ABNORMAL NORMAL STATE of the HIGH-Tc's
(and WHY USE HIGH MAGNETIC FIELDS?)

LOW - DIMENSIONAL TRANSPORT
and QUASI-PARTICLE CONFINEMENT

REVEALING QUANTUM CRITICAL POINTS
in the HIGH-Tc PHASE DIAGRAM

SCIENCE, Correlated Electron Systems, 21 April 2000
"Advances in the Physics of High-Temperature Superconductivity"
"Quantum Criticality: Competing Ground States in Low Dimensions"
"Sources of Quantum Protection in High-Tc Superconductivity"

THE Theory of SUPERCONDUCTIVITY in the High-Tc Cuprates,

IN A "SEA" OF ANTIFERROMAGNETIC
WEAK LINKS"
"STRIPE "GRAINS"
CHARGE MOTION (hard). STRIPE MOTION (easy).
The PROs and CONs of CONFINEMENT


---There might be a more down-to-earth explanation. By an accident of band structure, the interplane hopping matrix element vanishes at the nodal point ($\pi/2, \pi/2$). If normal-state transport in underdoped cuprates is dominated by the Fermi arc near ($\pi/2, \pi/2$), then c-axis conductivity can be strongly suppressed.


---Confinement might have been observed in double-layer quantum Hall systems.


FRACTIONAL QUANTUM HALL EFFECT

J. P. Eisenstein and H.L. Stormer, Science 248 (1990) 1510

Fig. 1. A typical Hall bar sample. The structure is formed by depositing a metal strip on a substrate material. The current is confined to the metal strip by the magnetic field. The shaded regions are the areas where electrical contact is made to the 2-D electron gas.

Fig. 2. Composite view showing the Hall resistance $R_H$ and longitudinal resistance $R_T$ as a 2-D electron gas versus magnetic field. The diagonal dashed line passing through the $R_H$ trace represents the classical expected Hall resistance for the sample. For each of the plasmons in $R_H$ there is an associated minimum in $R_T$. The numbers give the value of $p/q$ determined from the value of $R_H$ on the plasmons. While some of the $p/q$ values are integers, the great majority are fractions. Note in particular the "1/3 state" at the far right. This most prominent example of the fractional quantum Hall effect exhibits a Hall plateau at $R_H = (h/3) / (1/3) = \text{Max}$. 33 JUNE 1990
LANDAU LEVELS in a magnetic field

Fig. 3. Three lowest Landau levels, \( j = 1, 2, 3 \), in a five-electron system. Each panel corresponds to a specific magnetic field, \( B \). The number of available states within each level is indicated. In the rightmost panel the magnetic field is high enough so that all five electrons may reside in the highest level. In the middle panel the field has been reduced to the value \( B_j \) where the lowest level is completely occupied and all higher levels are empty. This corresponds to the filling fraction \( \nu = 1 \). (In the left panel the field has been further reduced, forcing some electrons into the \( j = 3 \) Landau level.)
 picks up 2π phase shift when one electron circulates around another.

is odd under electron exchange.

The m = 1 Landau wavefunction.

The Siegel determinant wavefunction is the m = 1 Landau wavefunction.

The number of flux quanta equals the number of electrons.

At Landau level filling factor, ν = 1:

One electron in a magnetic field.
The ground state of Landau level filling factor \( v = 1/3 \).

The number of flux quanta is three times the number of electrons.

There are many equivalent permutations.

A possible configuration at Landau level filling factor \( v = 1/3 \).

A QUAUSPARICLES at $\nu = 1/3$ yield a local electric charge of $(+1/3)e$.

Is avoid by other electrons, yielding a local electric charge of $(+1/3)e$.

Is one extra flux quantum moving freely (which forms a first-order minimum).

A QUAUSHOLE at $\nu = 1/3$.

To guarantee a quasihole or quasiparticle at $\nu = 1/3$, Laughlin employs nesting and lowering operations.

QUAUSHOLE AND QUAUSPARICLES AT $\nu = 1/3$.
DOUBLE QUANTUM WELLS

\( n = 4 \times 10^8 \text{ cm}^{-2} \)

\( (\sim 160 \text{ Å between } e^-) \)

\( \Delta_{\text{SAS}} = 4 - 17 \text{ K} \)

\[ d = 168 - 191 \text{ Å} \]

WELL WIDTHS \( 140 \text{ Å} \)

BARRIER WIDTHS \( 28 - 51 \text{ Å} \)

\[ \lambda = \sqrt{\frac{\hbar}{eB}} = \frac{256}{\sqrt{B(\Omega)}} \text{ (Å)} \]

\[ \frac{e^2}{e\lambda} = 51 \sqrt{B(\Omega)} \text{ (K)} \]

SINGLE ELECTRON ENERGY LEVELS

Three relevant single electron energies:

- Cyclotron energy, \( \hbar \omega_c \)
- Zeeman energy, \( g^* \mu_B B_{\text{ext}} \)
- Symmetric-antisymmetric energy, \( \Delta_{\text{SAS}} \)

\[ v = n + \frac{\hbar}{eB} t \rightarrow \text{number of occupied energy levels} \]

\[ \nu = 4 \pi \omega_c \quad \nu = 4n + 2 \quad \nu = 0 \nu_B \]

\( \hbar \omega_c \quad g^* \mu_B B \quad \Delta_{\text{SAS}} \)
% 30% DECREASE IN D_N.

\[ d_B = 40 \text{ Å} \text{ SAMPLE} \]

\[ \text{NO QHE — } \]  
\[ \Delta \text{SAS CAP DESTROYED} \]

\[ \psi_{SM} \]  
\[ \psi_{STM} \]

\[ T = 0.3 \text{ K}, \]  
\[ n = 1.2 \times 10^{10} \text{ cm}^{-2} \]

\[ \text{BOEBINGER et al. } \text{PRL 45 (1980) 11, 391 (RC)} \]

\[ \text{NO QHE} \]

\[ \nu = 1 \]

\[ \text{QHE} \]

\[ d_B = 51 \text{ Å} \]

\[ 40 \text{ Å} \]

\[ 28 \text{ Å} \]

\[ \Delta_{SAS} / (e^2 / \epsilon_0) \]

\[ \text{PHASE DIAGRAM AT } \nu = \text{odd} \]

\[ \text{DO NOT AFFECT OVERLAYS ALONG THIS SURFACE} \]
\[ B = 0 \Rightarrow \text{LIKE 2 LOWEST SUBBANDS} \]

\[ n = 1 \]

\[ n = 0 \]

\[ \Delta_{\text{SAS}} \]

\[ \Delta_{\text{SAS}} \]

\[ \text{ENERGY, } E \]

\[ \text{MAGNETIC FIELD, } B \]

\[ n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \]

\[ n = \frac{2}{3}, \frac{4}{3}, \ldots \]

\[ n = \frac{3}{3}, \frac{5}{3}, \ldots \]

\[ \text{LIMIT OF } d_x \rightarrow 0 \quad \frac{d_x}{d} \rightarrow \infty \]

\[ = 1 \text{ IS TUNNELING GAP} \]
THEORETICAL APPROACHES to the HIGH-T_c CUPRATES

discussion from "Singular Quasiparticle Scattering in the Proximity of Charge Instabilities"
C. Castellani, C. Di Castro, and M. Grilli, PRL 75 (1995) 4650

--- Low dimensionality and correlations break down the Fermi liquid.
--- Formation of a two-dimensional Luttinger liquid. (Anderson)

--- Landau Fermi liquids serve as a suitable starting point for theory and
abnormal normal state arises from strong scattering processes at low
energy between the quasiparticles.

QCP --- Magnetic scattering in a nearly antiferromagnetic Fermi liquid is responsible for abnormal
normal state and superconducting pairing.

QCP --- Excitonic scattering leads to marginal Fermi liquid behavior.
Vazza, PRL 75 (1995) 898

QCP --- Singular scattering by gauge fields, arising from implementing the resonating-valence-bond
theory in the t-J model.

QCP --- Long-range Coulomb interactions near a charge instability (stripe formation) give rise to
singular scattering.
Castellani, Di Castro, and Grilli, PRL 75 (1995) 4650

QUANTUM
CRITICAL
POINT
in the
HIGH-T_c
CUPRATES

--- In the quantum critical region, temperature is the
only energy scale
controlling the physics.
--- Strong critical
fluctuations can mediate
singular interactions
between quasiparticles.
--- These singular
interactions can provide
both a strong pairing
mechanism and a source of
non-Fermi liquid behavior.

FIG. 1. Phase diagram of Tc (after Ref. [4]). The magnetic
LRO can be either spin-glass or Néel type, and is present only
at T=0. The boundaries of the QC region are $T - |g-g_c|$. 
SCALE-LESS and FEATURE-LESS SCATTERING RATE

dc resistivity

optical conductivity

\[ \frac{\rho_c}{\rho_{ab}} \times 10^{-5} \]

\[ \rho_{ab} (10^{-4} \Omega cm) \]

Figure 3.7a. Planar resistivity vs. T for a variety of materials. (b) Anisotropic resistivity of BISCO.

Optimally-doped Bi-2212, Martin, et al.
Optimally-doped YBCO, Schlesinger, et al.

\[ \frac{kT}{\epsilon \ll \max \left\{ \frac{kT}{\hbar}, \frac{\pi c}{R_{\text{el}}} \right\} } \]

\[ \gamma \sim \frac{\hbar}{m} \frac{1}{8\pi} \frac{1}{\text{INELASTIC}} \]

QUANTUM CRITICAL POINT in the HIGH-Tc CUPRATES

from "Universal Quantum-Critical Dynamics of Two-Dimensional Antiferromagnets"

Subir Sachdev and Jinwu Ye, PRL 69 (1992) 2411

--- A remarkable feature of the measured dynamic susceptibilities of the cuprates is that the frequency scale of the spin excitation spectrum is given simply by the absolute temperature (ie. independent of all microscopic energy scales, such as an antiferromagnetic exchange constant).

--- This is a very general property of finite-T quantum-critical spin fluctuations.
--- The coupling constant, g, depends on the ratios of JJ in H.
--- At g_c, the correlation length diverges with exponent \( \nu_c = \frac{\nu g_c}{g} \).
--- At finite T, the thermal length, \( \xi_T \sim T^{-\nu_c} \), is the scale at which deviations from T=0 behavior are first felt.

--- The Quantum Critical (QC) regime is defined by the inequality \( \epsilon_r < \epsilon \).
--- In the QC regime, the spin system notices the finite value of T before becoming sensitive to the deviation of g from g_c.
--- In the QC regime, dynamic spin correlations will be found to be remarkably universal.

FIG. 1. Phase diagram of \( H \) (after Ref. [4]). The magnetic LRO can be either spin-glass or Néel type, and is present only at T=0. The boundaries of the QC region are \( T \sim |g - g_c|^{\nu_c} \).

--- At JJJ in H.
--- At g_c, the correlation length diverges with exponent \( \nu_c 
--- At finite T, the thermal length, \( \xi_T \sim T^{-\nu_c} \), is the scale at which deviations from T=0 behavior are first felt.

--- The Quantum Critical (QC) regime is defined by the inequality \( \epsilon_r < \epsilon \).
--- In the QC regime, the spin system notices the finite value of T before becoming sensitive to the deviation of g from g_c.
--- In the QC regime, dynamic spin correlations will be found to be remarkably universal.
QUANTUM CRITICAL POINT in the HIGH-Tc CUPRATES
from "Tape Formation: A Quantum Critical Point for Cuprate Superconductors"

--- There is a Quantum Critical Point related to local charge segregation (stripe phase) near optimum doping.
--- Superconductivity is a stabilizing mechanism against fully-developed charge ordering.

--- EVIDENCE CITED
- Resistivity
- Neutron scattering
- Hall number
- Uniform susceptibility

see discussion and references in:
C. Castellani, C. Di Castro, and M. Grilli

--- IN THE NORMAL STATE OF A HIGH-Tc SUPERCONDUCTOR

--- I. SCDO single crystals

--- II. Material response

--- III. Conclusion
Insulator-Metal Crossover near Optimal Doping in $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_4$

*(an electron-doped high-$T_c$ cuprate)*

Fournier, Mohanty, Maiser, Darzens, Venkatesan, Lobb, Czjzek, Webb, Greene
PRL 81 (1998) 4720

**FIG. 1.** Resistivity $\rho_{xx}$ as a function of temperature for the $x$-axis oriented $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_4$ thin films in magnetic fields of 0 T (dashed lines), 6.7 T (thin lines), and 12 T (thick lines). (a) $x = 0.13$ and 0.14; (b) $x = 0.15$. The field is applied along the c axis.

**FIG. 2.** $\rho_{xx}$ below 10 K and at 12 T for some samples of Fig. 1: (a) and (b) $x = 0.14$; (c) and (d) $x = 0.15$; and (e) $x = 0.17$. In (e), the inset shows a magnified view of the metallic range.

**FIG. 3.** Phase diagram determined from resistivity data of $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_4$ thin film. Solid triangles and solid squares are $T_c$ and $T_{\text{I-I}}$ (defined in text), respectively. The solid lines are guides to the eye.
Metal-Insulator Crossover in the Low-Temperature Normal State of Bi$_2$Sr$_{2-x}$La$_x$CuO$_{6+\delta}$

S. Ono$^1$, Yoichi Ando$^{1,2}$, T. Murayama$^{1,2}$
F.F. Balakirev$^3$, J.B. Betts$^3$, G.S. Boebinger$^3$

$^1$ Central Research Institute of Electric Power Industry, Tokyo, Japan
$^2$ Dept. of Physics, Science University of Tokyo, Japan
$^3$ National High Magnetic Field Laboratory,
  Los Alamos National Laboratory, Los Alamos, New Mexico

$p$ (hole concentration per Cu)

0.11 0.13 0.16 0.19

$p$ vs $T$ (K)

$T_{\text{min}}$

$\chi$ (La concentration per Cu)

0 0.2 0.4 0.6 0.8 1

Resistivity [m$\Omega$cm] vs Temperature [K]

$x=0.84$

0.76

0.73

0.66

0.61

0.49

0.39

0.23

Samples grown at
Central Research Institute of Electric Power Industry, Tokyo, Japan
60 Tesla Pulsed Magnetic Fields

Maximum field: 79 T (60T routine)
Pulse duration: 60 msec
Pulse repetition rates: 2-3 pulses/hour (60T)
Energy in magnetic field: 1 stick of dynamite

Magnetic fields at
National High Magnetic Field Laboratory,
Los Alamos National Laboratory, Los Alamos, New Mexico

Suppressing Superconductivity

\[ x = 0.39 \]

\[ x = 0.84 \]
Revealing the Low-Temperature Normal State

![Graph showing the relationship between \( \rho_{ab} \) and \( T \)]

Estimating Hole Concentration in Bi\(_2\)Sr\(_{2-x}\)La\(_x\)CuO\(_6+\delta\)

![Graph showing the relationship between \( R_{H,e/V} \) and \( T \)]

Close-up of the Crossover from Insulating Behavior to Metallic Behavior

\[ \rho_{ab} / e^2 \ll k_B l = 11 \]

\[ \rho_{ab} / e^2 \approx \frac{h}{(e^2k_Bl)} \]

Similar Hall data for optimally-doped BSLCO, LSCO, TI-2201.

Estimate BSLCO hole concentration by comparing with LSCO.
Conclusions:

Metal-Insulator Crossover in the Low-Temperature Normal State
---occurs near p=1/8 in Bi$_2$Sr$_{2-x}$La$_x$CuO$_{6+y}$
---occurs near optimum doping in La$_{2+y}$Sr$_x$CuO$_4$

**Similarities between BSLCO and LSCO:**

*Insulator-to-Metal Crossover*
---observed when superconductivity is suppressed.
---occurs at relatively low normalized resistivities, k, l ~15.
*Insulating Behavior exhibits Log-T divergence.*

**Differences between BSLCO and LSCO:**

*Insulator-to-Metal Crossover occurs gradually in BSLCO.*
*BSLCO data suggest the underdoped regime contains a metallic state exhibiting unusual localization behavior.*