Modeling of signal transmission in granular materials

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Aim

To determine how much information about perturbations at one end of a granular material may be deduced from measurements at the other end.

System construction

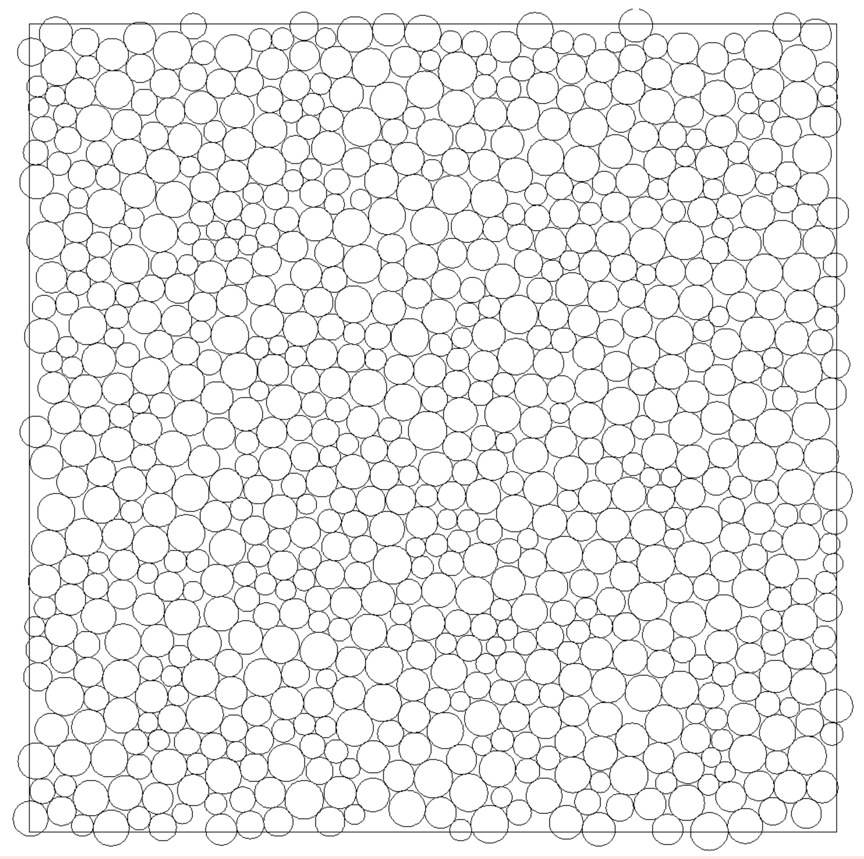

Figure 1: A typical pack, comprising 800 particles, with periodic boundary conditions applied at the box edges

A granular pack for further computations is formed by putting N circular particles in a large box with periodic boundary conditions, and reducing the box size until a jammed, isostatic state is achieved. The particles are then replaced by nodes located at particle centers, with bonds joining nodes representing touching particles.

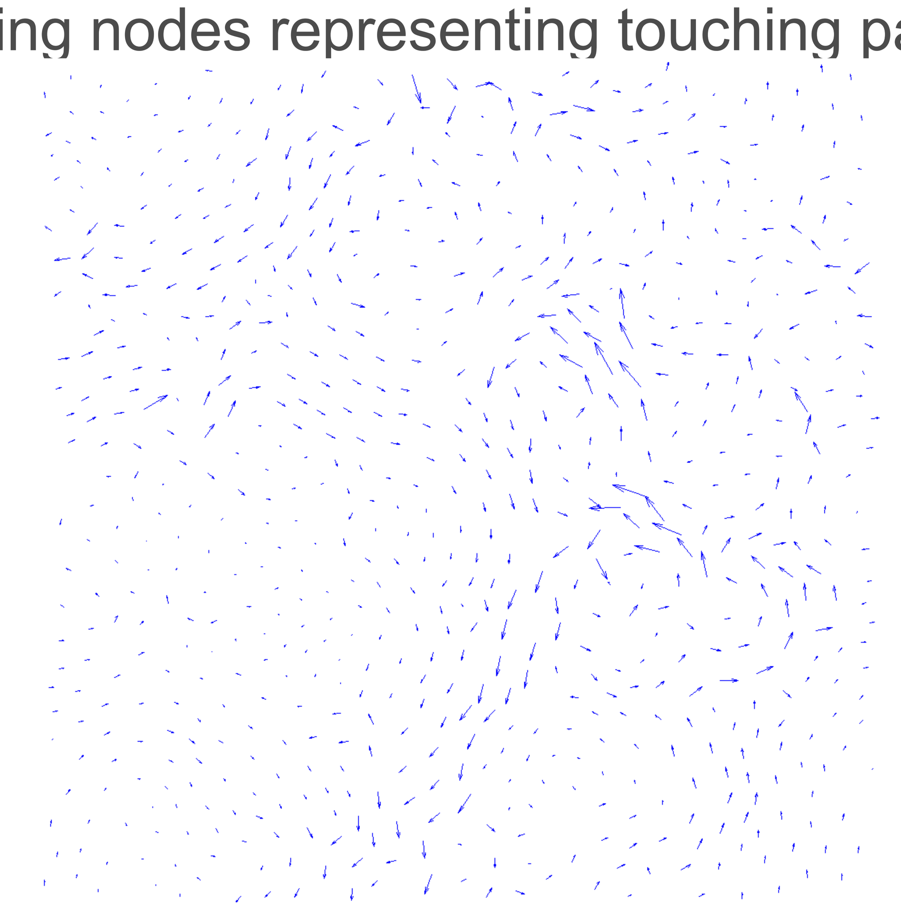

Figure 2: Displacements in an allowed mode of motion produced by removing one constraint from the packing of figure 1.

Cutting a bond introduces a mode in which the particles can move without changing the length of any bond. Such modes are long-ranged: each mode involves the movement of nodes all across the system.

Mode profiles

To model a physical sample of a granular material, all constraints crossing the periodic boundary condition on the left and right sides are removed. This introduces numerous free modes of motion. Many bases of the allowed motion could be formed, but it is useful to form a basis by sequentially finding modes maximally localised to the left and right sides of the system.

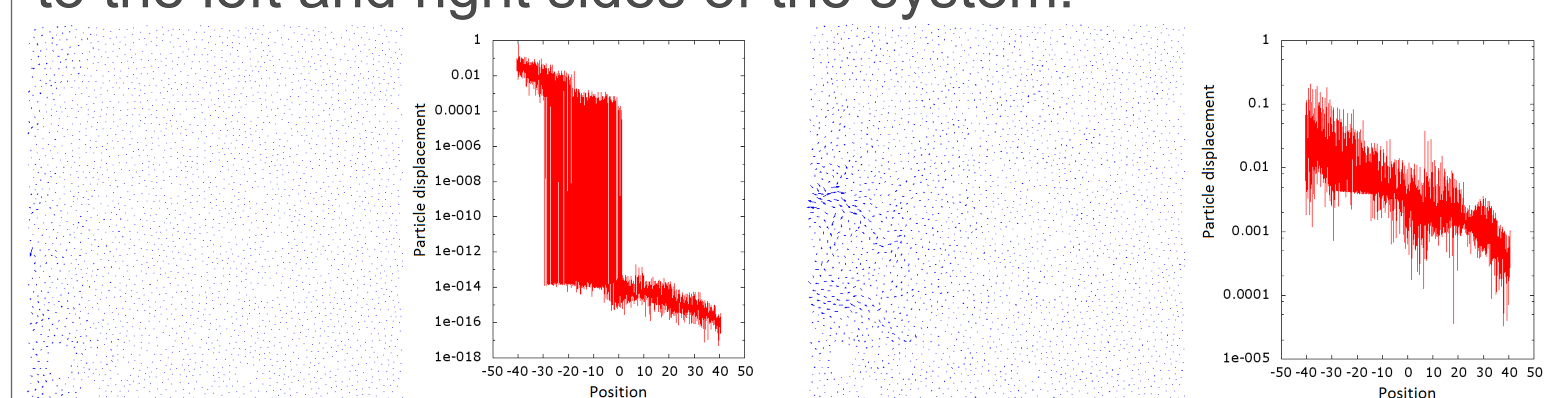

Figure 3: Left: a mode strictly localised to the left side of the system. Right: a mode exponentially localised to the left side of the system. Each mode is represented by a map of the displacements of all particles, and a graph of the magnitude of particle displacement vs position in the horizontal direction.

In this basis, some modes are strictly localised to a portion of the system, with displacements outside this portion comparable to numerical error. Others are exponentially localised to one side of the system, so that particles throughout the system move but with displacement that decays with position.

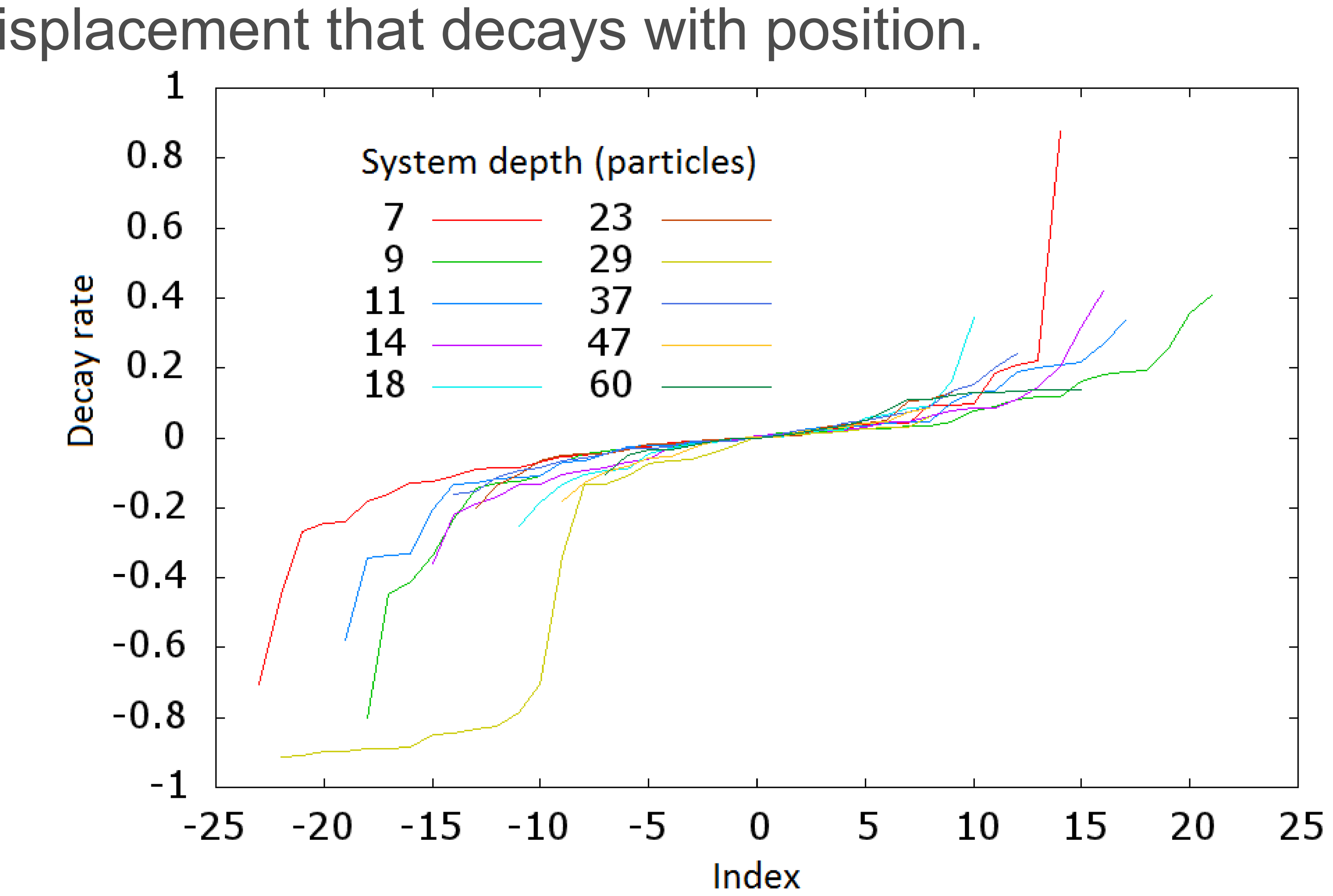

Figure 4: Decay rate of modes in the localised basis vs mode index, obtained simply by arranging modes in order of increasing decay rate. Modes with small index display a linear dependence of decay rate on index, just like modes in an elastic material.

Ordering the modes by this decay rate shows a region where a mode's decay rate is proportional to its index. This corresponds to the decay of disturbances to an elastic material.

We call the proportionality constant between decay rate and index the "mode disappearance rate". In an elastic material, the appropriate basis of perturbations is a set of sinusoidal functions. These disturbances have decay rates of the form $2\pi n/L$, where n is an integer and L is the width of the system. The mode disappearance rate for an elastic material is thus $2\pi/L$.

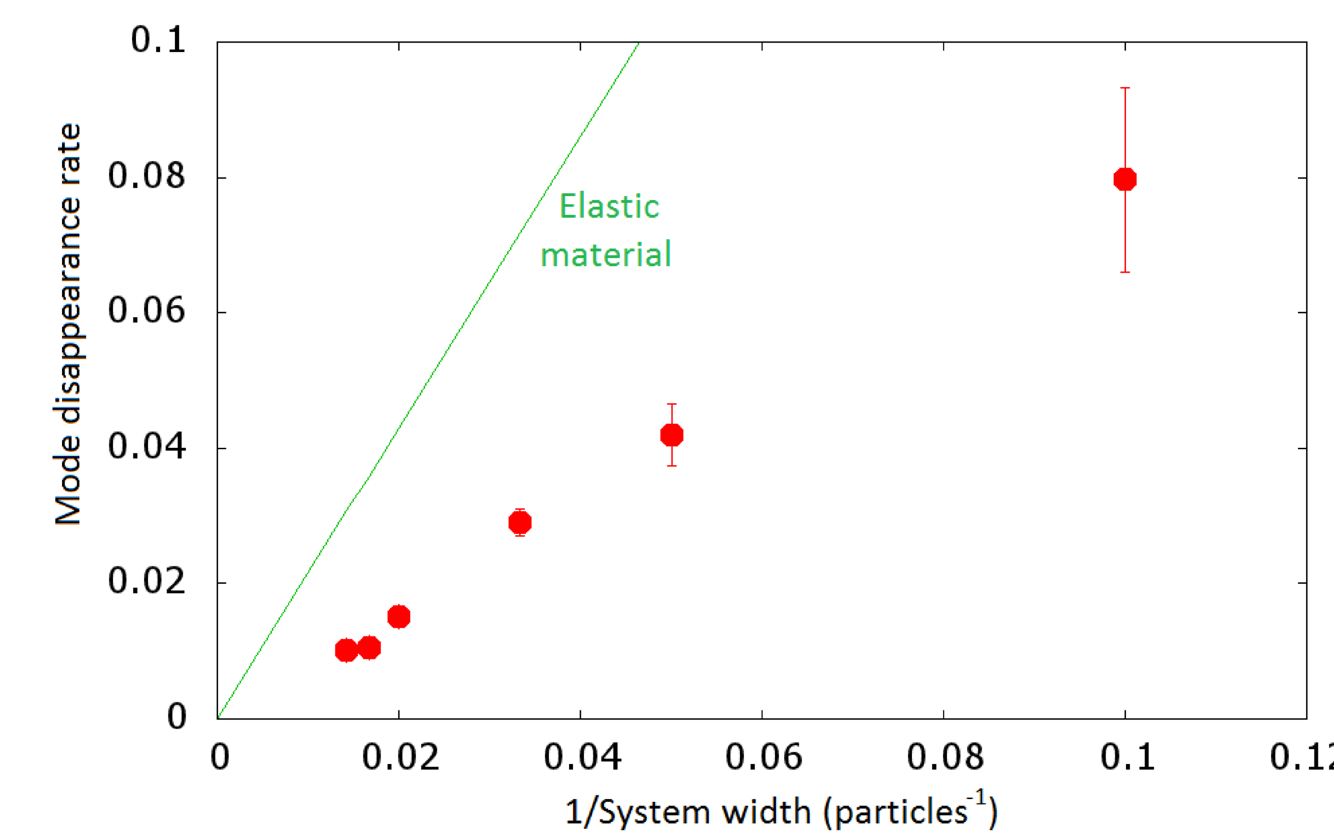

Figure 5: Mode disappearance rate vs the reciprocal of system width. Just as in the case of an elastic material, there is a linear relationship between these quantities, and the coefficient appears to be very similar.

In the granular system considered here, the mode disappearance rate is likewise inversely proportional to system width, with a proportionality constant of similar size. The exponential decay of displacement with position means that, if there is even a small uncertainty in measurement, a very limited number of pieces of information about a perturbation at one side of the system can be determined by measurements at the other side.

Conclusions

- Particle displacement decays with distance from the surface in a granular material in the same way as for an equivalent mode in an elastic material.
- This means that measurements at one end of a granular material give very limited information about perturbations at the other end.

Acknowledgements

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