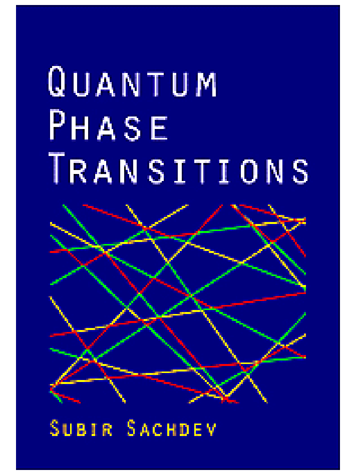
Magnetic phases and critical points of insulators and superconductors

• Colloquium article in *Reviews of Modern Physics*, July 2003, cond-mat/0211005.

• cond-mat/0109419

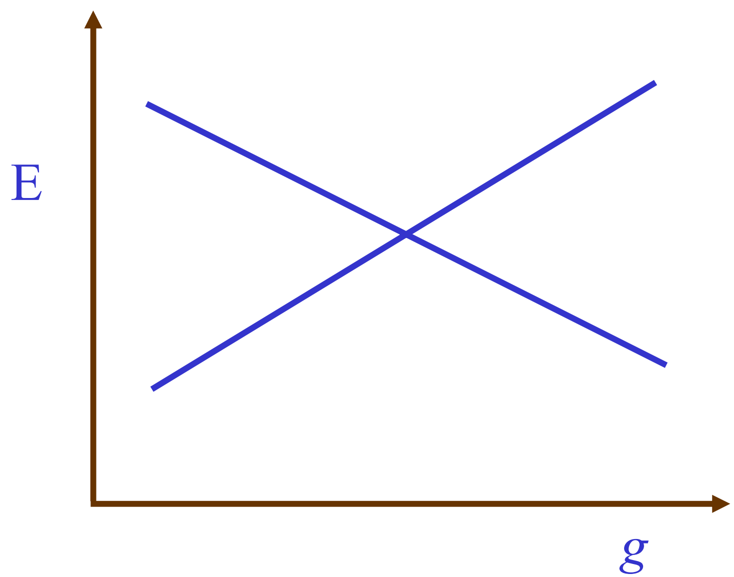


Quantum Phase Transitions
Cambridge University Press

Talks online:
Google Sachdev

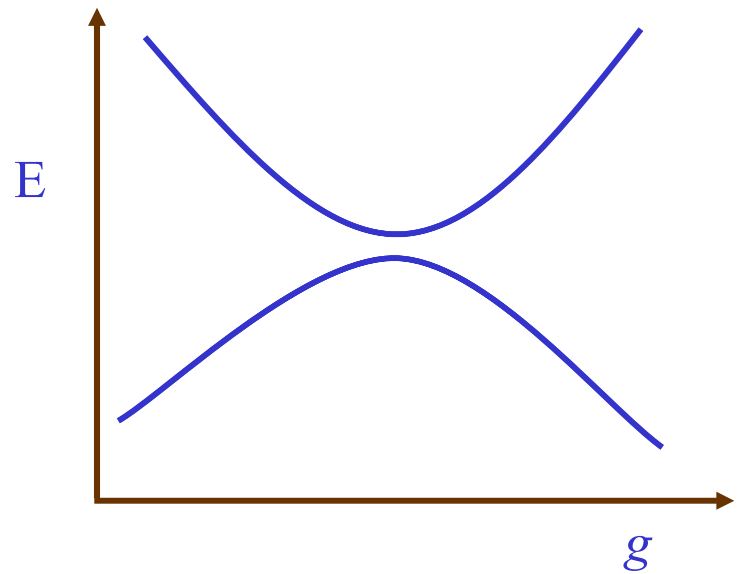
What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter *g*



True level crossing:

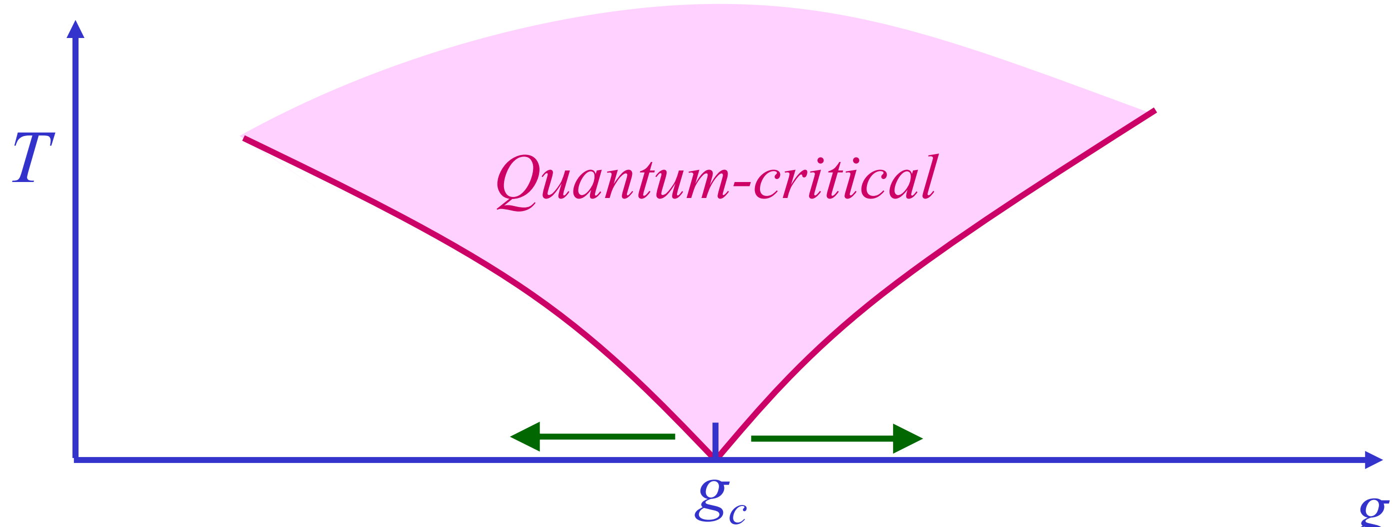
Usually a *first*-order transition



Avoided level crossing which becomes sharp in the infinite volume limit:

second-order transition

Why study quantum phase transitions ?



• Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

• Critical point is a novel state of matter without quasiparticle excitations

• Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$:
temporal and spatial scale invariance;
characteristic energy scale at other values of g: $\Delta \sim |g - g_c|^{z\nu}$

Outline

I. Quantum Ising chain

II. Coupled Dimer Antiferromagnet
- A. Coherent state path integral
- B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field
- Bose condensation of "triplons"

Single order parameter.

IV. Magnetic transitions in superconductors
- Quantum phase transition in a background Abrikosov flux lattice

V. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell.
- **Class A**: Compact U(1) gauge theory: collinear spins, bond order and confined spinons in $d=2$
- **Class B:** Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

Multiple order parameters.

VI. Conclusions

I. Quantum Ising Chain

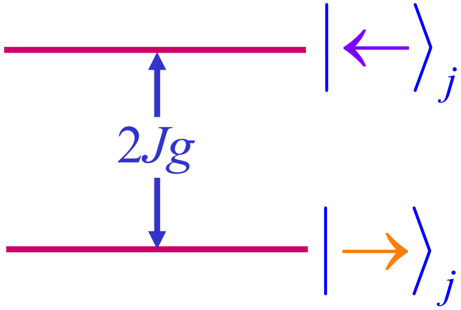
Degrees of freedom: $j = 1 \ldots N$ qubits, N "large"

$$\left|\uparrow\right\rangle_j , \left|\downarrow\right\rangle_j$$

or $$\left|\rightarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j + \left|\downarrow\right\rangle_j\right) , \quad \left|\leftarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j - \left|\downarrow\right\rangle_j\right)$$

Hamiltonian of decoupled qubits:

$$H_0 = -Jg \sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -J\sum_j \sigma_j^z \sigma_{j+1}^z$$

$$\Big(\left|\rightarrow\right\rangle_j \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_j \left\langle\rightarrow\right|\Big)\Big(\left|\rightarrow\right\rangle_{j+1} \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_{j+1} \left\langle\rightarrow\right|\Big)$$

Prefers neighboring qubits

are *either* $\left|\uparrow\right\rangle_j \left|\uparrow\right\rangle_{j+1}$ *or* $\left|\downarrow\right\rangle_j \left|\downarrow\right\rangle_{j+1}$

(not entangled)

Full Hamiltonian

$$H = H_0 + H_1 = -J\sum_j \left(g\sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$

leads to entangled states at g of order unity

Weakly-coupled qubits $(g \gg 1)$

Ground state:

$$|G\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle$$

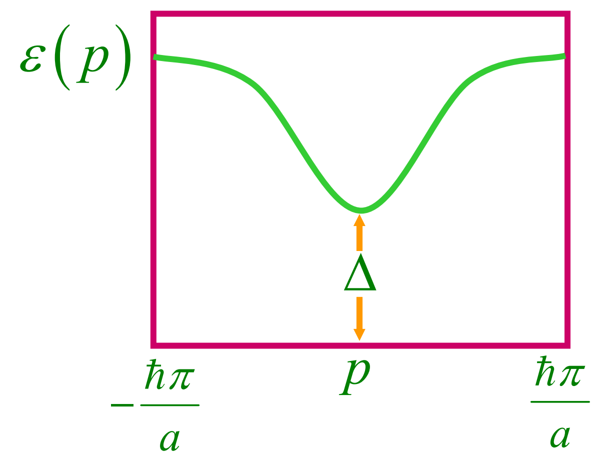
$$-\frac{1}{2g}|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\leftarrow\leftarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle - \cdots$$

Lowest excited states:

$$|\ell_j\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\underset{j}{\leftarrow}\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle + \cdots$$

Coupling between qubits creates "flipped-spin" ***quasiparticle*** states at momentum p

$$|p\rangle = \sum_j e^{ipx_j/\hbar} |\ell_j\rangle$$



Excitation energy $\varepsilon(p) = \Delta + 4J\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^{-1}\right)$

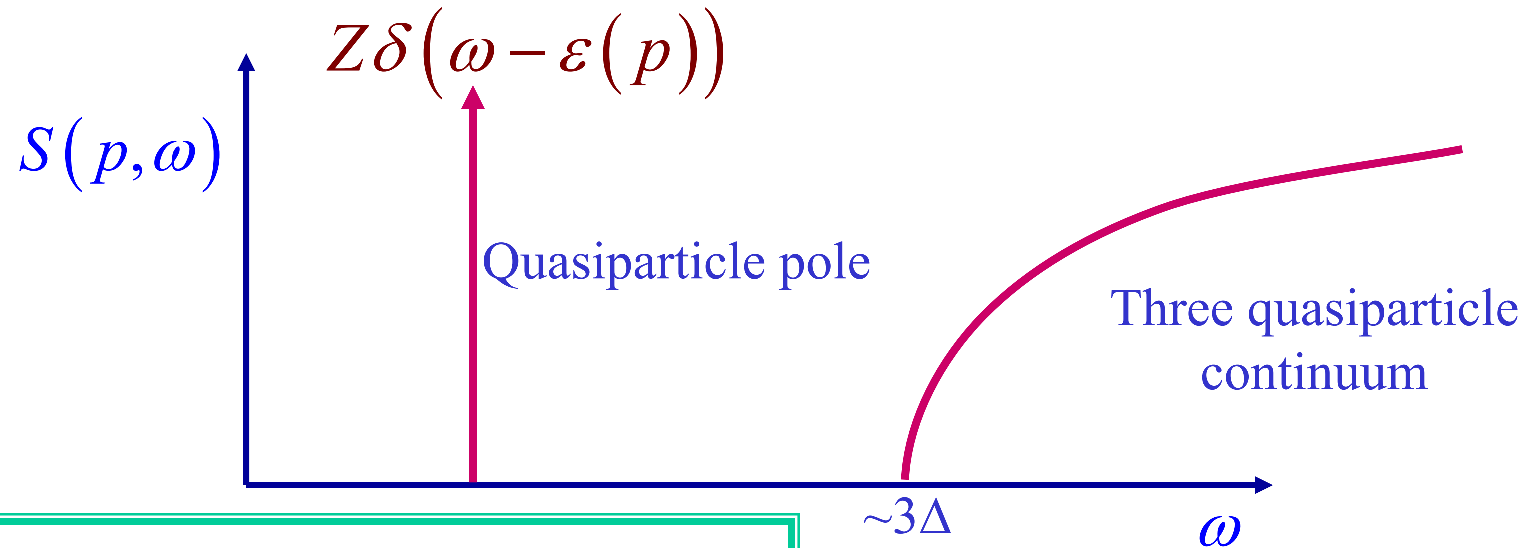
Excitation gap $\Delta = 2gJ - 2J + O\left(g^{-1}\right)$

Entire spectrum can be constructed out of multi-quasiparticle states

Dynamic Structure Factor $S(p,\omega)$:

Weakly-coupled qubits $(g \gg 1)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in $1/g$

At $T>0$, collisions between quasiparticles broaden pole to a Lorentzian of width $1/\tau_\varphi$ where the ***phase coherence time*** τ_φ is given by

$$\frac{1}{\tau_\varphi} = \frac{2k_B T}{\pi\hbar} e^{-\Delta/k_B T}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Strongly-coupled qubits $(g \ll 1)$

Ground states:

$$|G\uparrow\rangle = |\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle$$

$$-\frac{g}{2}|\cdots\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle - \cdots$$

Ferromagnetic moment
$N_0 = \langle G|\sigma^z|G\rangle \neq 0$

Second state $|G\downarrow\rangle$ obtained by $\uparrow \Leftrightarrow \downarrow$

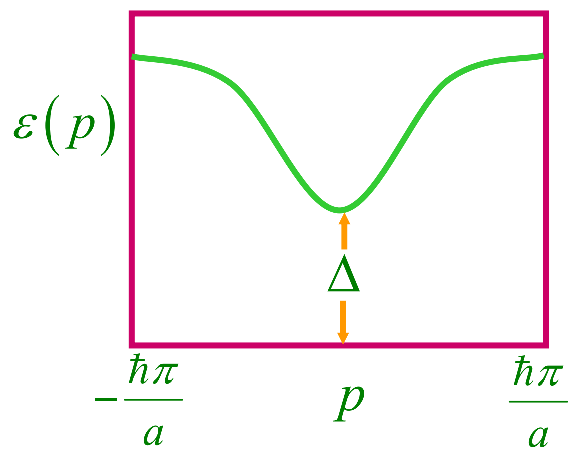
$|G\downarrow\rangle$ and $|G\uparrow\rangle$ mix only at order g^N

Lowest excited states: domain walls

$$|d_j\rangle = |\cdots\uparrow\uparrow\uparrow\uparrow\uparrow_j\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\rangle + \cdots$$

Coupling between qubits creates new "domain-wall" *quasiparticle* states at momentum p

$$|p\rangle = \sum_j e^{ipx_j/\hbar}|d_j\rangle$$



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Excitation gap $\Delta = 2J - 2gJ + O(g^2)$

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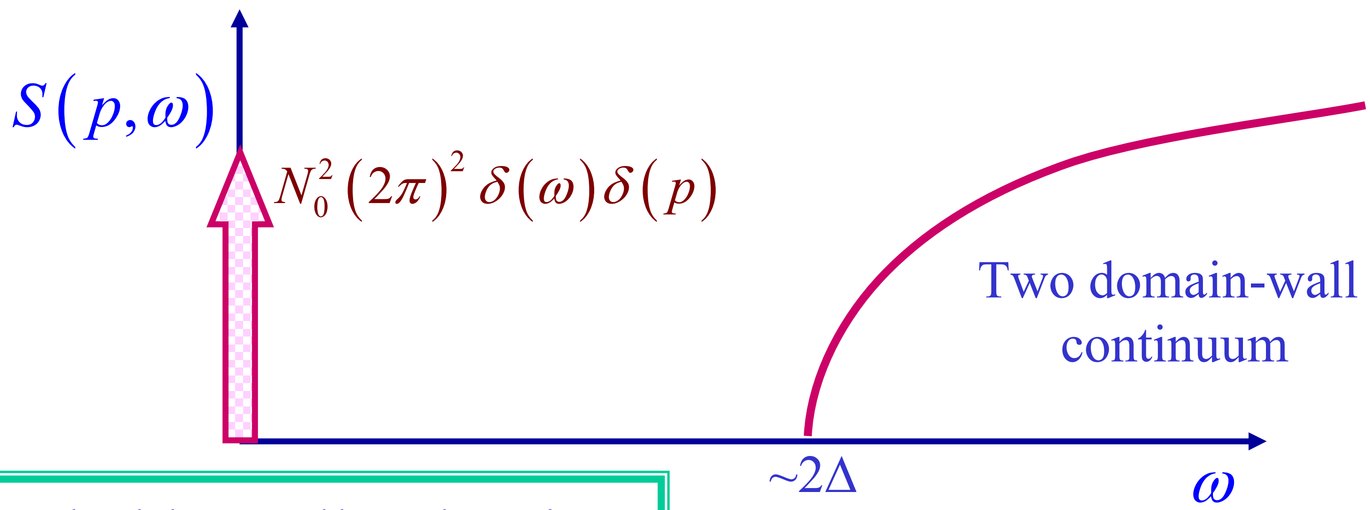


Structure holds to all orders in g

At $T > 0$, motion of domain walls leads to a finite ***phase coherence time*** τ_φ, and broadens coherent peak to a width $1/\tau_\varphi$ where $$\frac{1}{\tau_\varphi} = \frac{2k_B T}{\pi\hbar} e^{-\Delta/k_B T}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Entangled states at g of order unity

“Flipped-spin” Quasiparticle weight Z

$$Z \sim (g - g_c)^{1/4}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

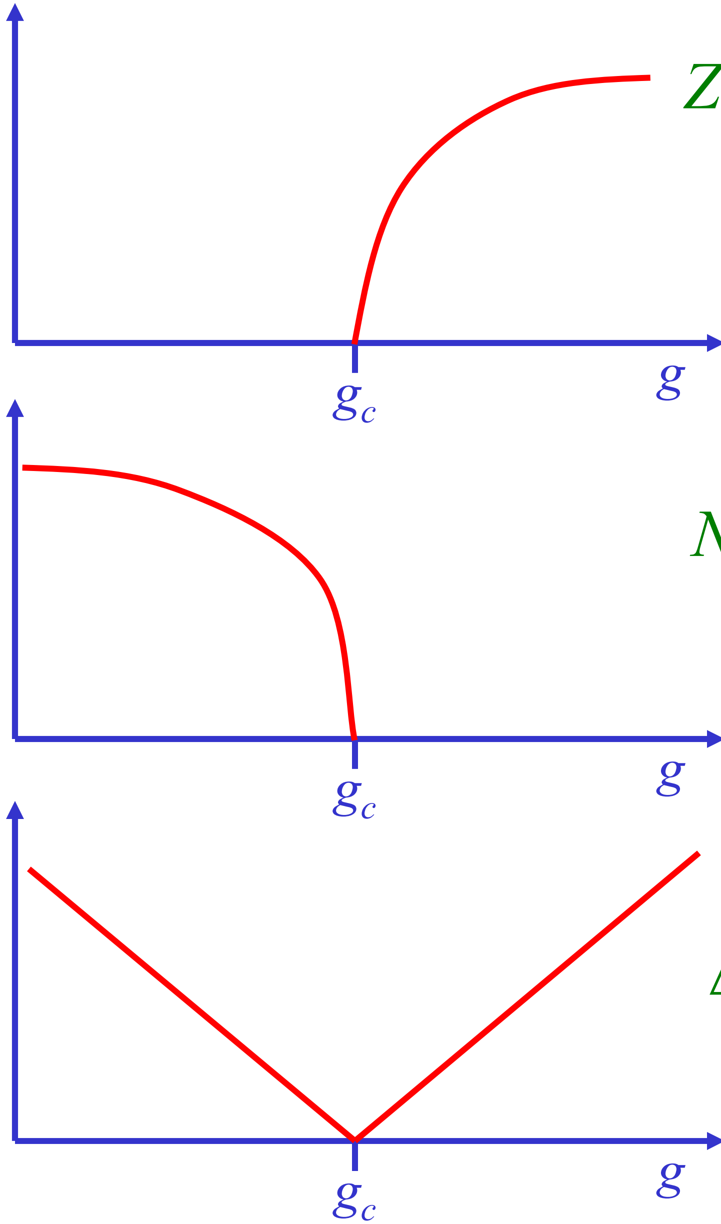
Ferromagnetic moment N_0

$$N_0 \sim (g_c - g)^{1/8}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

Excitation energy gap Δ

$$\Delta \sim |g - g_c|$$



Dynamic Structure Factor $S(p,\omega)$:

Critical coupling $(g = g_c)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)

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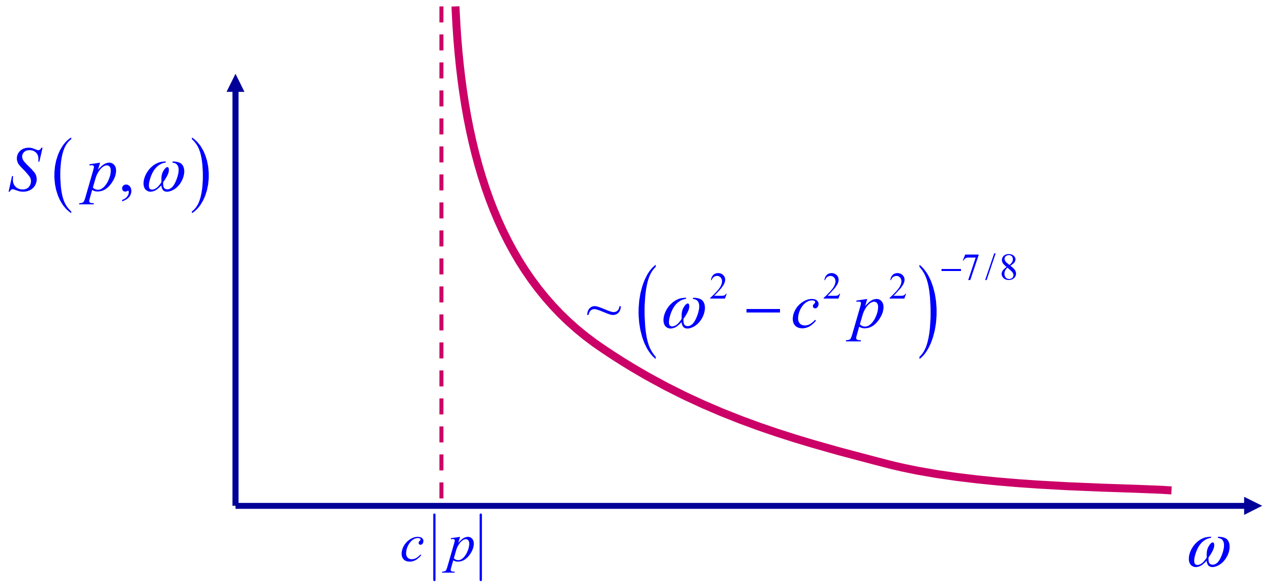
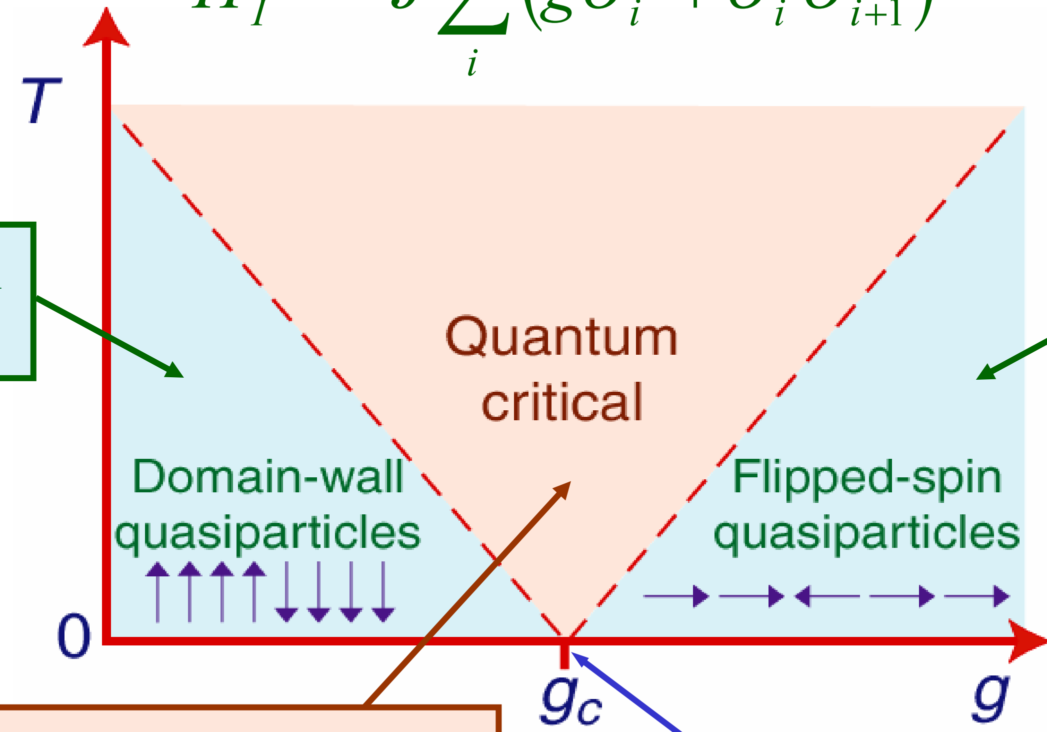


No quasiparticles --- dissipative critical continuum

$$H_I = -J \sum_i \left(g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics



Quasiclassical dynamics

$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4} \left(1 - i\omega / \Gamma_R + \ldots \right)}$$

$$\Gamma_R = \left(2 \tan \frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{\left| j - k \right|^{1/4}}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

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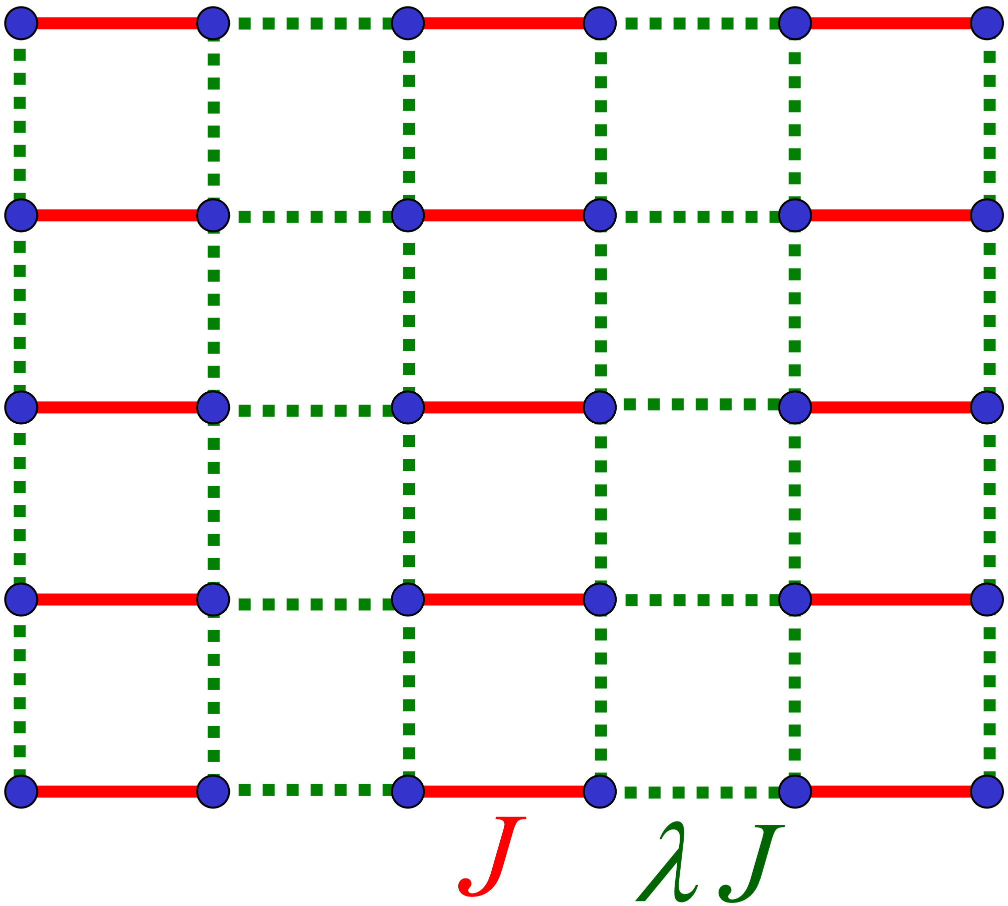
Multiple order parameters.

VI. Conclusions

II. Coupled Dimer Antiferromagnet

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



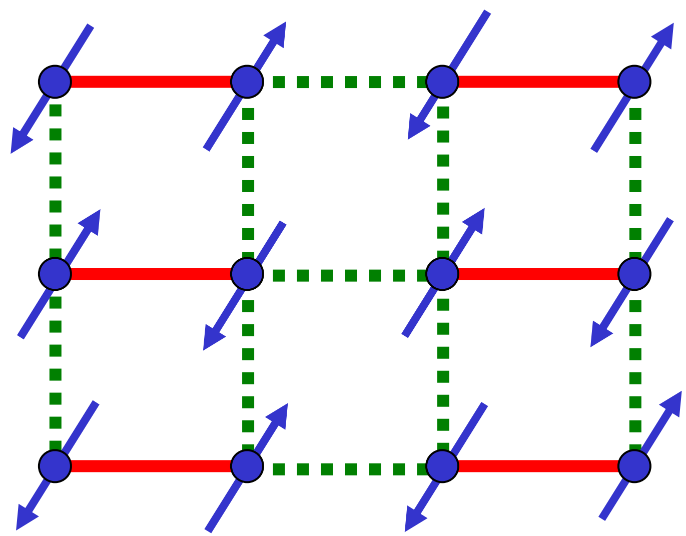
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



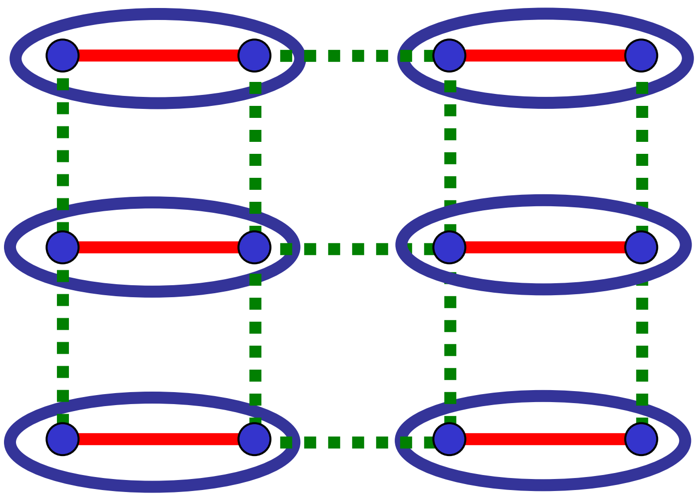
Ground state has long-range magnetic (Neel) order

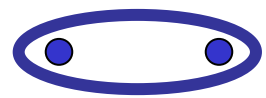
$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves (***magnons***) $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled dimers

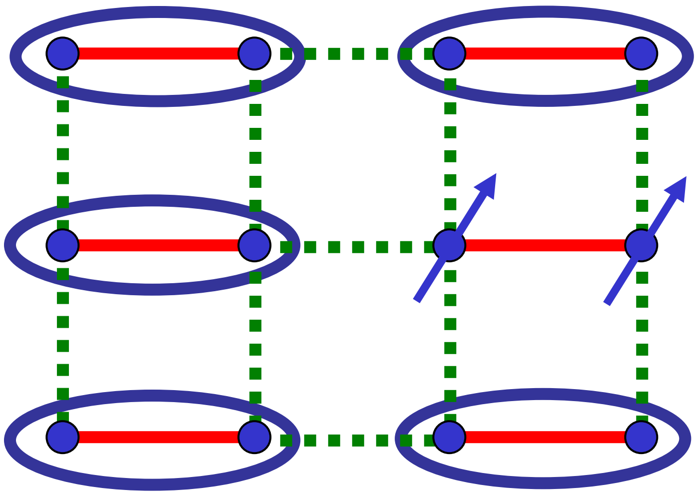


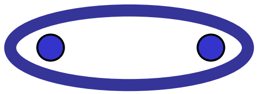
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

λ close to 0

Weakly coupled dimers





$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon*** (***exciton***, spin collective mode)

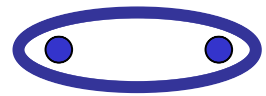
Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

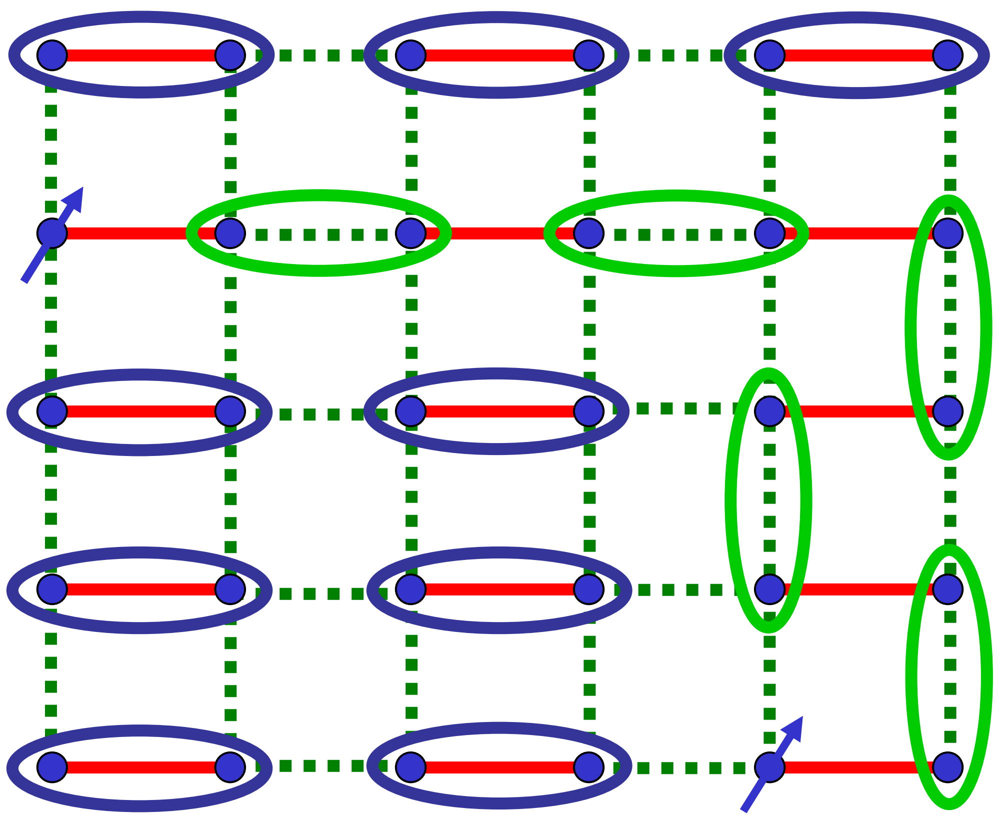
$\Delta \rightarrow$ spin gap

λ close to 0

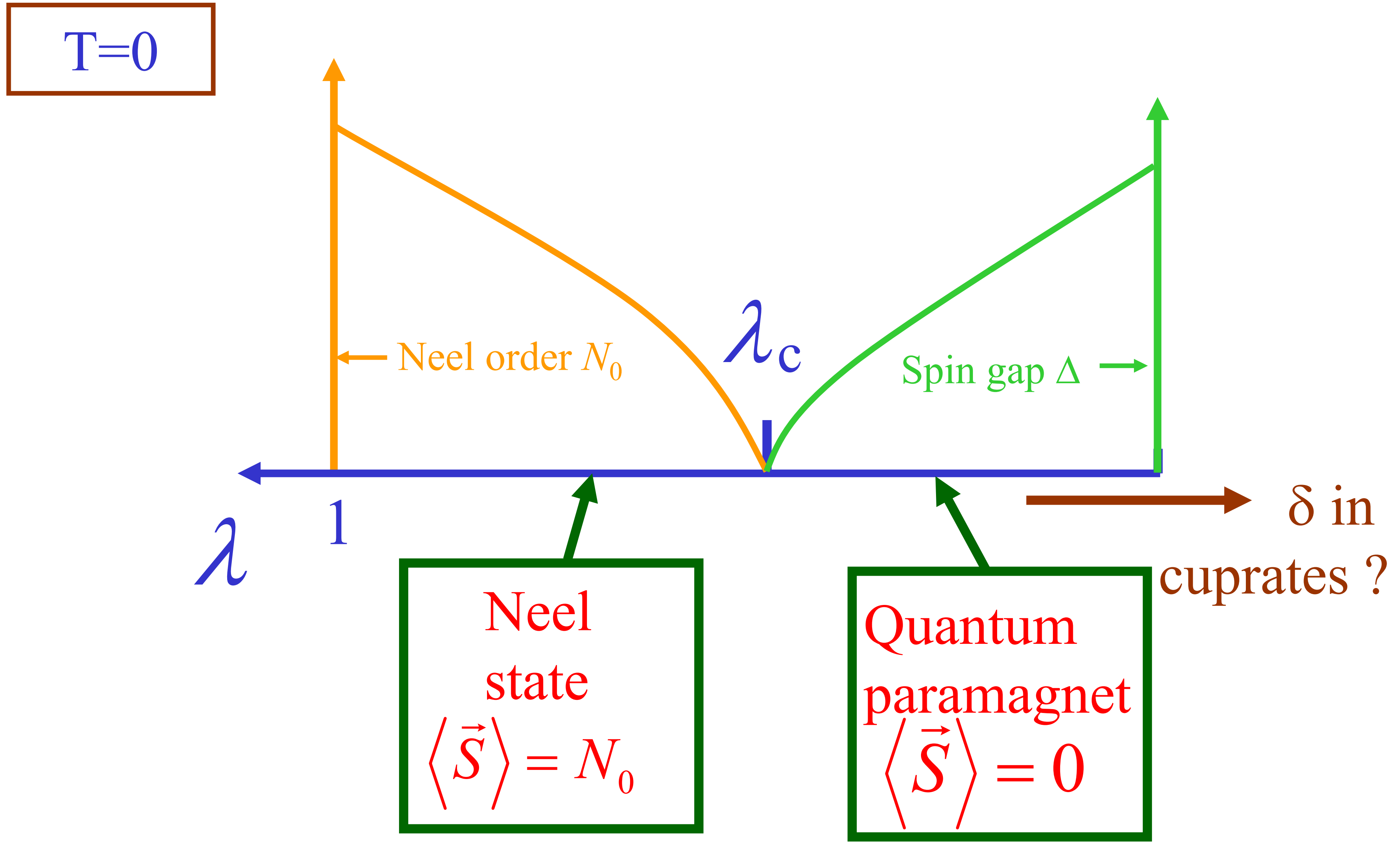
Weakly coupled dimers



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$



S=1/2 ***spinons*** are <u>*confined*</u> by a linear potential into a S=1 ***triplon***

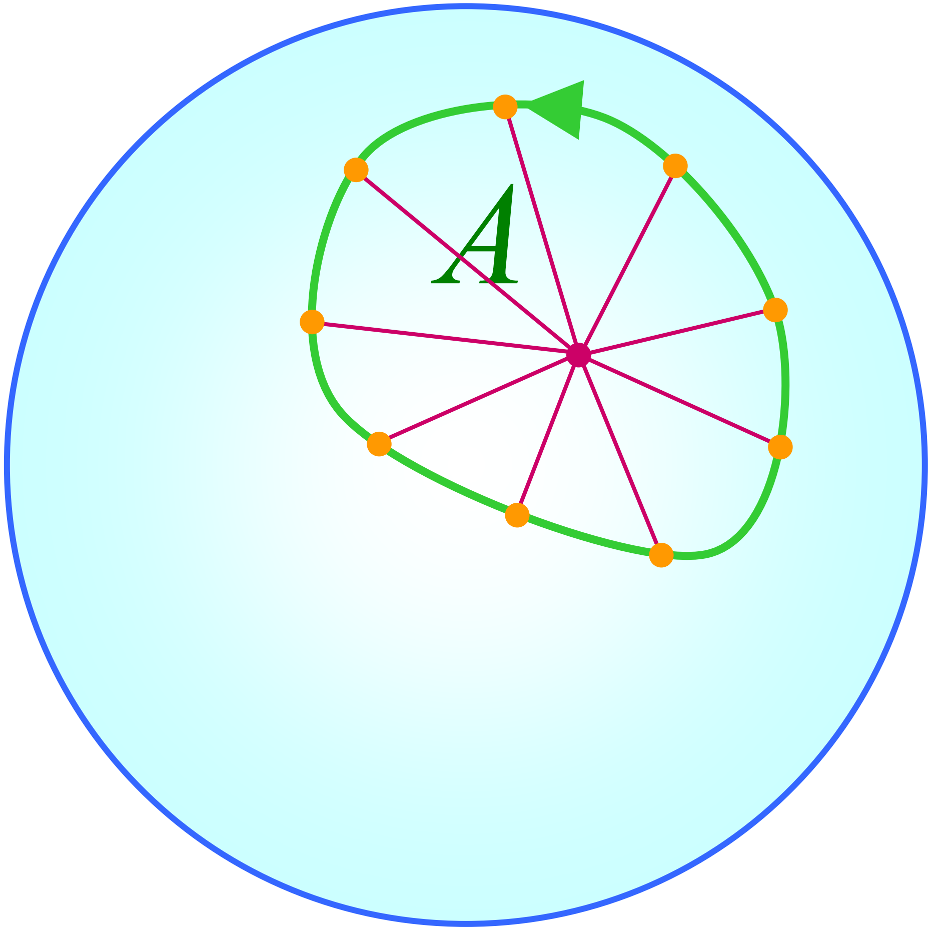


T=0
Neel order N_0
λ_c
Spin gap Δ
λ
1
δ in
cuprates ?
Neel
state
$\left\langle \vec{S} \right\rangle = N_0$
Quantum
paramagnet
$\left\langle \vec{S} \right\rangle = 0$

II.A Coherent state path integral

Path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$$e^{iSA}$$

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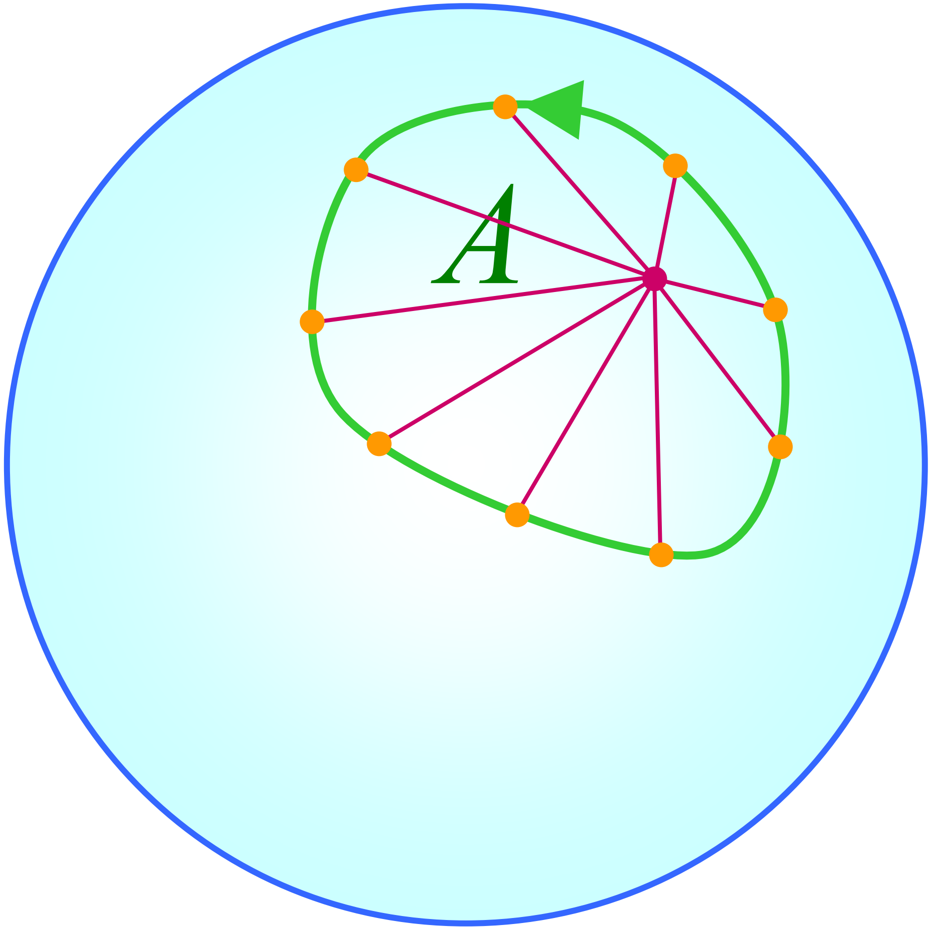

$$e^{iSA}$$

II.A Coherent state path integral

See Chapter 13 of *Quantum Phase Transitions*, S. Sachdev, Cambridge University Press (1999).

Path integral for a single spin

$$Z = \mathrm{Tr}\left(e^{-H[\boldsymbol{S}]/T}\right)$$

$$= \int \mathcal{D}\boldsymbol{N}(\tau)\,\delta\left(\boldsymbol{N}^2 - 1\right)\exp\left(-iS\int A_\tau(\tau)\,d\tau - \int d\tau\, H\left[S\boldsymbol{N}(\tau)\right]\right)$$

$A_\tau(\tau)\,d\tau =$ Oriented area of triangle on surface of unit sphere bounded by $\boldsymbol{N}(\tau), \boldsymbol{N}(\tau + d\tau)$, and a fixed reference $\boldsymbol{N}_0$

Action for lattice antiferromagnet

$$\boldsymbol{N}_j(\tau) = \eta_j \boldsymbol{n}(x_j, \tau) + \boldsymbol{L}(x_j, \tau)$$

$\eta_j = \pm 1$ identifies sublattices

$\boldsymbol{n}$ and $\boldsymbol{L}$ vary slowly in space and time

Integrate out $\boldsymbol{L}$ and take the continuum limit

$$Z = \int \mathcal{D}\boldsymbol{n}(x,\tau)\,\delta\left(\boldsymbol{n}^2 - 1\right)\exp\left(-iS\sum_j \int \eta_j A_\tau(x_j,\tau)\,d\tau - \frac{1}{2g}\int d^2x\,d\tau\left(\left(\partial_\tau \boldsymbol{n}\right)^2 + c^2\left(\nabla_x \boldsymbol{n}\right)^2 \right) \right)$$

$\eta_j = \pm 1$ identifies sublattices

$g < g_c \Leftrightarrow \lambda > \lambda_c$

$g > g_c \Leftrightarrow \lambda < \lambda_c$

Discretize spacetime into a cubic lattice

$a \rightarrow$ cubic lattice sites;

$\mu \rightarrow x, y, \tau;$

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right)\exp\left(\frac{1}{g}\sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2}\sum_a \eta_a A_{a\tau} \right)$$

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Integrate out $\boldsymbol{L}$ and take the continuum limit

$$Z = \int \mathcal{D}\boldsymbol{n}(x,\tau)\,\delta\left(\boldsymbol{n}^2 - 1\right)\exp\left(-iS\sum_j \int \eta_j A_\tau(x_j,\tau)\,d\tau - \frac{1}{2g}\int d^2x\,d\tau\left(\left(\partial_\tau \boldsymbol{n}\right)^2 + c^2\left(\nabla_x \boldsymbol{n}\right)^2\right)\right)$$

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Berry phases can be neglected for coupled dimer antiferromagent

(justified later)

Quantum path integral for two-dimensional quantum antiferromagnet
$\Leftrightarrow$ Partition function of a classical three-dimensional ferromagnet at a "temperature" g

Quantum transition at $\lambda=\lambda_c$ is related to classical Curie transition at $g=g_c$

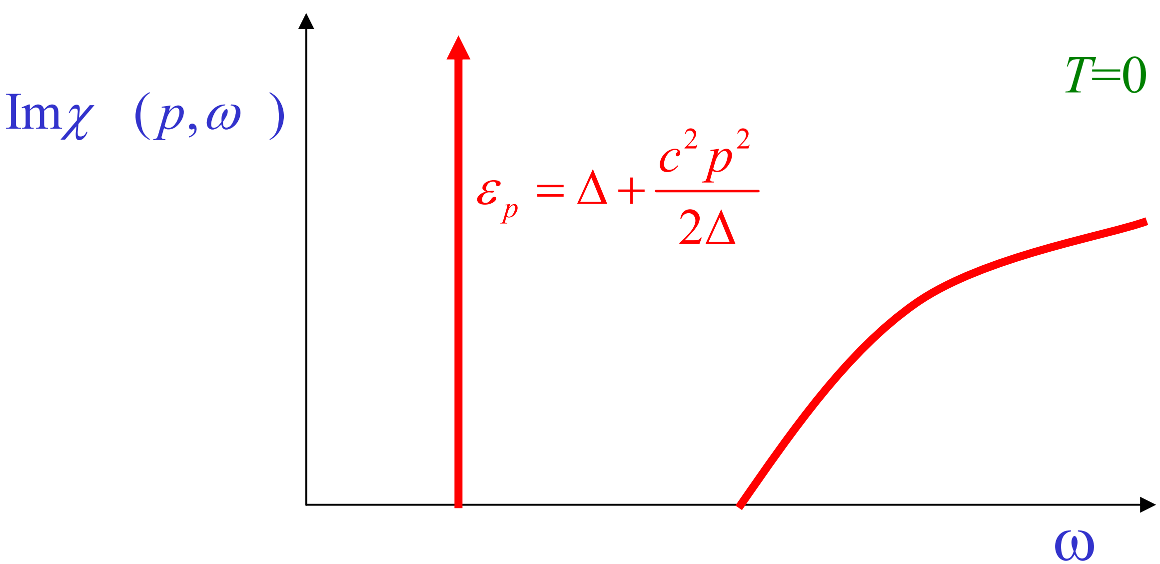
II.B Quantum field theory for critical point

λ **close to** λ_c **:** use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + \left(\lambda_c - \lambda\right)\phi_\alpha^2 \right) + \frac{u}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

Oscillations of ϕ_α about zero (for $\lambda < \lambda_c$)
→ spin-1 collective mode

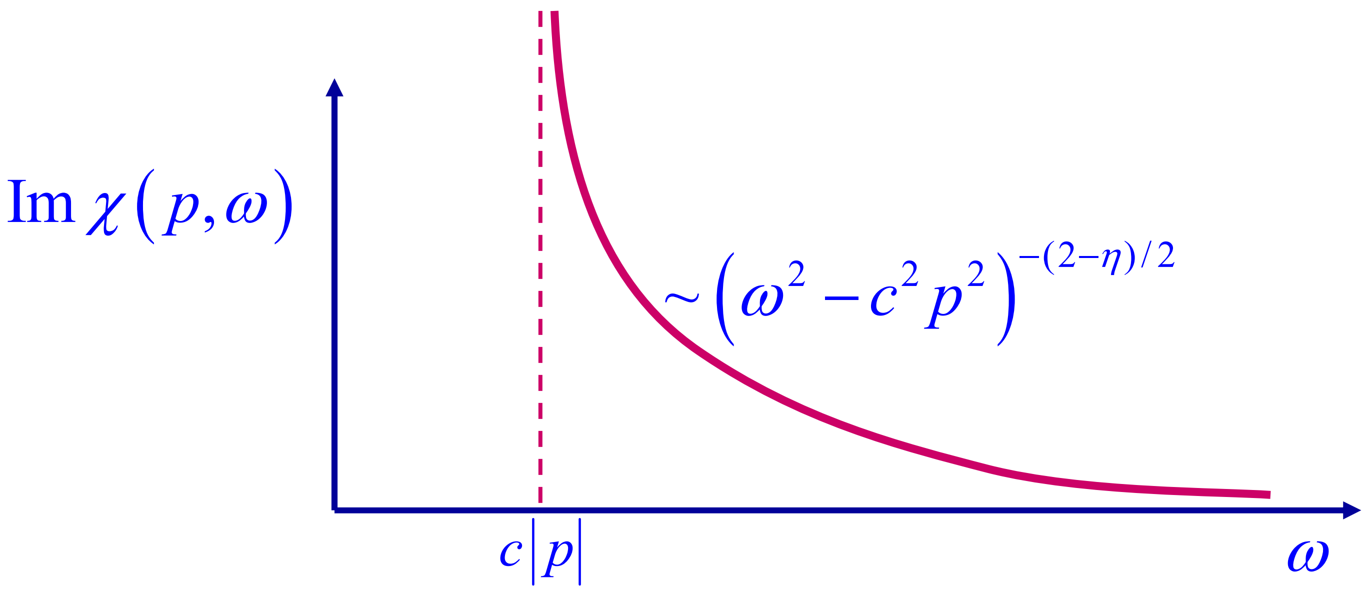


T=0 spectrum

$$\Delta = \sqrt{\lambda_c - \lambda}/c$$

Critical coupling $(\lambda = \lambda_c)$

Dynamic spectrum at the critical point



No quasiparticles --- dissipative critical continuum

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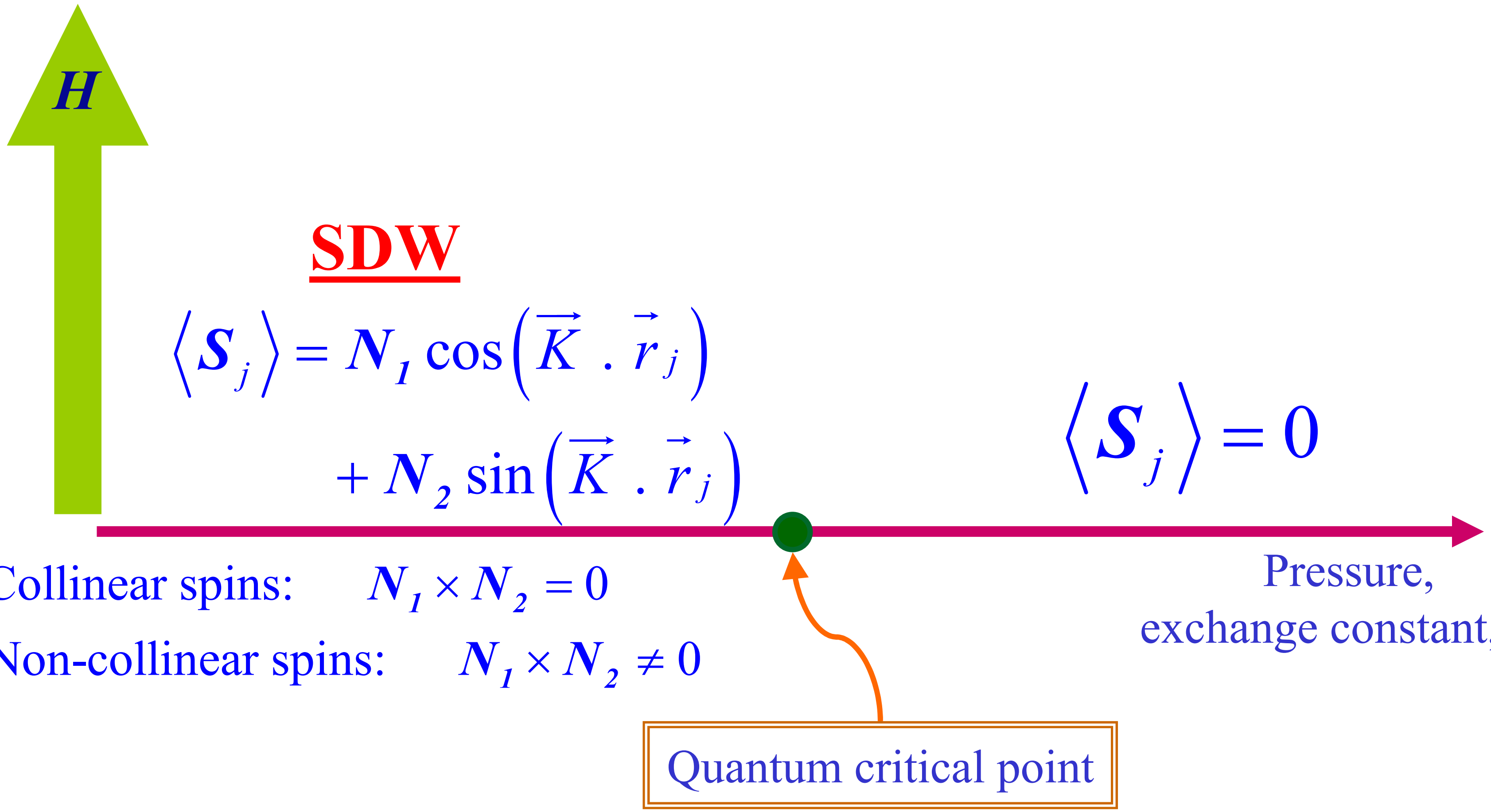
Multiple order parameters.

VI. Conclusions

$T=0$



SDW

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \,.\, \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \,.\, \vec{r}_j\right)$$

$$\langle \boldsymbol{S}_j \rangle = 0$$

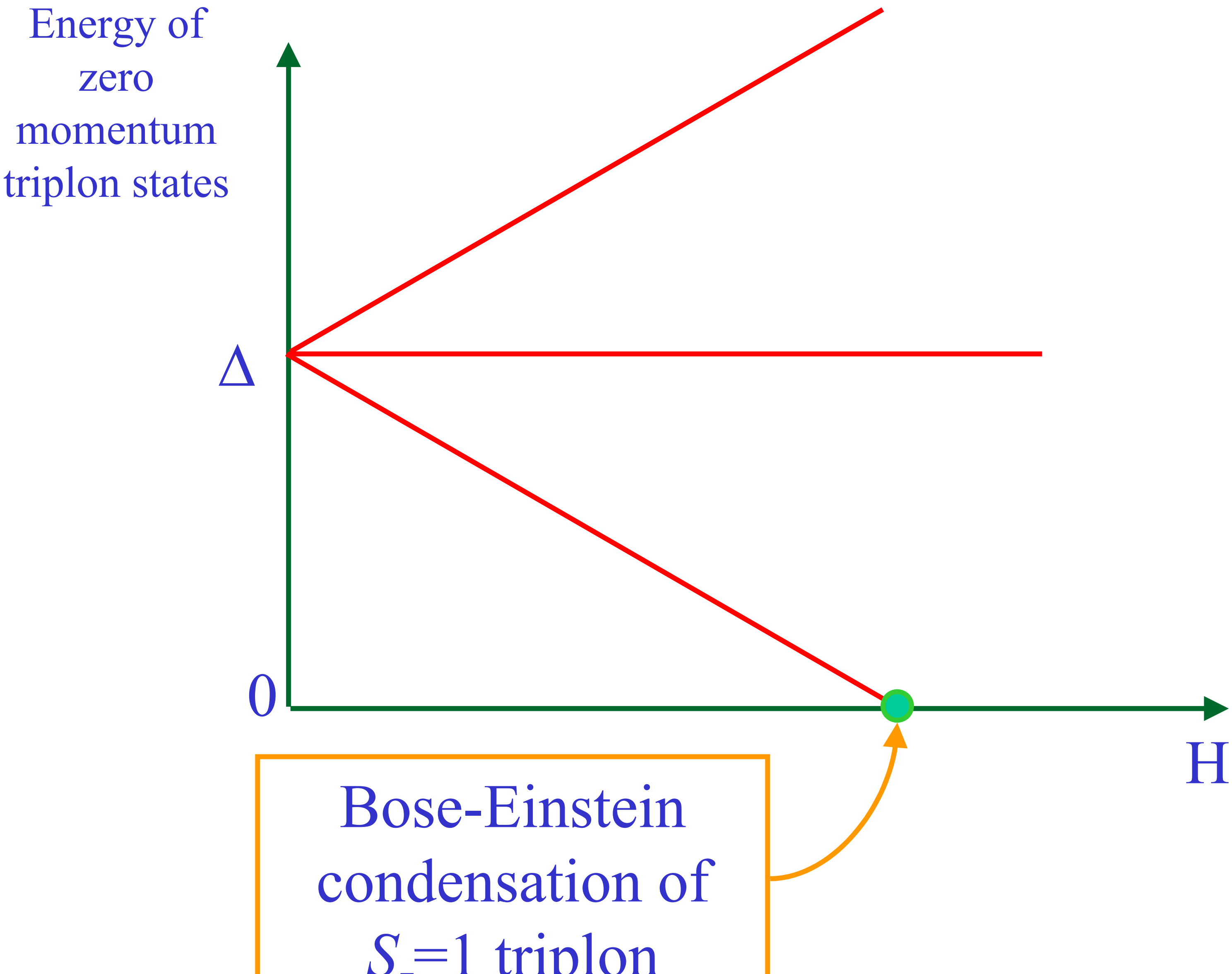
Collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 = 0$

Non-collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 \neq 0$

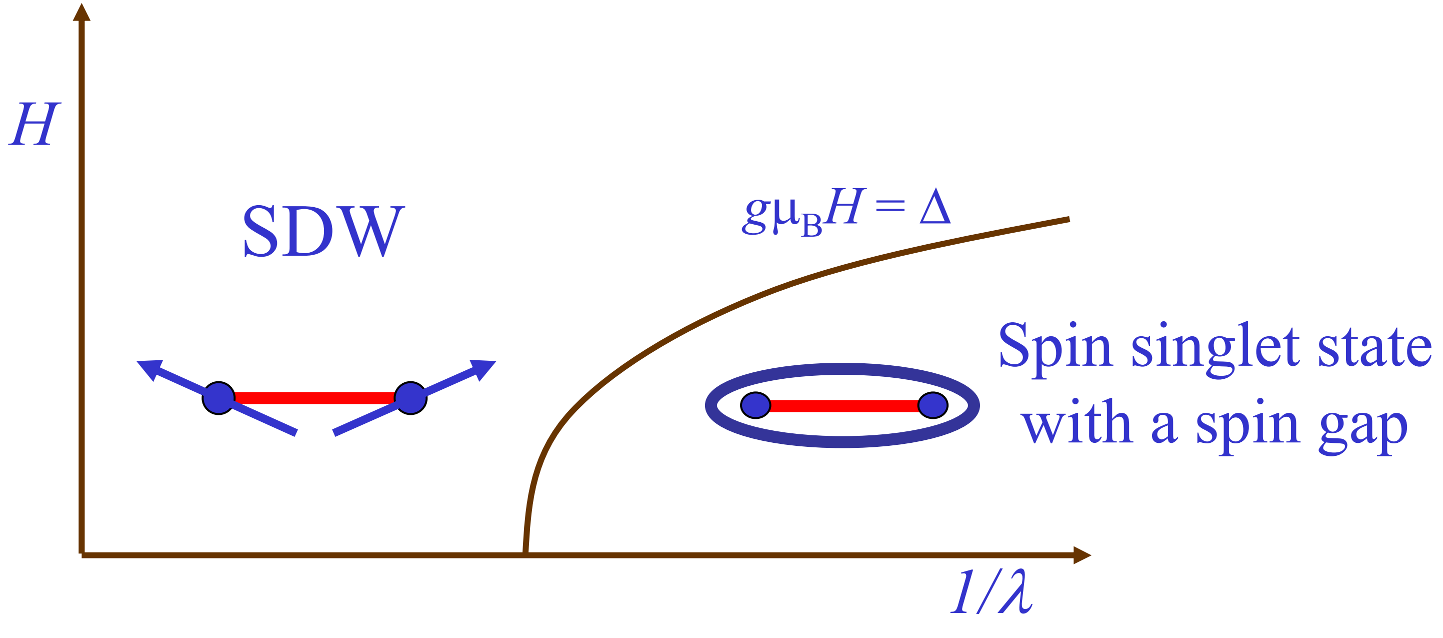
Evolution of phase diagram in a magnetic field

Both states are insulators

Effect of a field on paramagnet



III. Phase diagram in a magnetic field.



1 Tesla = 0.116 meV

Related theory applies to double layer quantum Hall systems at ν=2

III. Phase diagram in a magnetic field.

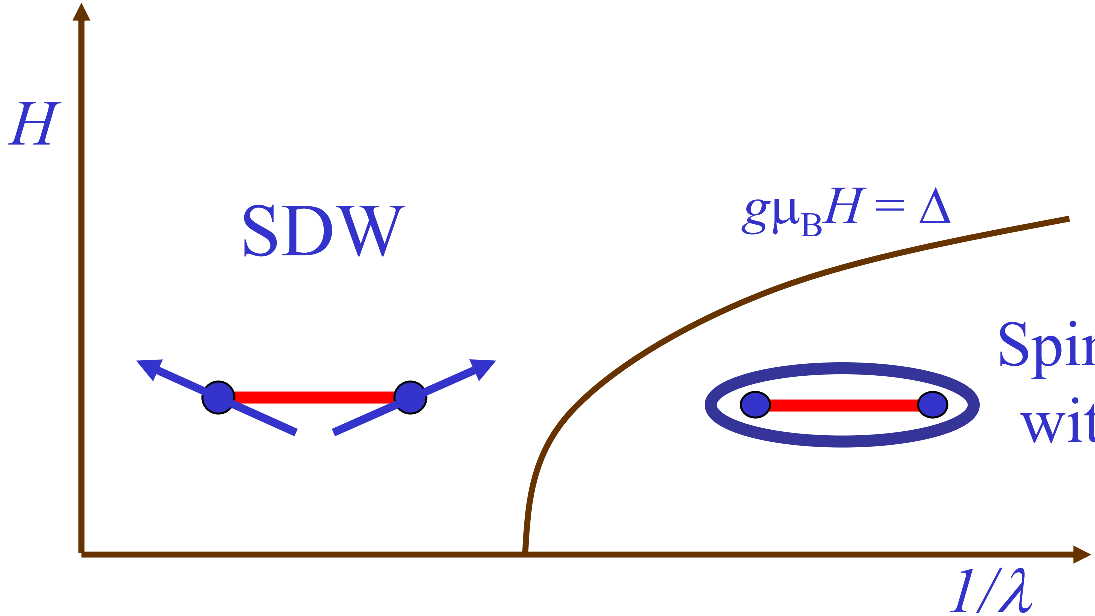
Zeeman term leads to a uniform precession of spins

$$\left|\partial_\tau \phi_\alpha\right|^2 \Rightarrow \left(\partial_\tau \phi_\alpha^* - i\varepsilon_{\alpha\sigma\rho} H_\sigma \phi_\rho\right)\left(\partial_\tau \phi_\alpha - i\varepsilon_{\alpha\beta\gamma} H_\beta \phi_\gamma\right)$$

Take H oriented along the z direction. Then

$$\left(\lambda_c - \lambda\right)\left(\phi_x^2 + \phi_y^2\right) \Rightarrow \left(\lambda_c - \lambda - H^2\right)\left(\phi_x^2 + \phi_y^2\right).$$

For $\lambda > \lambda_c$, $\phi_x \sim \sqrt{\lambda - \lambda_c + H^2}$, while for $\lambda < \lambda_c$, $H_c = \Delta \sim \sqrt{\lambda_c - \lambda}$



1 Tesla = 0.116 meV

Related theory applies to double layer quantum Hall systems at ν=2

III. Phase diagram in a magnetic field.

Zeeman term leads to a uniform precession of spins

$$\left|\partial_\tau \phi_\alpha\right|^2 \Rightarrow \left(\partial_\tau \phi_\alpha^* - i\varepsilon_{\alpha\sigma\rho} H_\sigma \phi_\rho\right)\left(\partial_\tau \phi_\alpha - i\varepsilon_{\alpha\beta\gamma} H_\beta \phi_\gamma\right)$$

Take H oriented along the z direction. Then

$$\left(\lambda_c - \lambda\right)\left(\phi_x^2 + \phi_y^2\right) \Rightarrow \left(\lambda_c - \lambda - H^2\right)\left(\phi_x^2 + \phi_y^2\right).$$

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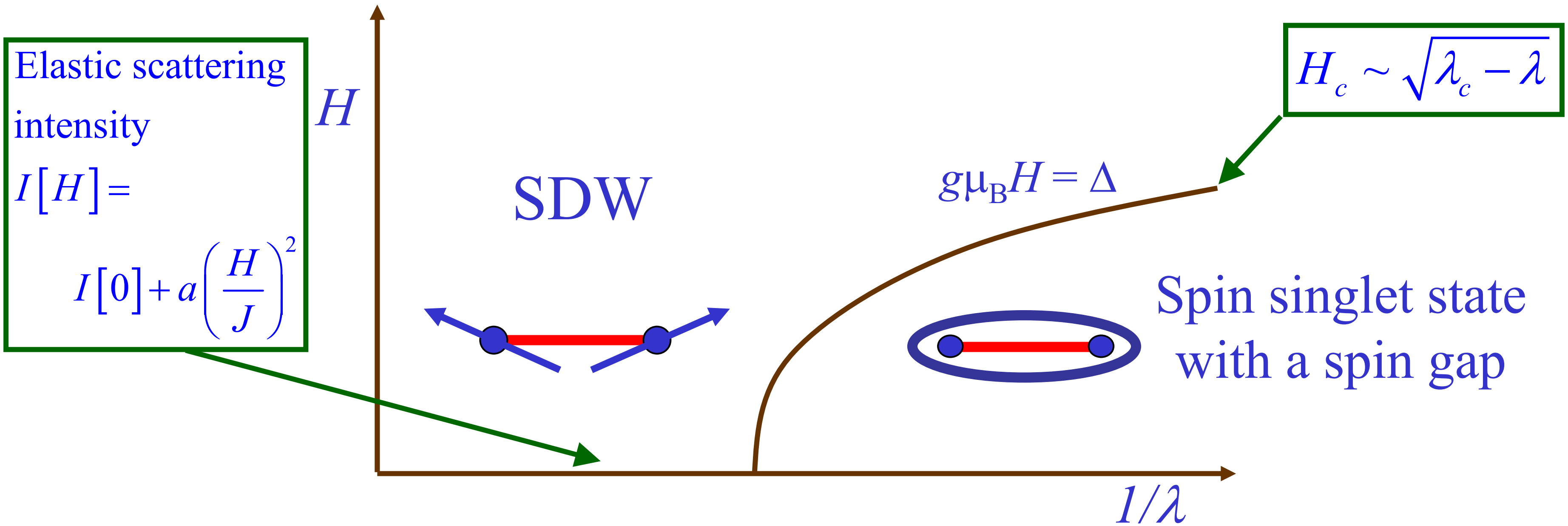

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Related theory applies to double layer quantum Hall systems at ν=2

Outline

I. Quantum Ising Chain

II. Coupled Dimer Antiferromagnet
  A. Coherent state path integral
  B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field
  Bose condensation of "triplons"

Single order parameter.

IV. Magnetic transitions in superconductors
  Quantum phase transition in a background Abrikosov flux lattice

V. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell.
  Class A: Compact U(1) gauge theory: collinear spins, bond order and confined spinons in d=2
  Class B: Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

Multiple order parameters.

VI. Conclusions

$T=0$

SDW

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$$

Collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 = 0$

Non-collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 \neq 0$

$$\langle \boldsymbol{S}_j \rangle = 0$$

Pressure,
carrier concentration,….

Quantum critical point

We have so far considered the case where both states are insulators

T=0

SC+SDW

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K}\,.\,\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}\,.\,\vec{r}_j\right)$$

Collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 = 0$

Non-collinear spins: $\boldsymbol{N}_1 \times \boldsymbol{N}_2 \neq 0$

SC

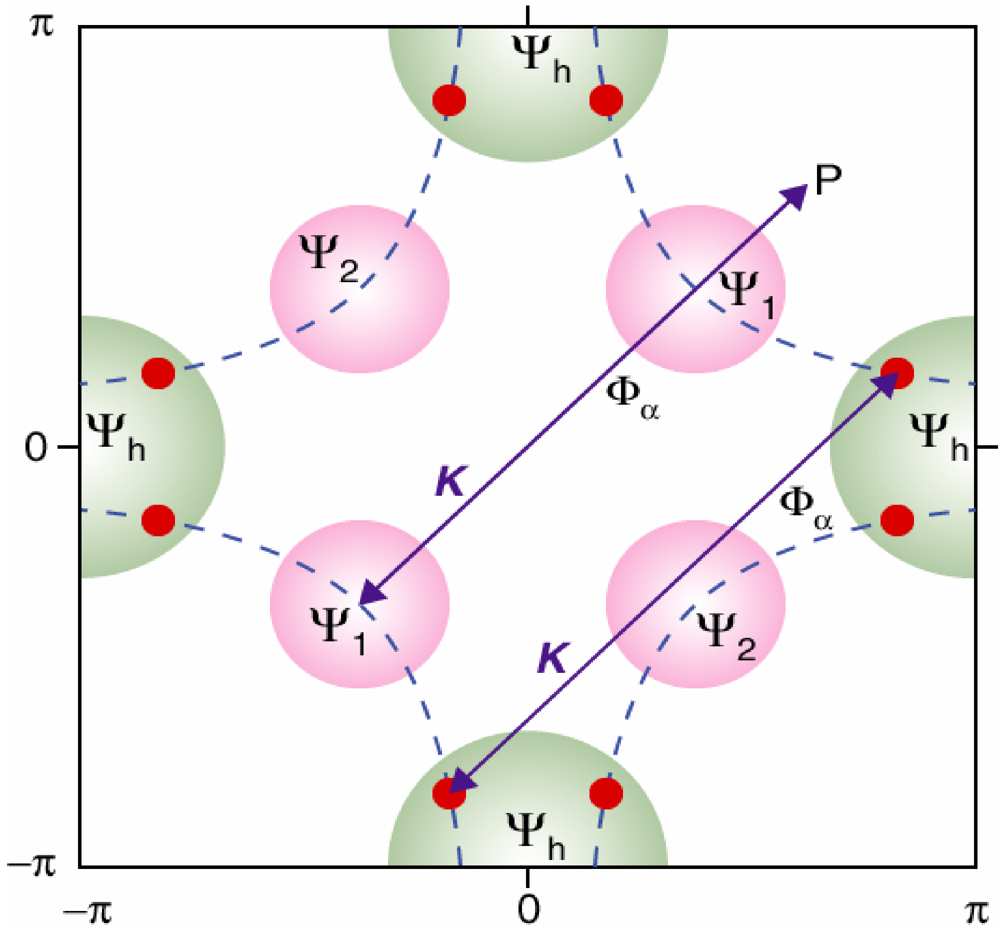
$$\langle \boldsymbol{S}_j \rangle = 0$$

Pressure,
carrier concentration,….

Quantum critical point

Now both sides have a “background” superconducting (SC) order

Magnetic transition in a *d*-wave superconductor



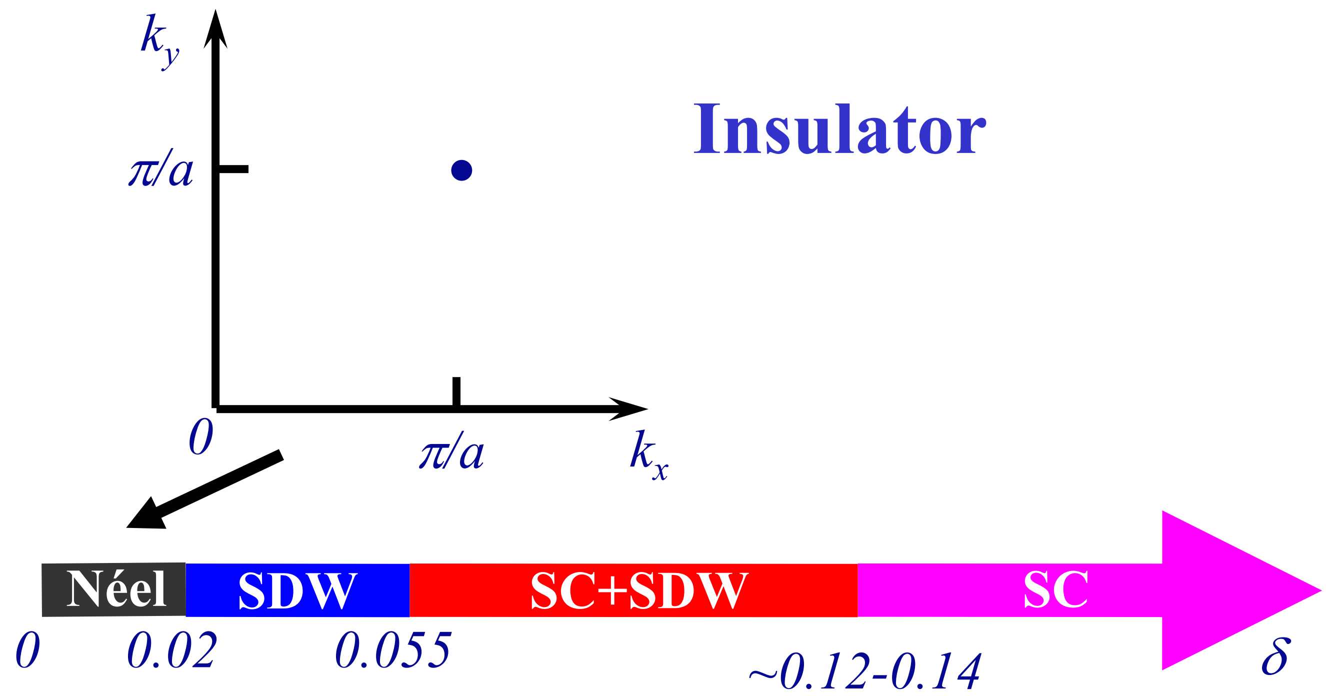
If $\vec{K}$ does not exactly connect two nodal points, critical theory is as in an insulator

Otherwise, new theory of coupled excitons and nodal quasiparticles

L. Balents, M.P.A. Fisher, C. Nayak, *Int. J. Mod. Phys.* B **12**, 1033 (1998).

Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

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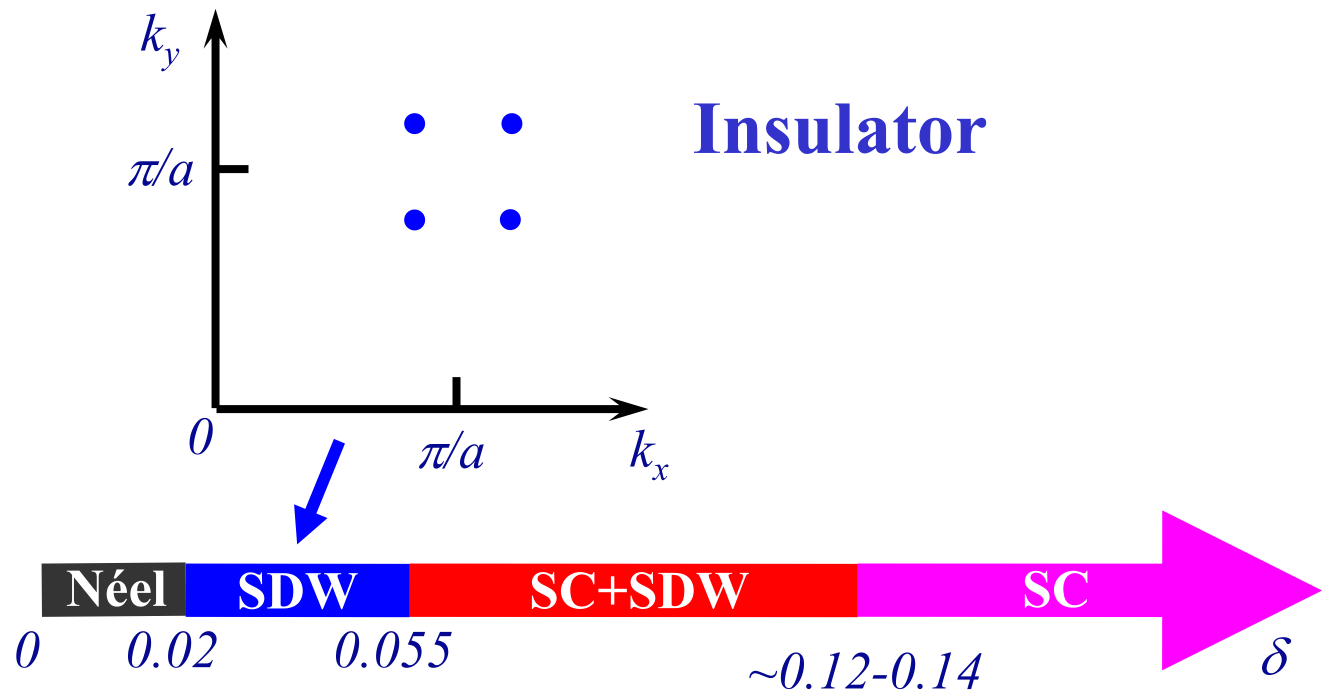

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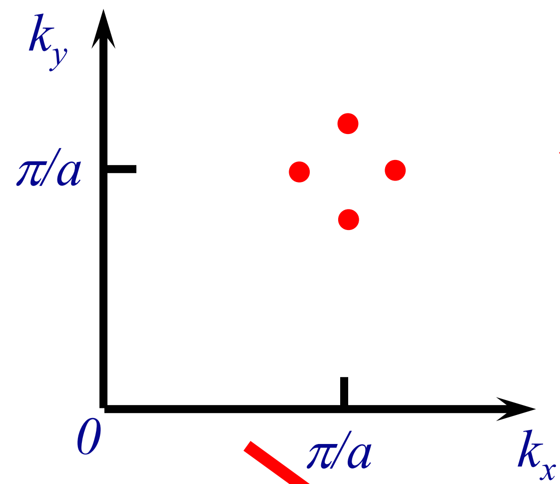
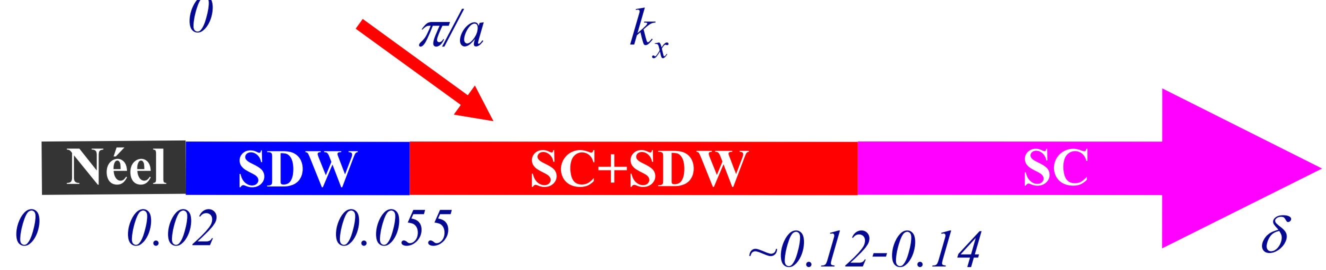


Superconductor with $T_{c,min}$ =10 K



(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al.*, *Phys. Rev*. B **54**, 7489 (1996).

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).

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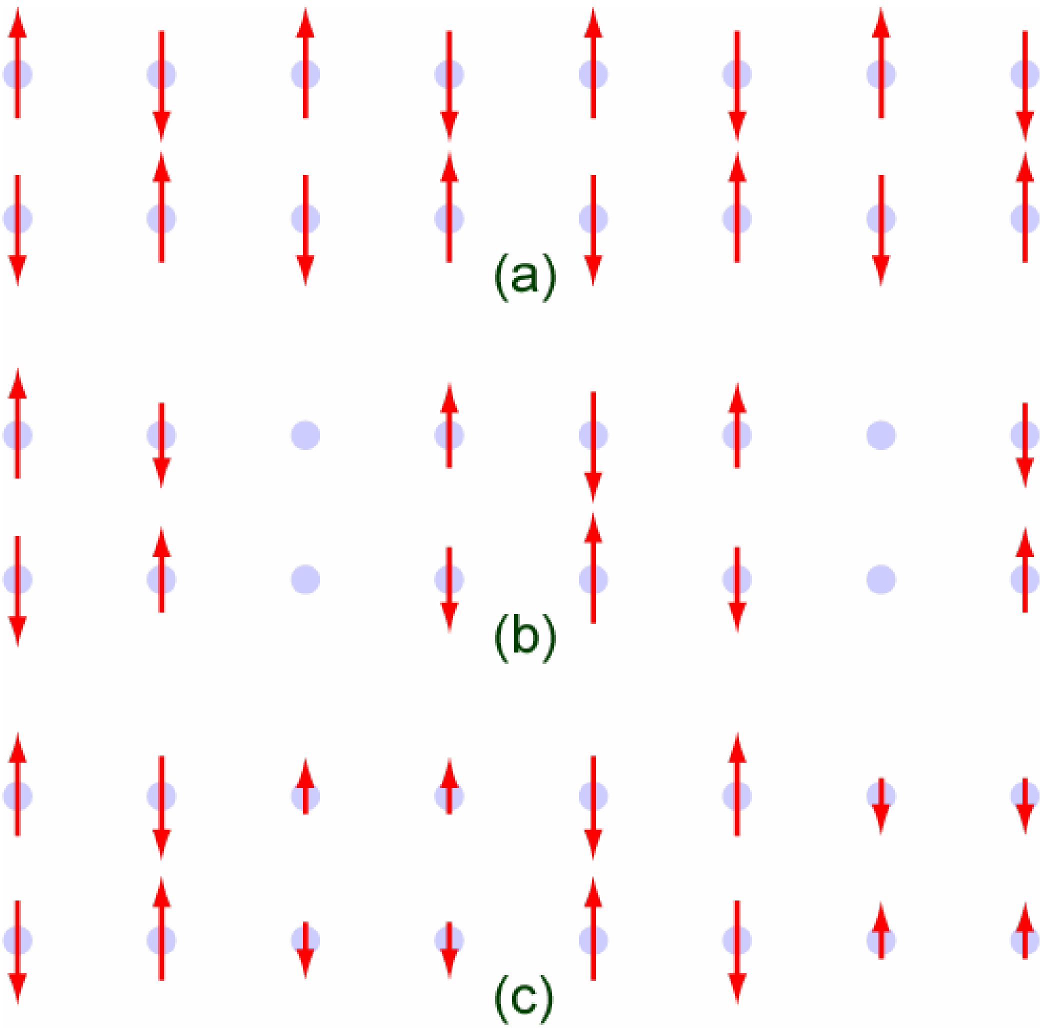
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

Collinear magnetic (spin density wave) order

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K}.\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}.\vec{r}_j\right)$$

Collinear spins



$$\vec{K} = (\pi, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$

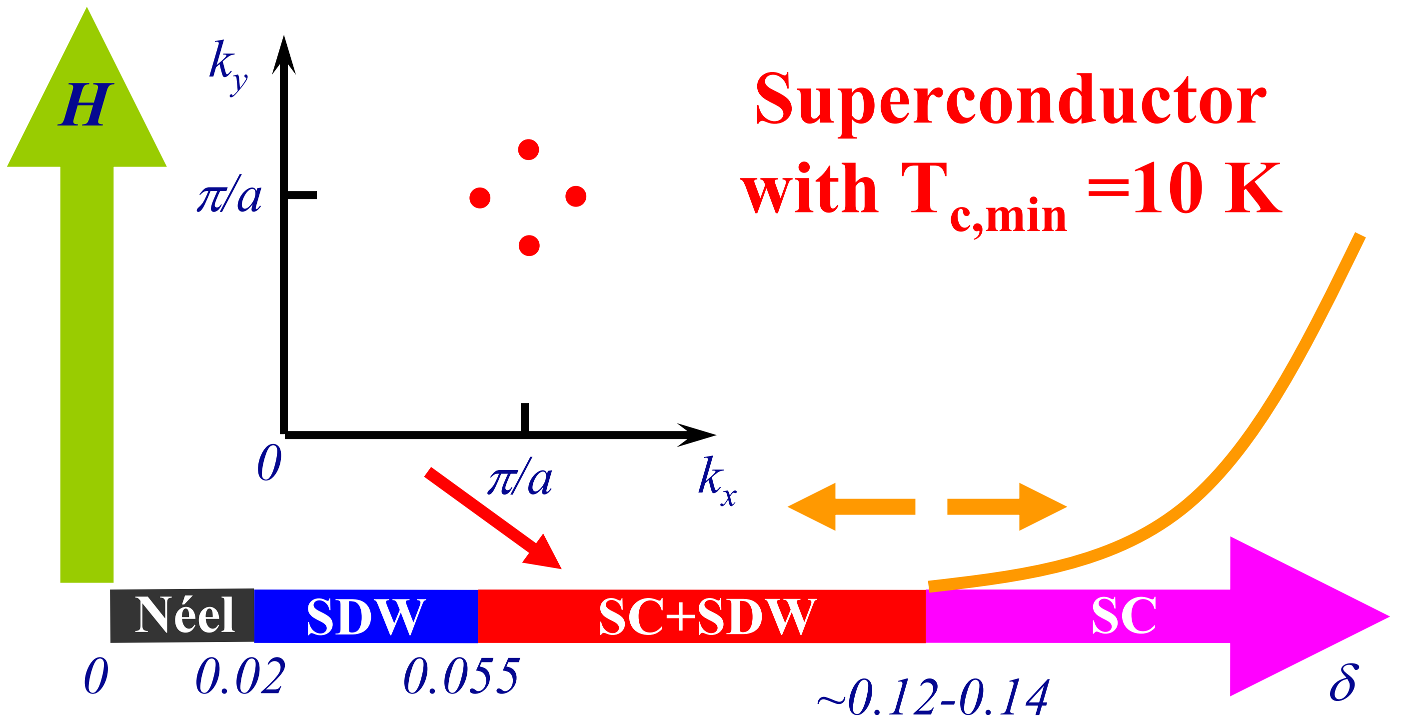
$$\vec{K} = (3\pi/4, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$

$$\vec{K} = (3\pi/4, \pi) \;\; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2} - 1\right) \boldsymbol{N}_1$$

Interplay of SDW and SC order in the cuprates

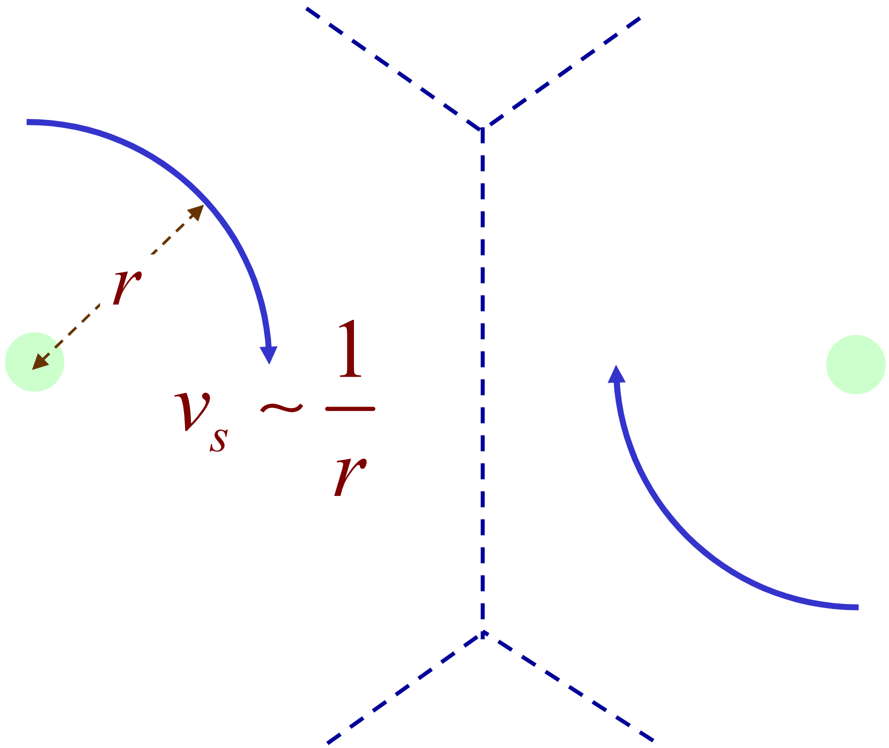
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

Dominant effect of magnetic field: Abrikosov flux lattice



Spatially averaged superflow kinetic energy

$$\sim \langle v_s^2 \rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

$$\Phi_\alpha = N_{1\alpha} + iN_{2\alpha}$$

Quantum theory for dynamic and critical spin fluctuations

$$S_b = \int d^2r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

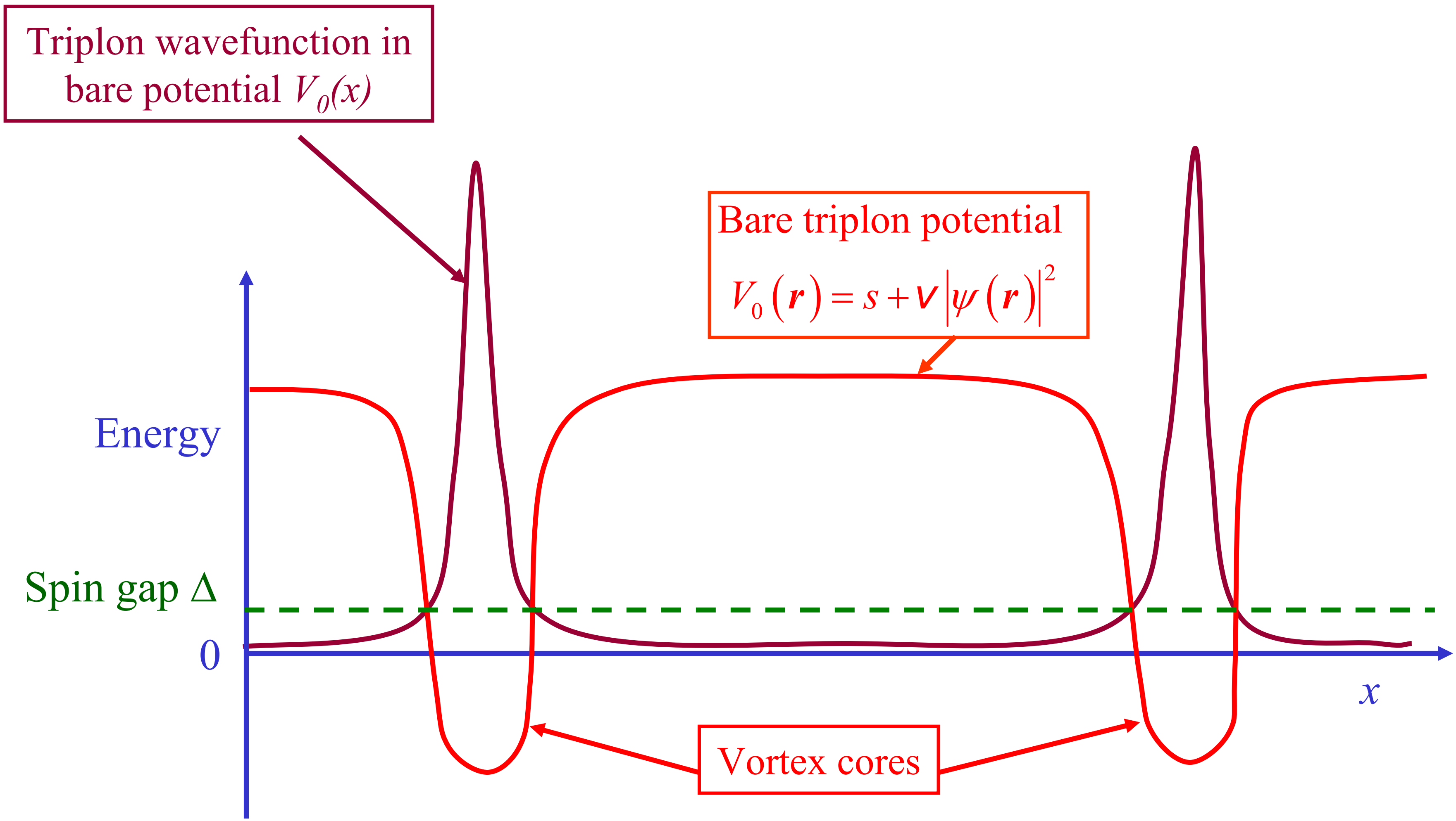
$$S_c = \int d^2r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

$$Z\left[\psi(r)\right] = \int D\Phi(r,\tau) e^{-F_{GL} - S_b - S_c}$$

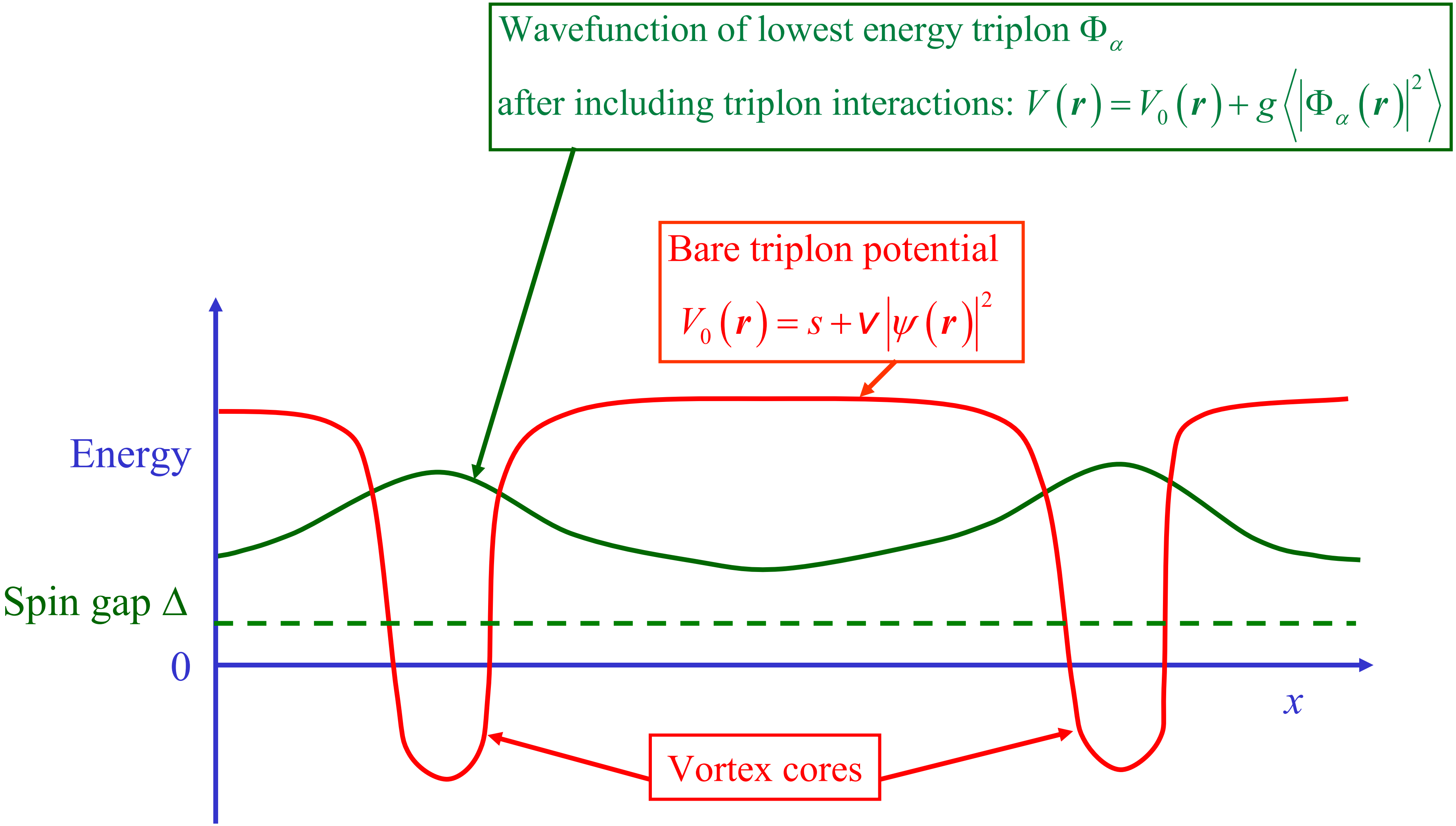
$$\frac{\delta \ln Z\left[\psi(r)\right]}{\delta\psi(r)} = 0$$

$$F_{GL} = \int d^2r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

Static Ginzburg-Landau theory for non-critical superconductivity



D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed **static** magnetism (with Δ=0) localized within vortex cores



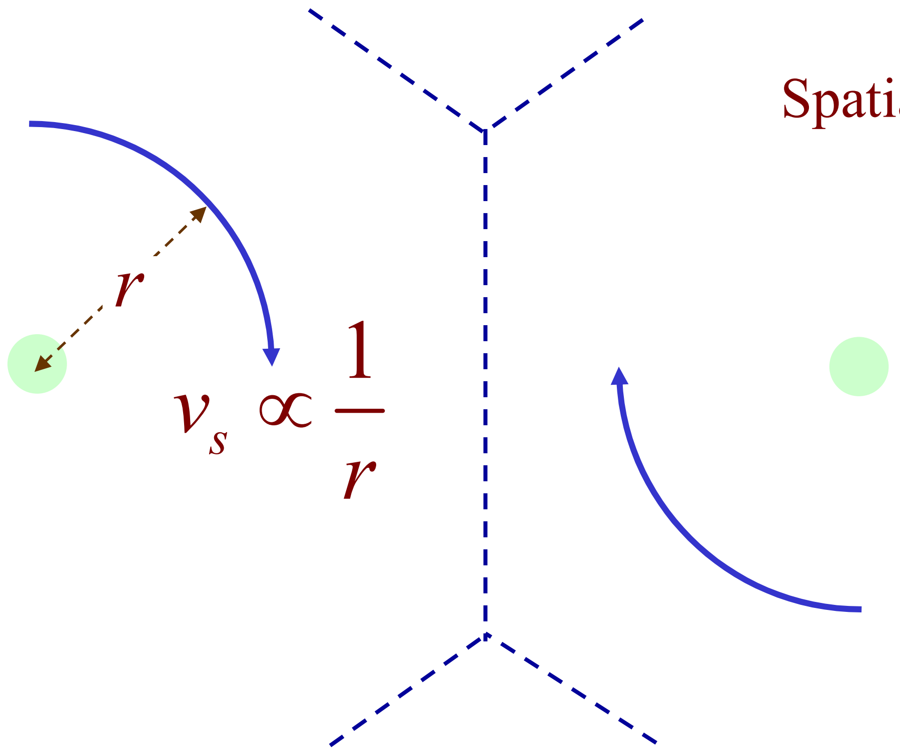
Strongly relevant repulsive interactions between excitons imply that triplons must be extended as $\Delta \to 0$.

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

A.J. Bray and M.A. Moore, *J. Phys.* C **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, *Phys. Rev. Lett.* **43**, 942 (1979).

Phase diagram of SC and SDW order in a magnetic field



Spatially averaged superflow kinetic energy

$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as a ***uniform*** effective "doping" δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C \frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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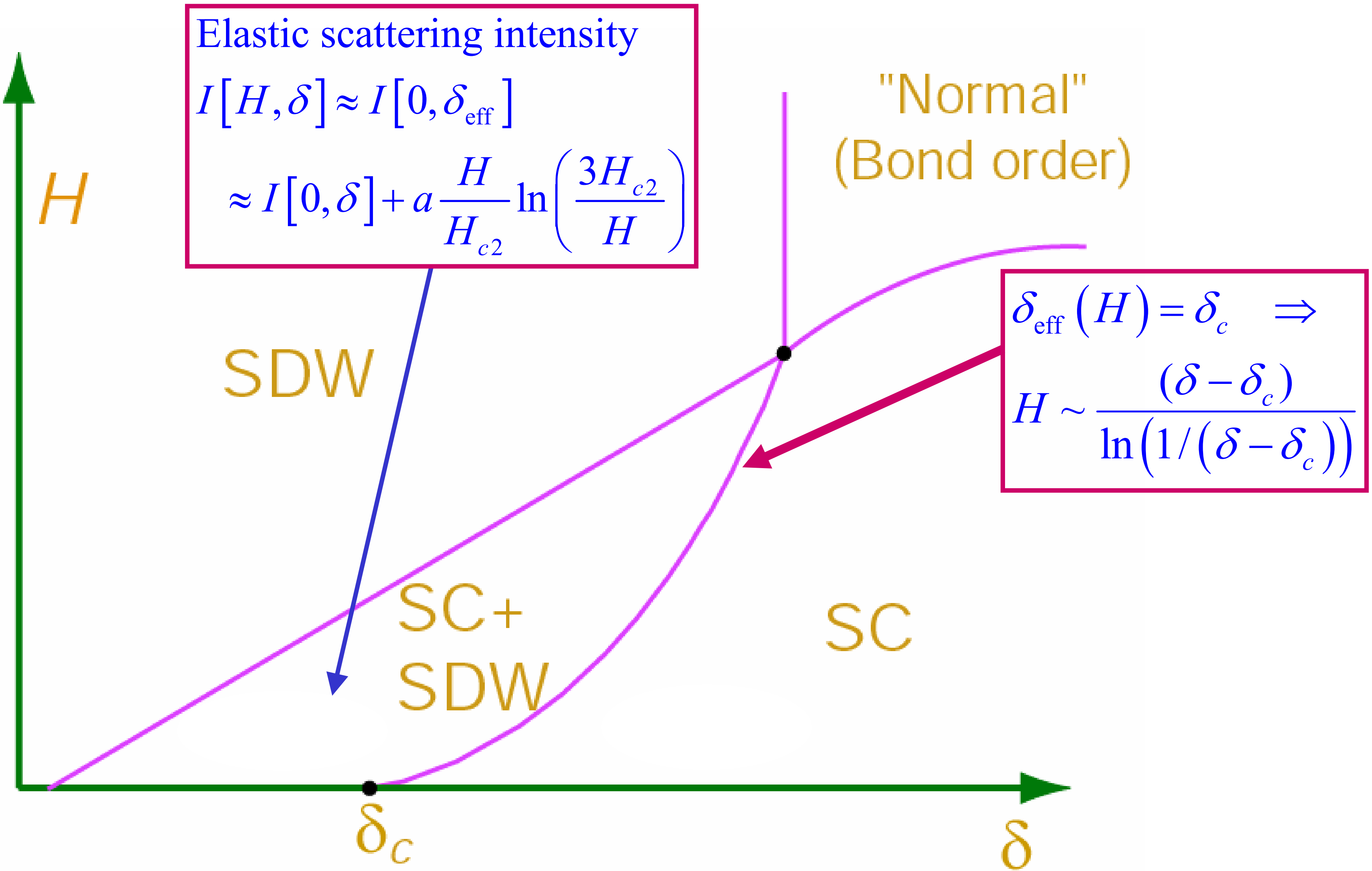
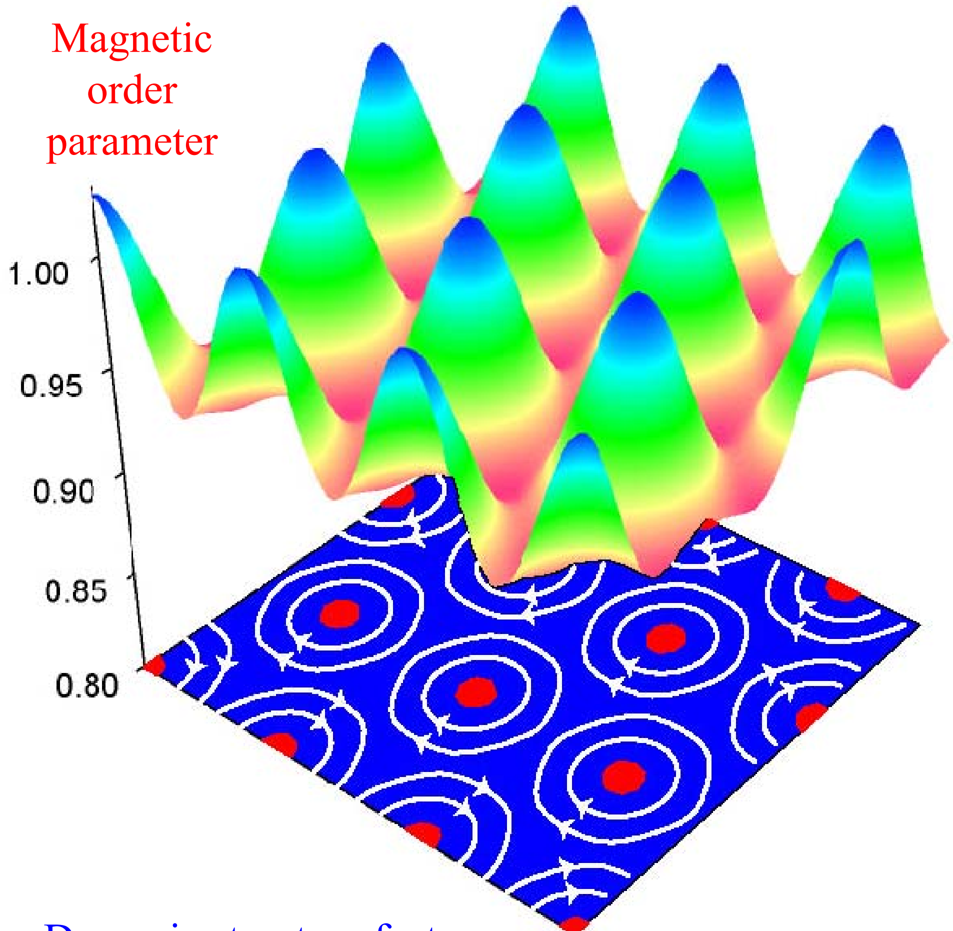


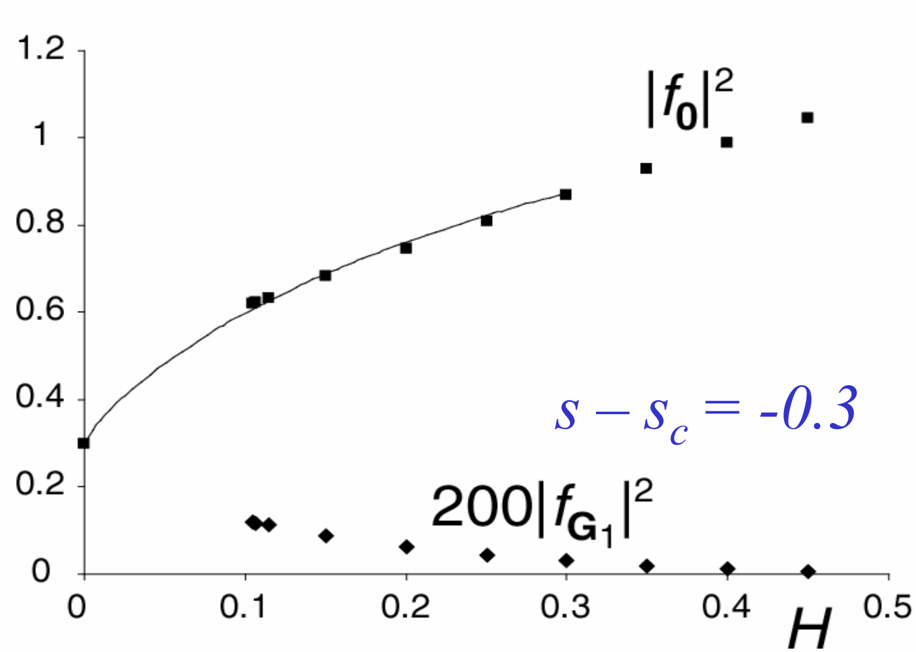
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



$$\delta \left| f_0 \right|^2 \propto H \ln(1/H)$$



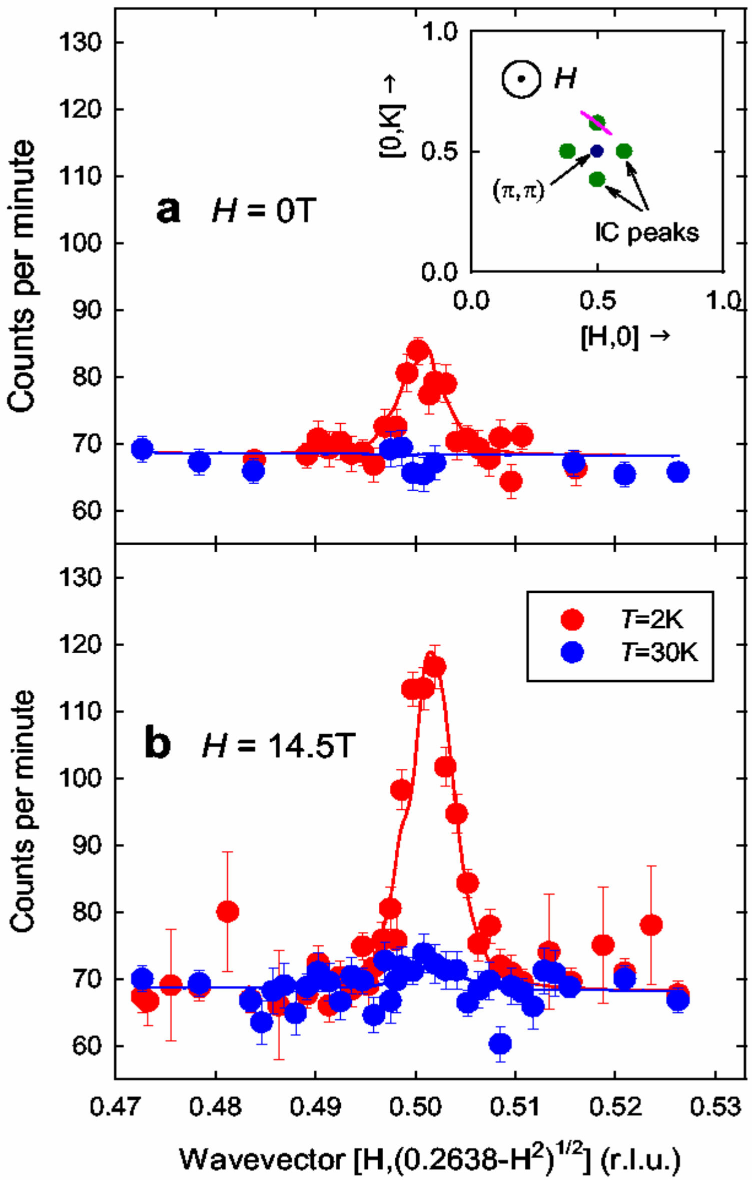
Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} \left| f_{\boldsymbol{G}} \right|^2 \delta(\boldsymbol{k} - \boldsymbol{G}) + \cdots$$

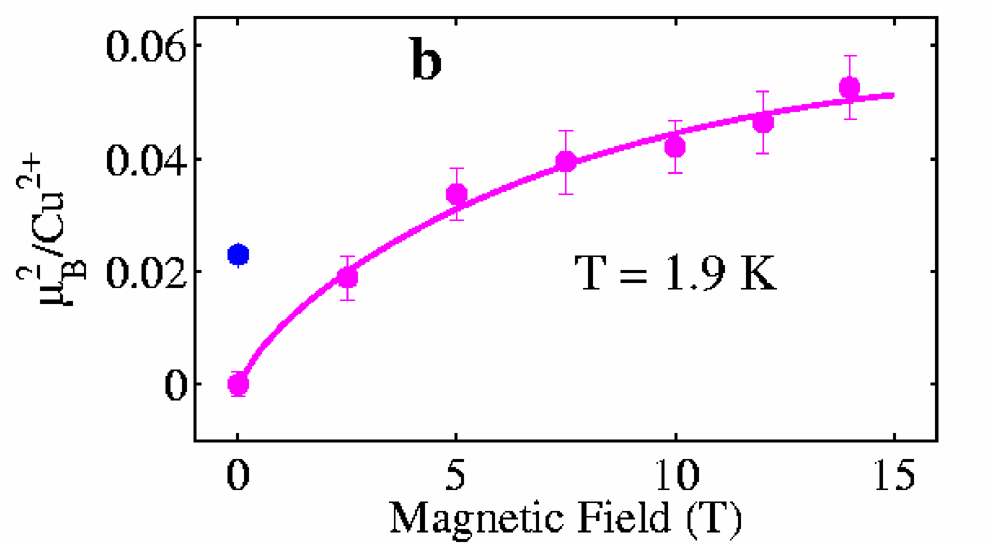
$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



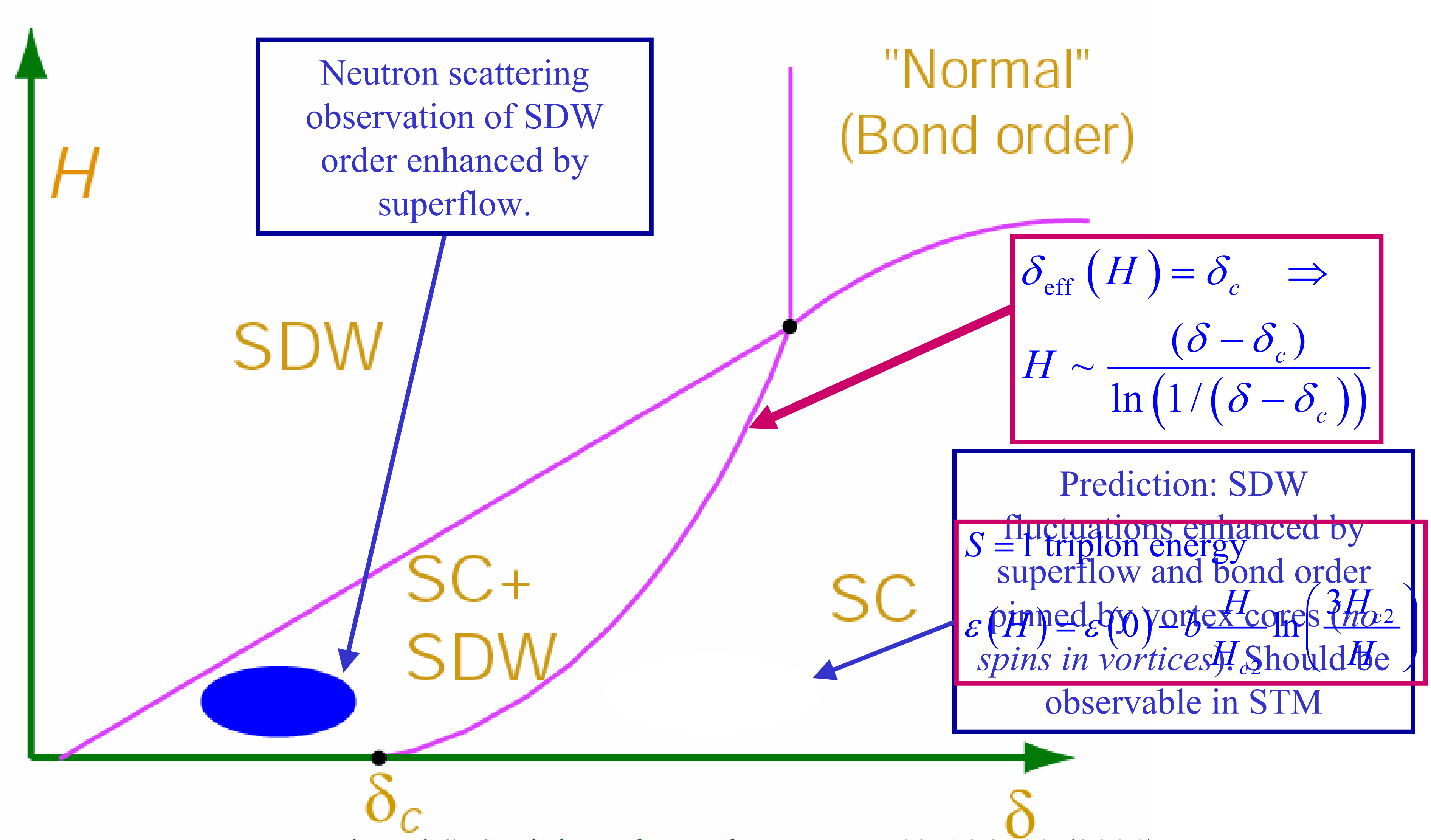
B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



Solid line - fit to : $I(H) = a\dfrac{H}{H_{c2}}\ln\left(\dfrac{H_{c2}}{H}\right)$

See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev*. B **62**, R14677 (2000).

Phase diagram of a superconductor in a magnetic field

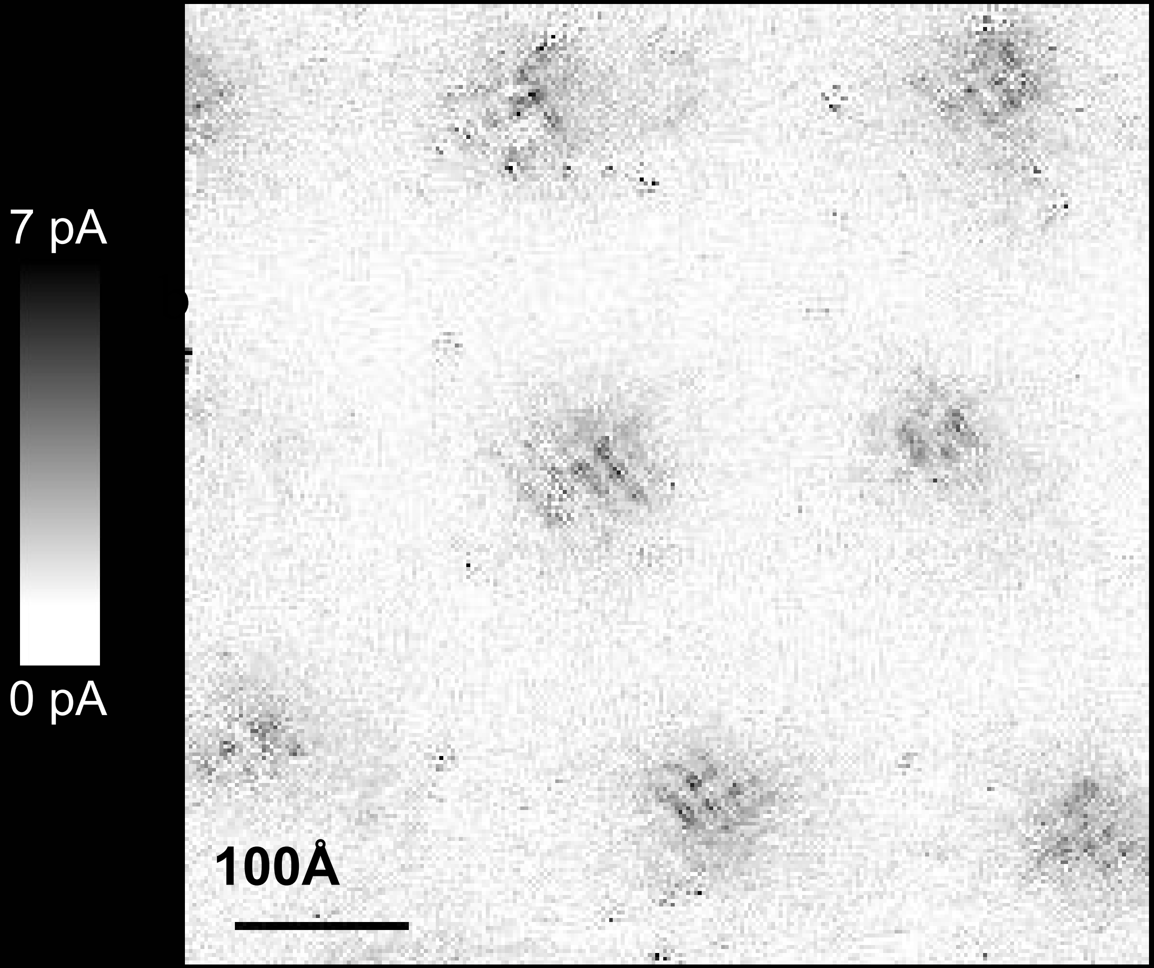


K. Park and S. Sachdev *Physical Review* B **64**, 184510 (2001);

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Y. Zhang, E. Demler and S. Sachdev, *Physical Review* B **66**, 094501 (2002).

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



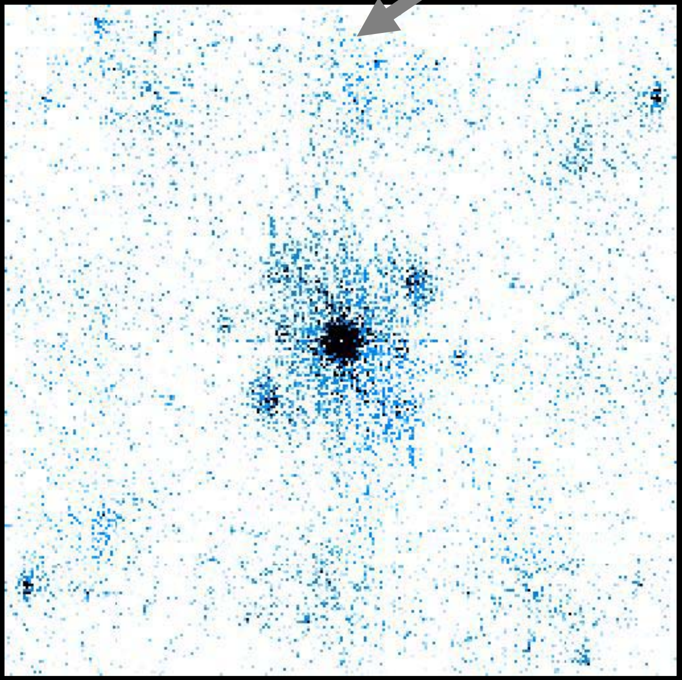
Our interpretation:
LDOS modulations are signals of bond order of period 4 revealed in vortex halo

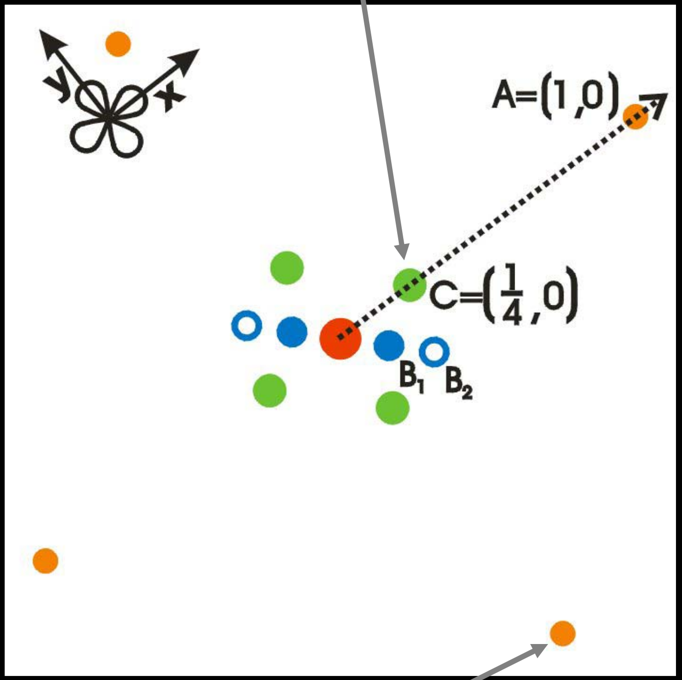
See also:
S. A. Kivelson, E. Fradkin, V. Oganesyan, I. P. Bindloss, J. M. Tranquada, A. Kapitulnik, and C. Howald, cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map

K-space locations of vortex induced LDOS



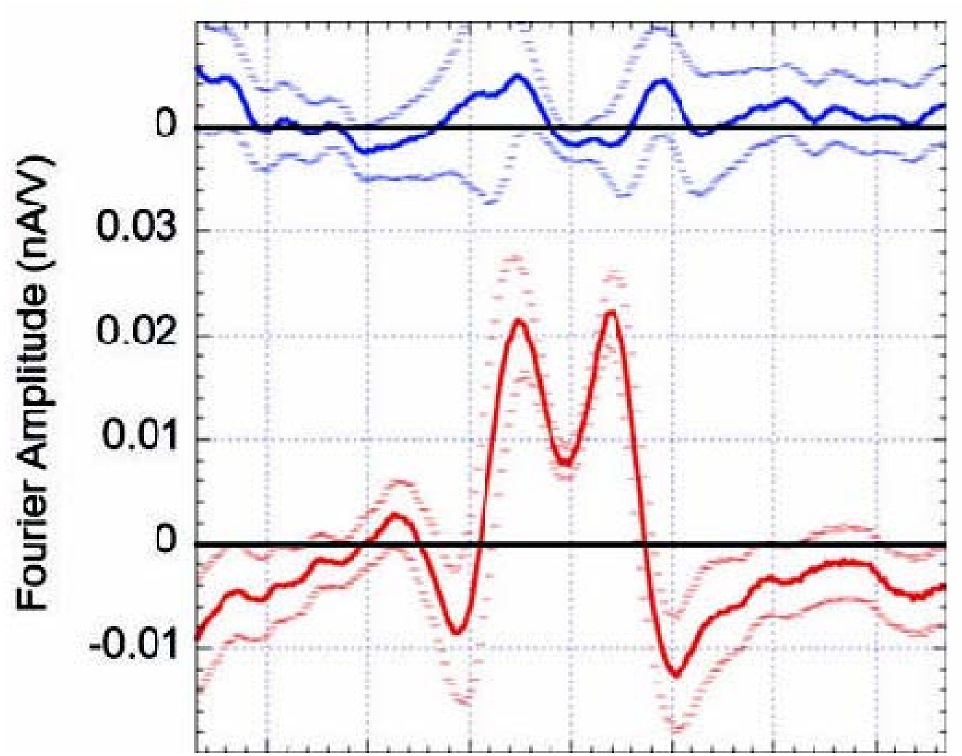


K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

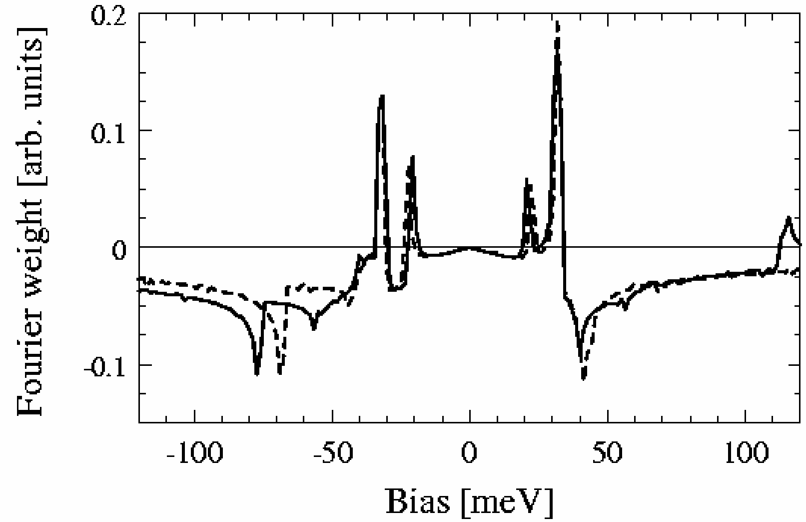
J. Hoffman *et al. Science*, **295**, 466 (2002).

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, *Phys. Rev.* B **67**, 014533 (2003).



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, *Phys. Rev.* B **66**, 104505 (2002); D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, *Phys. Rev.* B in press, cond-mat/0204011

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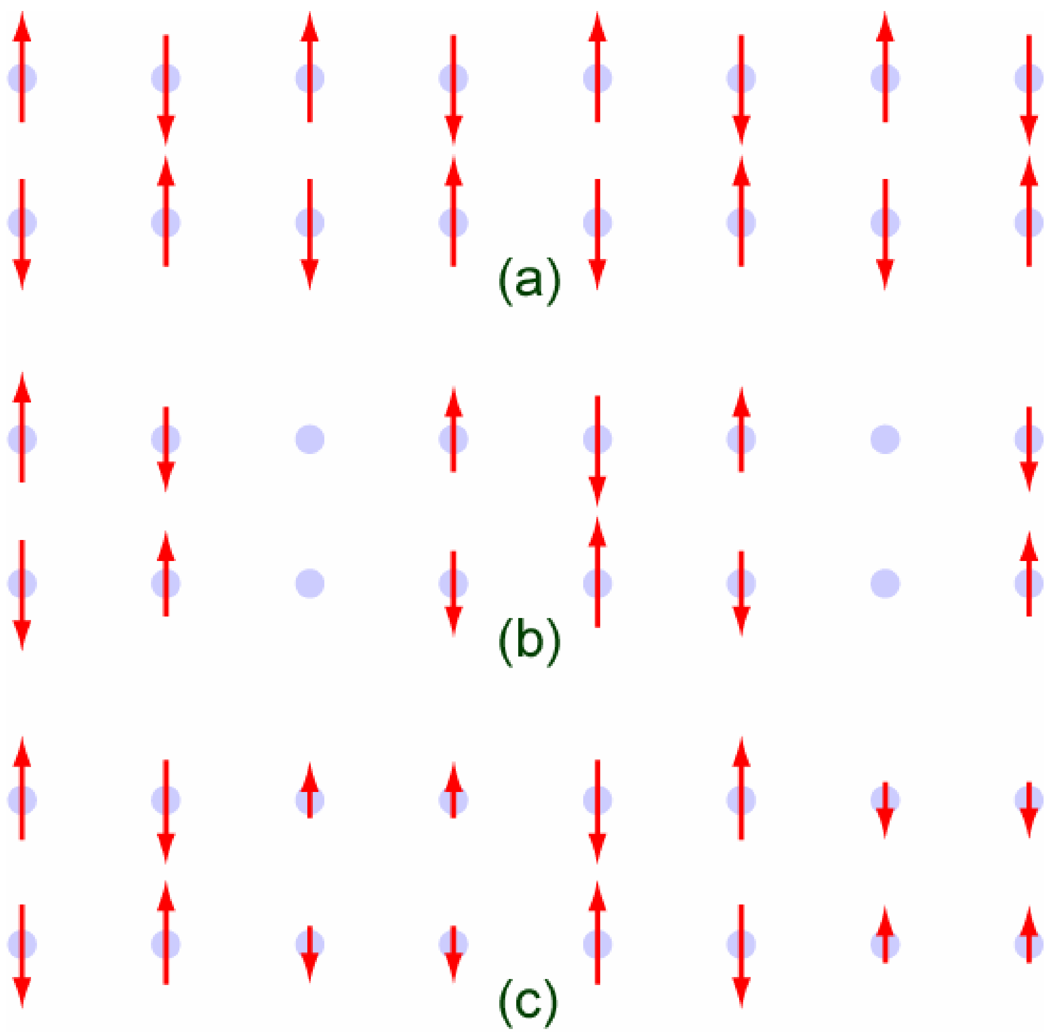
Multiple order parameters.

VI. Conclusions

V. Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class A. Collinear spins



$$\vec{K} = (\pi, \pi) \;;\; \mathbf{N}_2 = 0$$

$$\vec{K} = (3\pi/4, \pi) \;;\; \mathbf{N}_2 = 0$$

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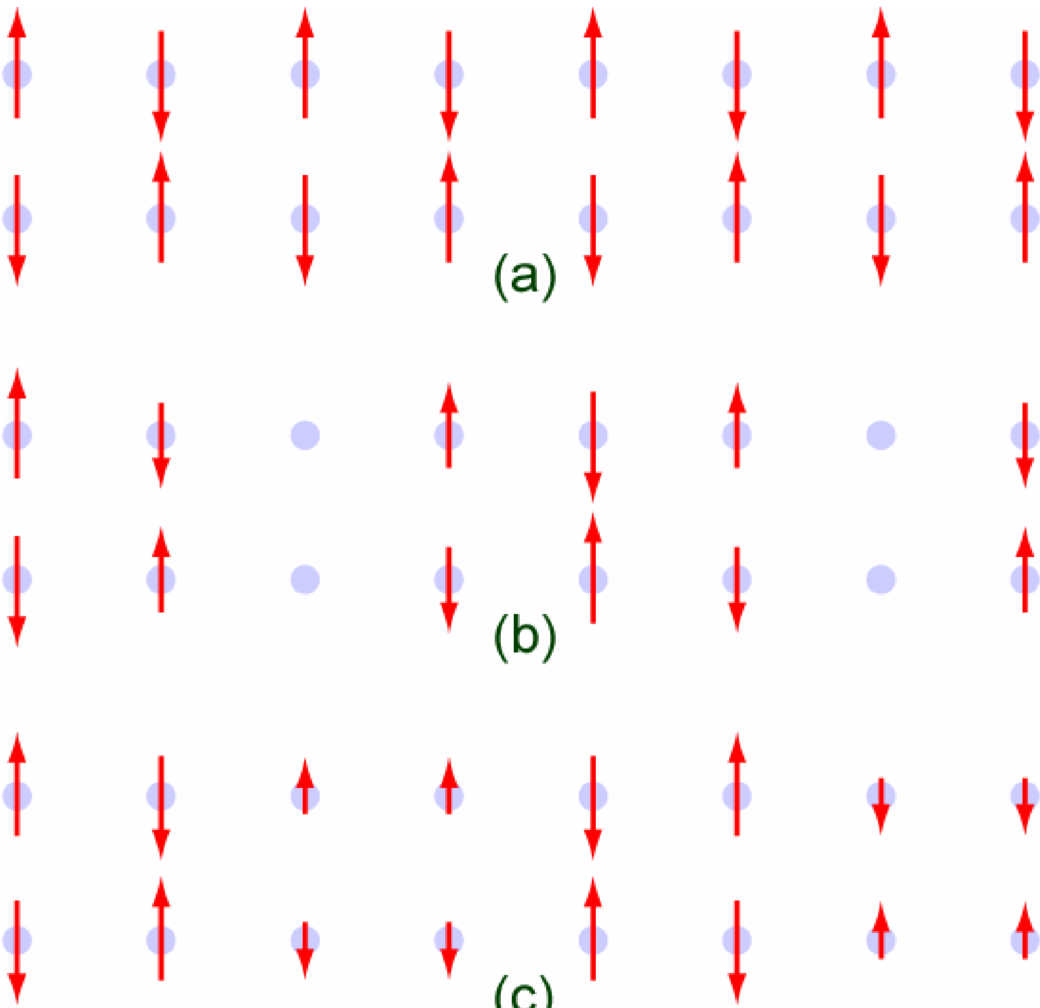
V. Order in Mott insulators

Magnetic order $\left\langle \mathbf{S}_j \right\rangle = \mathbf{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class A. Collinear spins



Key property

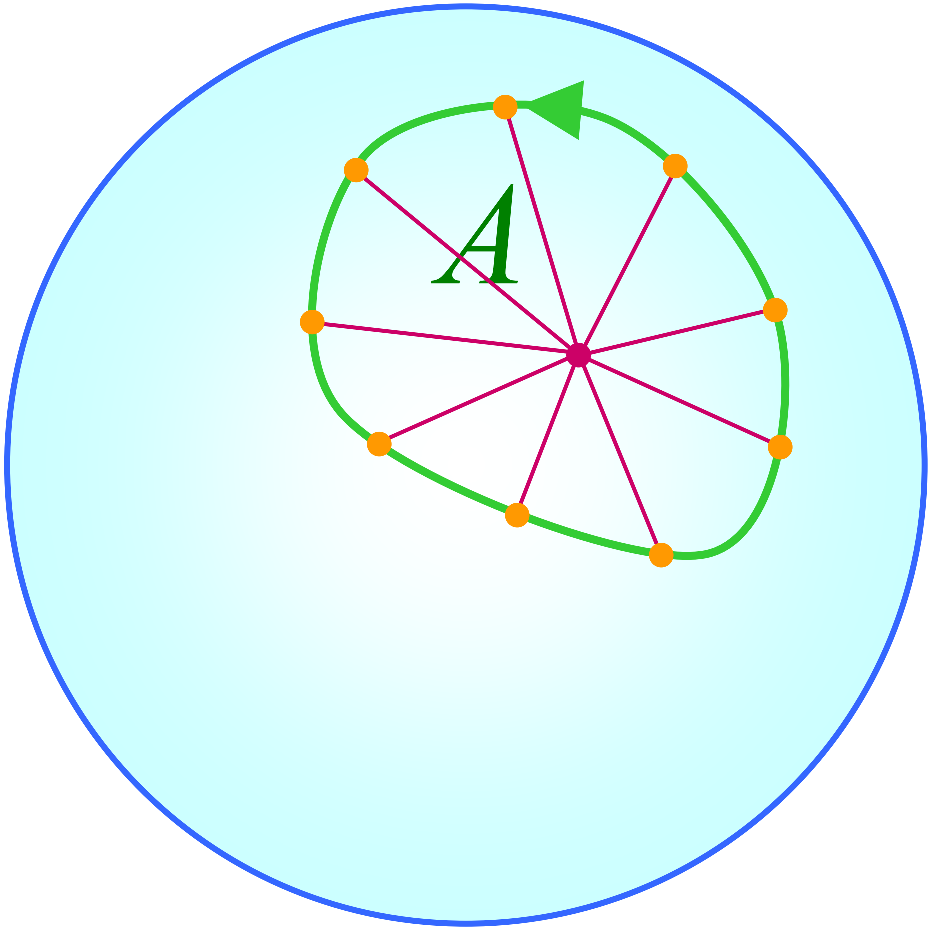
Order specified by a single vector $\mathbf{N}$.

Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector (S=1) quasiparticle excitation.

Class A: Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$$e^{iSA}$$

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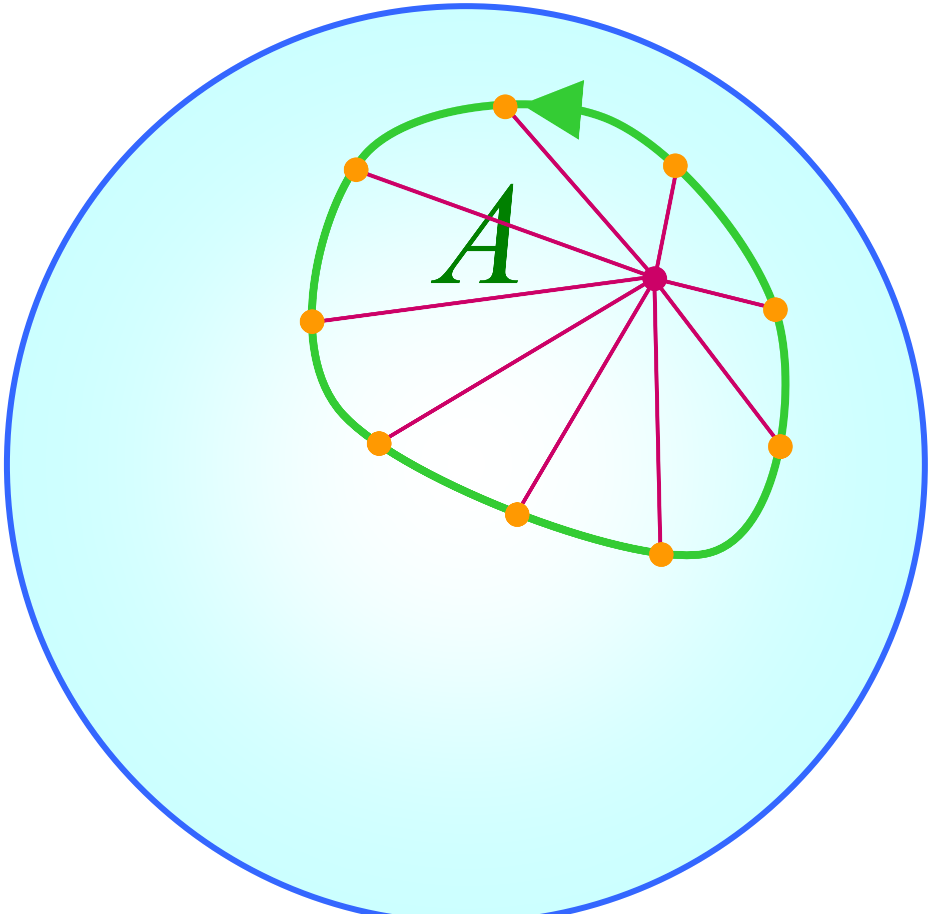


e^{iSA}

Class A: Collinear spins and compact U(1) gauge theory

S=1/2 square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two square sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

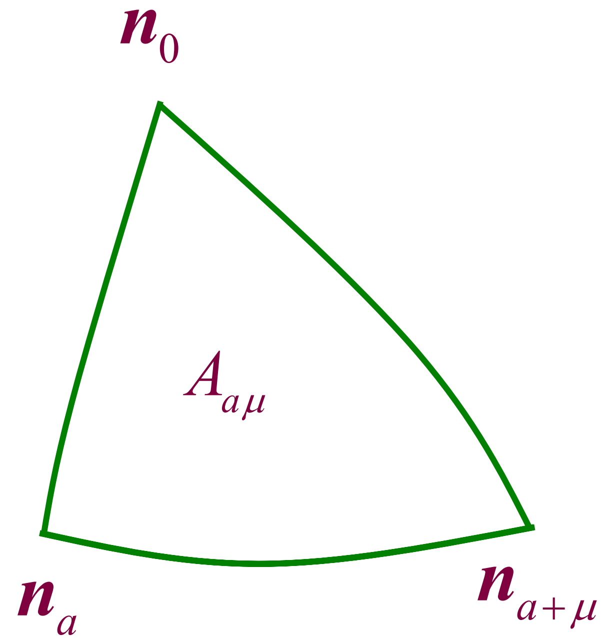
formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

Small g $\rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

Large g $\rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \boldsymbol{n}_a \rangle = 0$

Integrate out $\boldsymbol{n}_a$ ***to obtain effective action for*** $A_{a\mu}$

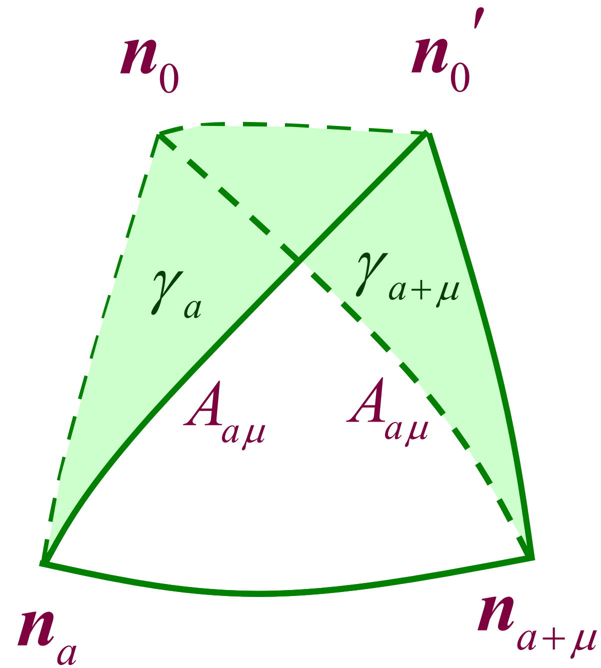


$\boldsymbol{n}_0$
$A_{a\mu}$
$\boldsymbol{n}_a$
$\boldsymbol{n}_{a+\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \to A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \to A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the large g phase

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2}\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

***d*=2**: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

***d*=3**: A deconfined phase with a gapless "photon" is possible.

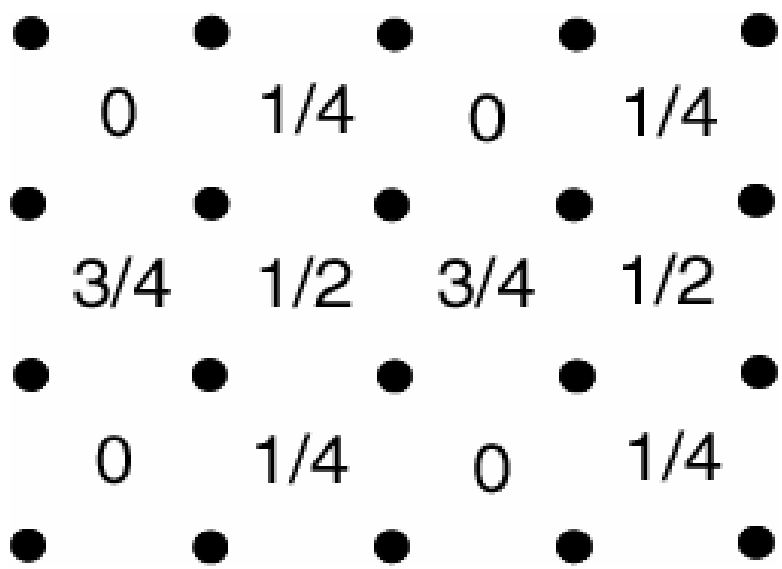
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields

$$Z = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2} \sum_{\bar{\jmath}} \left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}} \right)^2 \right)$$

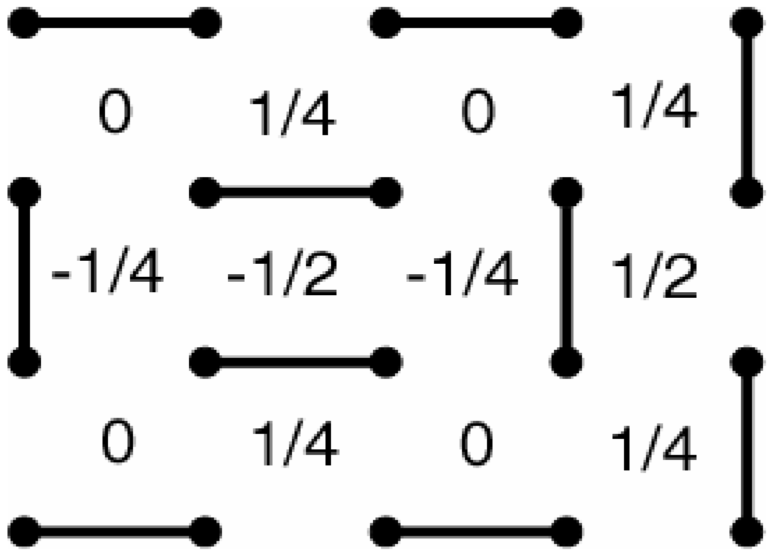
with $h_{\bar{\jmath}}$ integer.

Height model in 2+1 dimensions with 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

⇒ 2+1 dimensional height model is the path integral of the **Quantum Dimer Model**



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

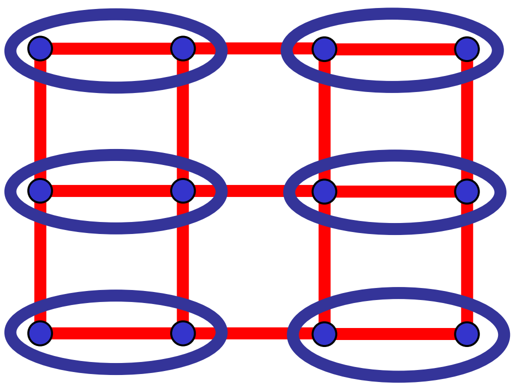
⇒ There is a definite average height of the interface

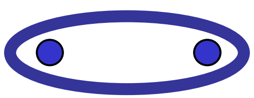
⇒ **Ground state has bond order.**

V. Order in Mott insulators

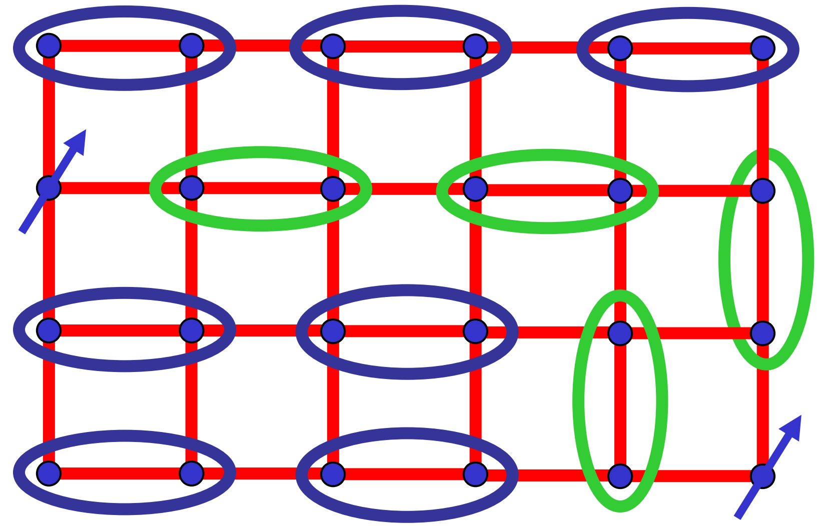
Paramagnetic states $\left\langle \mathbf{S}_j \right\rangle = 0$

Class A. Bond order and spin excitons in *d*=2





$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$



S=1/2 ***spinons*** are *confined* by a linear potential into a *S*=1 spin ***triplon***

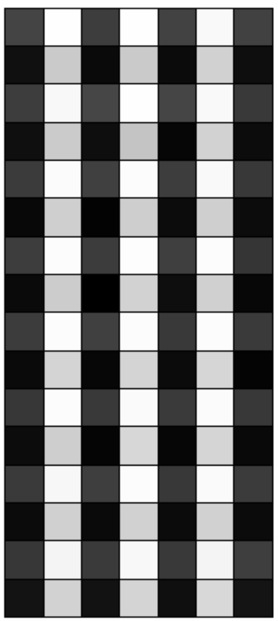
Spontaneous bond-order leads to vector *S*=1 spin excitations

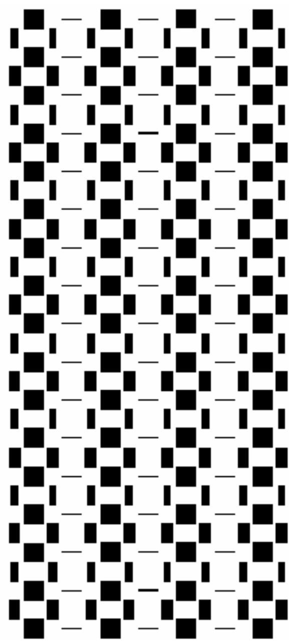
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

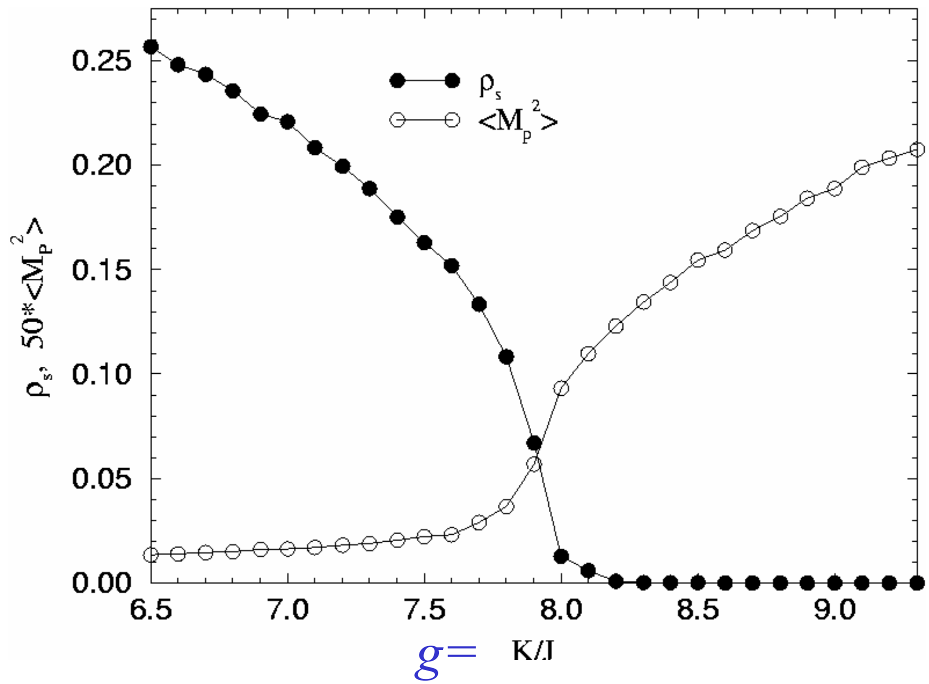
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry







$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

Outline

I. Quantum Ising Chain

II. Coupled Dimer Antiferromagnet
   A. Coherent state path integral
   B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field
   Bose condensation of "triplons"

Single order parameter.

IV. Magnetic transitions in superconductors
   Quantum phase transition in a background Abrikosov flux lattice

V. Antiferromagnets with an odd number of *S*=1/2 spins per unit cell
   Class A: Compact U(1) gauge theory: collinear spins, bond order and confined spinons in *d*=2
   Class B Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

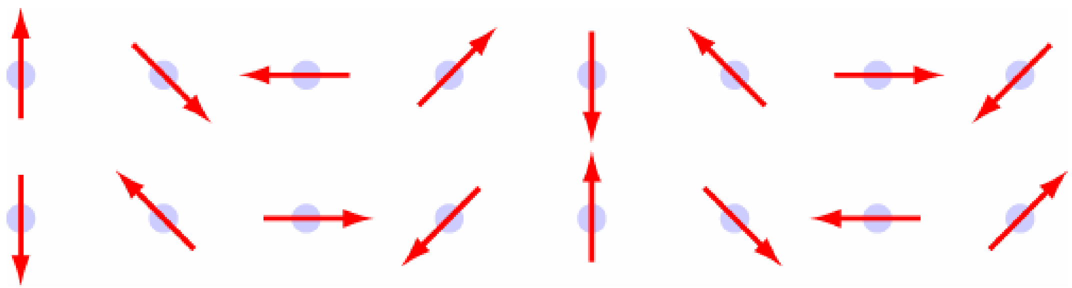
Multiple order parameters.

VI. Conclusions

V.B Order in Mott insulators

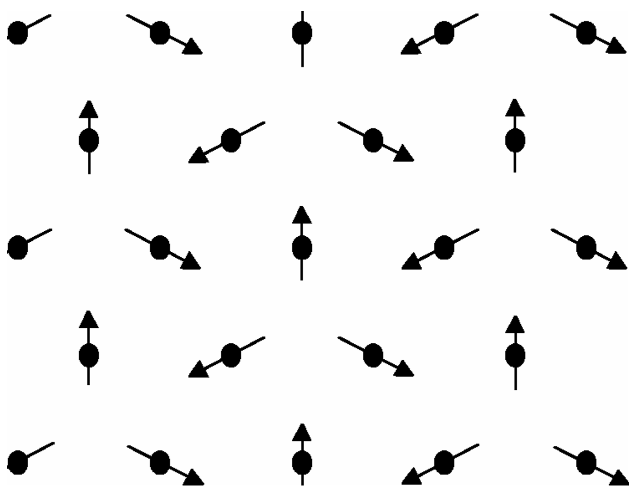
Magnetic order $\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class B. Noncollinear spins



$$\vec{K} = \left(3\pi/4, \pi\right)$$

(B.I. Shraiman and E.D. Siggia, *Phys. Rev. Lett.* **61**, 467 (1988))



$$\vec{K} = \left(4\pi/3, 4\pi/\sqrt{3}\right)$$

$$\boldsymbol{N}_2^2 = \boldsymbol{N}_1^2 \,,\ \boldsymbol{N}_1 \cdot \boldsymbol{N}_2 = 0$$

V.B Order in Mott insulators

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N_1} \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N_2} \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class B. Noncollinear spins

$$\boldsymbol{N_2}^2 = \boldsymbol{N_1}^2 \, , \, \boldsymbol{N_1} \cdot \boldsymbol{N_2} = 0$$

Solve constraints by expressing $\boldsymbol{N_{1,2}}$ in terms of two complex numbers $z_\uparrow \, , z_\downarrow$

$$\boldsymbol{N_1} + i\boldsymbol{N_2} = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2 z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign).

This spinor can become a S=1/2 spinon in paramagnetic state.

Order parameter space: S_3 / Z_2

Physical observables are invariant under the Z_2 gauge transformation $z_a \to \pm z_a$

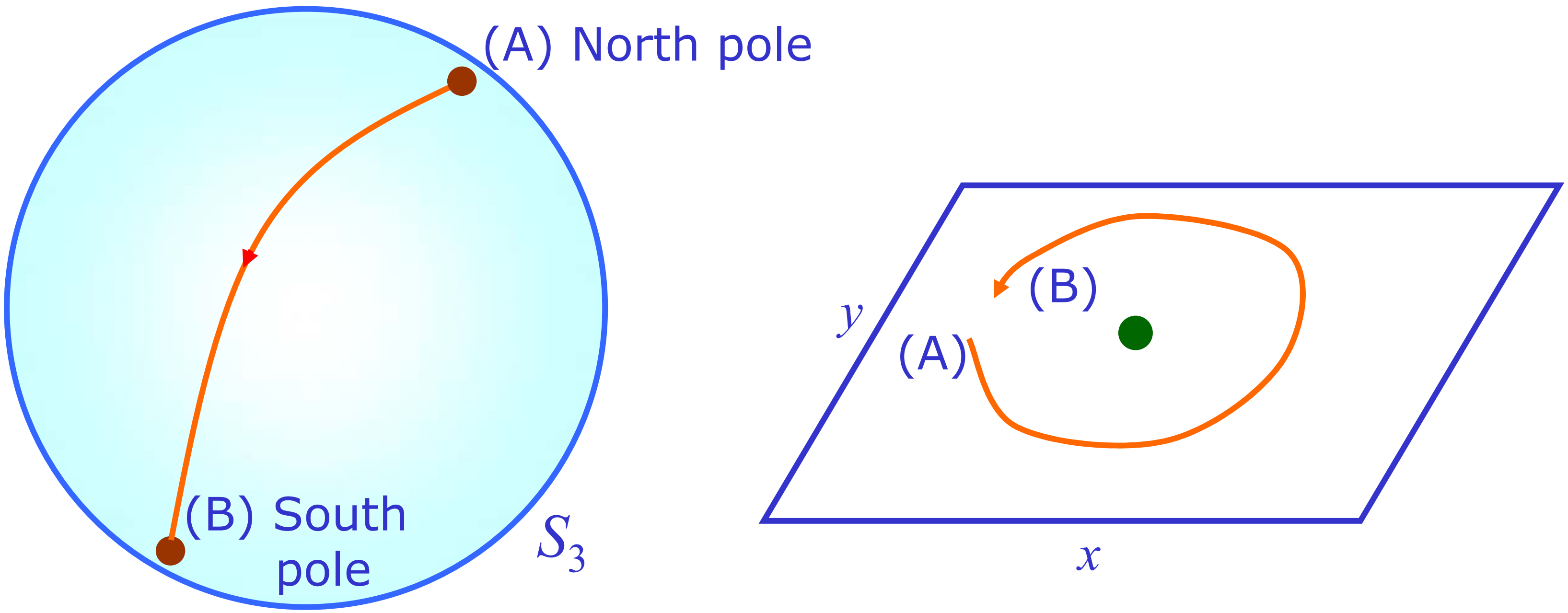
A. V. Chubukov, S. Sachdev, and T. Senthil *Phys. Rev. Lett.* **72,** 2089 (1994)

V.B Order in Mott insulators

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class B. Noncollinear spins

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (*visons*)



Such vortices (visons) can also be defined in the phase in which spins are "quantum disordered". A Z_2 spin liquid with deconfined spinons must have ***visons supressed***

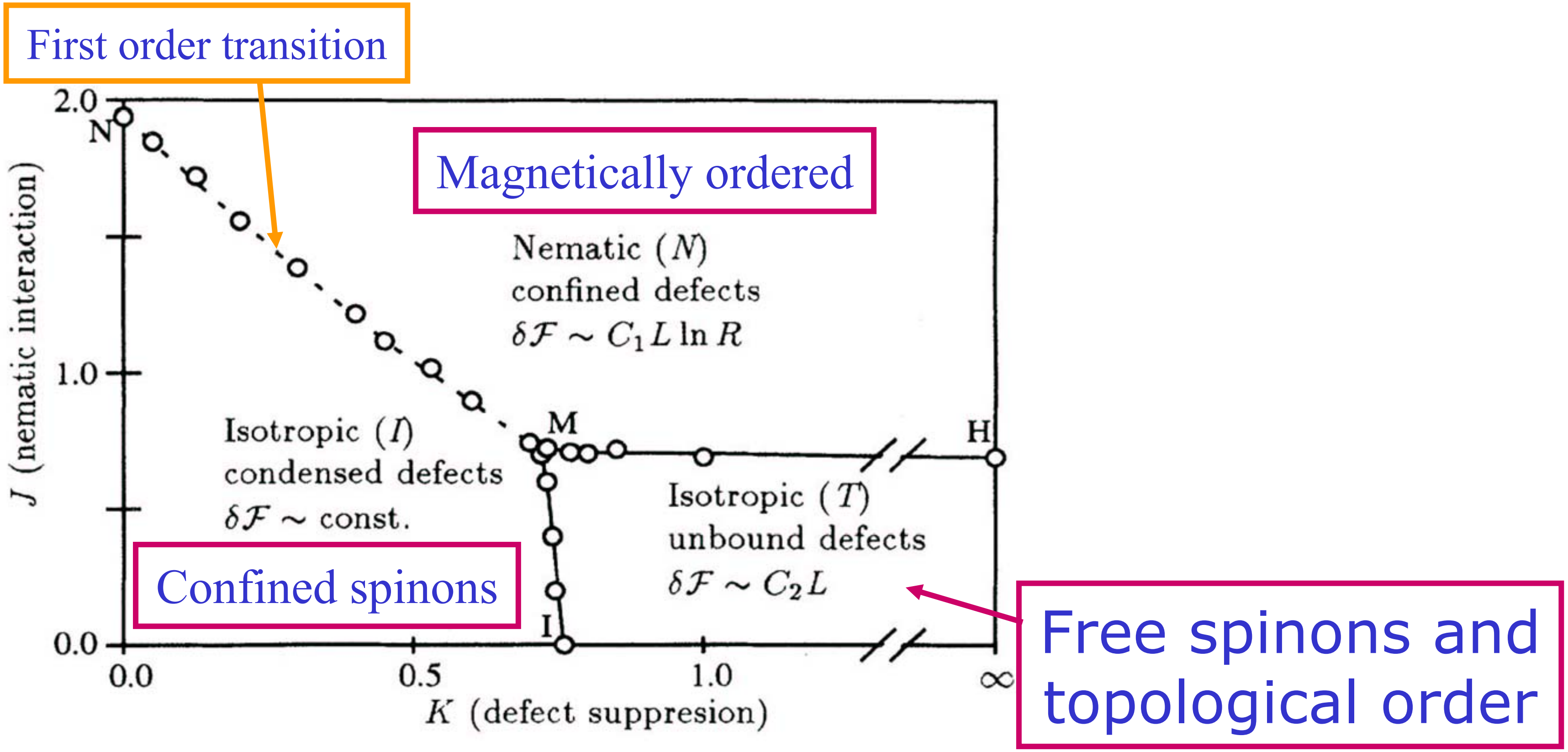
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Model effective action and phase diagram

$$S = -J \sum_{\langle ij \rangle} \sigma_{ij} z^*_{\alpha i} z_{\alpha j} + \text{h.c.} - K \sum_{\square} \prod_{\square} \sigma_{ij}$$

(Derivation using Schwinger bosons on a quantum antiferromagnet: S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991)).

$$\sigma_{ij} \rightarrow Z_2 \text{ gauge field}$$



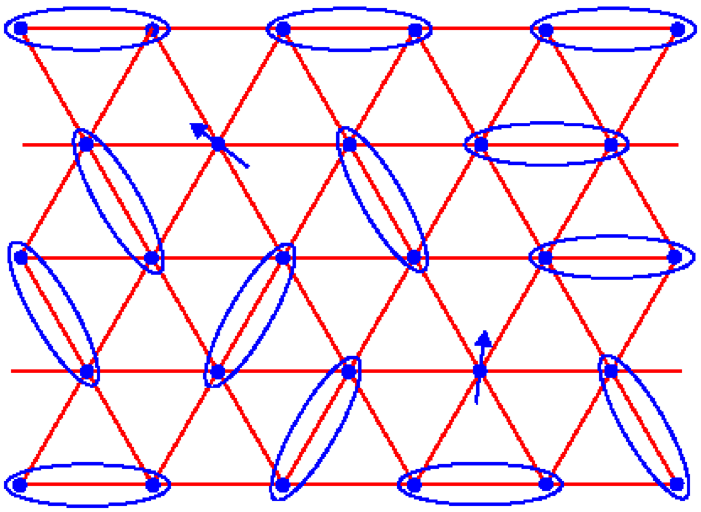
P. E. Lammert, D. S. Rokhsar, and J. Toner, *Phys. Rev. Lett*. **70**, 1650 (1993) ; *Phys. Rev.* E **52**, 1778 (1995). (For nematic liquid crystals)

V.B Order in Mott insulators

Paramagnetic states $\langle \mathbf{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons

A topologically ordered state in which vortices associated with $\pi_1(S_3/Z_2)=Z_2$ ["visons"] are gapped out. This is an RVB state with deconfined $S=1/2$ spinons z_a



N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991).
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).
A.V. Chubukov, T. Senthil and S. S., *Phys. Rev. Lett.***72**, 2089 (1994).
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).
G. Misguich and C. Lhuillier, *Eur. Phys. J*. B **26**, 167 (2002).
R. Moessner and S.L. Sondhi, *Phys. Rev. Lett*. **86**, 1881 (2001).

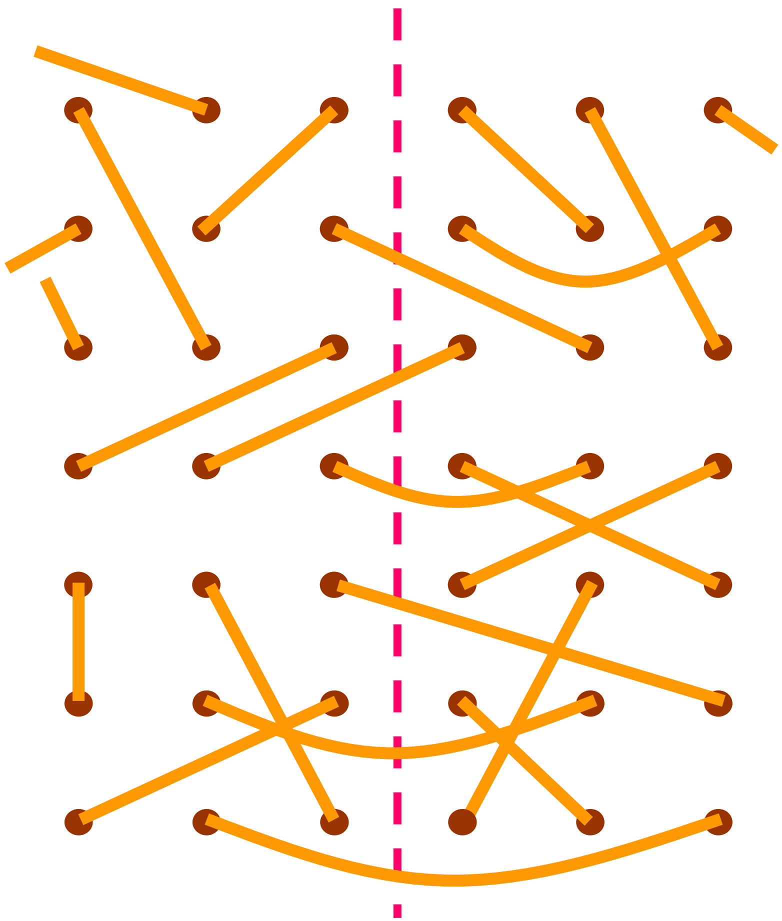
Recent experimental realization: Cs_2CuCl_4
R. Coldea, D.A. Tennant, A.M. Tsvelik, and Z. Tylczynski, *Phys. Rev. Lett*. **86**, 1335 (2001).

V.B Order in Mott insulators

Paramagnetic states $\left\langle \boldsymbol{S}_j \right\rangle = 0$

Class B. Topological order and deconfined spinons

Direct description of topological order with valence bonds



Number of valence bonds cutting line is conserved modulo 2. Changing sign of each such bond does not modify state. This is equivalent to a Z_2 gauge transformation with $z_a \rightarrow -z_a$ on sites to the right of dashed line.

D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988); N. Read and B. Chakraborty, *Phys. Rev.* B **40**, 7133 (1989).

V.B Order in Mott insulators

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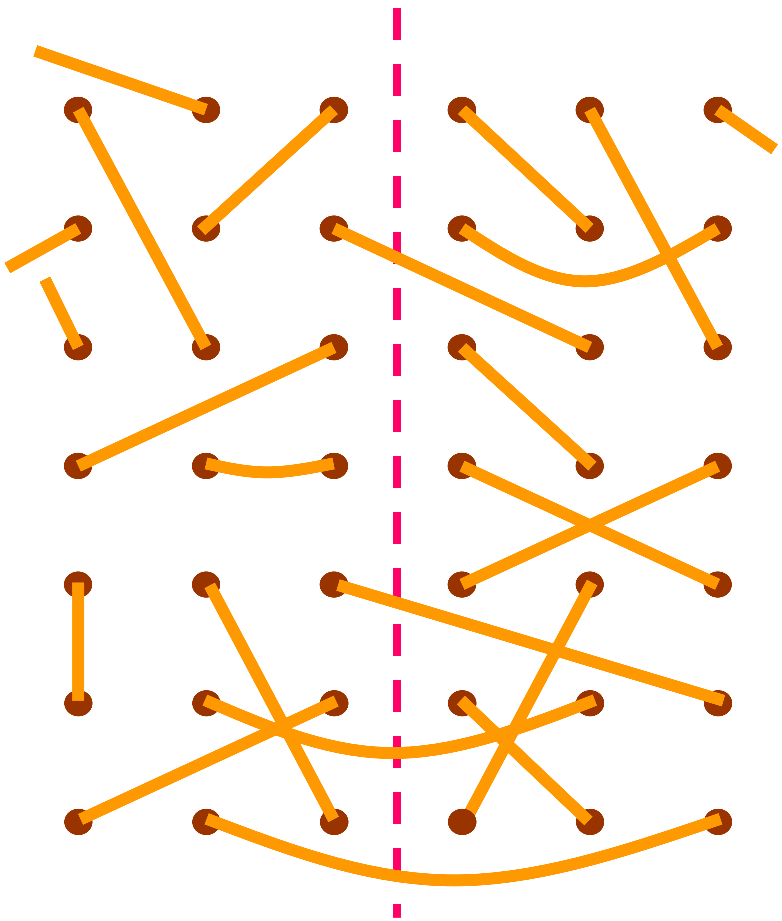

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D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988); N. Read and B. Chakraborty, *Phys. Rev.* B **40**, 7133 (1989).

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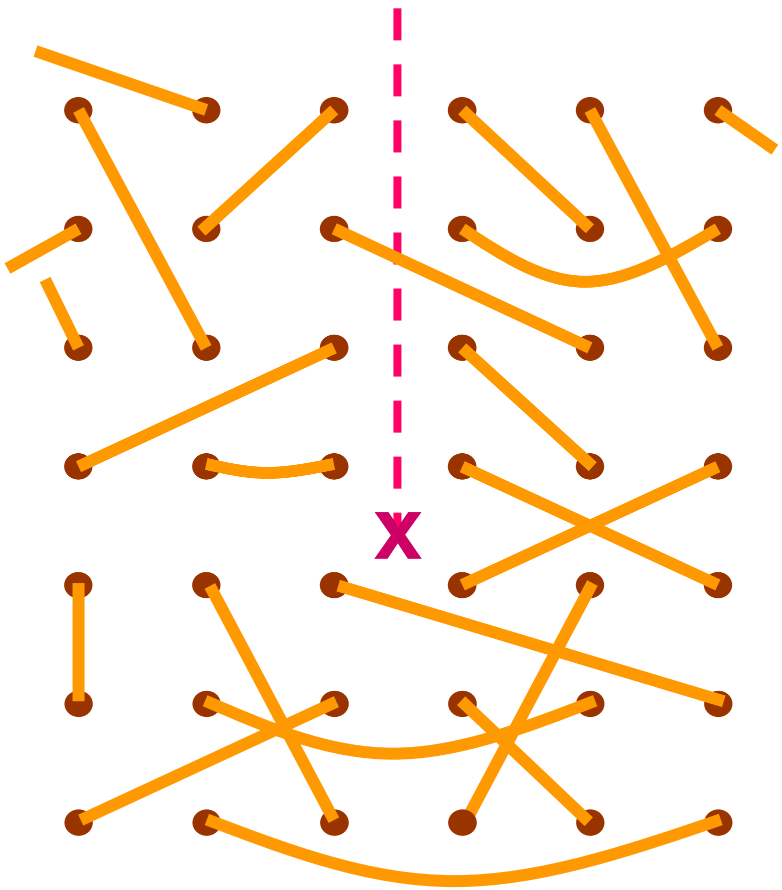


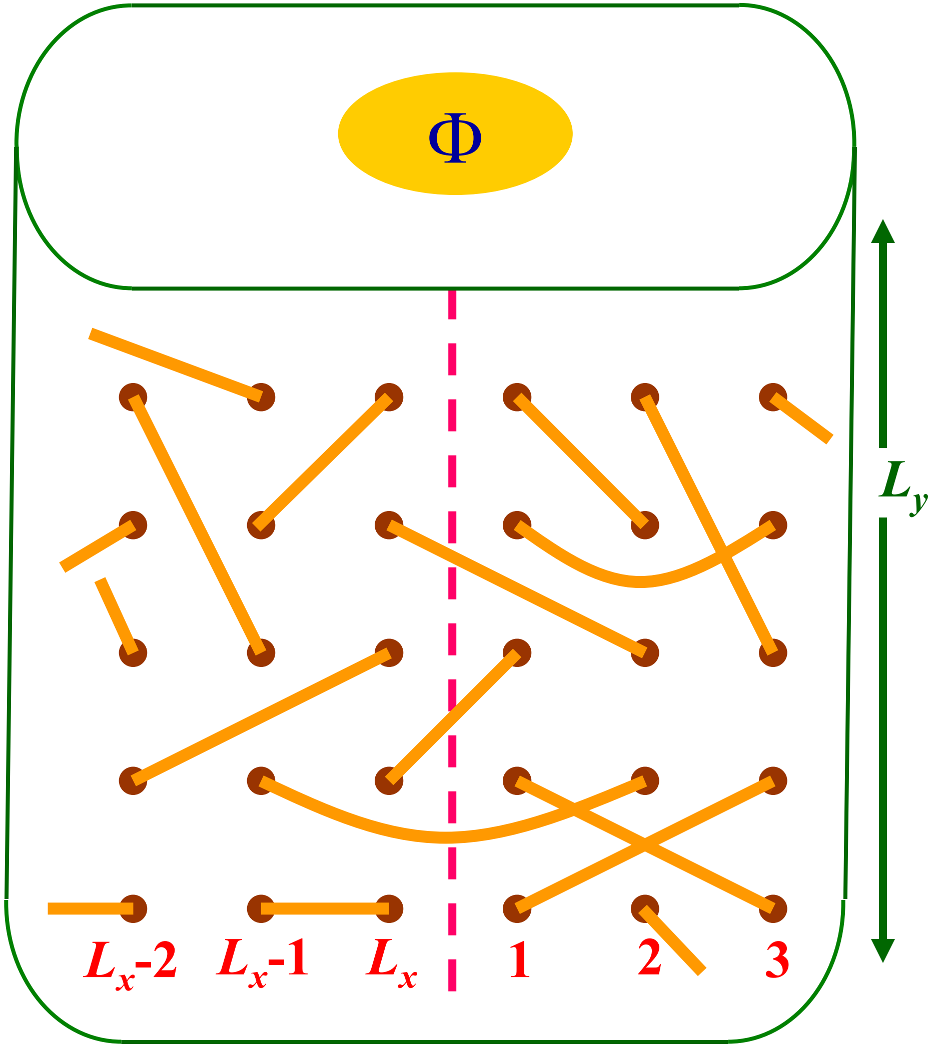
Terminating the line creates a plaquette with Z_2 flux at the X --- a ***vison***.

D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett*. **61**, 2376 (1988); N. Read and B. Chakraborty, *Phys. Rev*. B **40**, 7133 (1989).

Effect of flux-piercing on a topologically ordered quantum paramagnet

N. E. Bonesteel, *Phys. Rev.* B **40**, 8954 (1989).
G. Misguich, C. Lhuillier, M. Mambrini, and P. Sindzingre, *Eur. Phys. J.* B **26**, 167 (2002).

$$|D\rangle =$$

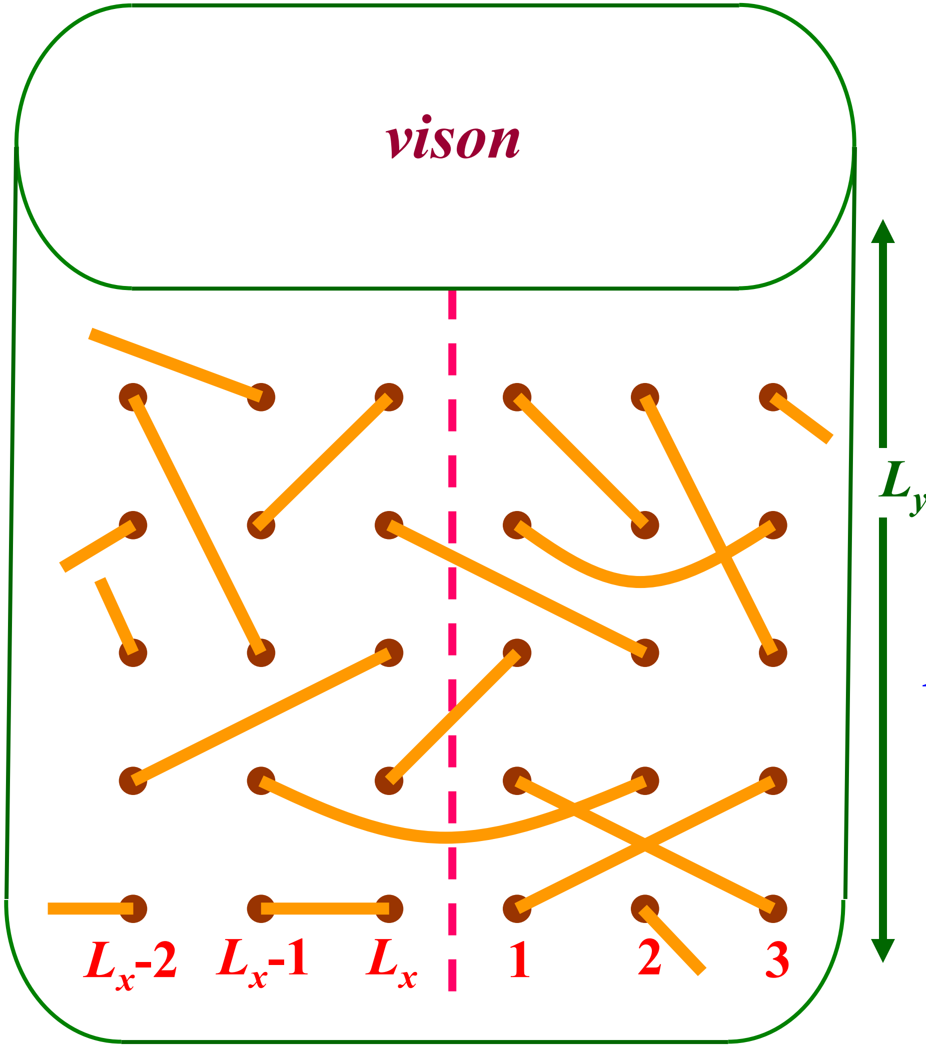


$$|\Psi\rangle = \sum_D a_D |D\rangle$$

Effect of flux-piercing on a topologically ordered quantum paramagnet

N. E. Bonesteel, *Phys. Rev.* B **40**, 8954 (1989).
G. Misguich, C. Lhuillier, M. Mambrini, and P. Sindzingre, *Eur. Phys. J.* B **26**, 167 (2002).

$|D\rangle =$



$$|\Psi\rangle = \sum_D a_D |D\rangle$$

After flux insertion $|D\rangle \Rightarrow$

$$(-1)^{\text{Number of bonds cutting dashed line}} |D\rangle;$$

Equivalent to inserting a ***vison*** inside hole of the torus.

This leads to a ground state degeneracy.

VI. Conclusions

I. Quantum Ising Chain

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   B. Quantum field theory near critical point

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   Bose condensation of “triplons”

Single order parameter.

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   Class B: Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

Multiple order parameters.

VI. Cuprates are best understood as doped class A Mott insulators.

Competing order parameters in the cuprate superconductors

1. Pairing order of BCS theory (SC)

(Bose-Einstein) condensation of *d*-wave Cooper pairs

Orders (possibly fluctuating) associated with proximate Mott insulator in class A

2. Collinear magnetic order (CM)

3. Bond/charge/stripe order (B)

(couples strongly to half-breathing phonons)

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000);
M. Vojta, Phys. Rev. B **66**, 104505 (2002).

Evidence cuprates are in class A

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity

J. M. Tranquada *et al*., *Phys*. *Rev*. B **54**, 7489 (1996).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys*. *Rev*. B **60**, 3643 (1999).
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys*. *Rev*. B **63**, 172501 (2001).

Evidence cuprates are in class A

• Neutron scattering shows collinear magnetic order co-existing with superconductivity

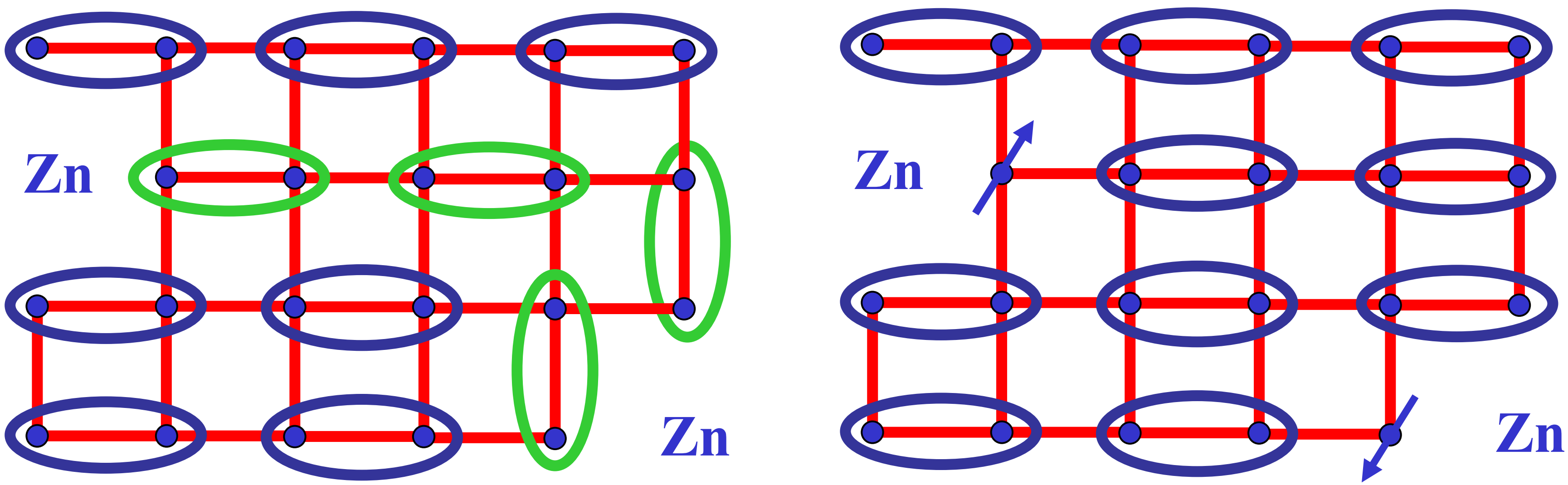
• Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect

S. Sachdev, *Physical Review* B **45**, 389 (1992)
N. Nagaosa and P.A. Lee, *Physical Review* B **45**, 966 (1992)
T. Senthil and M. P. A. Fisher, *Phys. Rev. Lett.* **86**, 292 (2001).
D. A. Bonn, J. C. Wynn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Nature* **414**, 887 (2001).
J. C. Wynn, D. A. Bonn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a S=1/2 moment

<u>Effect of static non-magnetic impurities (Zn or Li)</u>

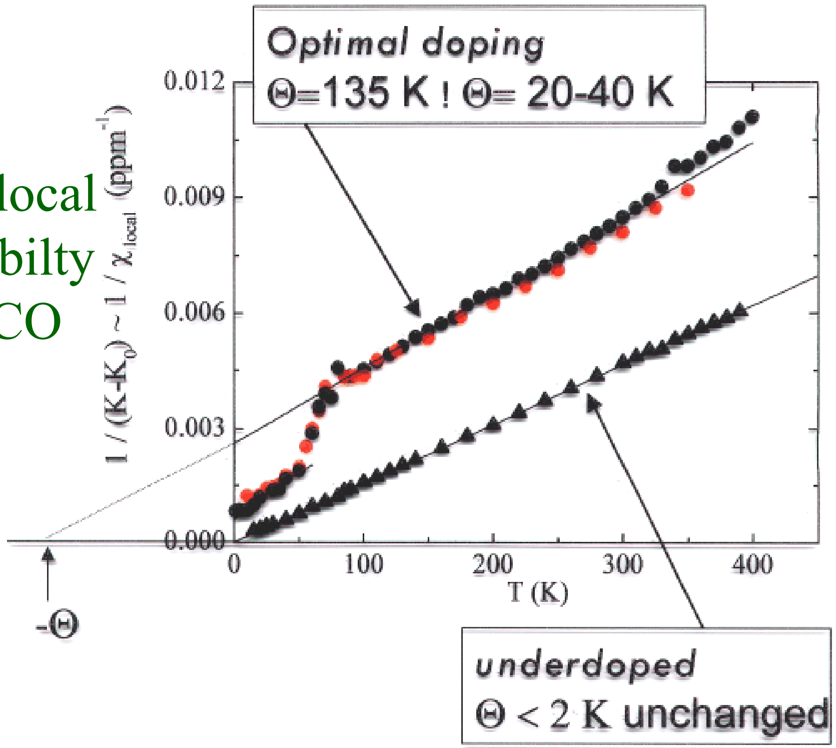


Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

Spatially resolved NMR of Zn/Li impurities in the superconducting state

Inverse local susceptibilty in YBCO



^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, *Phys. Rev. Lett*. **86**, 4116 (2001).

Measured $\chi_{impurity}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

This behavior does not emerge out of BCS theory.

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, *Physica* C **168**, 370 (1990).

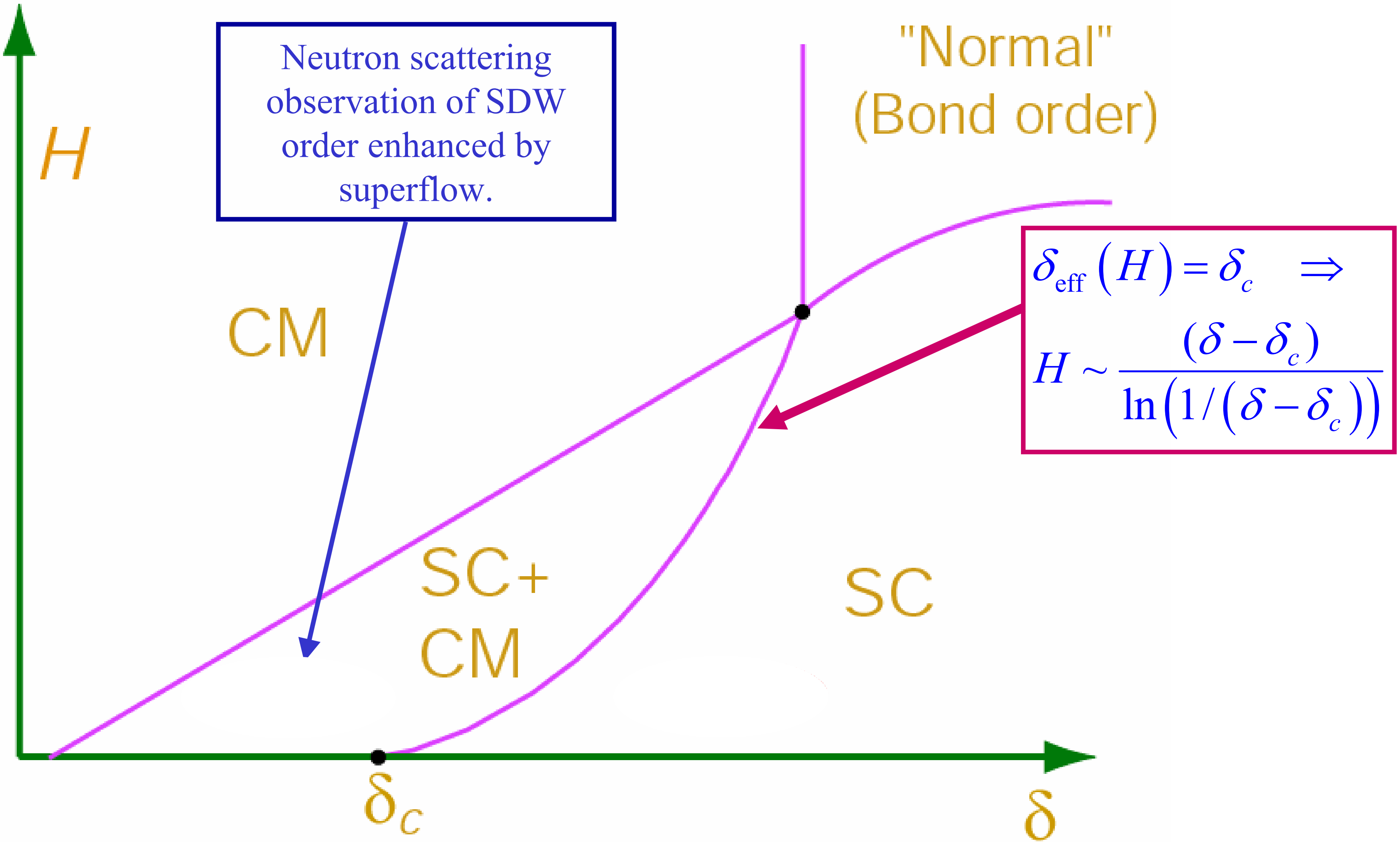
Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a S=1/2 moment

Evidence cuprates are in class A

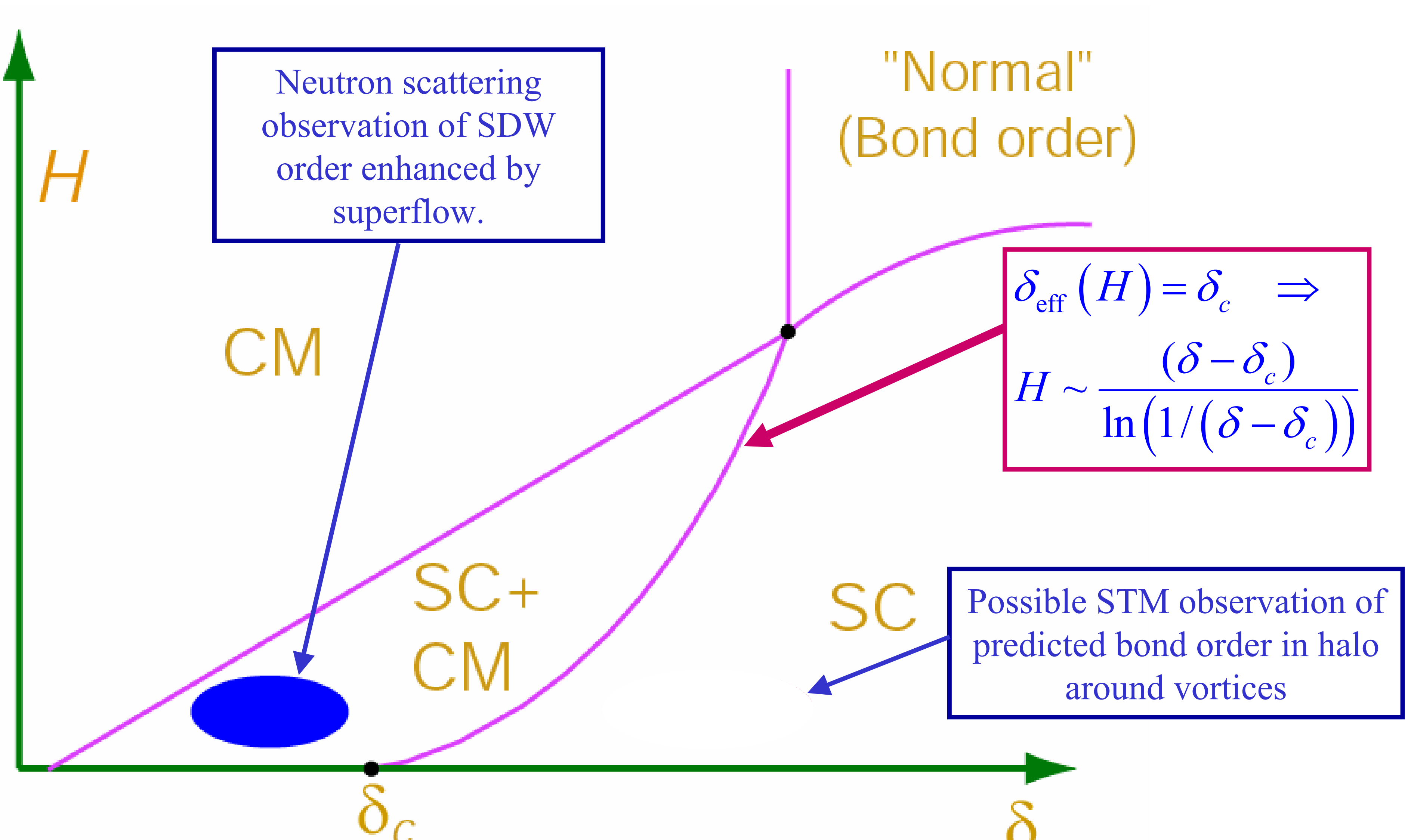
- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a S=1/2 moment
- Tests of phase diagram in a magnetic field

Phase diagram of a superconductor in a magnetic field



E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phase diagram of a superconductor in a magnetic field

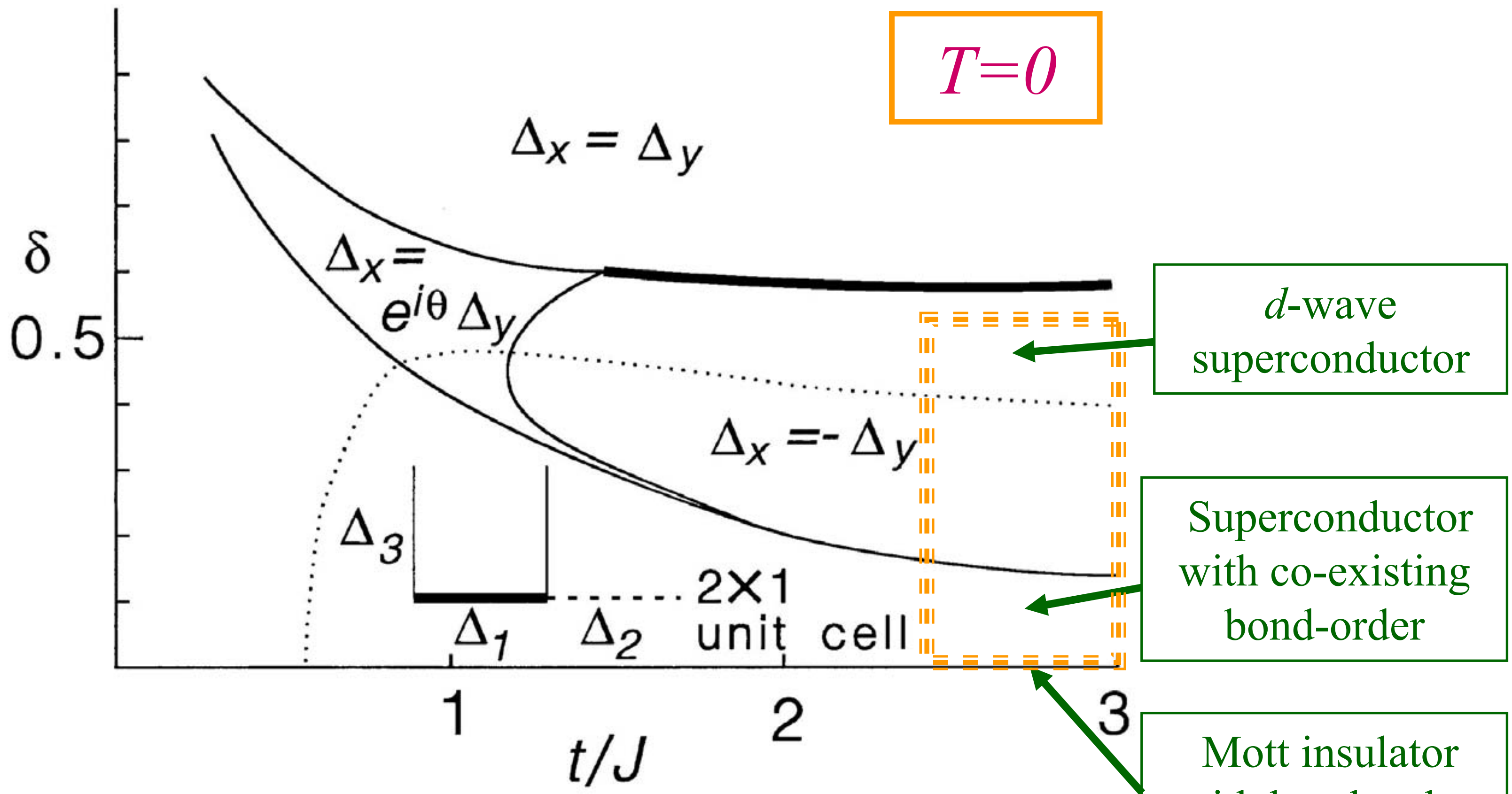


K. Park and S. Sachdev *Physical Review* B **64**, 184510 (2001);
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001);
Y. Zhang, E. Demler and S. Sachdev, *Physical Review* B **66**, 094501 (2002).

VI. Doping Class A

Doping a paramagnetic bond-ordered Mott insulator

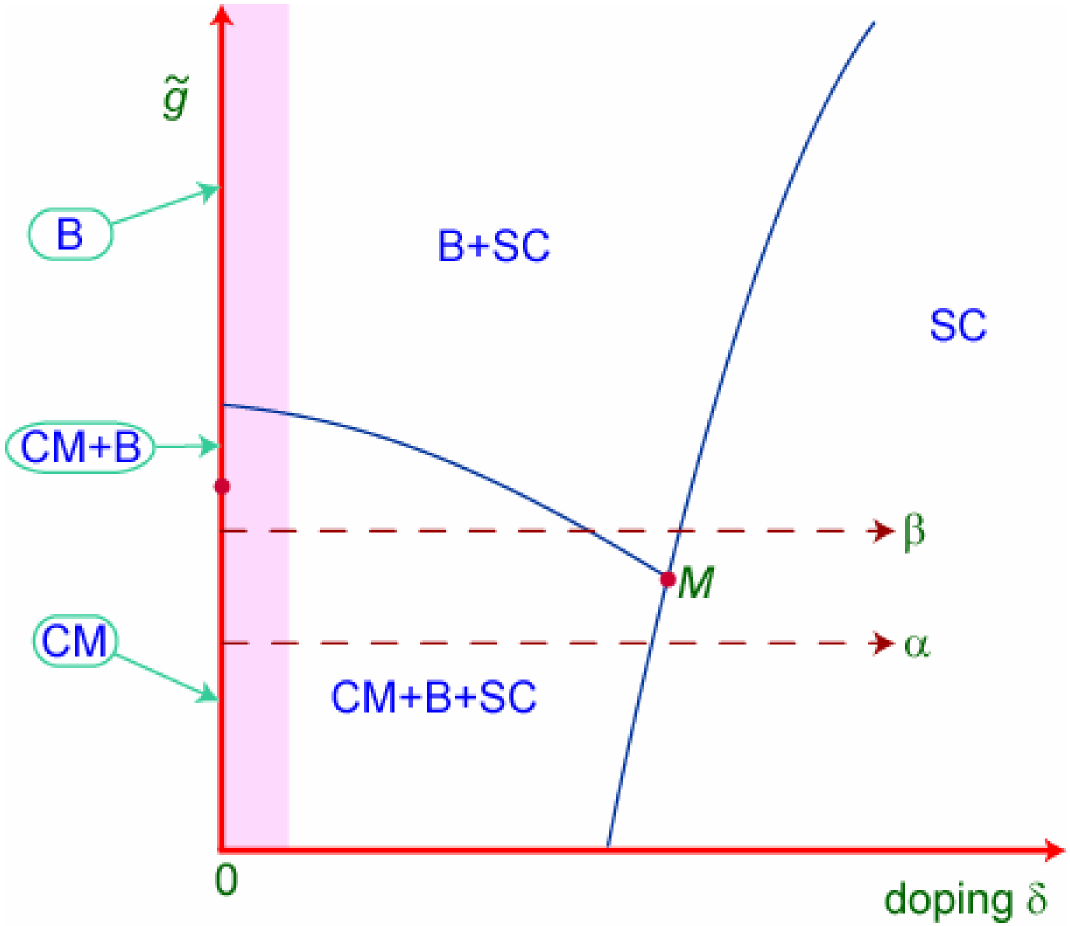
systematic Sp(N) theory of translational symmetry breaking, while preserving spin rotation invariance.

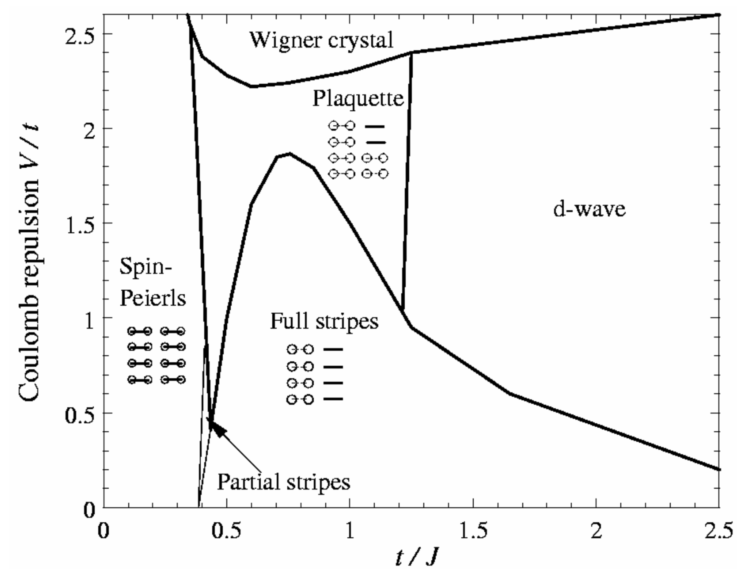


S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

A phase diagram

Vertical axis is any microscopic parameter which suppresses CM order





Microscopic theory for the interplay of bond (B) and *d*-wave superconducting (SC) order

- **Pairing order of BCS theory (SC)**
- **Collinear magnetic order (CM)**
- **Bond order (B)**

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000); M. Vojta, Phys. Rev. B **66**, 104505 (2002).