NONEQUILIBRIUM PHASES OF DIRTY DRIVEN PERIODIC MEDIA

M.C. Marchetti, Syracuse University

2001 BOULDER SCHOOL

Work done with Leon Balents (UCSB)
and Leo Radzihovsky (CU-Boulder)

REFERENCES (not a complete list!)

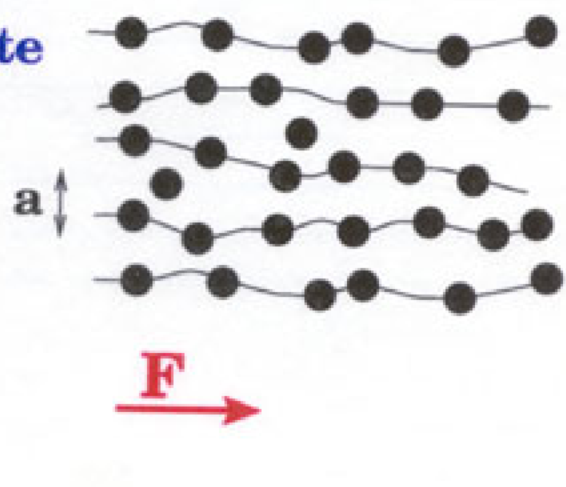
- L. Balents and M.P.A. Fisher, PRL 75, 4270 (1995).
- T. Giamarchi and P. Le Doussal, PRL 76, 3408 (1996).
- L. Balents, M.C. Marchetti and L. Radzihovsky, PRL 78, 751 (1997); PRB 57, 7705 (1998).
- T. Giamarchi and P. Le Doussal, PRB 57, 11356 (1998).
- K. Moon, R.T. Scalettar, and G. Zimanyi, PRL 77, 2778 (1996)
- F. Pardo et. al, Nature 396, 348 (1998)

OUTLINE

- Framework for the classification of nonequilibrium ordered states (order parameters & phases)

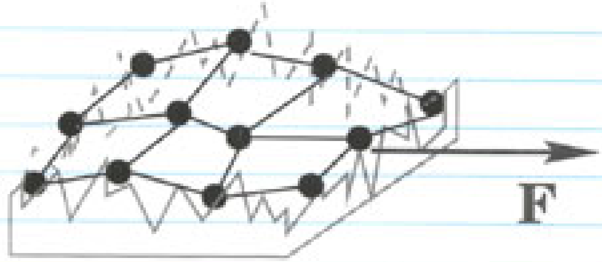
- Long-wavelength/low frequency properties:
  nonequilibrium hydrodynamics of driven lattices
  ⟶ host of new noneq. terms

- Full periodicity never recovered due to static disorder
  Moving ordered smectic as
  a generic noneq. ordered state
  ⟶ Experiments
  in vortex lattices



OUTLINE

- lattice pinned by quenched impurities fixed in space
  → pinned glass



  + constant driving force

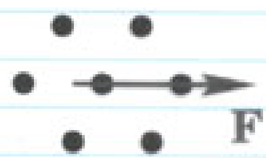
Does the lattice reorder at large drives?

- 1. Generalization of the equilibrium framework for classifying phases: translational & temporal order.

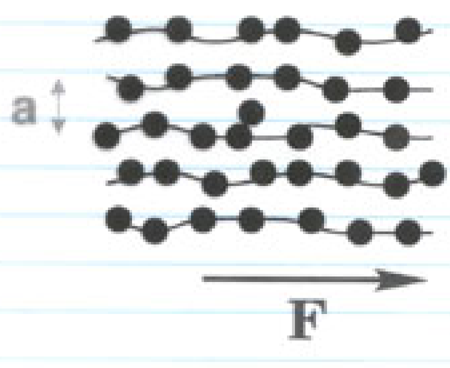
  2. Derivation of far-from-equilibrium hydrodynamics.
  → noneq. "noise"

- 1. Full periodicity not recovered due to quenched disorder

  2. Drive breaks rotational symmetry
  --> longitudinal & transverse order
  may not be recovered simultaneously

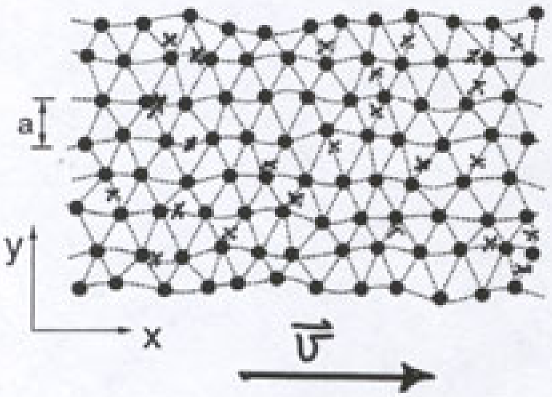


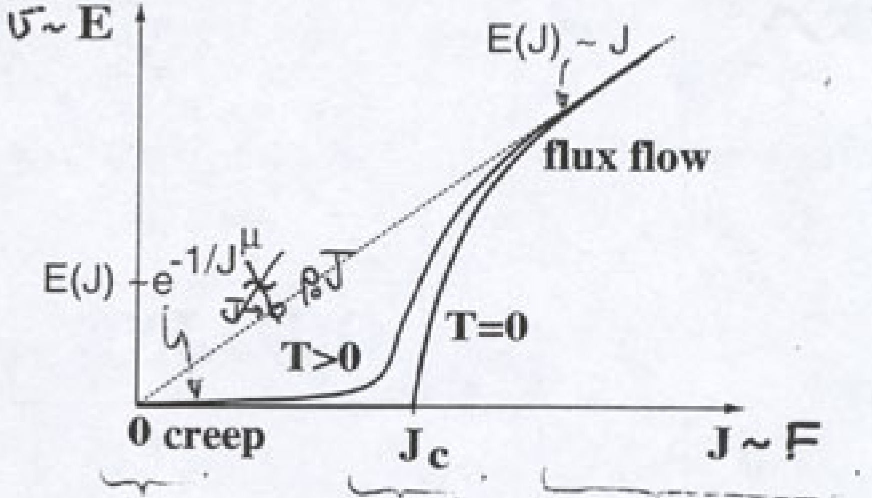
Moving transverse smectic
as a generic noneq. ordered state



→ Experiments
in vortex lattices

• **DRIVEN DYNAMICS:**





near equilibrium (Balents, D.S. Fisher '97 + L.R)

critical depinning transition (D.S. Fisher + Narayan) Middleton '90 '92

strongly driven (Balents '97 Marchetti + Li (Scheidle + Vinok

– Important questions about strongly driven regime:

- ∗ hydrodynamic modes and equation of motion
- ∗ E(J) (unique or hysteretic?)
- ∗ phases (classifications, correlation function,...)
- ∗ phase transitions (universality classes and order parameters)

– Earlier work

∗ *Experiments*:

Neutron scattering show increase in ξ_T (Yaron, et al. '94)

Transport show ordering above dV/dI peak (Bhattacharya, et.al. '93)

∗ *Simulations*:

Decrease in dislocation/disclination defect density with increasing drive (Shi+Berlinsky '91, Koshelev+Vinokur '94)

∗ *Theory*:

Current-driven crystallization at $T_{eff} = T + T_{sh}(J)$, with $T_{sh} \sim \Delta/J^2$ (Koshelev+Vinokur '94)

Moving Bragg glass with $u_x \approx 0$ (ordered), u_y rough and pinned → *finite transverse critical current* (Giamarchi+LeDoussal '96)

Slide courtesy of L. Radzihorsky

Koshelev & Vinokur, 1994:

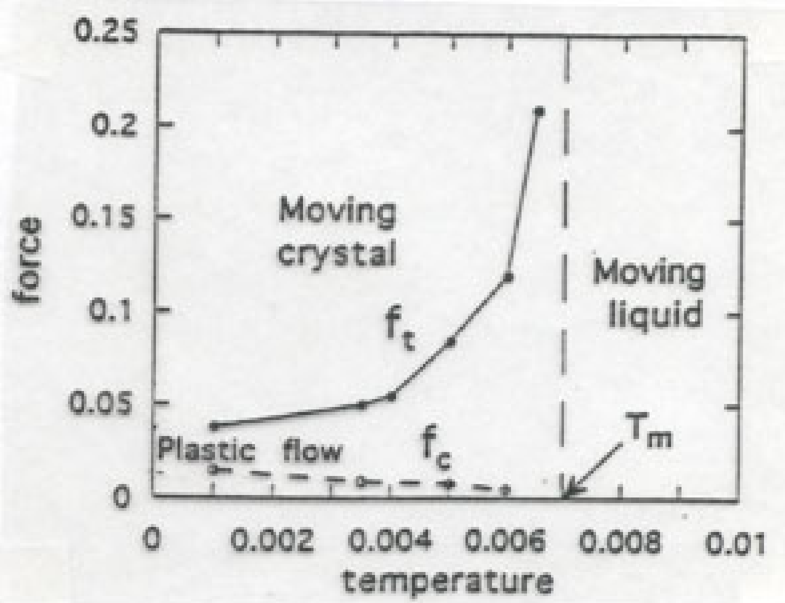
Used perturbation theory in disorder/drive (cf. Schmid & Hauger, 1973) to study both the driven liquid and the driven lattice.

They argued that at large drives disorder acts like a "thermal shaking" in the liquid:

$$T_{sh} \sim \frac{\Delta}{F^2} \qquad T_{eff} = T + T_{sh}(F)$$

When T_{eff} (F) ~ $T_{Melting}$ the moving liquid becomes unstable

current-driven crystallization into an ordered state ?



GENERAL ISSUE: (CDWs, Wigner crystals, mag. bubble arrays, ...)

Periodic structure
+ Drive
+ Disorder

→

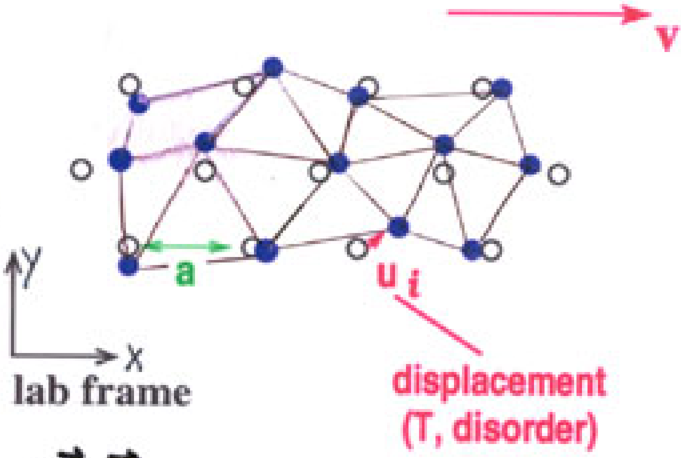
- Complex dynamics
- Noneq. PT between pinned and sliding states
- ordered sliding states?

GOAL: Nonequilibrium Phase Diagrams

CLASSIFICATION of PHASES & ORDER PARAMETERS

coarse-grained density of a moving lattice:

$$\rho(\vec{r},t) = \sum_i \delta(\vec{r} - \vec{r}_i^{\,eq} - \vec{u}_i - \vec{v}t)$$



$$\rho(\vec{r},t) = \underbrace{\rho_0}_{1/a^2} + \frac{1}{2}\sum_{\underset{\text{RLV}}{\vec{Q}}} \underbrace{\rho_Q(\vec{r},t)}_{\text{ORDER PARAMETERS}}\, e^{i\vec{Q}\cdot\vec{r}}$$

time oscillations

$$\underbrace{\rho_{\vec{Q},0}(\vec{r},t)}_{\text{amplitude} \approx \text{constant}} \, e^{i\vec{Q}\cdot(\underbrace{\vec{u}(\vec{r},t)}_{\text{elastic deformations}} + \vec{v}t)}$$

amplitude ≈ constant
no topological defects

elastic deformations
described by the **displacement field** in the lab frame

- small deformations ---> elastic approx. ok at large scales
- large deformations ---> elastic model breaks down:
  instability of the ordered phase

EXPERIMENTALLY (neutron scatt., flux decorations):

$$S(\vec{q},t) = \langle \rho(\vec{q},t)\rho(-\vec{q},0)\rangle = \sum_{\vec{Q}} \int_{\vec{r}} e^{i\vec{r}\cdot(\vec{q}-\vec{Q}) + i\vec{Q}\cdot\vec{v}t} \underbrace{\langle \rho_Q(\vec{r},t)\rho_Q^*(\vec{0},0)}_{}$$

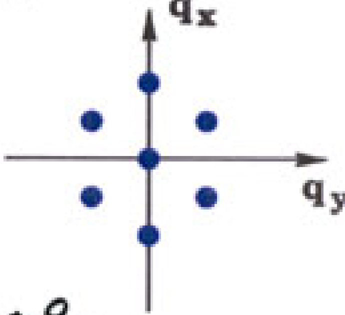
ORDER PARAMETER CORRELATIONS:

$$C_{\vec{Q}}(\vec{r},t) = \rho_0^2 \langle e^{i\vec{Q}\cdot[\vec{u}(\vec{r},t) - \vec{u}(\vec{0},0)]}$$

TRANSLATIONAL ORDER (t=0):

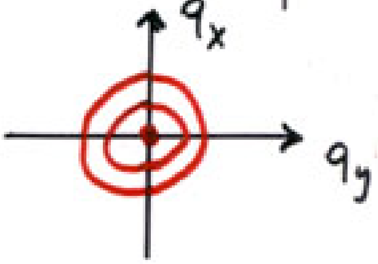
$S(\vec{q},0) \sim \sum_{Q} \int_{r} e^{i r\cdot(\vec{q}-\vec{Q})} C_Q(r,$

- $C_Q(r,0) \sim \text{const.}$ — **Bragg peaks at Q's**
  LRO

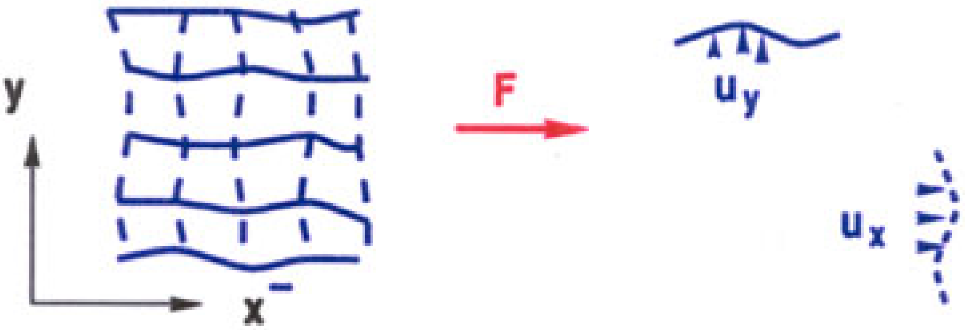


- $C_Q(r,0) \sim r^{-\eta}$ — **power-law peaks**
  QLRO

- $C_Q(r,0) \sim \exp(-r)$ — **diffuse rings**
  SRO (liquid)



But the driving force breaks the rotational symmetry and longitudinal and transverse order may not be recovered simultaneously ---> SMECTIC ORDER



TEMPORAL ORDER ($t \neq 0$):

$S(q,\omega) \sim \int_{t} e^{it(\omega - \vec{Q}\cdot\vec{v})} C_Q(\vec{r},t$

Balents and M.P.A. Fisher, 1995 (CDW)

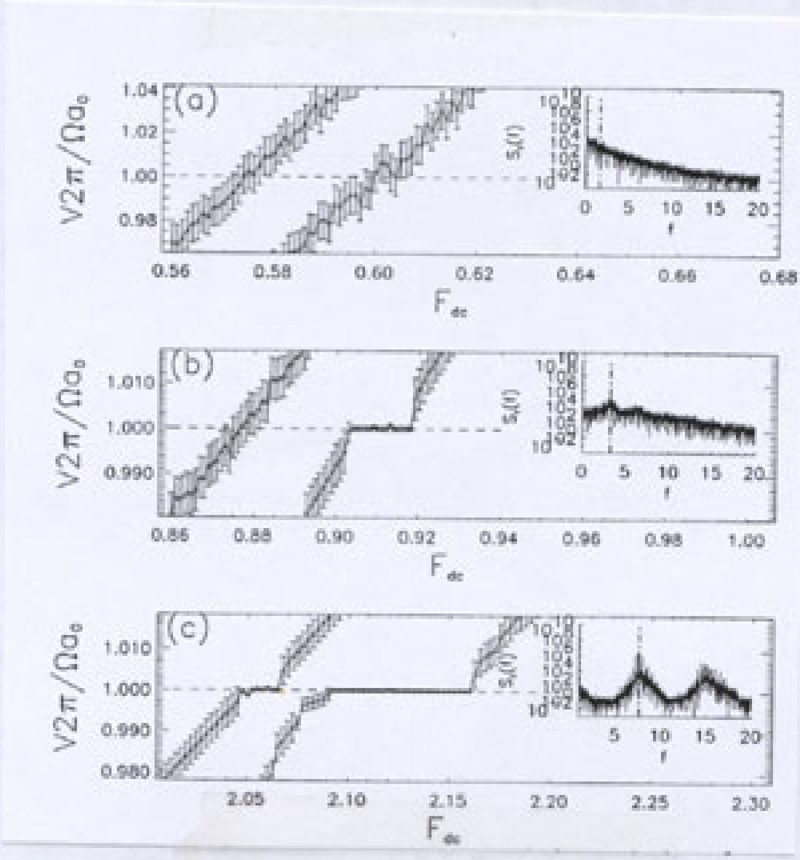
- $C_Q(r,t) \to \text{const.}$ — **LR temporal order at $\omega_Q = Q\cdot v$**

  $S(q,\omega) \sim \delta(\omega - \omega_Q)$

Complete mode-locking to an arbitrarily weak periodic drive at ω_Q

- $C_Q(r,t) \to t^{-\eta}$ — **QLR temporal order**
- $C_Q(r,t) \to \exp(-t)$ — **SR temporal order**

Kolton et al, 2000
2d driven vortices



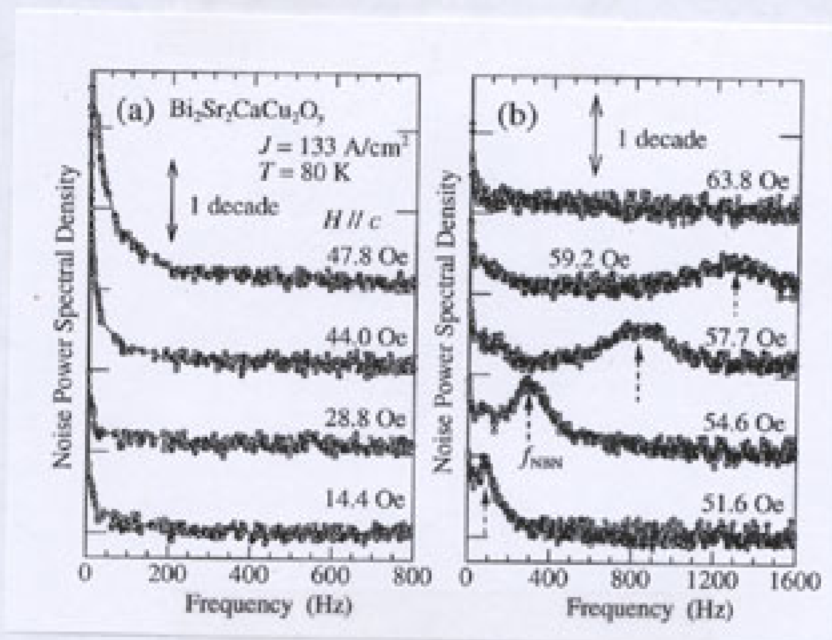
plastic

smectic

transverse solid

Togawa et al., PRL 2000
BSCCO

conduction noise spectrum

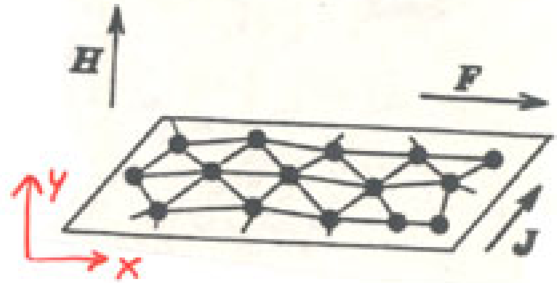


BBN

some NBN

DRIVEN (FLUX) LATTICE

Microscopic mass/spring Hamiltonian coupled to a disordered substrate



$$H = \frac{1}{2}\sum_{i,j} K_{\alpha\beta}(r_{ij}^{eq})\, u_i^\alpha u_j^\beta + \sum_i V_{DIS}(\vec{r}_i^{\,eq} + \vec{u}_i)$$

overdamped dyamics

$$\gamma\,\partial_t \vec{u}_i = -\frac{\delta H}{\delta \vec{u}_i} + \vec{F} + \vec{\eta}_i(t)$$

thermal noise

$\langle \vec{\eta}_i(t)\cdot\vec{\eta}_j(0)\rangle = 2\gamma T\,\delta(t$

Noneq. "hydrodynamics" for the continuum displacement field $\vec{u}(\vec{r},t)$ derived by integrating out short wavelength modes

Balents, MCM & Radzihovs
Giamarchi & Le Doussal
Scheidl & Vinokur

→ MANY NEW NONEQ. TERMS (no FDT)

$$\boxed{\gamma\big(\underbrace{\partial_t + \vec{v}\cdot\vec{\nabla}}_{\text{CONVECTION}}\big)\vec{u} = \underbrace{c\,\nabla^2\vec{u}}_{\text{ELASTICITY}} + \underbrace{\vec{F}_P[\vec{r},t;\vec{u}]}_{\text{PINNING}} + \vec{F} + \vec{\eta}(\vec{r},t) + \lambda(F)(\vec{\nabla}\cdot\vec{u})^2 + \dots}$$

$$\vec{v} = \langle \partial_t \vec{u}\rangle \simeq \vec{F}/\gamma$$

$\langle \eta_i(\vec{r},t)\,\eta_j(\vec{0},0)\rangle = 2\gamma k_0 T\,\delta(\vec{r})$

PINNING FORCE

$$\vec{F}_p(\vec{r},t;\vec{u}) = \vec{F}_p^{\,EQ} + \vec{F}_p^{\,NEQ}$$

EQUILIBRIUM PART

$$H_{DIS} = \int_{\vec{r}} V(\vec{r})\, \underbrace{\rho(\vec{r};\vec{u})}_{\text{density of the lattice}}$$

$$\langle V(\vec{r})V(0)\rangle = \Delta\,\delta(\vec{r})$$

$$\vec{F}_p^{\,EQ} = -\frac{\delta H_{DIS}}{\delta \vec{u}}$$

gradient of the disorder potential

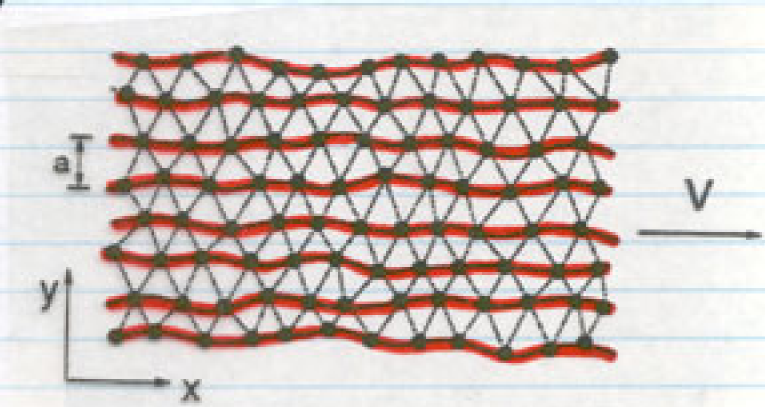
$$= \rho_0 V(\vec{r}) \sum_{\vec{Q}} i\vec{Q}\, e^{i\vec{Q}\cdot(\vec{r}-\vec{u}-\vec{v}t)}$$

lattice periodicity

fast oscillation at large $\vec{v}$.

There are TRANSVERSE ($\perp \vec{F}$) components that are STATIC ($\vec{Q}\cdot\vec{v}=0$)

(Giamarchi + Le Doussal, 1996)



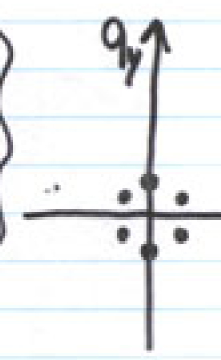
- $F_{px} \sim e^{i\vec{Q}\cdot\vec{v}t} \sim 0$

  $\Rightarrow$ LRO along x

- $F_{py} \sim \cos(Q_y y)$

  $\Rightarrow$ periodicity and QLRO along y

ANISOTROPIC SLIDING SOLID



NONEQ. PART OF $\vec{F}_p$

- NOT GRADIENT OF A POTENT
- VANISHES WHEN $F=0$

STATIC RANDOM DRAG FROM SPATIAL INHOMOGENEITIES IN THE DEFECT DENSITY:

$$\langle F_{pi}^{NEQ}(\vec{r})\, F_{pj}^{NEQ}(\vec{0})\rangle = g_{ij}\,\delta(\vec{r})$$

KRUG, 1995

CDW $\begin{cases} \text{BALENTS \& FISH} \\ \text{CHEN et al.} \end{cases}$

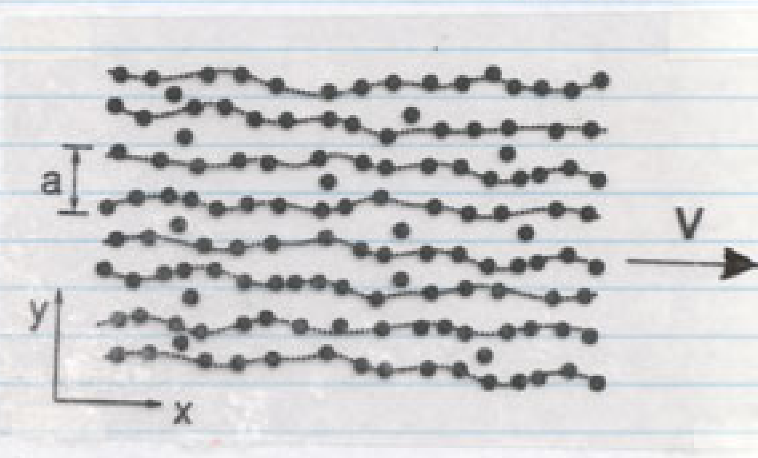
$$g \approx \left(\frac{\Delta}{v}\right)^2$$

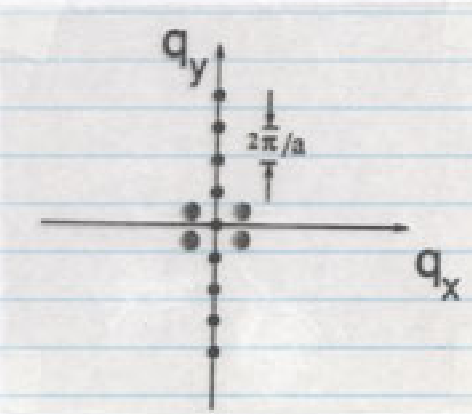
This static disorder always destroys the periodicity along the direction of motion $\Longrightarrow$ $\boxed{\text{MOVING SMECTIC}}$

SRO along x $\quad d=2,3$

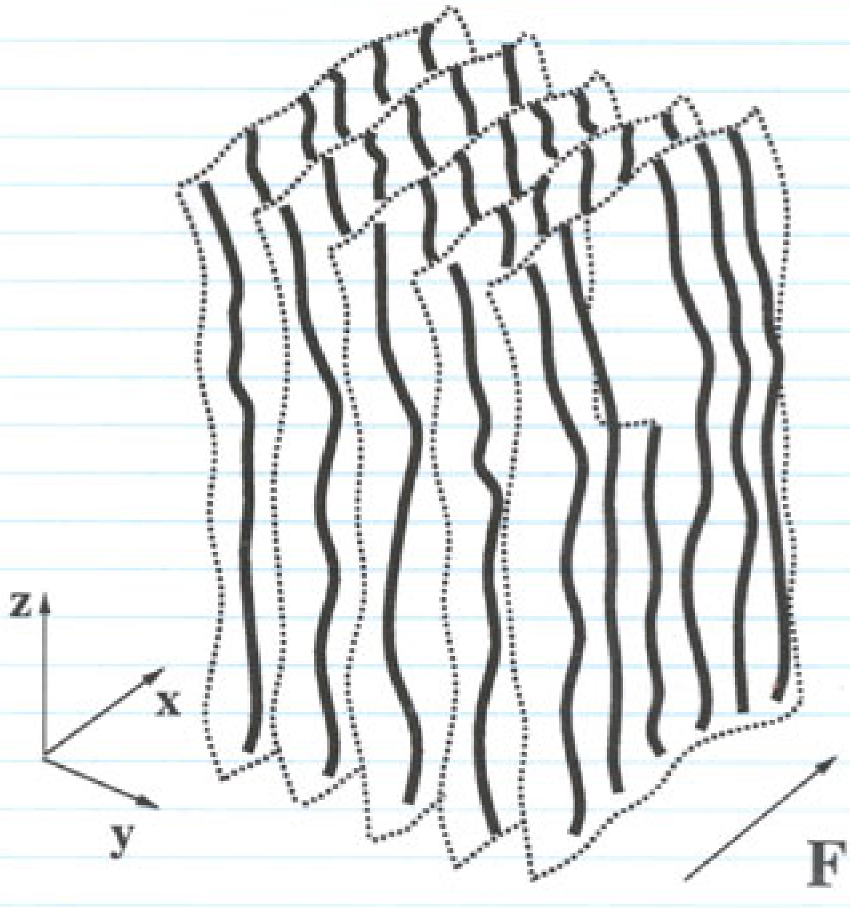
STACK OF SLIDING LIQUID LAYERS

- elasticity along y preserved (no unbound dislocations)
  → channels and PHONON MODE u_y
- unbinding of dislocations along x → liquid
  CONSERVED DENSITY ρ

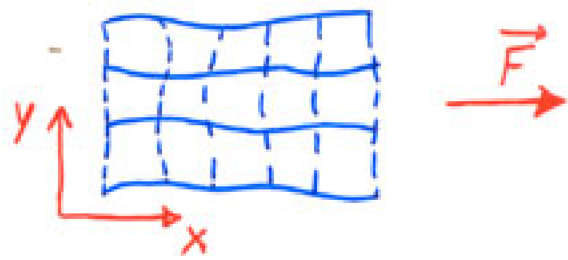




LINE SMECTIC



LINEARIZED THEORY



$$\vec{u} = \tilde{\vec{u}} + \vec{v}\,t$$

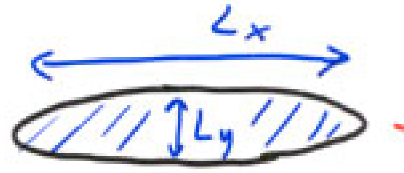
$$\gamma(\partial_t + v\partial_x)u_i = c_{66}\nabla^2 u_i + c_{11}\partial_i \vec{\nabla}\cdot\vec{u} + F_i^{NEQ}(\vec{r}) + \delta_{iy}F_y^{EQ}(\vec{r}, u_y)$$

NEGLECT

$$\langle F_y^{EQ}(\vec{r})\,F_y^{EQ}(\vec{0})\rangle \simeq \Delta\,\delta(\vec{r})$$

$$\langle F_i^{NEQ}(\vec{r})\,F_j^{NEQ}(\vec{0})\rangle = g_{ij}\,\delta(\vec{r}) \qquad g \sim \left(\frac{\Delta}{v}\right)^2$$

ANISOTROPY: $q_x \sim c\,q_y^2/\gamma v$



- $$\langle[\Delta u_x]^2\rangle \sim g\,\frac{y^{3-d}}{\gamma v c_{66}}\,\mathcal{F}\!\left(\frac{|x|c_{66}}{\gamma v y^2}\right)$$ controlled by noneq. random drag and shear modes

- $$\langle[\Delta u_y]^2\rangle \sim \Delta\,\frac{y^{3-d}}{\gamma v c_{11}}\,\mathcal{F}\!\left(\frac{|x|c_{11}}{\gamma v y^2}\right)$$ controlled by equilibrium "periodic" force and compressional modes

$$\langle[\Delta u_x]^2\rangle \sim y^{3-d}$$

SRO in $d<3$

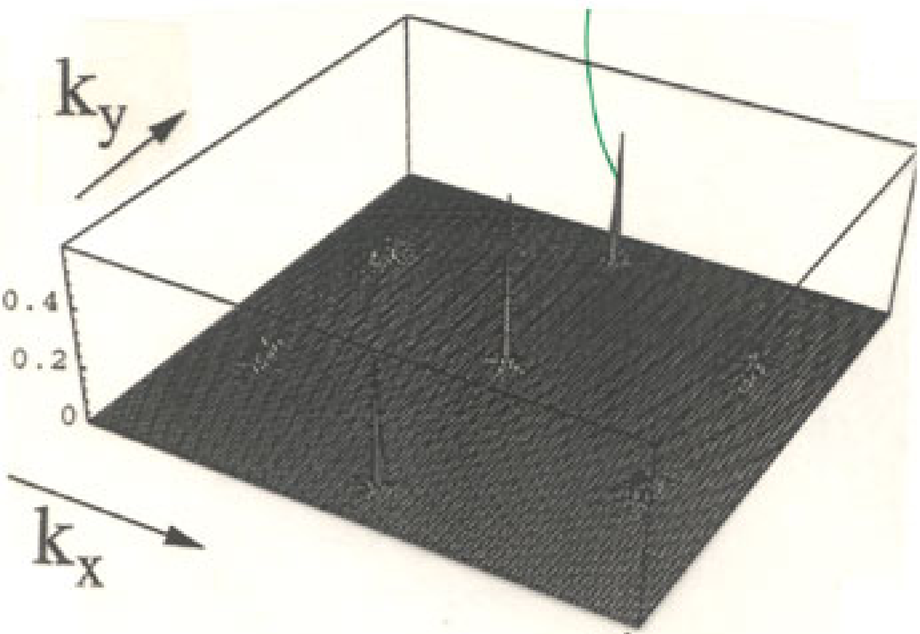
elastic theory breaks down

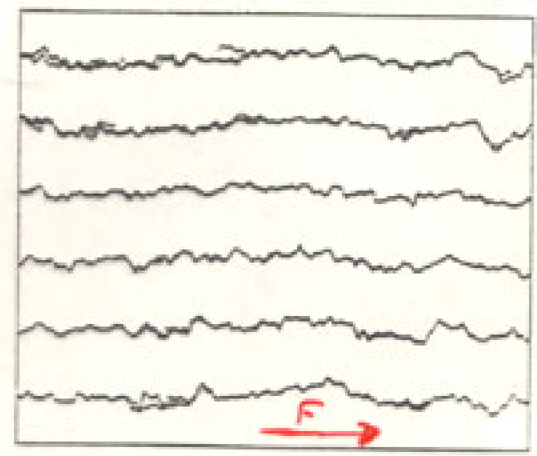
$\longrightarrow$ LIQUID

2d SIMULATIONS

by MOON, SCALETTAR, ZIMANYI
PRL 1996

(also RYU et al., SPENCER et al.)





Algebraic Bragg peaks along k_y

CHANNELS, but with LONGITUDINAL PHASE SLIPS

EXPERIMENTS

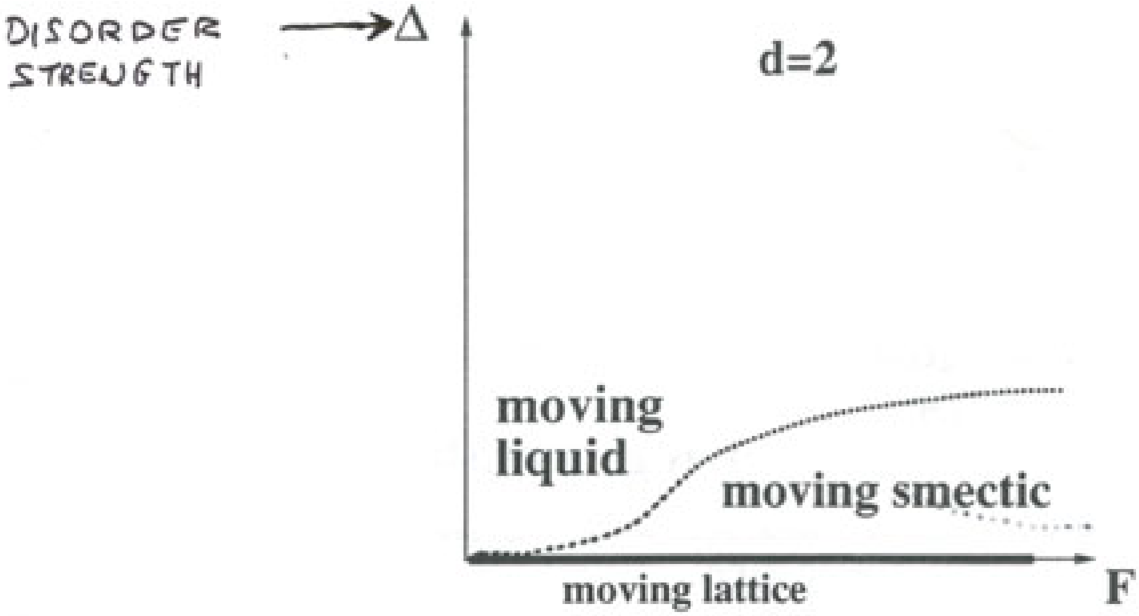
decorations of current-driven flux lattices

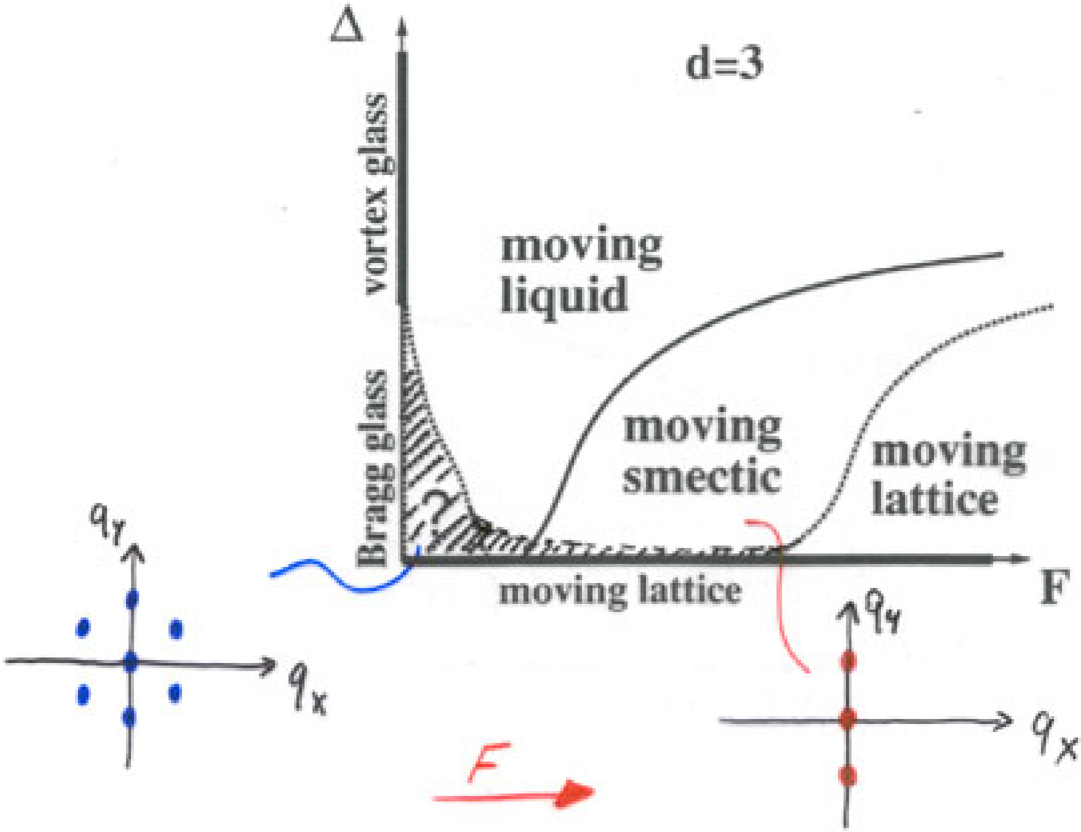
Marchevsky et al. (PRL 1997) → channels

Pardo et al. → MOVING SMECTIC

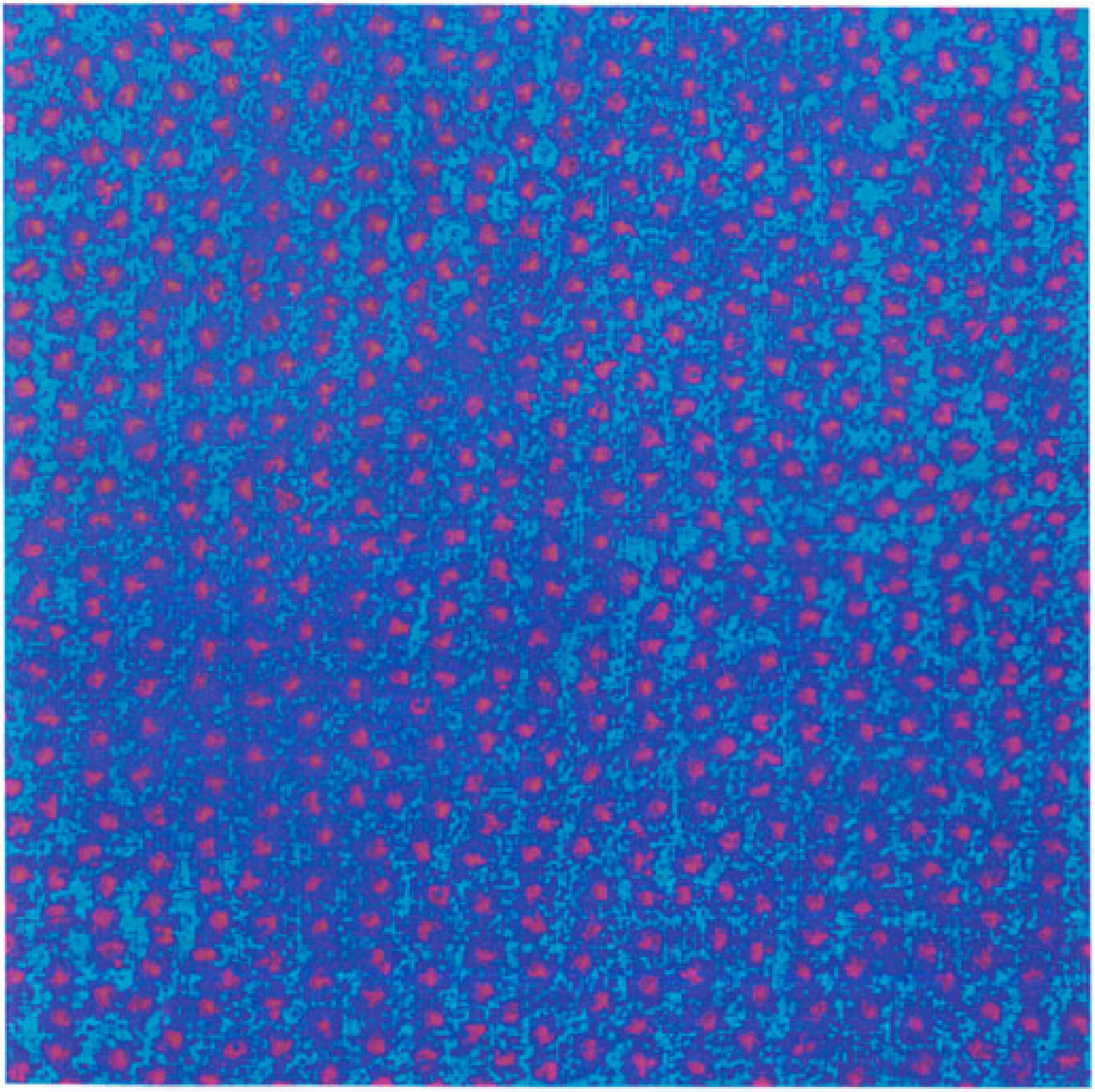
SCHEMATIC PHASE DIAGRAMS

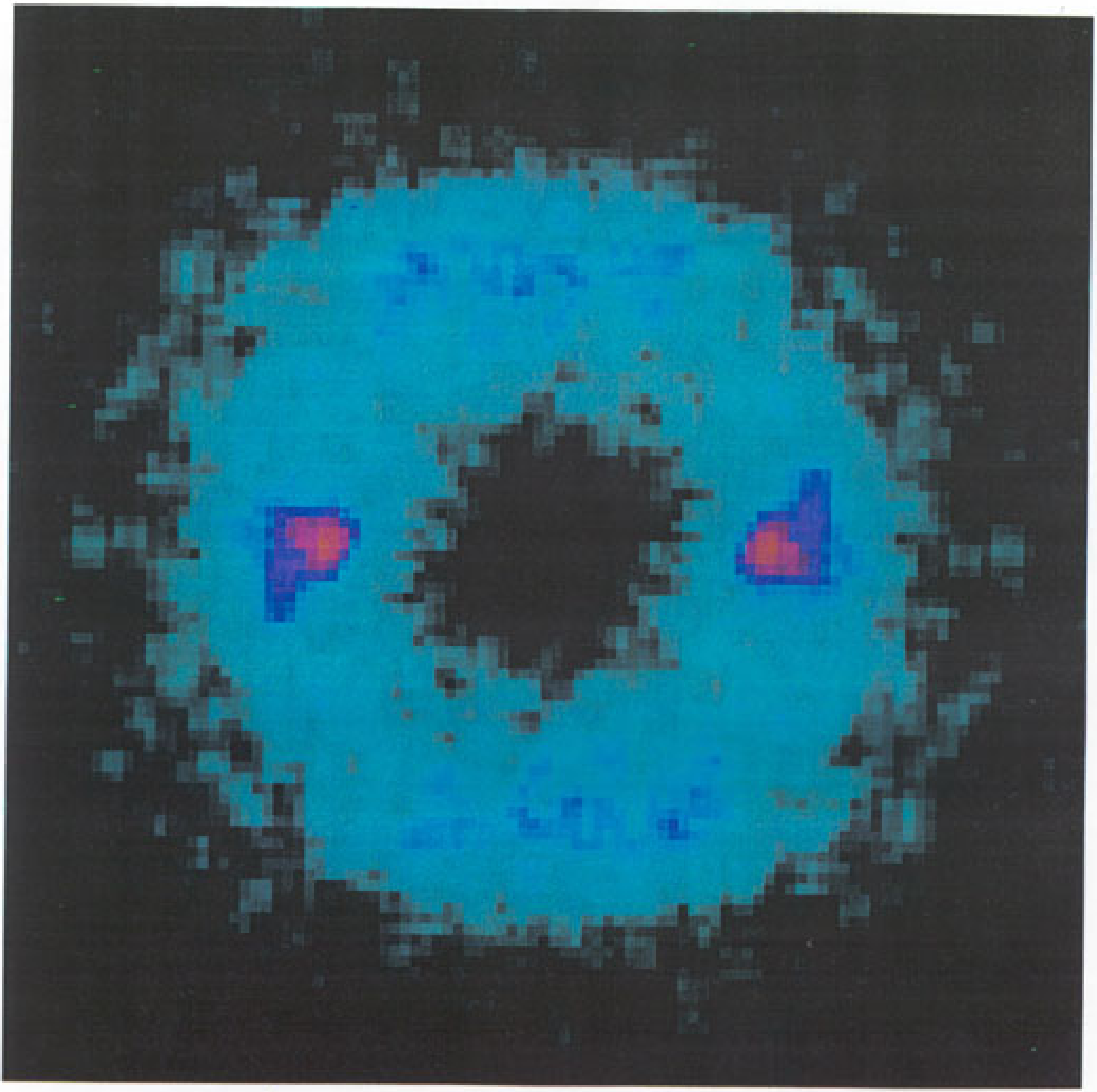
RG analysis of a model smectic



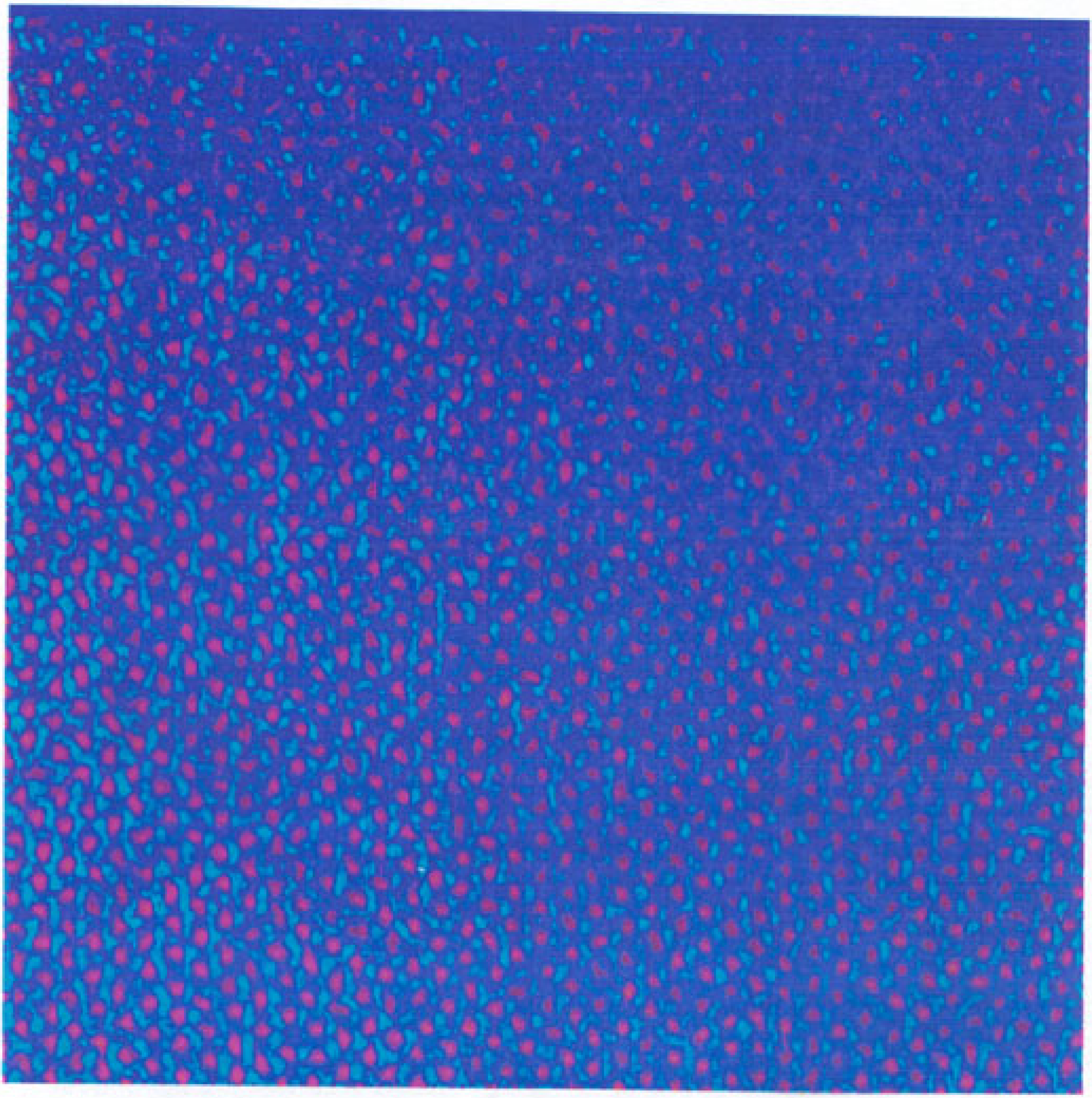


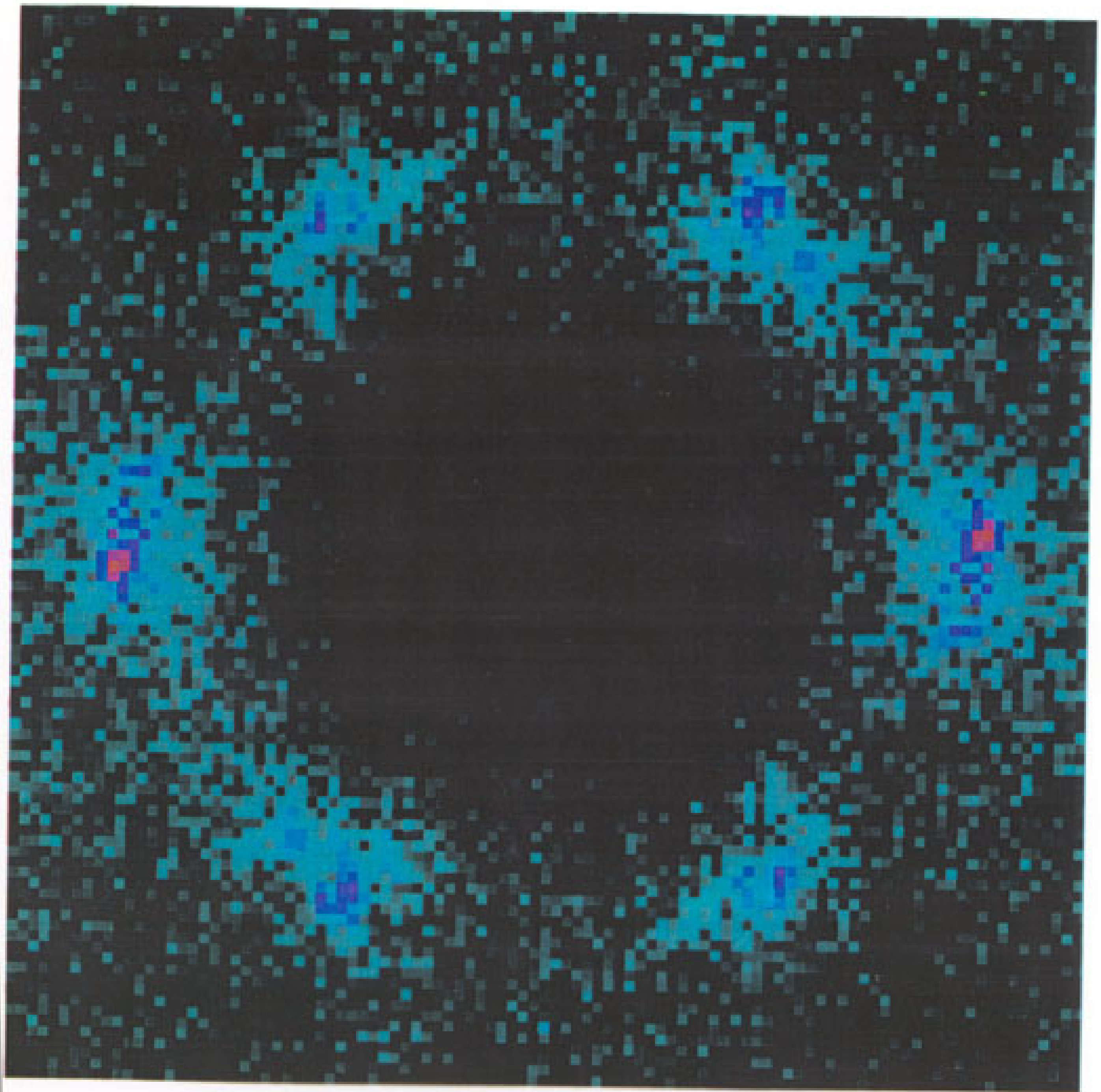
F. Pardo et al. , submitted to Nature NbSe$_2$, 4K , 5G





20G

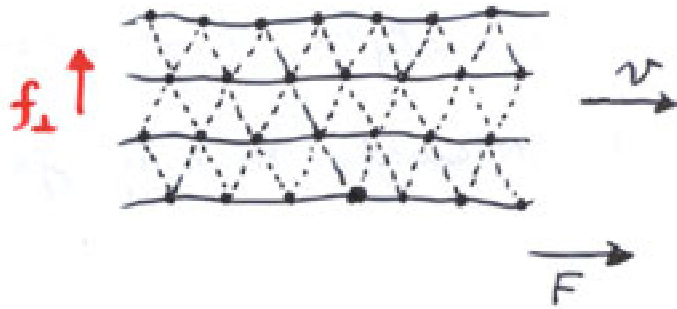




TRANSVERSE RESPONSE

Giamarchi & Le Doussal suggested that the driven Bragg glass would have at $T=0$ a (PRL 1996)

TRANSVERSE CRITICAL FORCE $f_{\perp c}$

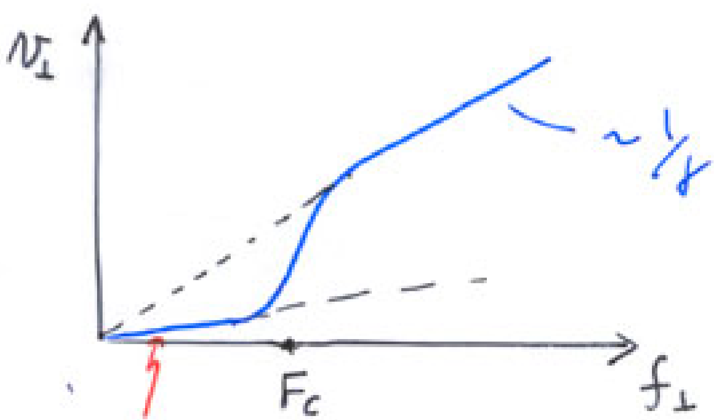


Both Bragg glass and smectic may have $f_{\perp c} \neq 0$ at $T=0$, **BUT**:

- $T=0$ not a stable fixed point
- density fluctuations are not negligible → permeation
  $\mu_{PERM} \sim e^{-U_\Delta/T}$

$T \neq 0$ smectic transverse response is LINEAR:

(neglecting permeation mo



renormalized by disorder $\sim \frac{1}{\gamma} e^{-(c/\gamma v a)^2/T}$

$d = 2 + \varepsilon$

Single-harmonic approximation for $\Delta(u)$:

$$\gamma(\partial_t + v\partial_x)u = (k_{\parallel}\partial_x^2 + k_{\perp}\nabla_{\perp}^2)u + F_0(\vec{r}) + F_1(\vec{r})\cos(y-u) + \eta(\vec{r},t)$$

$$\langle F_0(\vec{r})F_0(\vec{0})\rangle = \Delta_0\,\delta(\vec{r})$$

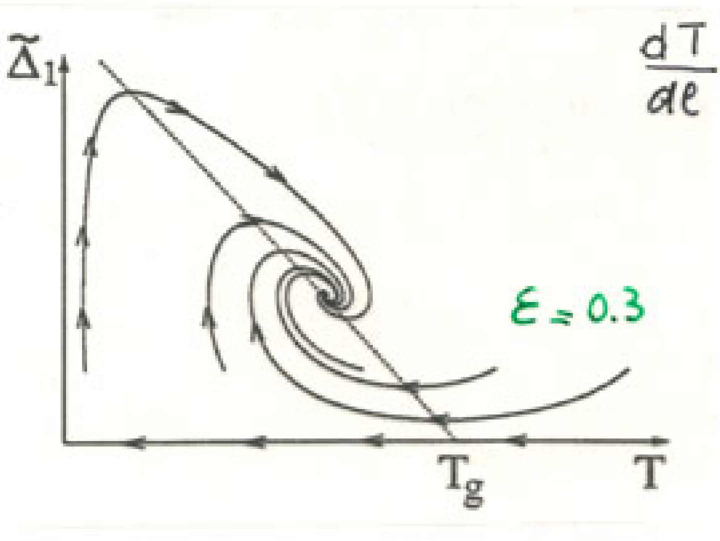
$$\langle F_1(\vec{r})F_1(\vec{0})\rangle = \Delta_1\,\delta(\vec{r})$$

- Δ_0 is strongly relevant, but it can be incorporated by shifting u (cf. Narayan & Fisher, 1992)
- RG equations:

$$\frac{d\Delta_1}{d\ell} = (3 - d - T)\Delta_1 - \frac{1}{2}\Delta_1^2$$

$$\frac{dT}{d\ell} = \left(2 - d + \frac{\Delta_1}{2}\right)T$$

disorder-induced "heating"

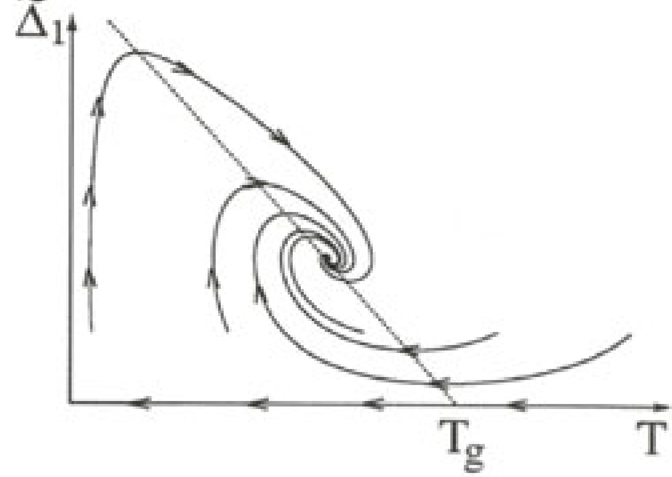


T_g: glass transition of a stationary 1+1 dim lattice (Cardy & Ostlund, 1982)

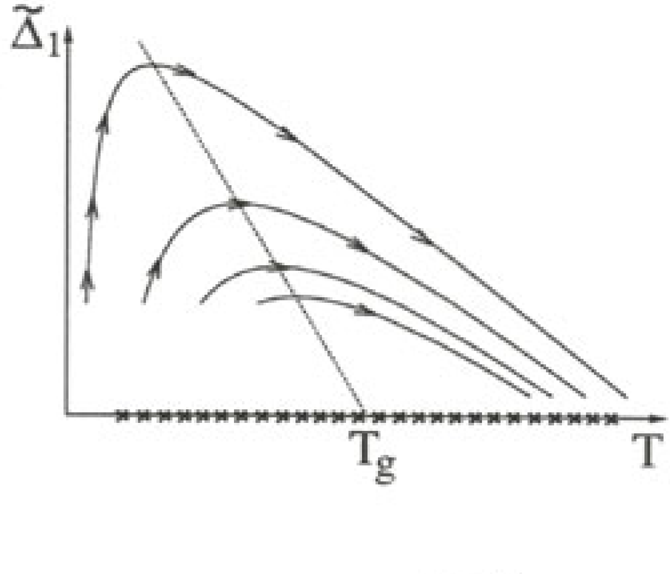
$d = 2$: $T \neq 0,\ \Delta_1 = 0$ stable fixed point

thermal moving smectic with $T_{eff}(T, \Delta)$

RG Flow Diagrams for
$d = 2 + \epsilon$



$d = 2 + \epsilon$, $\epsilon = 0.3$
for a set of initial conditions with $\overline{\Delta}_1(0) = 0.2$ and $\overline{T}(0)$ ranging from 0.05 to 0.85, in increments of 0.2.



$d = 2$
for a set of initial conditions with $\overline{\Delta}_1(0) = 0.2$ and $\overline{T}(0)$ ranging from 0.05 to 0.65, in increments of 0.2.

RENORMALIZATION GROUP ANALYSIS
of a simplified smectic model

Balents, MCM, Radzihovsky
cond-mat/9707302

Neglect coupling of layer displacement u_y to density:

$$\gamma\left(\partial_t + v\partial_x\right)u_y = K_{\parallel}\partial_x^2 u_y + K_\perp \nabla_\perp^2 u_y + \underbrace{F_p(\vec{r},u_y)} + \underbrace{\eta(\vec{r},t)}$$

"static" pinning force

thermal noise

$$\langle F_p(\vec{r},u_y)\,F_p(\vec{0},u_y')\rangle = \Delta(u_y-u_y')\,\delta(\vec{r})$$

PERIODIC with SMECTIC layer periodicity

$$\langle \eta(\vec{r},t)\,\eta(\vec{0},0)\rangle = 2\gamma T\,\delta(\vec{r})\,\delta(t)$$

- **functional RG for d=3-ε:** Narayan & Fisher, 1992
  Use MSR formalism to transform the stochastic eqn. of motion into a field theory, then integrate out the fast modes perturbatively in the disorder.

- **conventional RG in d=2+ε**
  single-harmonic approximation for $\Delta(u_y)$

> **Because of the lack of FDT, T is not irrelevant as indicated by naive power counting.
> The T=0 fixed point is unstable and is replaced by a finite temperature fixed point.** (2d)

$\boxed{d = 3 - \varepsilon}$

FUNCTIONAL RG $\Delta(u;\ell)$

$$\frac{\partial \Delta}{\partial \ell} = \Big[(3-d) + \underbrace{T\frac{\partial^2}{\partial u^2}}\Big]\Delta - \frac{1}{2}\Delta''(u)\left[\Delta(u) - \Delta(0)\right]$$

$$\frac{\partial T}{\partial \ell} = \Big[(2-d) - \underbrace{\left(\frac{1}{2} - \frac{\varkappa}{1+2\varkappa}\right)}\Delta''(0)\Big]\, T$$

these two terms cancel exactly in equilibrium due to FDT

$\varkappa \sim 1/v^2 \to \infty$ as $v = 0$

smooths out the "cusps" in $\Delta(u)$

$T = 0$ F.P. is unstable

$\Longrightarrow$ new $T \neq 0,\ \Delta \neq 0$ F.P. $T^* \sim (\ln \varepsilon)^{-1}$