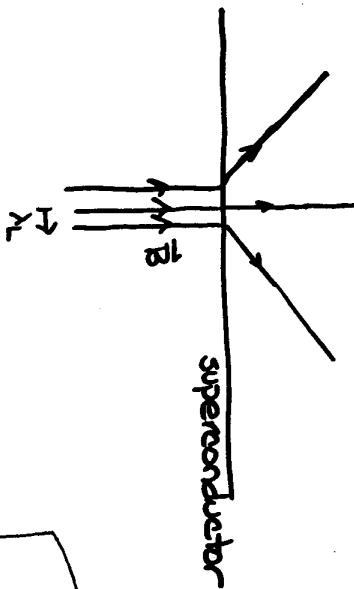
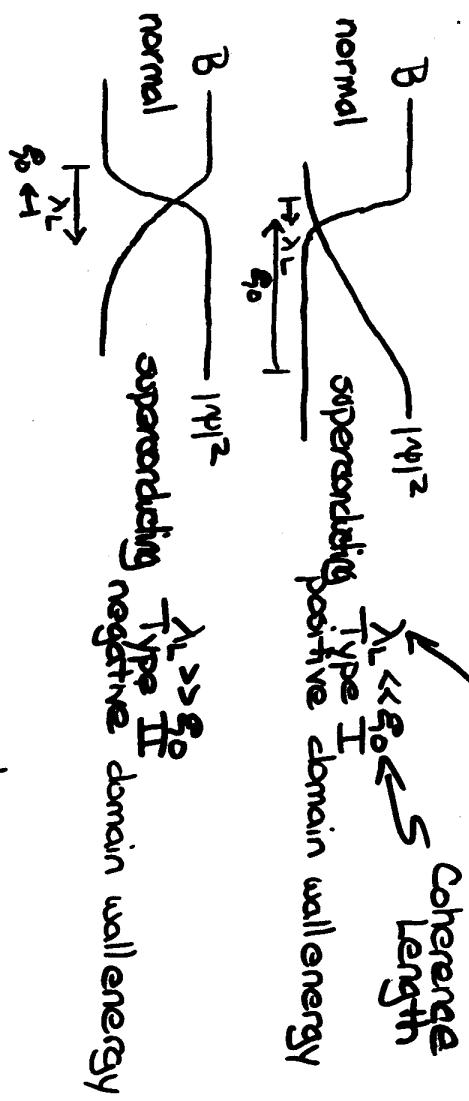


(2)

## Wall Energy

London Penetration Depth



Moler  
Lecture 3

(1)

## Single-Vortex Structure

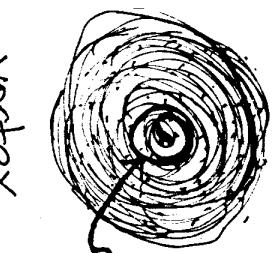
- Normal core model
- STM of  $NbSe_2$  and related theory importance of band structure
- Theoretical work on d-wave vortices
- STM on high- $T_c$
- Is g field-dependent? µSR

(4)

## Supercurrent Distribution

fluxoid quantization

$$\Phi + \frac{m^*c}{e^*} \oint \vec{v}_s \cdot d\vec{\ell} = n \Phi_0$$



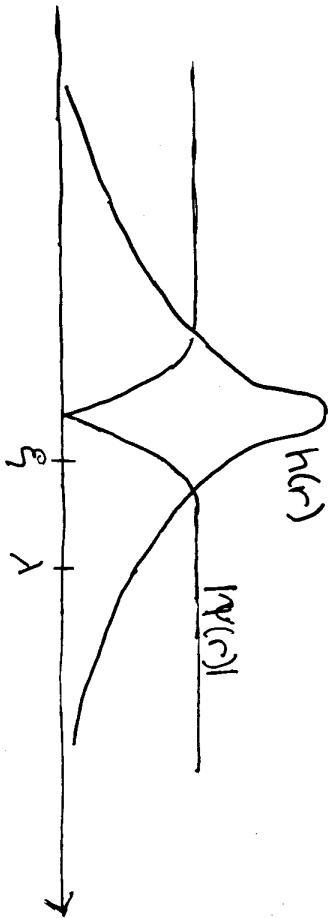
path of integration  
in vortex interior - negligible  $\Phi$   
vortex

$$\frac{m^*c}{e^*} v_s(r) 2\pi r \approx n \Phi_0 = \frac{hc}{2e}$$

$$v_s = \frac{\kappa}{m^*r}$$

(3)

## Vortex Structure in Ginzburg-Landau



requires numerical solution for arbitrary  $\kappa \equiv \lambda/e$

(6)

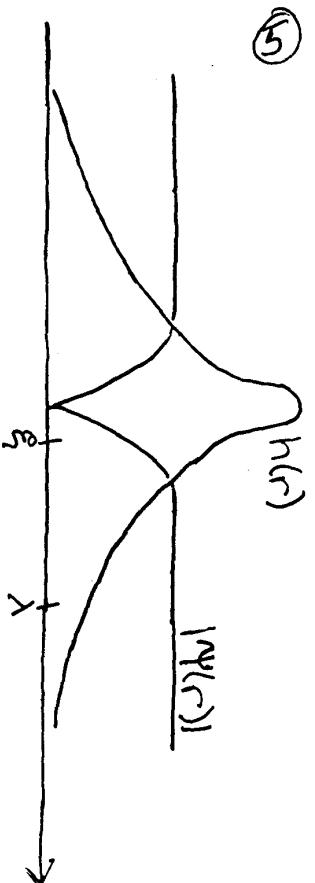
How large is the core?

Estimation #1

- Doppler shift of excitation energies (semiclassical)  
 $E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2} \rightarrow E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2} + \hbar \vec{k} \cdot \vec{v}_s$

- Taking  $v_s = \frac{\hbar}{m^* r}$ , we find  $E_{\vec{k}}(r) = 0$  for  $r \approx \xi_1$

K>>1



$$|\psi(r)| \approx \tanh \frac{C r}{\xi_1} \quad \text{where } C \text{ is } \mathcal{O}(1)$$

for  $r \gg \xi_1$ , use London model to find  $h(r)$

$$h(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right) \rightarrow \frac{\Phi_0}{2\pi\lambda^2} \left(\frac{\pi}{2} \frac{\lambda}{r}\right)^{1/2} e^{-r/\lambda} \quad \text{larger } r$$

$$\approx \frac{\Phi_0}{2\pi\lambda^2} \left[ \ln \frac{\lambda}{r} + 0.12 \right] \quad \xi_1 \ll r \ll \lambda$$

must be cut off for  $r \approx \xi_1$   
 $\Rightarrow \xi_1 \sim \lambda/r$

(8)

## Vortex Core Structure



$\frac{\hbar^2}{2m\epsilon^2}$  I  
 $\sim \frac{\Delta^2}{E_F}$

quasiparticle bound states  
lowest energy excitation  
 $\sim 10^{-4} \Delta \ll kT$  for  
coherence length  $\sim 0.1 \mu m$

Caroli, de Gennes, Matricon 1964  
Bogoliubov - de Gennes

also for intermediate  $\kappa$  values:  
Bardeen, Kummel, Jacobs, Tewordt  
1969  
hand-waving picture:  
"normal core" of radius  $\xi$

(7)

How large is the core? Estimation #2

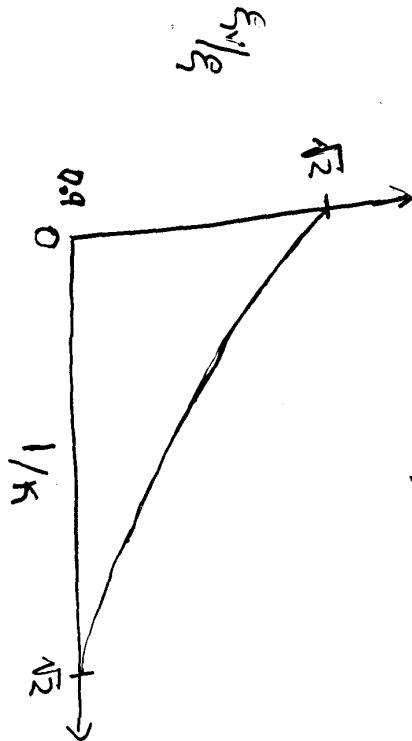
GL - Clem 1974

$R = (r^2 + \xi_V^2)^{1/2}$        $r$  = cylindrical radius  
 $\xi_V$  = variational parameter

$$b_2(r) = \frac{\Phi_0}{2\pi\lambda\xi_V} \frac{K_0(R/\lambda)}{K_1(\xi_V/\lambda)}$$

compare to London model for  $\kappa \gg 1$

$$b_2^{(L)}(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0(r/\lambda)$$

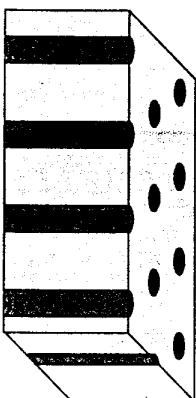


### Scanning-Tunneling-Microscope Observation of the Abrikosov Flux Lattice and the Density of States near and inside a Fluxoid

H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, Jr., and J. V. Waszczak  
*AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974*  
 (Received 28 October 1988)

The Abrikosov flux lattice is imaged in NbSe<sub>3</sub> by tunneling into the superconducting gap edge with a low-temperature scanning-tunneling microscope. The tunneling conductance into a single vortex core is strongly peaked at the Fermi energy, suggesting the existence of core states or core excitations. As one moves away from the core, this feature evolves into a density of states which is consistent with a BCS superconducting gap.

PACS numbers: 74.50.Ir, 61.16.Di



$$C = \gamma_n T \frac{H}{H_{c2}}$$

Vortex Cores  $\approx$  Cylinders of Normal Metal

### Traditional Mixed-State Specific Heat

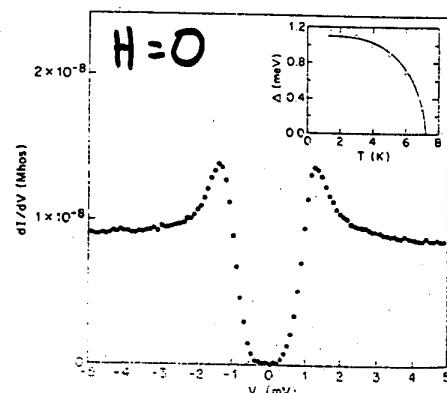


FIG. 1.  $dI/dT$  vs  $T$  for NbSe<sub>3</sub> and 0-T applied magnetic field used to determine the gap at 1.45 K. Inset: The gap vs temperature and the corresponding BCS fit.

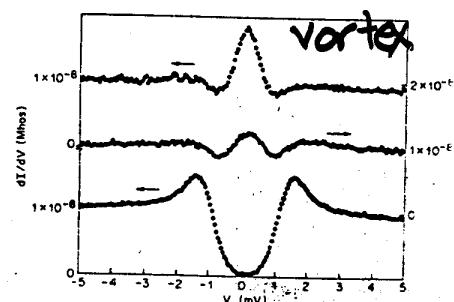
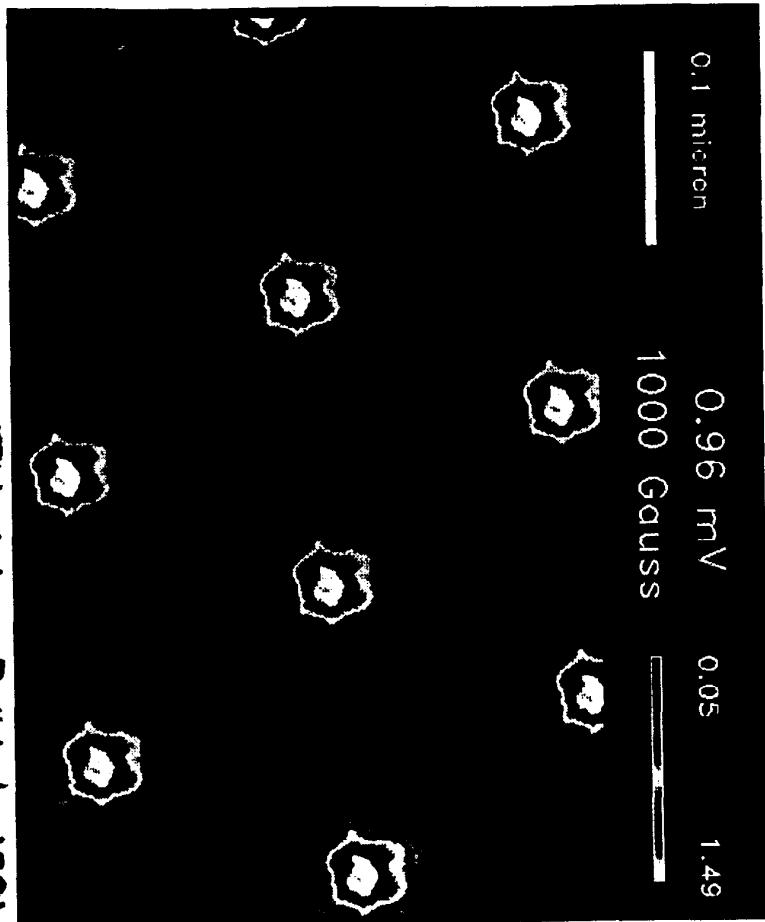


FIG. 3.  $dI/dV$  vs  $V$  for NbSe<sub>3</sub> at 1.85 K and a 0.02-T field, taken at three positions: on a vortex, about 75 Å from a vortex, and 2000 Å from a vortex. The zero of each successive curve is shifted up by one quarter of the vertical scale.

- C. Caroli, P. G. deGennes, and J. Matrice, Phys. Lett. **9**, 307 (1964).
- A. L. Fetter and P. C. Hohenberg, *Superconductivity* ed. by Parks (1969).



(11)

NbSe<sub>3</sub>

## STM spectroscopy of vortex cores and the flux lattice

H.F. Hess, R.B. Robinson and J.V. Waszczak  
AT&T Bell Laboratories, Murray Hill, NJ 07974, USA

The plenary talk was given by H.F. Hess.

(12)

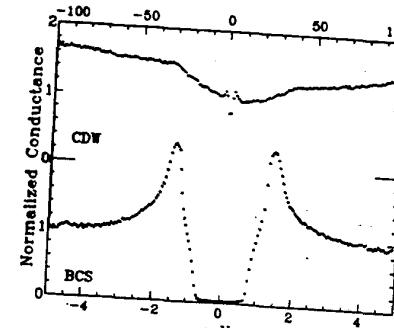
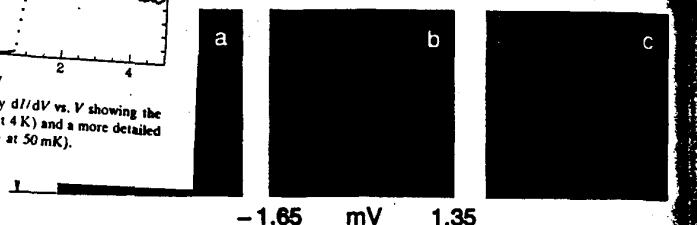


Fig. 1. Tunneling spectra given by  $dI/dV$  vs.  $V$  showing the CDW gap at 35 mV (upper curve at 4 K) and a more detailed view of the BCS gap (lower curve at 50 mK).

**B = 250 Gauss**

**$\theta = 15^\circ$**

**$\theta = 30^\circ$**



-1.65 mV 1.35

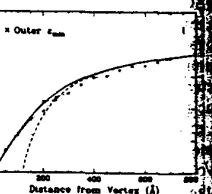
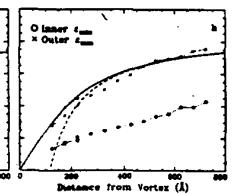
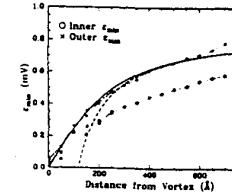
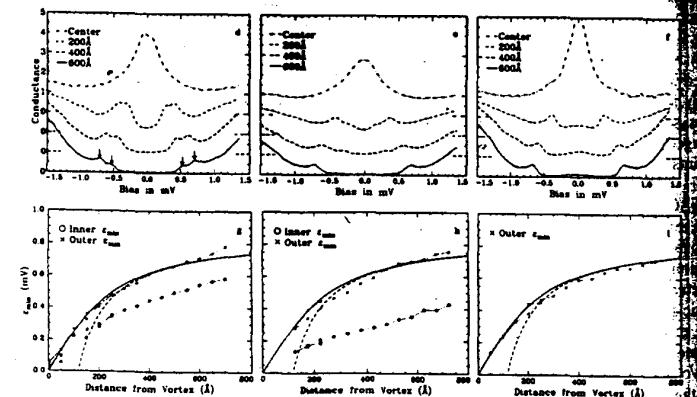


Fig. 6. (a), (b) and (c): Grey scale image of  $dI/dV(V, r)$  showing how it evolves along the three lines sketched in fig. 5(a). horizontal scale is bias voltage ranging from -1.65 to +1.35 mV. The vertical scale corresponds to 1000 Å sampling line width. vortex positioned about 250 Å from the bottom. This line may not intersect the vortex exactly but should miss the center by more than 25 Å resulting in some variation of the center spectra. The magnetic field is about 250 G. The grey scale refers normalized conductance where 3.0 is white and 0 is black. (d), (e) and (f): Slices of this data set at four radii displaying a dot sub-gap peak. (g), (h) and (i): The energy of the sub-gap peak vs. radius at three angles. The outer-most or higher-energy peak not strongly angle sensitive, while the inner peak is sensitive and collapses to zero at 30°. The dashed and solid lines represent eqs. (4) and (5), respectively.

## Six-fold symmetry vortex core

(14)

- Empirically: not from lattice
- different orientations at different biases
- undiminished at larger separations

Possible effects

- atomic crystal structure (dominates)
- gap anisotropy?

(13)

## Improved Bogoliubov Equations Calculations (BGS)

J.D. Shore et al 1989

- zero bias peak at vortex center
- Peak splits away from center

F. Gygi and M. Schlüter 1990

- confirmed zero-bias peak
- more stringent calculations
- included scattering

Physical origin of sub-gap peaks:

$$\varepsilon = \mu \frac{1}{k_F} \frac{d\Delta}{dr} + \mu \hbar \omega_L$$

$\mu$  = angular momentum

## Star-Shaped Local Density of States around Vortices in a Type-II Superconductor

Nobuhiko Hayashi,<sup>a</sup> Masanori Ichioka,<sup>b</sup> and Kazushige Machida<sup>c</sup><sup>a</sup>Department of Physics, Okayama University, Okayama 700, Japan

(Received 6 June 1996)

The electronic structure of vortices in a type II superconductor is analyzed within the quasiclassical Eilenberger framework. The possible origin of a sixfold "star" shape of the local density of states, observed by scanning tunneling microscope (STM) experiments on NbSe<sub>3</sub>, is examined in the light of the three effects: the anisotropic pairing, the vortex lattice, and the anisotropic density of states at the Fermi surface. Outstanding features of split parallel rays of this star are well explained in terms of an anisotropic s-wave pairing. This reveals not only a rich internal electronic structure associated with a vortex core, but also unique ability of the STM spectroscopy. [S0031-9007(96)01546-3]

PACS numbers: 74.60.Ec

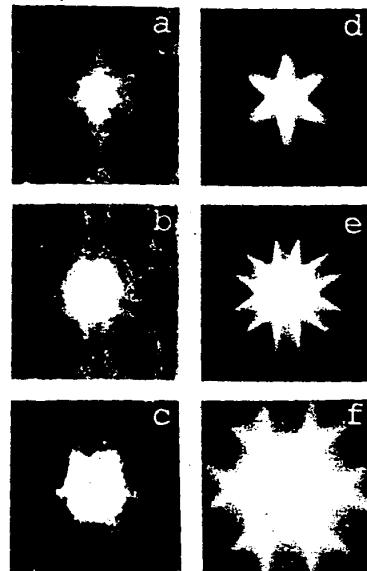
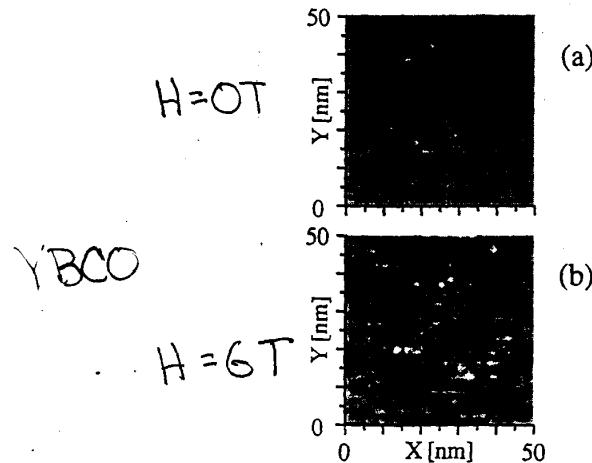


FIG. 1. Tunneling conductance images observed by Hess *et al.* at 0.1 T for the bias voltage 0.0 mV (a), 0.24 mV (b), 0.48 mV (c), where 1759 Å × 1759 Å is shown (also see Refs. [5,6]). The nearest-neighbor direction of the vortex lattice is the horizontal direction. The LDOS images calculated for  $E = 0$  (d), 0.2 (e), and 0.32 (f), where  $6\xi \times 6\xi$  is shown.

## Tunneling spectroscopy and STS observation of vortices on high temperature superconductors

Ø. Fischer, Ch. Renner, I. Maggio-Aprile, A. Erb, E. Walker, B. Revaz and J.-Y. Genoud.

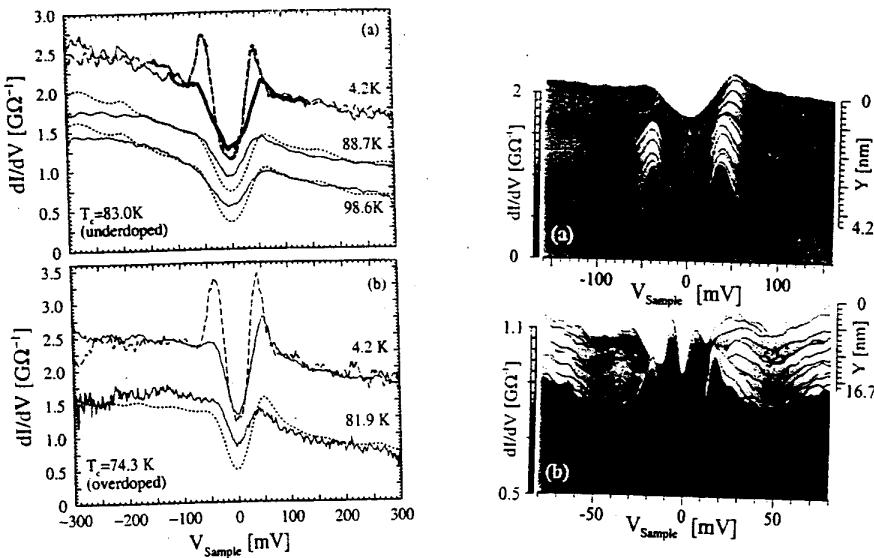
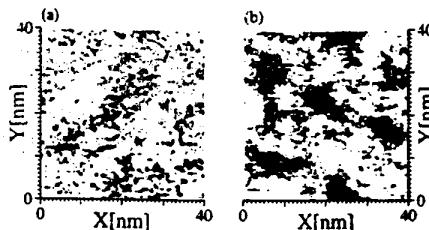
Département de Physique de la Matière Condensée, Université de Genève  
24, Quai E.-Ansermet, CH-1211 Genève 4, Switzerland

17

Observation of the Low Temperature Pseudogap in the Vortex Cores of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Ch. Renner,<sup>1</sup> B. Revaz,<sup>1</sup> K. Kadokawa,<sup>2</sup> I. Maggio-Aprile,<sup>1</sup> and Ø. Fischer<sup>1</sup><sup>1</sup>DPMC, Université de Genève, 24, Quai Ernest-Ansermet, 1211 Genève 4, Switzerland<sup>2</sup>University of Tsukuba, Institute of Materials Science, Tsukuba, 305 Ibaraki, Japan

(Received 3 December 1997)

Vortex cores in under- and overdoped  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  are studied by local probe tunneling spectroscopy. At the center of the cores, we find a gaplike structure at the Fermi level which scales with the superconducting gap, but no quasiparticle bound states. This low temperature pseudogap is intimately related to the superconducting gap and shows striking similarities with the normal state pseudogap measured above  $T_c$ . A possible interpretation is that both pseudogap structures reflect the same "normal" state containing phase incoherent excited pair states. [S0031-9007(98)05816-5]

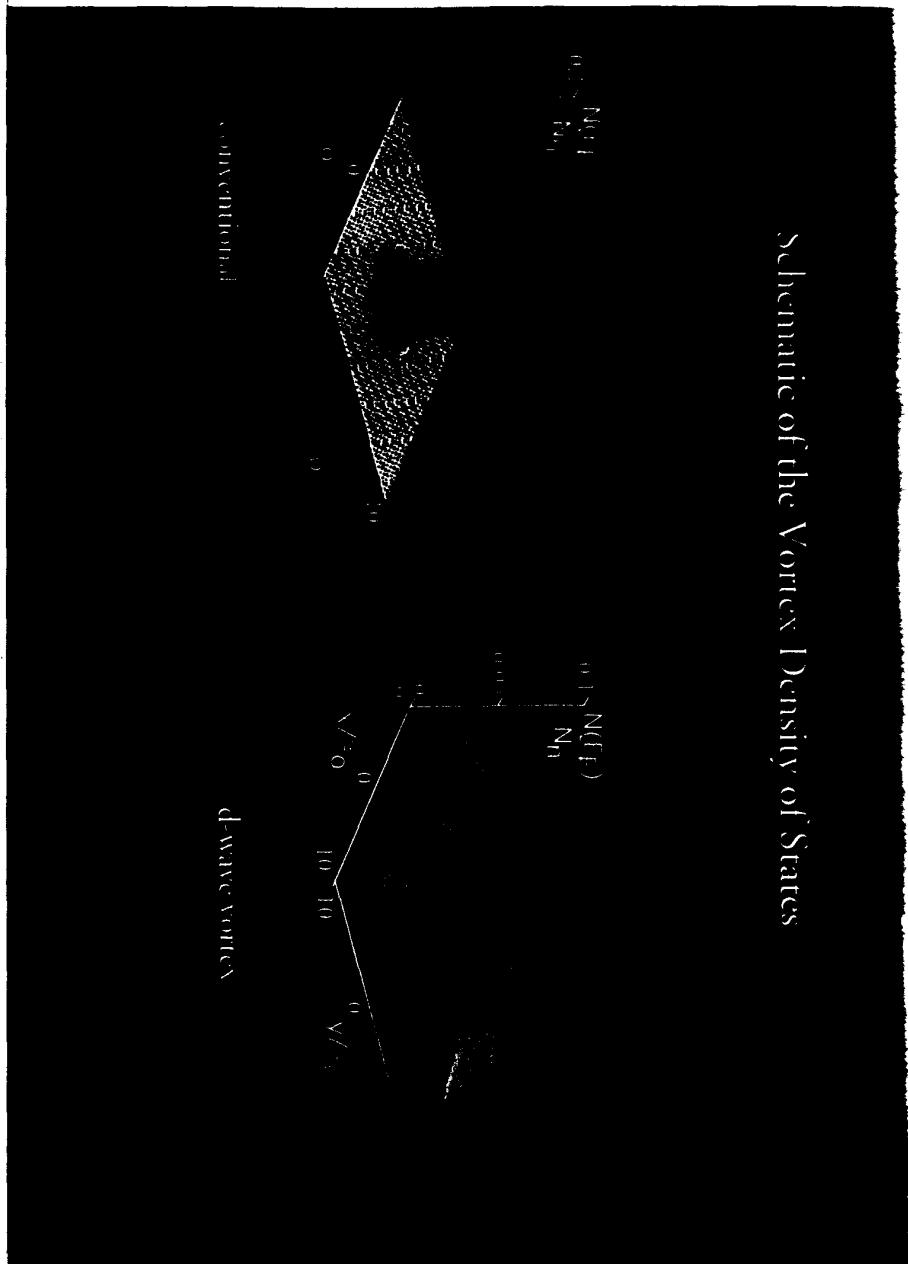
High-T<sub>c</sub> STM

- anisotropy of  $E_a$  and  $E_b$  clearly observed in YBCO vortex cores (Berkeley)
- evidence suggesting pseudogap in vortex cores of BSCCO (Geneva)
- quasiparticle bound states are ubiquitous in BSCCO, even good BSCCO, and trap vortices (Berkeley)
- talk to Kristine Lang at this summer school

18

Schematic of the Vortex Density of States

(20)



(19)

Density of states at the Fermi Level  
in a "d-wave Vortex" Jolovik 1993

$$E(\vec{k}) = \sqrt{\vec{\varepsilon}_k^2 + \Delta_k^2}$$

for delocalized states outside the vortex core:

$$N_{\text{deloc}}(E_F) = 2 \int \frac{d^3 k}{(2\pi)^3} \int d^2 r \delta \left[ E(\vec{k}) + m_e \vec{V}_F \cdot \vec{v}_s \right]$$

$$= \int \frac{dk_z}{2\pi^2 \gamma(k_z)} \sum_n \int d^2 r |m_{e\vec{v}_s} \cdot \vec{k}_{n\perp}|$$

$\uparrow_n$  indicates a nodal direction

$$\Rightarrow N_{\text{deloc}}(E_F, r) \sim 1/r$$

for better calculations

see Kallin and coworkers  
Tesanovic

Measurement of the Fundamental Length Scales in the Vortex State of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.60}$ 

J. E. Sonier, J. H. Brewer, R. F. Kiefl, D. A. Bonn, S. R. Dunsiger, W. N. Hardy, Ruixing Liang, W. A. MacFarlane, R. I. Miller, and T. M. Riseman\*

TRIUMF, Canadian Institute for Advanced Research and Department of Physics and Astronomy,  
University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

D. R. Noakes, C. E. Stronach, and M. F. White, Jr.

Department of Physics, Virginia State University, Petersburg, Virginia 23806  
(Received 3 June 1997)

The internal field distribution in the vortex state of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.60}$  is shown to be a sensitive measure of both the magnetic penetration depth  $\lambda_{ab}$  and the vortex-core radius  $r_0$ . The temperature dependence of  $r_0$  is found to be weaker than in the conventional superconductor  $\text{NbSe}_2$  and much weaker than theoretical predictions for an isolated vortex. The effective vortex-core radius decreases sharply with increasing  $H$ , whereas  $\lambda_{ab}(H)$  is found to be much stronger than in  $\text{NbSe}_2$ . [S0031-9007(97)04251-8]

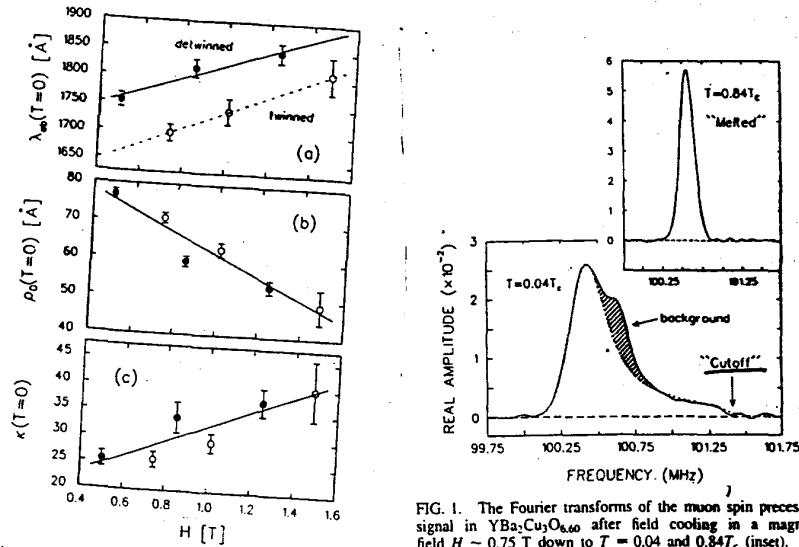


FIG. 4. The field dependence of (a)  $\lambda_{ab}$ , (b)  $\rho_0$ , and (c)  $\kappa_{ab} = \lambda_{ab}/\xi_{ab}$  extrapolated to  $T = 0$  in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.60}$ . The data for S1 (twinned) are shown as open circles whereas the data for S2 (detwinned) are shown as solid circles.

Expansion of the vortex cores in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.65}$  at low magnetic fields

J. E. Sonier,\* R. F. Kiefl, J. H. Brewer, D. A. Bonn, S. R. Dunsiger, W. N. Hardy, R. Liang, and R. I. Miller  
TRIUMF, Canadian Institute for Advanced Research and Department of Physics and Astronomy, University of British Columbia,  
Vancouver, British Columbia, Canada V6T 1Z1

D. R. Noakes and C. E. Stronach  
Department of Physics, Virginia State University, Petersburg, Virginia 23806  
(Received 27 October 1998)

Muon-spin-rotation spectroscopy has been used to measure the effective size  $r_0$  of the vortex cores in optimally doped  $\text{YBa}_2\text{Cu}_3\text{O}_{6.65}$  as a function of temperature  $T$  and magnetic field  $H$  deep in the superconducting state. While  $r_0$  at  $H = 2$  T is close to 20 Å and consistent with that measured by scanning tunneling microscopy at 6 T, we find a striking increase in  $r_0$  at lower magnetic fields, where it approaches an extraordinarily large value of about 100 Å. This suggests that the average value of the superconducting coherence length  $\xi_{ab}$  in cuprate superconductors may be much larger than previously thought at low magnetic fields in the vortex state. [S0163-1829(99)50702-9]

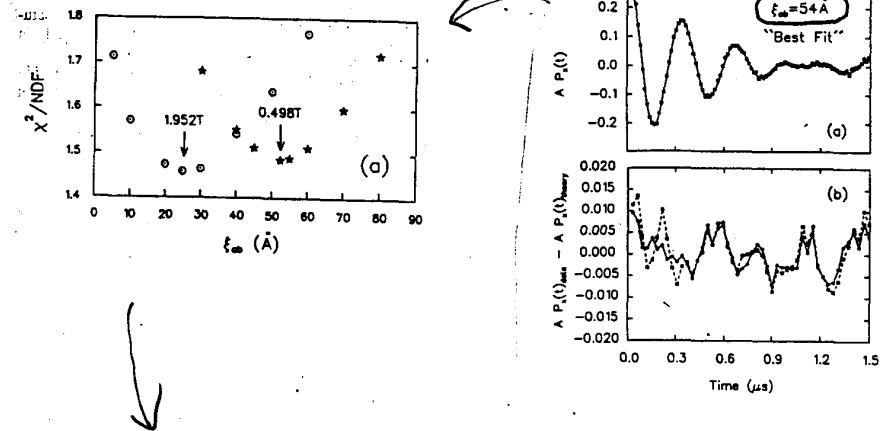


FIG. 1. The Fourier transforms of the muon spin precession signal in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.60}$  after field cooling in a magnetic field  $H \sim 0.75$  T down to  $T = 0.04$  and  $0.84$  T, (inset). The dashed curve is the Fourier transform of the simulated muon polarization function which best fits the data and the shaded region is the residual background signal.

