Noncollinear spin transfer in Co/Cu/Co multilayers (invited)

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This article has two parts. The first part uses a single point of view to discuss the *reflection* and *averaging* mechanisms of spin transfer between current-carrying electrons and the ferromagnetic layers of magnetic/nonmagnetic heterostructures. The second part incorporates both effects into a matrix Boltzmann equation and reports numerical results for current polarization, spin accumulation, magnetoresistance, and spin-transfer torques for Co/Cu/Co multilayers. When possible, the results are compared quantitatively with relevant experiments. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1446123]

I. INTRODUCTION

In 1996, Slonczewski[1] and Berger[2] pointed out that an electric current that flows perpendicularly through a magnetic multilayer can exert a torque on the magnetic moments of the heterostructure. The torque arises because a polarized electron in a nonmagnet feels a large exchange field when it propagates into a ferromagnet. For at least two distinct reasons, this interaction induces a transfer of spin angular momentum (and hence a torque) between the current-carrying electrons and the ferromagnetic layers of the heterostructure.

One source of spin transfer, the *reflection mechanism*, occurs because the reflection coefficient for electrons incident on a magnetic/nonmagnetic interface is spin dependent. The spin content of the reflected and transmitted wave functions differ (in general) so, inevitably, angular momentum is gained or lost to the magnetization in the immediate vicinity of the interface. A second source of spin transfer, the *averaging mechanism*, occurs because the spins of electrons transmitted into a ferromagnet from a nonmagnet precess around the magnetization of the ferromagnet. On account of this precession, the component of the total conduction electron spin transverse to the magnetization averages to zero when summed over all electrons. Since total angular momentum is conserved, the ferromagnetic moments gain what the electrons lose.

Motivated by theoretical considerations of this kind, and prior experimental indications of current-induced magnetic excitations,[3] groups at Cornell[4,5] and Orsay[6] recently demonstrated that the relative magnetization of the cobalt layers in Co/Cu/Co trilayer structures (Fig. 1) can be switched by passing an electric current through the structure. The observed asymmetry of the switching with respect to the direction of current flow is indicative of the effect of spin transfer torques (rather than an effect of a current-induced magnetic field).

The theoretical treatment of this problem is complicated by the fact that the magnetizations of the ferromagnetic layers are necessarily not collinear.[7–9] In this article, we use a Boltzmann equation to compute current polarization, spin accumulation, magnetoresistance, and spin transfer torques in Co/Cu/Co heterostructures. This approach is restricted to Ohmic transport, but it permits us to treat situations where the interface resistance does not necessarily dominate the transport and also where the layer thicknesses are less than relevant mean-free paths.[10] That is, we can treat spacer layers of arbitrary thickness. Our main results are: (1) spin–flip scattering in the external leads is sufficient to polarize the current; (2) the two sources of spin transfer torque identified above combine in a natural way; (3) the magnitude of the torque depends on the reflection coefficients, the spin-dependent conductivity of the ferromagnets, and the layer thicknesses; (4) the dependence of the magnetoresistance on the angle between the two ferromagnetic magnetization vectors is not exactly $\sin^2\theta/2$; and (5) satisfactory quantitative agreement is found with the magnetoresistance data of Katine *et al.*[5] but not with the data of Grollier *et al.*[6]

II. OBSERVABLES AND PARAMETERS

This section defines the observables we use to discuss transport and spin transfer. We also give the numerical values of the parameters used in our quantitative calculations for thin Co layers embedded in bulk-like Cu. Several of the most relevant observables involve incoherent sums of quantities that are defined quantum mechanically for each electron. One familiar example is the electron number current density

$$\mathbf{j}(\mathbf{r}) = -\frac{i\hbar}{2m}\sum_{\sigma}\left[\psi_{\sigma}^{*}\nabla\psi_{\sigma} + \text{h.c.}\right]. \tag{1}$$

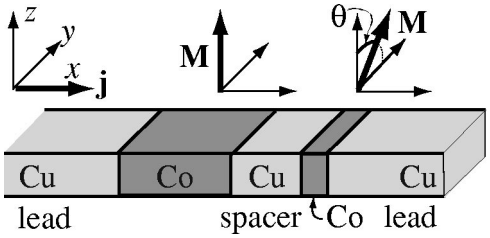

FIG. 1. Co/Cu/Co multilayer with noncollinear magnetizations.

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Less familiar is the current density of spin angular momentum

$$\mathbf{Q}(\mathbf{r}) = -\frac{i\hbar^2}{4m}\sum_{\sigma\sigma'}[\psi^*_\sigma(\mathbf{r})\boldsymbol{\sigma}_{\sigma,\sigma'}\otimes\nabla\psi_{\sigma'}(\mathbf{r}) + \text{h.c.}]. \quad (2)$$

The gradient of this quantity at any point in space is the local torque/volume exerted by the electrons on the rest of the system. Discontinuities are local torques/area exerted by the electrons. As the product notation indicates, $\mathbf{Q}$ is a tensor because $\boldsymbol{\sigma}$ has a direction and $\nabla\psi$ has a direction.

It is particularly useful to define a current polarization vector $\mathbf{p}(\mathbf{r})$ by contracting the space part of Q with the number current density:

$$\mathbf{p}(\mathbf{r}) = \frac{2}{\hbar}\frac{\mathbf{Q}(\mathbf{r})\cdot\mathbf{j}(\mathbf{r})}{|\mathbf{j}(\mathbf{r})|^2}. \quad (3)$$

For a distribution of electrons, $\mathbf{Q}(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ should each be computed separately and then contracted. For a completely polarized current, $\mathbf{p}$ is a unit vector that points in the direction of the polarization. The length of $\mathbf{p}$ is the up spin current minus the down spin current, all divided by the total current for up and down defined with respect to the direction of $\mathbf{p}$. Notice that the current polarization and density polarization (magnetization) need not lie in the same direction or have the same magnitude. We discuss such an example below.

We will also have occasion to discuss the voltage drops ΔV that occur over various portions of the sample. To be precise about our usage of this symbol, it is important to recall that both the electric field $\mathbf{E}$ and gradients of the density deviation from equilibrium $\delta n(\mathbf{r})$ lead to electric current flow. In this context, it is usual[11] to define an electrochemical potential

$$\bar{\mu}(\mathbf{r}) = \left[\frac{2\pi^2 v_F\hbar}{k_F^2}\right]\delta n(\mathbf{r}) - eV(\mathbf{r}), \quad (4)$$

as the combination that enters the transport equations. Here, $\mathbf{E} = -\nabla V(\mathbf{r})$. Of course, $V(\mathbf{r})$ and $\delta n(\mathbf{r})$ are related by the Poisson equation. However, as far as the transport equations are concerned, it does not matter how the two are distributed. Therefore, we are free to choose an approximate solution of Poisson's equation for which there is no charge accumulation, $\delta n(\mathbf{r}) = 0$, and interpret $\Delta\bar{\mu}/e$ as the voltage change ΔV. This is what we have done in this article.

On the other hand, the electric field does not couple to the deviation of the magnetization from its equilibrium value. This is called the spin accumulation, $\delta\mathbf{m}(\mathbf{r})$. Gradients of the spin accumulation lead to spin currents.

The numerical results we report in the next section were obtained by solving a matrix Boltzmann equation (see Appendix) appropriate to each portion of the heterostructure shown in Fig. 1 (leads, ferromagnets, and spacer layer). The reflection and averaging mechanisms of spin transfer are included automatically when we match the solutions together suitably using a generalization of the boundary conditions described in Ref. 12. The details will be given elsewhere.

We make several simplifying approximations which are not intrinsic to the Boltzmann equation method. We assume that all Fermi surfaces are spheres of the same size. Minority and majority electrons in the ferromagnets have different conductivities due to different Fermi velocities (effective masses) and different scattering rates. We also assume that the interface resistance is due to specular reflection instead of diffuse scattering. We parameterize the reflection amplitudes in terms of dimensionless parameters α_σ, chosen to give the correct interface resistances,[13] in the form

$$|R_\sigma(\mathbf{k})|^2 = \frac{\alpha_\sigma k_F^2}{\alpha_\sigma k_F^2 + k_x^2}, \quad (5)$$

for an electron with wave vector $\mathbf{k}$ and spin $\sigma = \uparrow,\downarrow$ incident on an interface with normal $\hat{\mathbf{x}}$. For simplicity, we choose $R_\sigma(\mathbf{k})$ to be real for all the calculations reported in this article.[14] Measured values of the interface resistance for Co/Cu (Refs. 15 and 16) are consistent with calculated results from first principles in the specular limit,[13,17] but they are also consistent with calculations in the diffuse limit.[18]

Resistances extracted from experiments performed at Michigan State[15,16] were used to determine the parameters we use to model Co/Cu structures. These include the mean free paths for Cu ($\lambda = 110$ nm) and Co ($\lambda_\uparrow = 16.25$ nm and $\lambda_\downarrow = 6$ nm) as well as the reflectivities for Co/Cu interfaces ($\alpha_\uparrow = 0.051$ and $\alpha_\downarrow = 0.393$). The spin–flip mean free path for Cu ($\lambda_{\uparrow\downarrow} = v_F\tau_{\uparrow\downarrow} = 2000$ nm) was taken from the spin-diffusion length $\sqrt{\lambda\lambda_{\uparrow\downarrow}}$ extracted from a different set of experiments on multilayers grown electrochemically.[19] The layer thicknesses were taken from the experiments done by Katine *et al.*[5] These are $t_{\text{Co}}(1) = 10.0$ nm, $t_{\text{Cu}} = 6.0$ nm, and $t_{\text{Co}}(2) = 2.5$ nm.

III. RESULTS

A. Current polarization by spin–flip scattering

Inside a ferromagnetic metal like Co, Ohm's law ($\mathbf{j}_\sigma = \sigma_\sigma\mathbf{E}$) guarantees that the current is naturally *polarized* ($j_\uparrow \neq j_\downarrow$) because the conductivities for majority and minority spin electrons are different ($\sigma_\uparrow \neq \sigma_\downarrow$), while both spin types feel the same electric field $\mathbf{E}$. By the same argument, the current is naturally *unpolarized* in a nonmagnetic metal like Cu because $\sigma_\uparrow = \sigma_\downarrow = \sigma$. However, for a heterostructure like the one shown in Fig. 2(a)–a thin ferromagnetic film sandwiched between two nonmagnetic leads—the steady-state current polarization can deviate (locally) from its preferred bulk behavior in the presence of spin accumulation.

To see this, suppose first that spin–flip scattering is absent. In that case, the number densities of up and down spin electrons are conserved separately and the two spin types conduct electricity in parallel. Moreover, in *steady state*, the up and down spin currents (and the current polarization) are time independent and spatially uniform everywhere. For a layer of Co of thickness t sandwiched between two Cu leads, each of length L, a simple series resistor model for the two spin channels conducting in parallel gives the polarization of the current as

$$\frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow} = \frac{t(\sigma_\uparrow - \sigma_\downarrow)}{L(4\sigma_\uparrow\sigma_\downarrow/\sigma) + t(\sigma_\uparrow + \sigma_\downarrow)}. \quad (6)$$

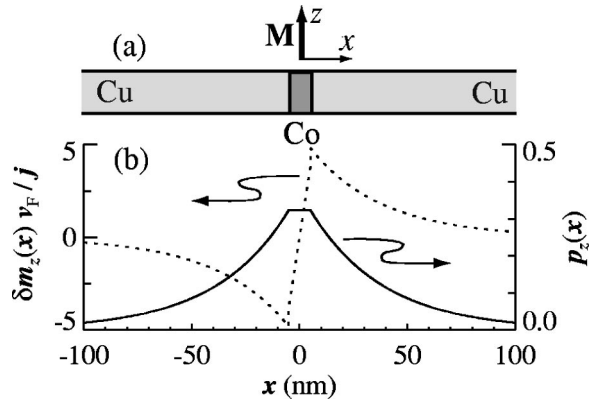

FIG. 2. Current polarization for a single ferromagnetic layer. (a) A thin Co layer embedded between two semi-infinite Cu leads; (b) Current polarization (solid line) and spin accumulation (dotted line) for a single Co layer embedded in Cu. The spin accumulation, defined as a density rather than a magnetization, is put in a dimensionless, scaled form by dividing by the ratio of the current to the Fermi velocity.

This formula shows that the current is unpolarized in the limit that the leads become infinitely long ($L\to\infty$).

Now introduce spin–flip scattering in the leads. The current polarization can vary spatially in this case because only the *sum* of the up and down spin currents is conserved. This is shown as the solid curve in Fig. 2(b) where $\mathbf{p}=p_z\hat{\mathbf{z}}$. Note that the current in the ferromagnet is polarized and the current in the leads (far from the interfaces) is unpolarized. In between, $p_z(x)$ varies on a scale set by the spin diffusion length. Therefore, the presence of spin–flip scattering[20] allows the system to accommodate as much as possible to the ''polarization desires'' of both the ferromagnet and the nonmagnet (as determined by their intrinsic conductivities). Nonzero values of the dashed curve in Fig. 2 identify portions of space where the spin density deviates from its equilibrium value, i.e., spin accumulation. As mentioned earlier, the gradient of this quantity contributes to the current polarization.

Returning to the solid curve, the fact that $p_z(x)$ is symmetrical around the origin tells us that the steady state current distribution is equally polarized on both sides of the ferromagnetic layer. This means that no torque acts on the magnet. On the other hand, the nonzero gradient of $p_z(x)$ elsewhere tells us that distributed torques act throughout the leads where spin–flip scattering occurs. These torques are equal and opposite at points which are symmetrically disposed with respect to the thin film. This means that current flow in this system with a single ferromagnetic layer induces a bending stress in the entire structure. In essence, the conduction electrons transfer angular momentum from one lead to the other. This interesting result motivates us to look into the mechanisms of spin transfer in more detail.

B. Spin transfer by reflection

The fate of a polarized electron incident on a ferromagnet depends on the angle between the electron spin moment and the magnetization direction of the magnet. We can encode this effect of quantum mechanical exchange most concisely using spin-dependent reflection and transmission coefficients R_σ and T_σ. This has been discussed qualitatively by Waintal *et al.*[8] Here, we focus on the scattering state for a polarized electron in a nonmagnet ($x<0$) that is incident on a ferromagnet ($x>0$). If the incident electron spin points in an arbitrary direction ($\theta\,\phi$) with respect to the permanent magnetization, we can write its wave function in the form

$$\psi=e^{-i\phi/2}\cos(\theta/2)|\psi_{\mathbf{k}\uparrow}\rangle+e^{i\phi/2}\sin(\theta/2)|\psi_{\mathbf{k}\downarrow}\rangle. \tag{7}$$

Here,

$$\begin{aligned}|\psi_{\mathbf{k}\uparrow}\rangle&=(e^{ikx}+R_\uparrow e^{-ikx})|\uparrow\rangle \quad x<0\\ &=T_\uparrow e^{ik_\uparrow x}|\uparrow\rangle \quad x>0\\ |\psi_{\mathbf{k}\downarrow}\rangle&=(e^{ikx}+R_\downarrow e^{-ikx})|\downarrow\rangle \quad x<0\\ &=T_\downarrow e^{ik_\downarrow x}|\downarrow\rangle \quad x>0,\end{aligned} \tag{8}$$

are scattering states in a majority/minority basis. Inserting Eq. (7) into Eq. (3) gives the incident current polarization as

$$\mathbf{p}_{\rm inc}=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta). \tag{9}$$

It is straightforward (but tedious) to compute the corresponding quantities $\mathbf{p}_{\rm refl}$ and $\mathbf{p}_{\rm tr}$ from the transmitted and reflected waves generated by Eq. (7). We omit them here and focus instead on the extreme case where $R_\uparrow=1$ and $R_\downarrow=0$ for an incident electron with a spin pointed in the *y* direction (the magnetization in the *z* direction). In this case, the incident spin current polarization is $\mathbf{p}_{\rm inc}=(0,1,0)$. Only up spins are reflected, so the reflected spin current polarization is $\mathbf{p}_{\rm refl}=(0,0,1)$. Only down spins are transmitted, so the transmitted spin current polarization is $\mathbf{p}_{\rm tr}=(0,0,-1)$.

Note first that the *z* component of $\mathbf{p}_{\rm inc}$ is the same as the *z* component of $\mathbf{p}_{\rm refl}+\mathbf{p}_{\rm tr}$. The numerical value happens to be zero in this case, but the stated equality is a general result. Nothing very interesting happens to the component of the electron spin that is parallel to the quantization axis of the ferromagnet. By contrast, the transverse component of the spin angular momentum does change. From Newton's law (and Ehrenfest's theorem), this is possible only if the magnetization exerts a torque on the conduction electron spins. For other angles and other reflection amplitudes, the amount of transferred angular momentum is more complicated, but it is nonzero in general.

From the sentence below Eq. (2) and using Eq. (3), the torque exerted on the permanent magnetization at $x=0$ due to this reflection mechanism is

$$\mathbf{N}_R=A\frac{\hbar}{2}(\mathbf{p}_{\rm inc}j_{\rm inc}-\mathbf{p}_{\rm tr}j_{\rm tr}-\mathbf{p}_{\rm refl}j_{\rm refl})_\perp, \tag{10}$$

where, A is the cross sectional area of the interface and the currents $j_{\rm inc}$, $j_{\rm refl}$, and $j_{\rm tr}$ are taken to be positive. The subscript $\perp$ reminds us that this vector is transverse to the magnetization. We have chosen $\mathbf{M}=M\hat{\mathbf{z}}$, so the torque lies in the $x-y$ plane specifically, the *y* direction for our simplified example.

C. Spin transfer by averaging

The averaging mechanism of spin transfer is also a consequence of the exchange interaction. But, it is completely distinct from the reflection mechanism. To see this, observe

first that the incident electron wave function Eq. (7) in the nonmagnet is a coherent superposition of two degenerate spinors with the same wave vector. When this electron enters the ferromagnet, the majority and minority spin components at the Fermi surface no longer share the same wave vector. As a result, the electron spin precesses rapidly in real space.[21] The spatial precession frequencies vary rapidly over the Fermi surface so, when we sum over all current-carrying electrons, the transverse component of the total conduction electron spin averages to zero. In other words, an ensemble of electrons that enters a ferromagnetic layer with a *nonzero* transverse component of the current polarization, exits the layer with *zero* transverse component. From the change, the torque the ensemble exerts on the permanent magnetization is $N_A=\frac{1}{2}\hbar A j_{\rm tr}(\mathbf{p}_{\rm tr})_\perp$. This ''averaging'' torque cancels part of the ''reflection'' torque Eq. (10) so the net spin-transfer torque is

$$\mathbf{N}=\mathbf{N}_R+\mathbf{N}_A=A\frac{\hbar}{2}(\mathbf{p}_{\rm inc}j_{\rm inc}-\mathbf{p}_{\rm refl}j_{\rm refl})_\perp . \tag{11}$$

The net torque manifests itself in a discontinuity in the transverse angular-momentum current; the latter is zero inside the ferromagnet. For an electron ''beam'' with current density j, this torque is

$$\mathbf{N}=Aj\frac{\hbar}{2}[1-R_\uparrow R_\downarrow](\sin\theta\cos\phi,\sin\theta\sin\phi,0), \tag{12}$$

if each electron is described by Eq. (7). We remind the reader that $R_\uparrow$ and $R_\downarrow$ are both real in our calculation. Also, since both reflection and averaging contribute to the torque, there can be extreme cases where only one or the other contributes. For example, only the reflection mechanism contributes if $R_\uparrow=1$ and $R_\downarrow=0$. Conversely, only the averaging mechanism contributes if $R_\uparrow=0$ and $R_\downarrow=0$. Both happen to give the same numerical result for these particular cases [$R_\uparrow R_\downarrow=0$ in Eq. (12) for both cases]. Finally, it is worth noting that the product $R_\uparrow R_\downarrow$ in Eq. (12) is closely related to the *mixing conductance* $G_{\uparrow\downarrow}$ used in the ''circuit'' theory of Ref. 7.

D. Noncollinear transport in a trilayer

We now apply all the above to a trilayer structure modeled after the experiments of Refs. 5 and 6. The one-dimensional geometry is shown schematically in Fig. 3(a). For simplicity, we first consider a situation where the magnetization of the left ferromagnet points along $\hat{\mathbf{z}}$ and the magnetization of the right ferromagnet points along $\hat{\mathbf{y}}$. We omit spin–flip scattering in the spacer layer because its thickness is small compared to $\lambda_{\uparrow\downarrow}$. Solving the Boltzmann equation, we see from image (b) that the ''voltage drop'' is largest across the interfaces (because the interface resistance is large) but not at all negligible across the layers themselves. The relative slopes of the lines in the Co and Cu layers reflects their relative resistivities.

Figures 3(c) and 3(d) show the current polarization along the magnetizations directions of the left and right ferromagnets, respectively. Both are discontinuous at the interface with the misaligned ferromagnet. This discontinuity is the origin of the torque exerted on the respective magnetizations.

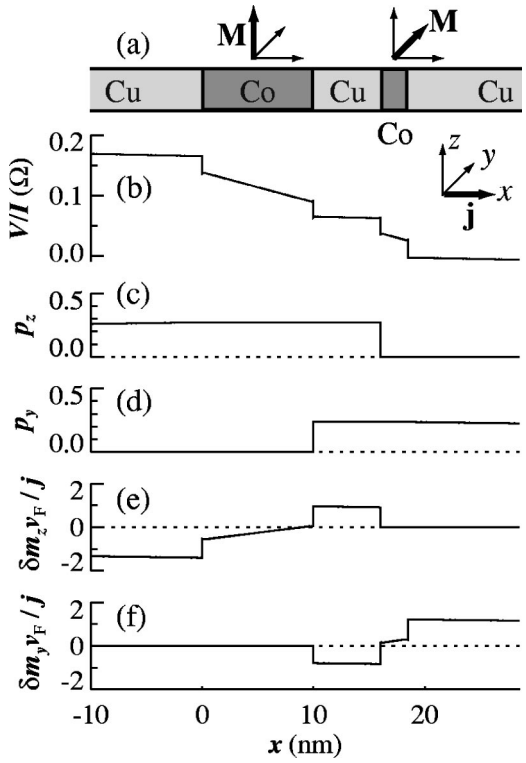

FIG. 3. Voltage, current polarization, and spin accumulation for a trilayer. (a) A heterostructure with two Co layers, an interposed Cu layer, and two semi-infinite Cu leads. Electron current flows in the x direction and the left magnetization is in the z direction and the right is in the y direction. (b) Shows the voltage drop (electrochemical potential) through the structure. (c) and (d) z and y components of the current polarization, respectively; (e) and (f) z and y components of the spin accumulation (see Fig. 2), respectively.

Moreover, as in the single-layer problem, $\mathbf{p}(x)$ decreases (too slowly to be seen in this plot due to the long spin-diffusion length) toward zero in each lead as $x\to\pm\infty$. This again corresponds to a distributed torque in each lead. Thus, for electron flow from left to right through the multilayer, the conduction electrons extract angular momentum from the lattice of the left lead (by spin–flip scattering) and deposit an equal amount of angular momentum into the magnetization of the right ferromagnet. A similar transfer occurs between the right lead and the magnetization of the left ferromagnet. The transfers are in the same direction, as pointed out in Ref. 1, so that the current induces the two Co layers to ''pinwheel'' in the same direction.

Images (e) and (f) show the spin accumulation along the respective magnetization directions that is required for consistency with the calculated current polarizations. The obvious discontinuities in spin accumulation across the interfaces are due to the large spin dependence of the interface resistances. From images (c) and (d), the polarization of the current in the Cu spacer layer is roughly in the $\hat{\mathbf{z}}+\hat{\mathbf{y}}$ direction inside the spacer layer, while from images (e) and (f), the polarization of the density in the Cu spacer layer is roughly in the $\hat{\mathbf{z}}-\hat{\mathbf{y}}$ direction. The two are not collinear.

E. Angular dependence of resistance and torque

For the structure illustrated in Fig. 3, Fig. 4 shows the dependence of the resistance and the torque on the angle θ

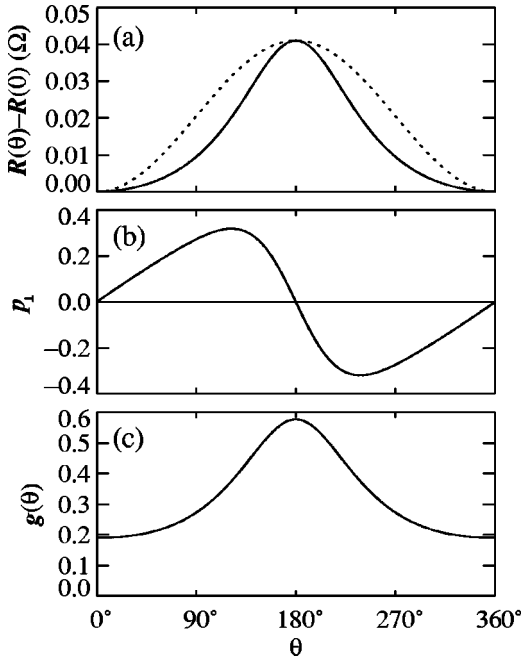

FIG. 4. Magnetoresistance and torque. (a) Shows the change in resistance as a function of the relative angle between the two magnetizations (solid curve). The dotted curve is proportional to $\sin^2\theta/2$. The resistance has been computed from the values in the text and a cross-sectional area of $\pi 65^2$ nm^2; (b) shows the transverse current polarization on the right ferromagnetic layer; (c) shows the same quantity divided by $-\sin\theta$.

between the magnetizations of the two ferromagnets. Image (a) shows that while the magnetoresistance varies roughly like $\sin^2\theta/2$, there are significant deviations. Several authors[22] find similar deviations using a fully quantum mechanical treatment. Our results show that nonsinusoidal behavior occurs already at the semiclassical level if spin noncollinearity is treated properly.

Our computed value of $R(180)-R(0)$ is about half of the value measured by Katine *et al.*[5] Possible sources of this discrepancy are (1) experimental uncertainty in the multilayer cross-sectional area (on the order of 40%); (2) material differences in the structures grown at Cornell and Michigan State; and (3) the treatment of the leads in the calculation. In our results, the leads have a higher resistance for parallel alignment than for antiparallel alignment. The current is largely unpolarized in the latter case, but not in the former, and there is extra resistance associated with the spin–flip scattering that polarizes the current. We suspect that the wider leads used in the experiment would reduce this effect leading to better agreement between the calculation and the measured results. Even though a series resistor model is not justified for layers thinner than the relevant mean free paths, we find from such a model, with no resistance in the leads, a factor of two increase in the difference in resistance, in much better agreement with the measured result.

We have carried out similar calculations to compare with the results of Grollier *et al.*[6] That comparison is much less satisfactory. Using their experimental geometry and the same transport parameters, we compute $R(180)-R(0)$ to be about 0.006 Ω. This is much larger than the experimental value of about 0.001 Ω. Moderate changes in the Co and Cu layer thicknesses do not change the results very much because the interfaces dominate the physics. We could bring the calculation into agreement if there was much less spin dependence to the interface resistance or if the cross-sectional area of the multilayer was much different than the quoted value. In addition to the possible sources of error discussed above, it is also possible that the ferromagnetic layers are not uniformly magnetized in either the parallel or the antiparallel state.

Figure 4(b) shows the transverse part of the current polarization at the interface with the left ferromagnetic layer. Again, this curve deviates significantly from simple $\sin\theta$ behavior. The deviation is highlighted in image (c) which shows

$$g(\theta) = -p_\perp(\theta)/\sin\theta. \tag{13}$$

We find that the deviations for the transverse current polarization track the deviations for the magnetoresistance as we vary material parameters. These deviations are quite pronounced, even for completely symmetric structures. The maxima in the transverse currents do not occur for perpendicular alignment of the magnetizations, but rather happen nearer to antiparallel alignment. For a symmetric structure with magnetizations perpendicular to each other, the current polarization is only 45° away from the magnetization. The current polarization becomes perpendicular to the magnetizations as they become antiparallel, but the amount of polarization decreases to zero in that limit. This is significant because the torque on the left ferromagnetic layer is proportional to the transverse part of the spin current incident on the interface.

The magnitude of the torques we compute are consistent with those that cause reversal in experiment, but direct comparison is difficult. As has been pointed out by others,[5,23] it is not simply a matter of computing when some energy barrier is overcome. The torque is zero in both the parallel and antiparallel configurations so fluctuations away from these orientations can be amplified by the current-induced torque. The other sources of torque—magnetostatics, magnetocrystalline anisotropy, and external fields—lead to precession. The damping tends to reduce the amplitude of the precession, counteracting the effects of the current-induced torque. At some point, the current becomes high enough that a complicated reversal occurs.

IV. SUMMARY

We have used a matrix version of the Boltzmann equation to study perpendicular transport in submicron multilayers where the magnetizations of the ferromagnetic layers point in different directions. Spin–flip scattering in the leads ensures that a polarized current flows. The boundary conditions for the Boltzmann equation incorporate the reflection and averaging mechanisms of spin transfer discussed by Slonczewski and Berger. As a result, the conduction electrons and the magnetic moments of the ferromagnets exert torques on one another. Using material parameters extracted from experiment, we computed the magnetoresistance, spin accumulation, current polarization, and magnetization

torques for Co/Cu/Co structures similar to those used in experiments. The transport data were compared quantitatively with data obtained at Cornell and Orsay and reasons were suggested to explain some discrepancies between theory and experiment. The magnitudes of the computed torques were comparable to the torques that induce magnetization reversal in the experiments.

ACKNOWLEDGMENT

A.Z. gratefully acknowledges support from National Science Foundation Grant No. DMR-9531115.

APPENDIX: THE BOLTZMANN EQUATION

The semiclassical Boltzmann equation is a standard approach to transport calculations that lies between a fully quantum calculation and classical drift-diffusion theory. In the portions of space occupied by ferromagnetic layers, we use Fert's ''two-current'' description,[24] where $f_\uparrow(\mathbf{k},\mathbf{r})$ and $f_\downarrow(\mathbf{k},\mathbf{r})$ describe the occupancy of up and down spin electrons in the phase space volume $d\mathbf{r}\,d\mathbf{k}$. Specifically, we restrict the wave vectors $\mathbf{k}$ to lie on the Fermi surface and let $g_\sigma(\mathbf{k},\mathbf{r})$ denote the *change* in the occupancy of electrons of spin type σ that occurs when we apply an electric field $\mathbf{E}$ to the system. In the linearized relaxation-time approximation, $g_\sigma(\mathbf{k},\mathbf{r})$ satisfies the stationary Boltzmann equation:

$$\mathbf{v}_{\mathbf{k}\sigma}\cdot\frac{\partial g_\sigma(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-e\mathbf{E}\cdot\mathbf{v}_{\mathbf{k}\sigma}=-\frac{g_\sigma(\mathbf{k},\mathbf{r})-\overline{g}_\sigma(\mathbf{r})}{\tau_\sigma},\tag{A1}$$

where $\mathbf{v}_\mathbf{k}$ is the velocity of a state on the Fermi surface, τ_σ is the spin-dependent relaxation time, and $\overline{g}_\sigma(\mathbf{r})$ is the average of $g_\sigma(\mathbf{k},\mathbf{r})$ over the Fermi surface.

We assume that nonmagnetic leads carry current into or out of each ferromagnet. In that case, each lead can share the quantization axis defined by the ferromagnet to which it is connected. A spin–flip scattering term

$$\frac{g_\uparrow(\mathbf{k},\mathbf{r})-g_\downarrow(\mathbf{k},\mathbf{r})}{\tau_{\uparrow\downarrow}},\tag{A2}$$

for both spin types is included in Eq. (A1) where appropriate.

The nonmagnetic spacer layer must be treated differently because the noncollinearity of the two ferromagnets induces a spin polarization in the spacer whose direction generally varies in both real space and reciprocal space. To treat this situation, we use a 2×2 Hermitian occupation matrix $\mathbf{f}(\mathbf{k},\mathbf{r})$ in place of the functions $f_\sigma(\mathbf{k},\mathbf{r})$.[25] In particular, at a point where the natural spin quantization axis points in the direction $(\theta\,\phi)$, a convenient representation for $\mathbf{f}$ is

$$\mathbf{f}=U(\mathbf{k},\mathbf{r})\begin{pmatrix}f_\uparrow(\mathbf{k},\mathbf{r}) & 0\\ 0 & f_\downarrow(\mathbf{k},\mathbf{r})\end{pmatrix}U^\dagger(\mathbf{k},\mathbf{r}),\tag{A3}$$

where

$$U(\mathbf{k},\mathbf{r})=\begin{pmatrix}\cos(\theta/2)e^{-i\phi/2} & -\sin(\theta/2)e^{-i\phi/2}\\ \sin(\theta/2)e^{i\phi/2} & \cos(\theta/2)e^{i\phi/2}\end{pmatrix},\tag{A4}$$

is the usual rotation matrix for spinors. We have suppressed the $\mathbf{k}$ and $\mathbf{r}$ dependence of θ and ϕ for simplicity. Then, since the matrix $\mathbf{g}$ is related to $\mathbf{f}$ as g_σ is related to f_σ, we describe[25] the transport in the spacer layer using the matrix analog of Eq. (A1):

$$\mathbf{v}_\mathbf{k}\cdot\frac{\partial\mathbf{g}(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-e\mathbf{E}\cdot\mathbf{v}_\mathbf{k}\mathbf{I}=-\frac{\mathbf{g}(\mathbf{k},\mathbf{r})-\overline{\mathbf{g}}(\mathbf{r})}{\tau}.\tag{A5}$$

In this equation, $\mathbf{I}$ is the 2×2 identity matrix and τ is the relaxation time in the nonmagnetic spacer.

[1] J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
[2] L. Berger, Phys. Rev. B **54**, 9353 (1996).
[3] M. Tsoi, A. G. M. Jansen, J. Bass, W. C. Chiang, M. Seck, V. Tsoi, and P. Wyder, Phys. Rev. Lett. **80**, 4281 (1998); J. Z. Sun, J. Magn. Magn. Mater. **202**, 157 (1999); J. E. Wegrove, D. Kelly, P. Guitienne, Y. Jaccard, and J. P. Ansermet, Europhys. Lett. **45**, 626 (1999).
[4] E. B. Myers, D. C. Ralph, J. A. Katine, R. N. Louie, and R. A. Buhrman, Science **285**, 867 (1999).
[5] J. A. Katine, F. J. Albert, R. A. Buhrman, E. B. Myers, and D. C. Ralph, Phys. Rev. Lett. **84**, 3149 (2000).
[6] J. Grollier, V. Cros, A. Hamzic, J. M. George, H. Jaffrés, A. Fert, G. Faini, J. Ben Youssef, and H. Legall, Appl. Phys. Lett. **78**, 3663 (2001).
[7] A. Brataas, Yu. V. Nazarov, and G. E. W. Bauer, Phys. Rev. Lett. **84**, 2481 (2000); Eur. Phys. J. B **22**, 99 (2001).
[8] X. Waintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph, Phys. Rev. B **62**, 12317 (2000).
[9] D. H. Hernando, Yu. V. Nazarov, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **62**, 5700 (2000).
[10] This semiclassical approximation works for layers thinner than relevant mean-free paths when both hot-electron effects due to ballistic transport and interference effects due to multiple reflections are not important.
[11] See, for example, H. B. Callen, *Thermodynamics* (Wiley, New York, 1960), Sec. 17.2.
[12] D. R. Penn and M. D. Stiles, Phys. Rev. B **59**, 13338 (1999).
[13] M. D. Stiles and D. R. Penn, Phys. Rev. B **61**, 3200 (2000).
[14] We have checked that no qualitative changes occur when R_σ is chosen so that $R_\uparrow R_\downarrow$ is purely imaginary. For a first-principles calculation of the complex reflection amplitudes for a Cu/Co interface, see K. Xia, P. J. Kelly, G. E. W. Bauer, A. Brataas, and I. Turek, e-print cond-mat/0107589.
[15] P. Holody, Ph. D. thesis, Michigan State University, 1996 (unpublished).
[16] J. Bass and W. P. Pratt, J. Magn. Magn. Mater. **200**, 274 (1999).
[17] K. M. Schep, J. B. A. N. van Hoof, P. J. Kelly, G. E. W. Bauer, and J. E. Inglesfield, J. Magn. Magn. Mater. **177**, 1166 (1998).
[18] K. Xia, P. J. Kelly, G. E. W. Bauer, I. Turek, J. Kudrnovsky, and V. Drchal, Phys. Rev. B **63**, 064407 (2001).
[19] L. Piraux, S. Dubois, A. Fert, and L. Belliard, Eur. Phys. J. B **4**, 413 (1998).
[20] We do not include spin-flip scattering in the magnetic layers because their thicknesses are much shorter than the spin diffusion length. If the opposite were true, spin–flip scattering in the Co (only) would be sufficient to polarize the current. In that case, the transition region between the unpolarized leads and saturation polarization in the Co would occur at the ends of the magnetic layers.
[21] Spatial precession occurs when we superpose two wave functions with the same energy but different wave vectors. The more familiar temporal precession occurs when we superpose two wave functions with the same wave vector but different energies.
[22] K. Wang, S. Zhang, and P. M. Levy, Phys. Rev. B **54**, 11965 (1996); A. Vedyayev, N. Ryzhanova, B. Dieny, P. Dauguet, P. Gandit, and J. Chaussy, *ibid.* **55**, 3728 (1997).
[23] See, e.g., J. Miltat, G. Albuquerque, A. Thiaville, and C. Vouille, J. Appl. Phys. **89**, 6982 (2001); J. Z. Sun, Phys. Rev. B **62**, 570 (2000); Ya. B. Bazaliy, B. A. Jones, and S. C. Zhang, J. Appl. Phys. **89**, 6793 (2001); e-print cond-mat/0009034.
[24] A. Fert, J. Phys. C **2**, 1784 (1969); T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).
[25] G. Baym and C. Pethick, *Landau–Fermi-Liquid Theory* (Wiley, New York, 1991). For an application to spin accumulation near a domain wall, see E. Simanek, Phys. Rev. B **63**, 224412 (2001).