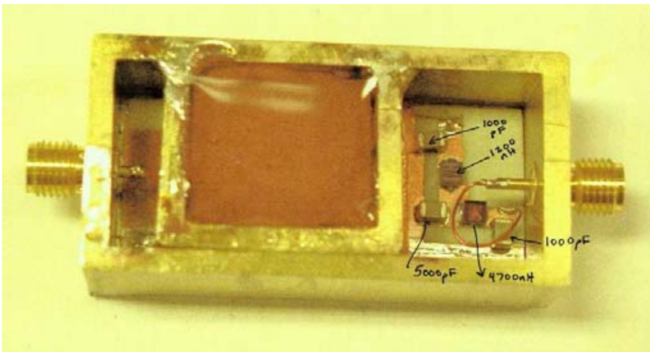
Challenges

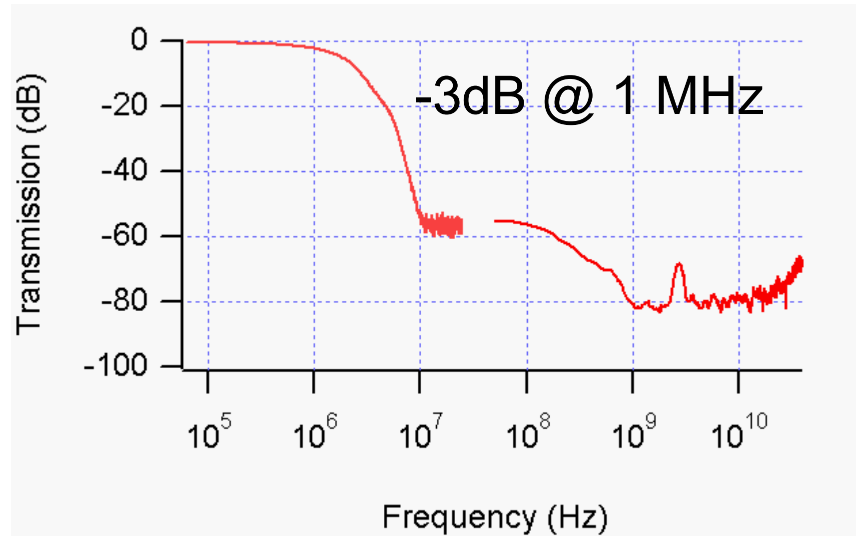
- Fully coherent microwave measurements
- High speed data acquisition
- Precision circuit design (remember L_J ~ 300pH)
  → special capacitors
- Ultra-low noise wide-band electronics
  → new cryo filters

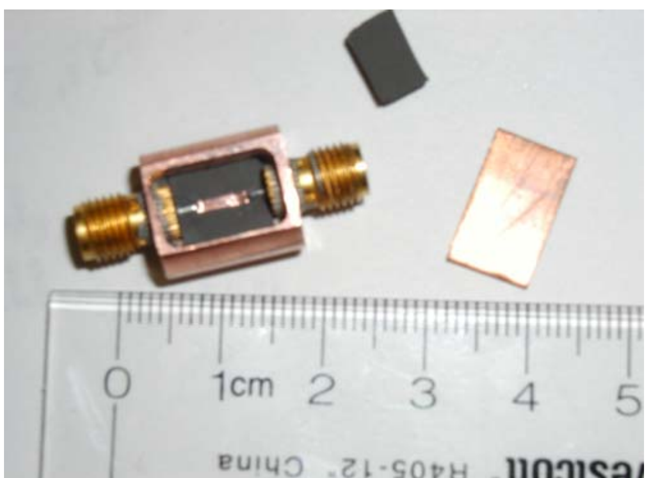
JUNCTION + MICROWAVE CAPACITOR

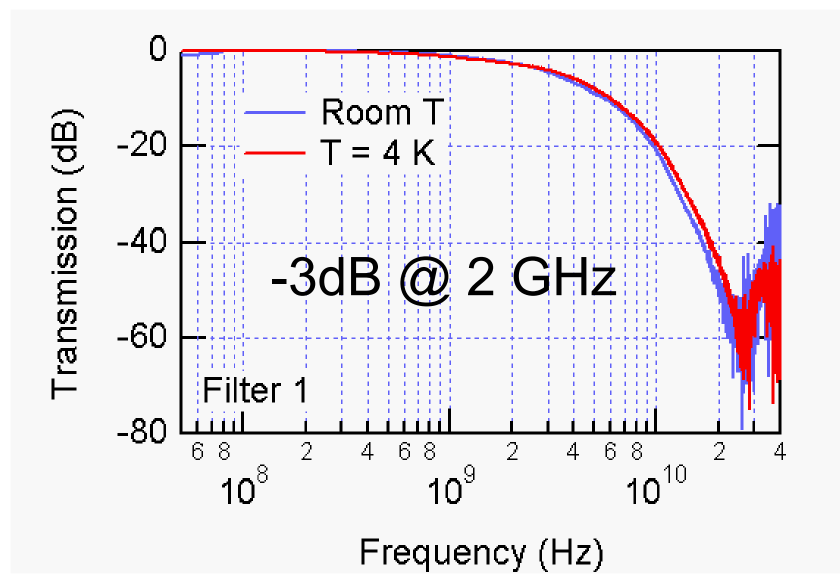


HIGH FREQUENCY CRYOGENIC FILTERING

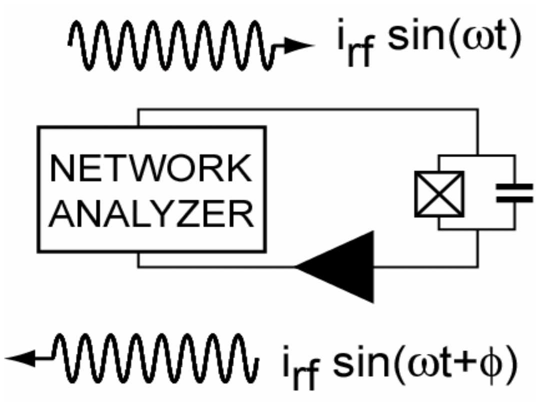


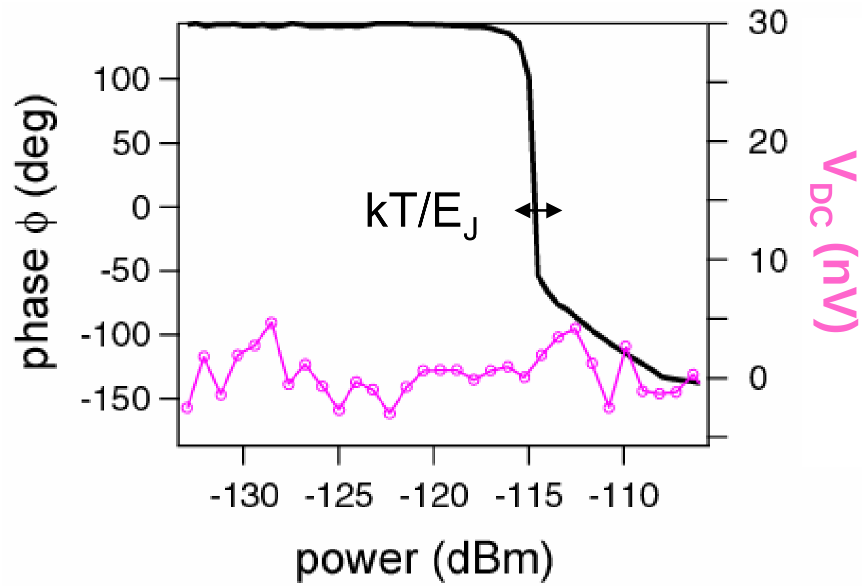


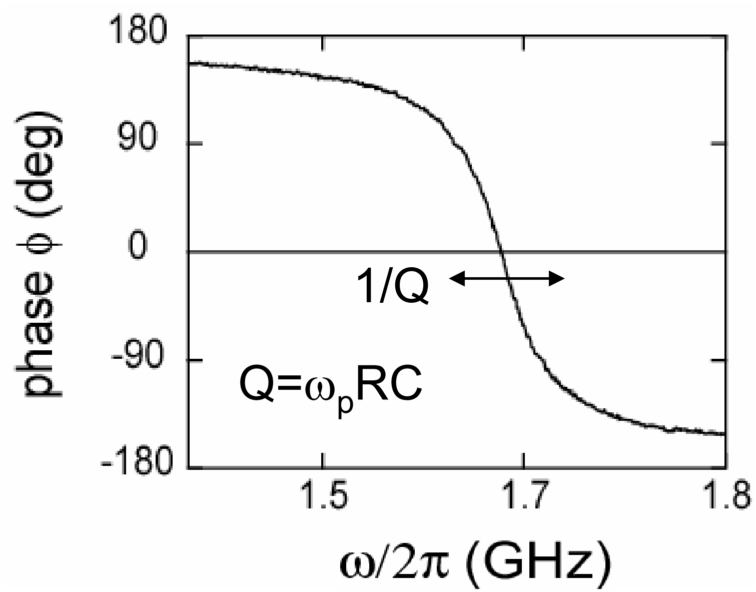


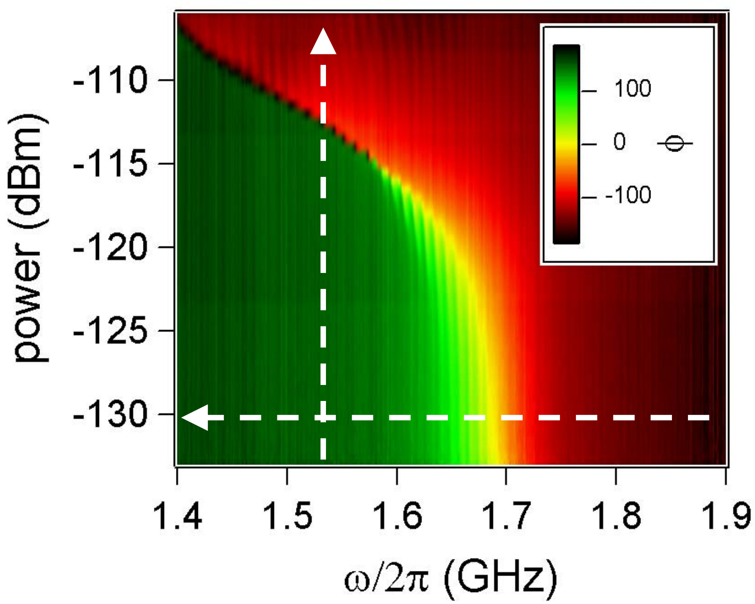


RF CURRENT BIAS III: Plasma Resonance

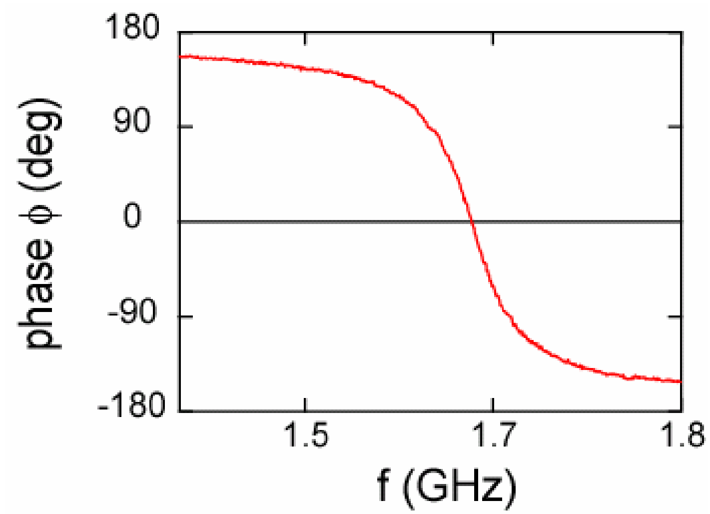


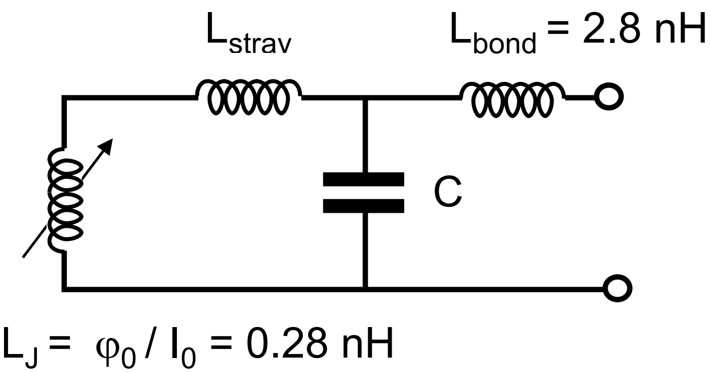


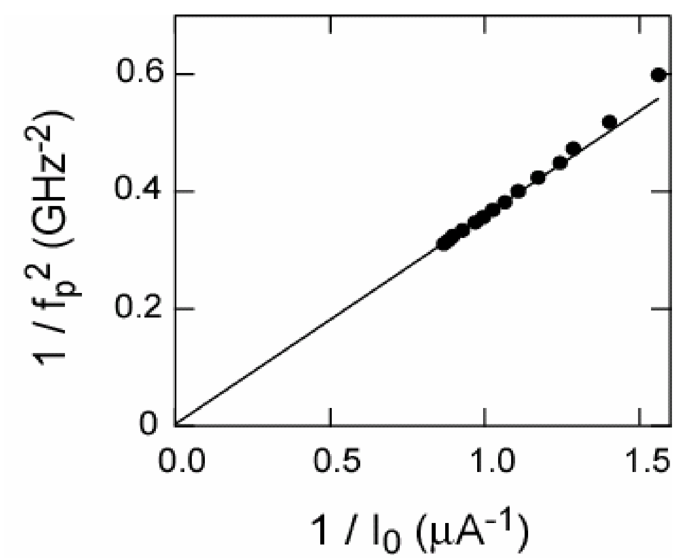




STRAY REACTANCES







$$\frac{1}{4\pi^2 f_p^2} = \frac{\hbar}{2e} C \frac{1}{I_0} + C\, L_{stray}$$

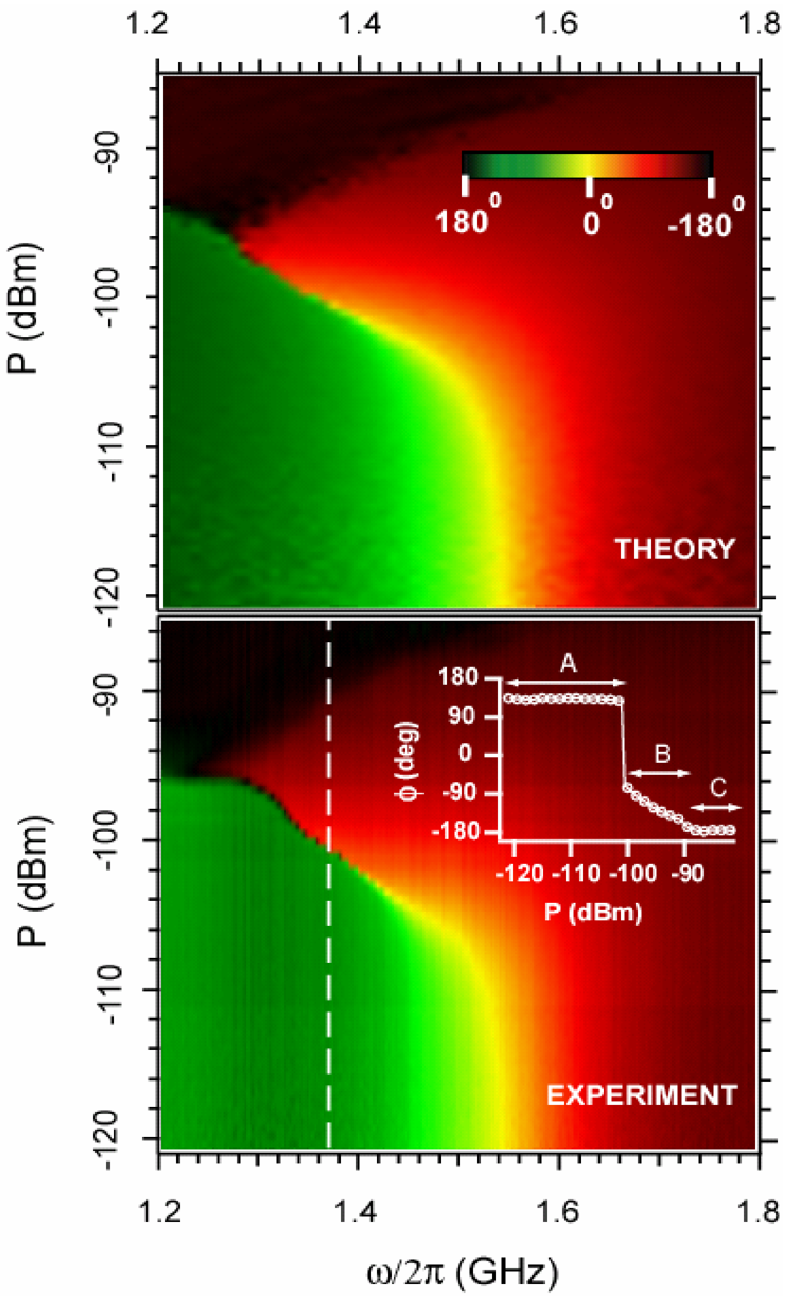
L_{stray} = 0.003 nH

C = 27.3 pF

PHASE DIAGRAM: EXP & THY IN GOOD AGEEMENT

- Dark region corresponds to well-jumping

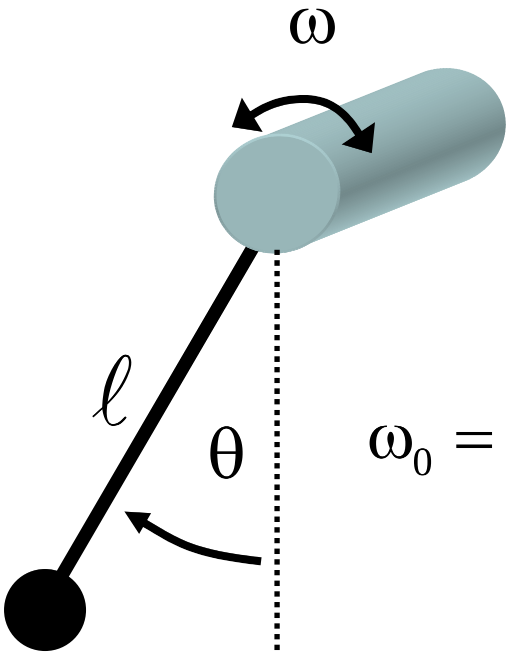
- All parameters in prediction measured experimentally!



THE DRIVEN PENDULUM

$\omega < \omega_0$

Regime	Drive	Dynamical States
harmonic	weak (θ_{max}<<1)	1
bifurcation	medium (θ_{max}~$\pi/4$)	2
chaotic	strong (θ_{max}>2π)	>>> 2



$$\omega_0 = 2\pi\sqrt{\frac{g}{\ell}}$$

$I_0^{-1} \leftrightarrow l$

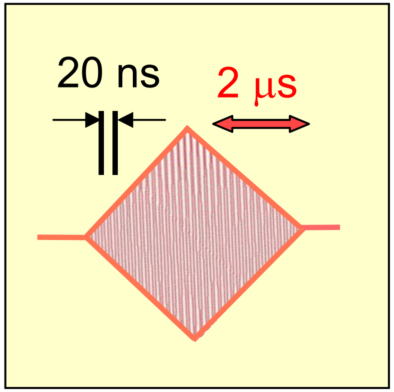
$C_J^{-1} \leftrightarrow g$

$\omega_p \leftrightarrow \omega_0$

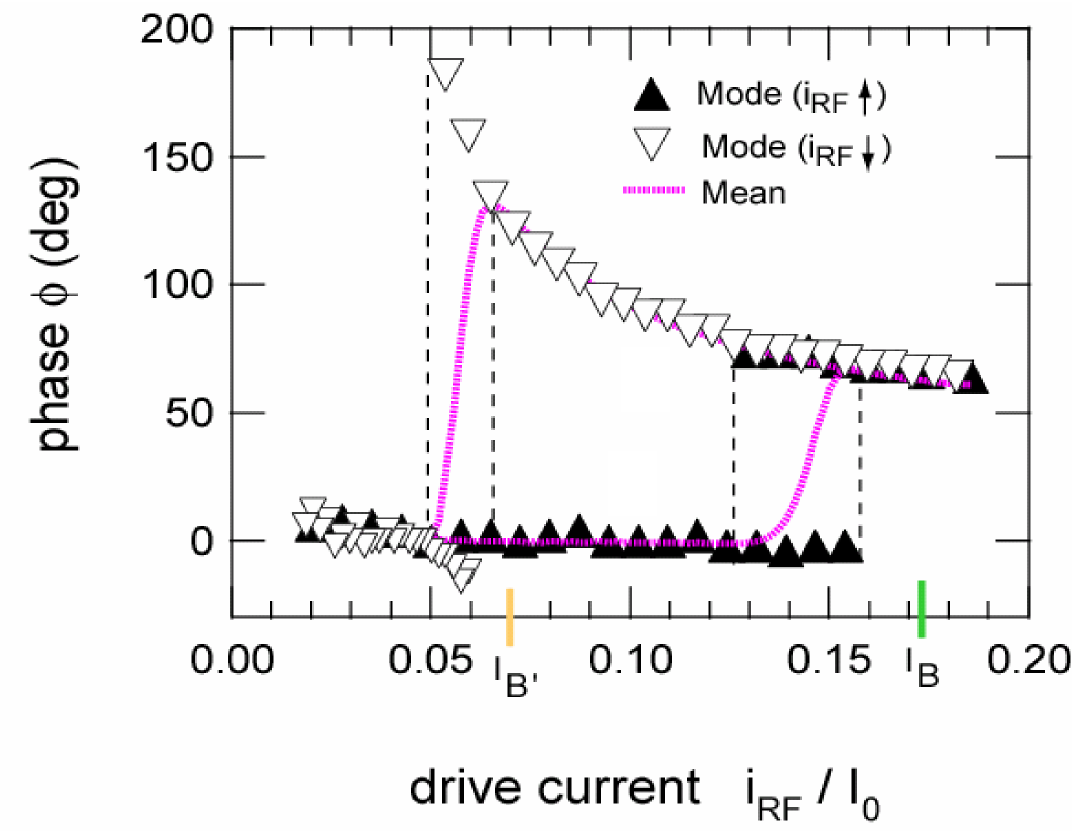
$\delta \leftrightarrow \theta$

$V \leftrightarrow \dot{\theta}$

DYNAMICS OF DYNAMICAL SWITCHING



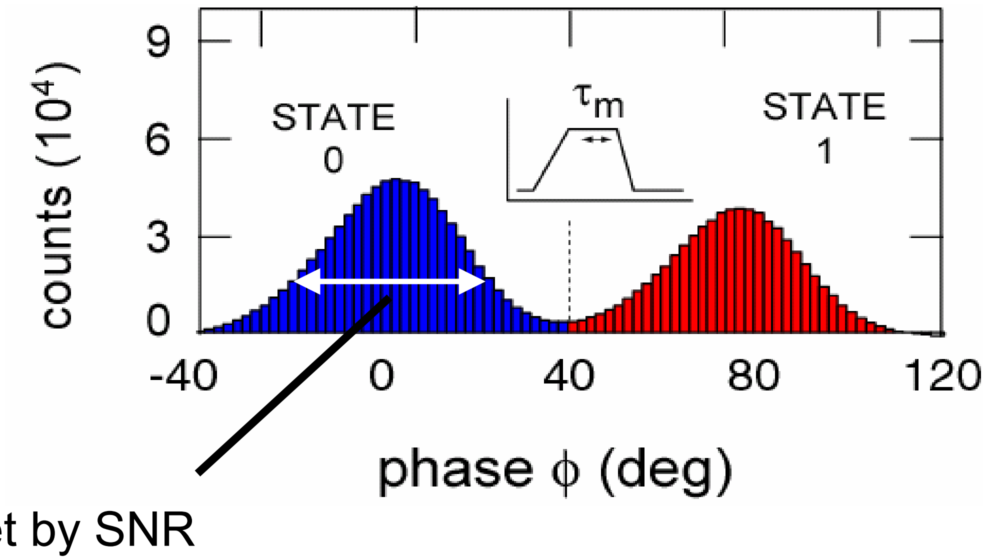
- N = 600,000
- $\Delta\phi$ = 74 deg
- Hysteresis
  I_B and $I_{B'}$ correct !



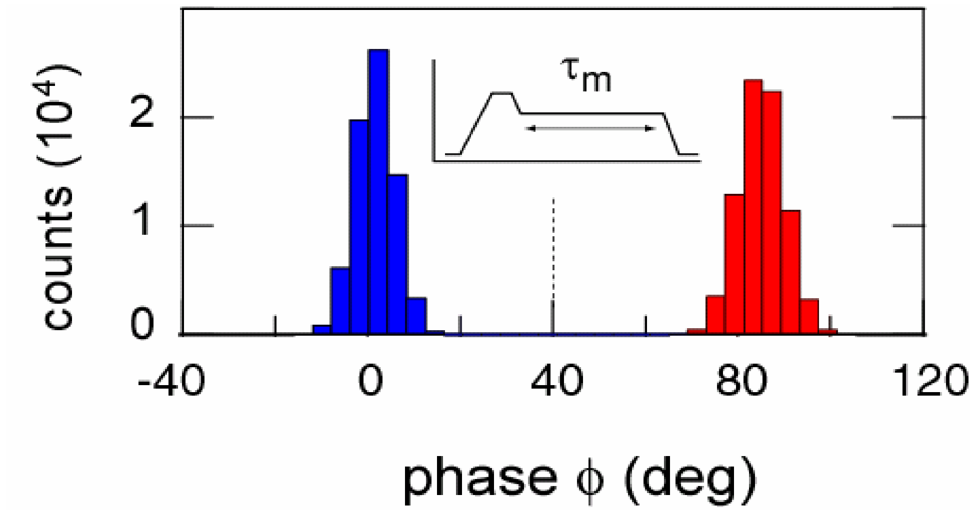
$$I_B = \left[\frac{16}{3\sqrt{3}} \alpha^{3/2} (1-\alpha)^{3/2} \right] I_0 \qquad \alpha = \left(1 - \frac{\omega}{\omega_p} \right) = 0.122$$

SWITCHING HISTOGRAMS

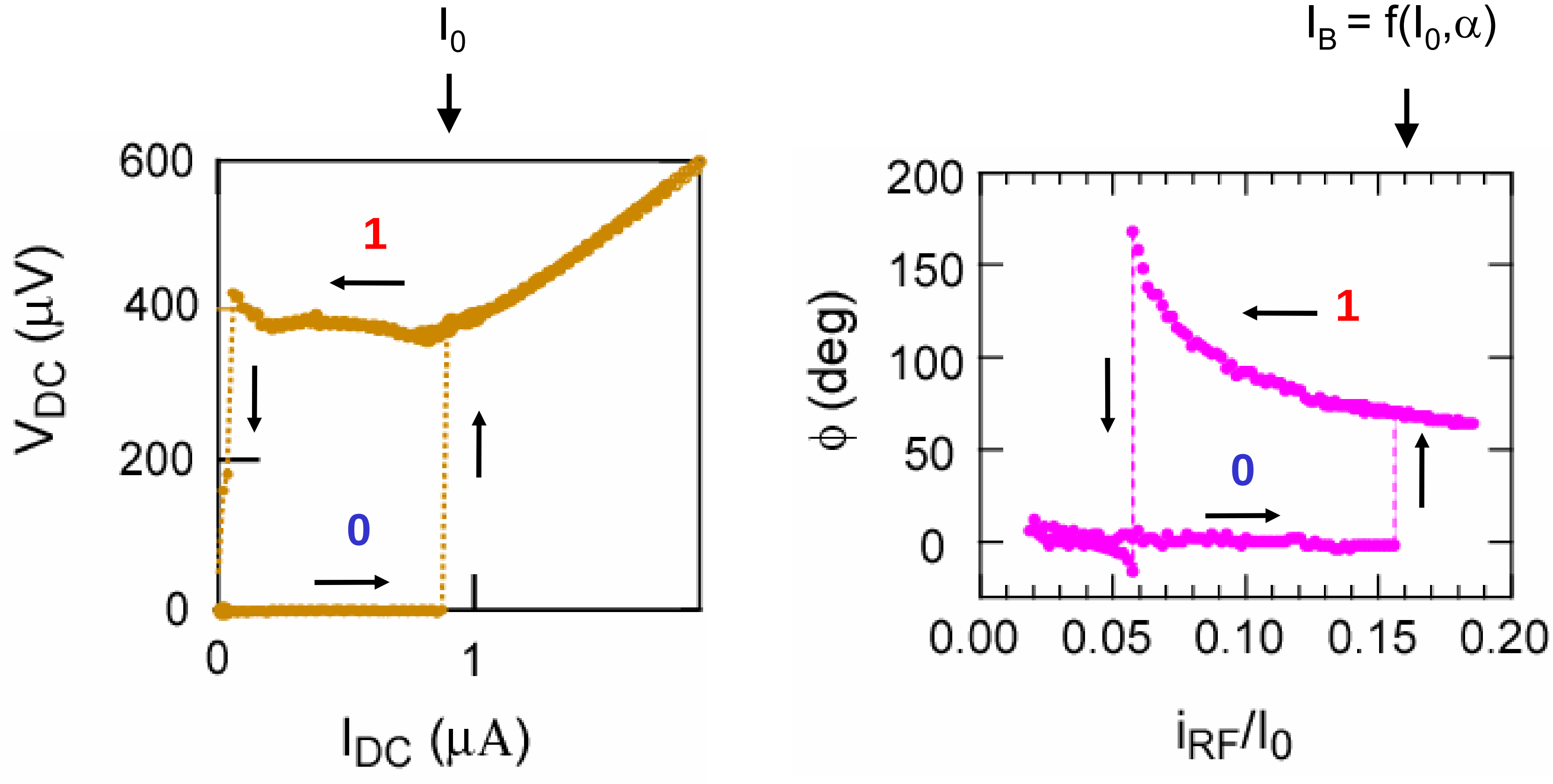
- No latching
- 40 ns rise + 20 ns settle
- τ_m = 20 ns



- Latching
- 40 ns rise + 20 ns settle
- τ_m = 300 ns



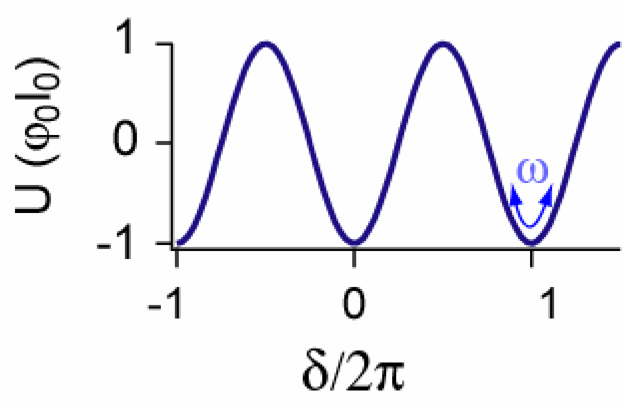
DC vs. AC



$I_{DC} \leftrightarrow i_{RF}$
$V_{DC} \leftrightarrow \phi$
$R_s \leftrightarrow \alpha$

ATTRACTORS

0



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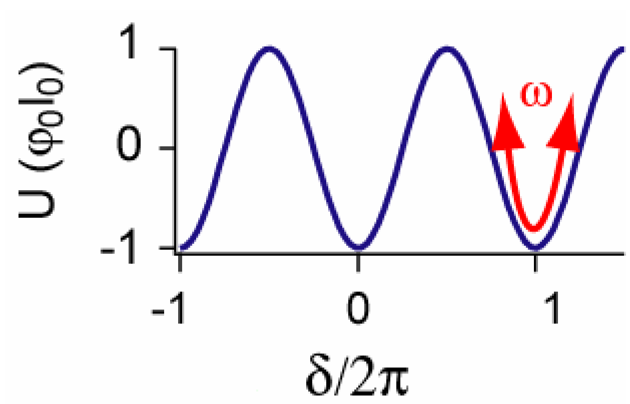
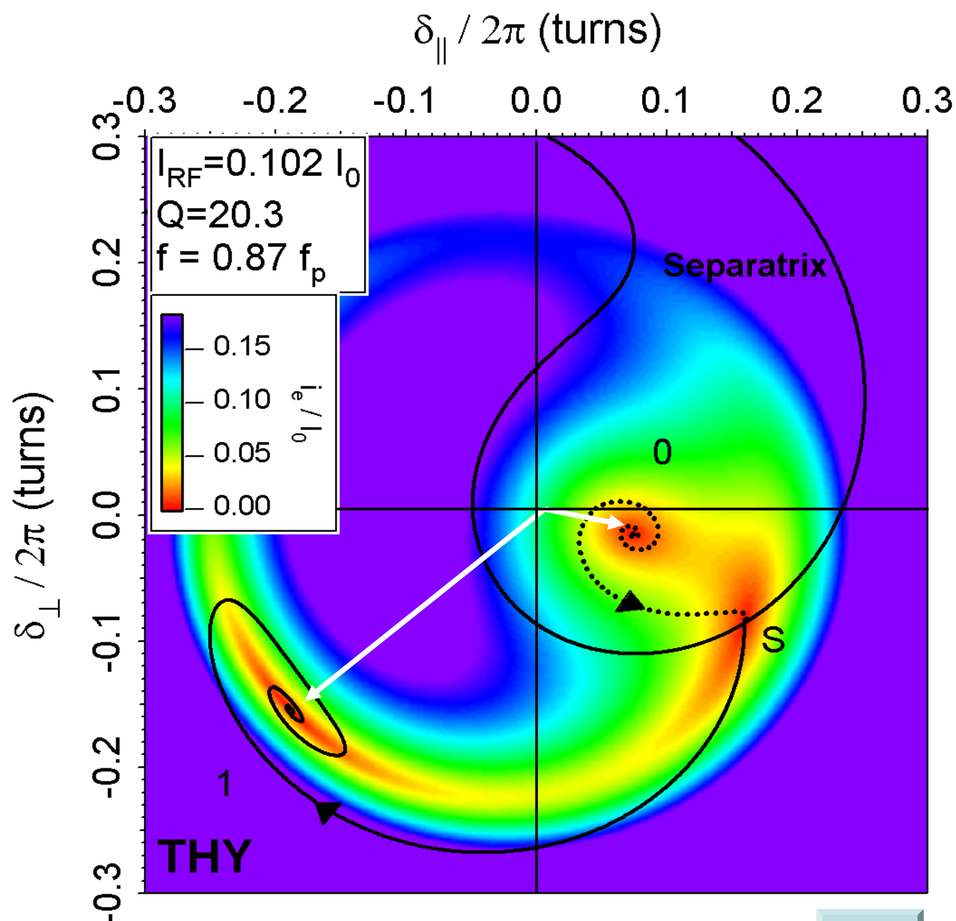
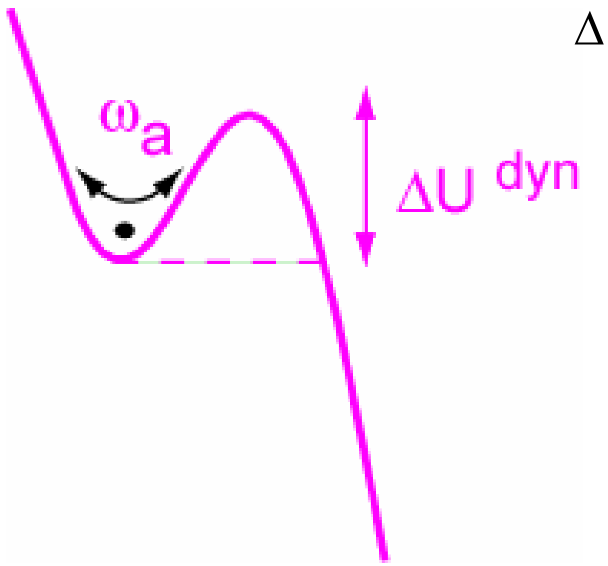




$$\delta(t) = \delta_{\parallel} \sin(\omega t) + \delta_{\perp} \cos(\omega t)$$

1-D METAPOTENTIAL



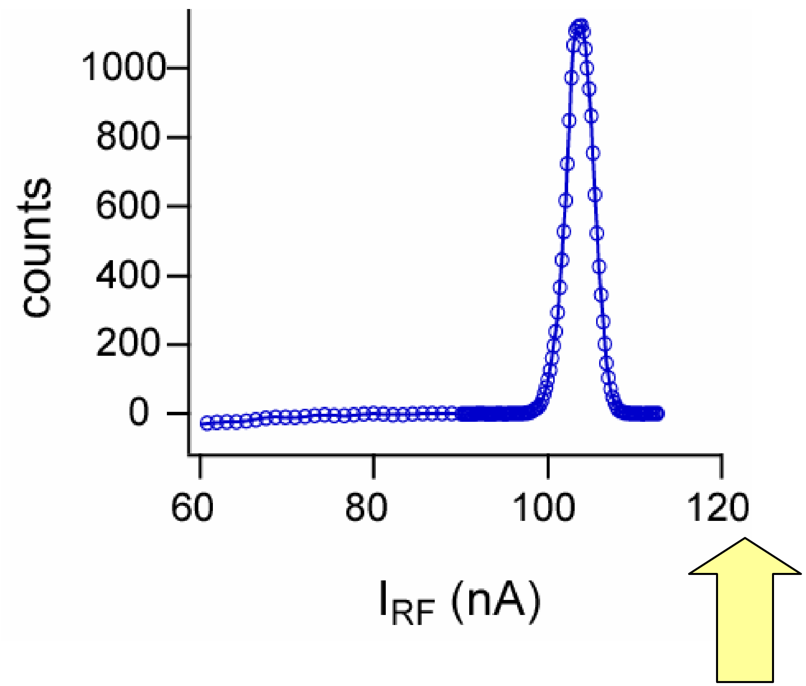
$$\Delta U^{dyn}(I_0, i_{RF}, \alpha) = \underbrace{\left[\frac{64\hbar}{18e\sqrt{3}} I_0\, \alpha (1-\alpha)^3\right]}_{u_0^{dyn} \approx 10\,\mathrm{K}} \left(1 - \left(\frac{i_{RF}}{I_B}\right)^2\right)^{3/2}$$

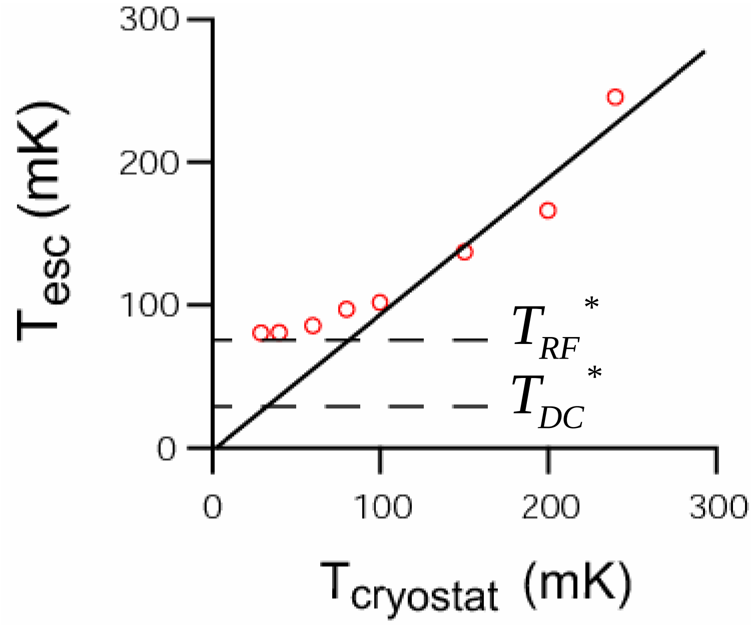
$$\omega_a = \underbrace{\left[\frac{4}{3\sqrt{3}} RC (\alpha\omega_p)^2\right]}_{\omega_{a0} \approx (2\pi)\bullet 350\mathrm{MHz}} \left(1 - \left(\frac{i_{RF}}{I_B}\right)^2\right)^{1/2}$$

$$\Gamma_{0\to1}^{dyn} = \frac{\omega_a}{2\pi} \exp\left(-\frac{\Delta U^{dyn}}{kT}\right)$$

$$I_B = \left[\frac{16}{3\sqrt{3}} \alpha^{3/2} (1-\alpha)^{3/2}\right] I_0 \approx 0.1 I_0$$

ESCAPE TEMPERATURE





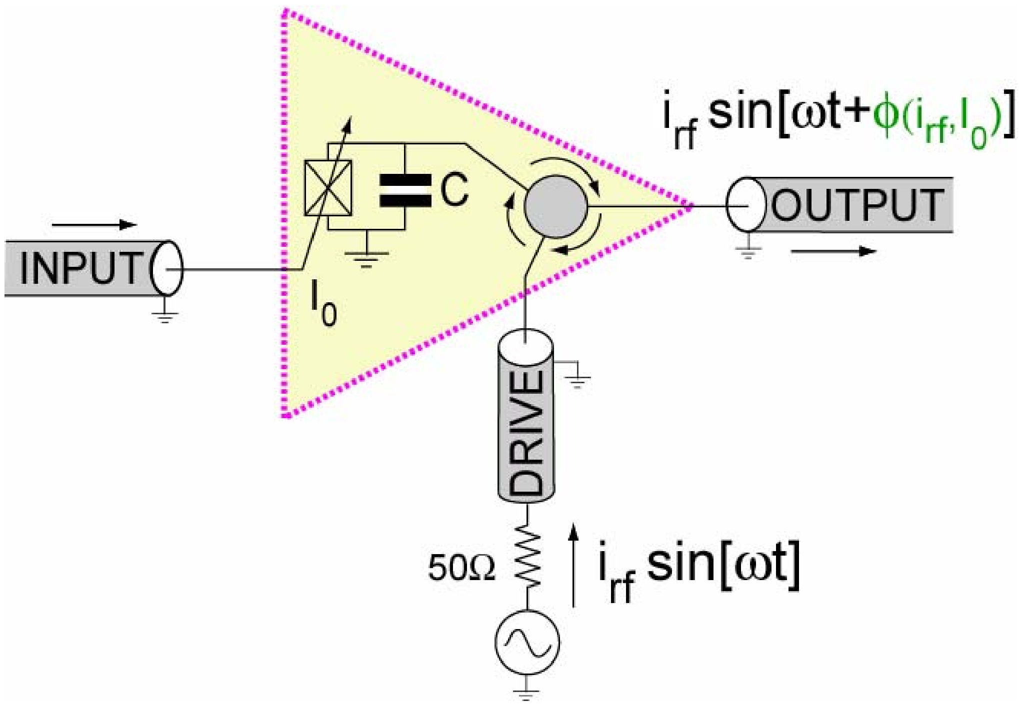
$I_b \sim 0.1\ I_0$

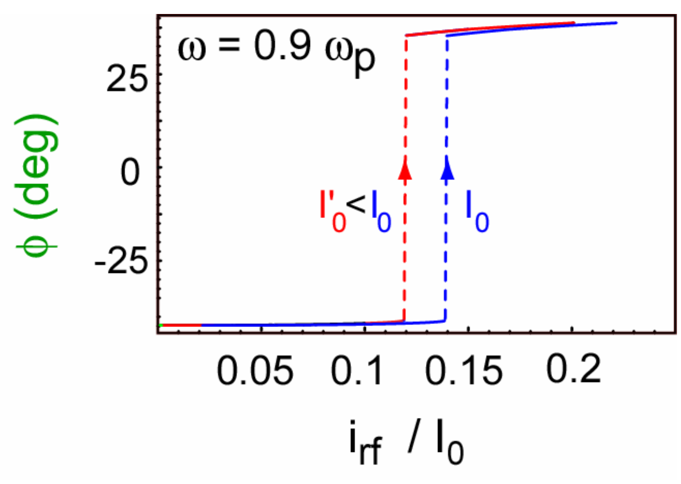
$$T_{RF}^* = \frac{\hbar\omega}{k}$$

$$T_{DC}^* = \frac{\hbar\omega_p}{7.2k}$$

Quantum Saturation
at Different Temperature !

JOSEPHSON BIFURCATION AMPLIFIER



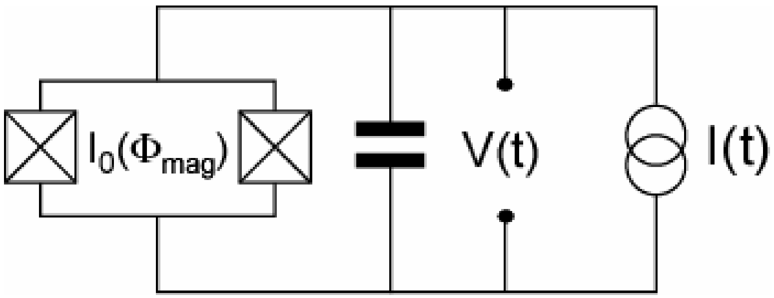


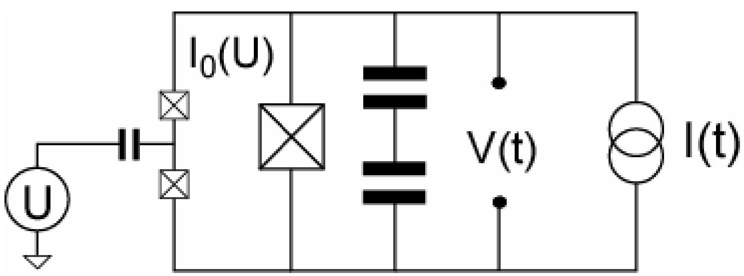
JBA: INPUT COUPLES TO I_0

- $\phi\ (i_{rf},I_0)$
- $P_{switch}\ (i_{rf},I_0)$

- minimal backaction
- no on-chip dissipation

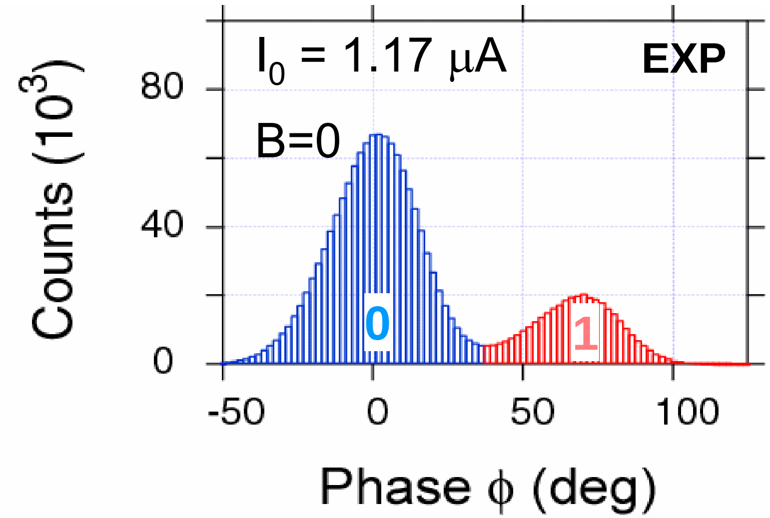
AMPLIFICATION

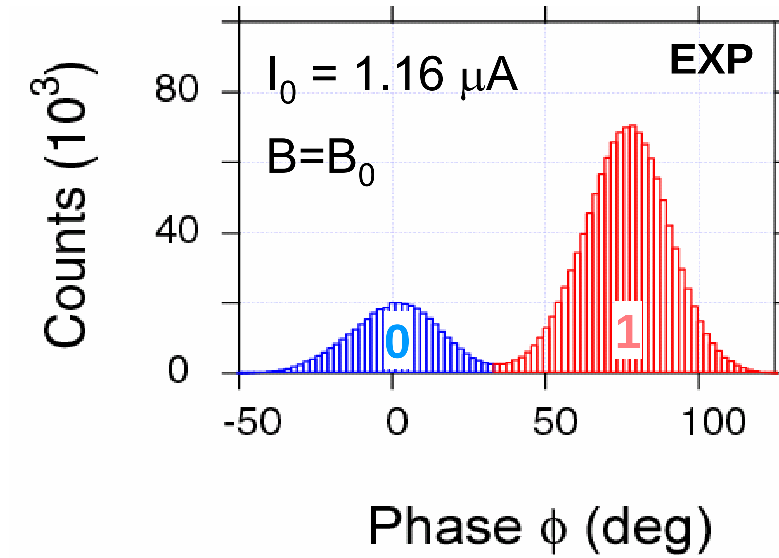


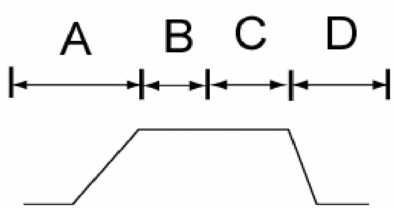


DEVICE	INPUT CIRCUIT	OPERATION
SQUID	Flux Loop	DC Switching
SQUID JBA	Flux Loop	RF Bifurcation
Quantronium	Single Cooper Pair Transistor	DC Switching
Quantronium + JBA	Single Cooper Pair Transistor	RF Bifurcation

SQUID JBA







A+D: τ_{dead} = 50 ns
B: $\tau_{register}$ = 20 ns
C: τ_{record} = 20 ns

10 MHz Rep. Rate

- fidelity = 80% @ 250mK for ΔI_0 = 10nA
- **predict > 95% @ T=60mK (single shot)**

SUMMARY

- RF DRIVEN JOSEPHSON JUNCTION
  - non-linear plasma resonance
  - metastable states
  - escape dynamics

- NOVEL QUANTUM SATURATION

- BIFURCATION AMPLIFICATION
  - observe predicted sensitivity
  - SQUID JBA
  - QUBIT READOUT → Next Lecture