Lecture #4: Measuring Social Behavior
(or: Emotions & Math)

Slides → We're good at measuring quantities from images now

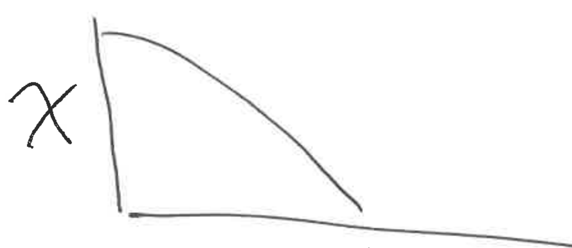
→ We've also made a lot of progress in understanding how to quantify behavior of individual animals

→ Social Behavior is still a mess!

Vicsek: $\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{V}_i(t)\Delta t$ $\quad (\vec{v}_i = v_0 \langle \cos\theta_i, \sin\theta_i \rangle)$

$\theta_i(t+1) = \langle \theta_j(t) \rangle_{n.n.} + \eta_i(t)$

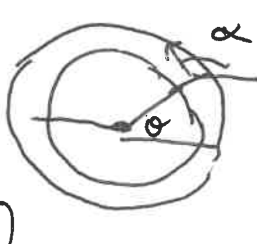
→ Measure polarization $\chi = |\langle e^{i\theta_j} \rangle_j|$



Buhl:

$\vec{X}_i(t+1) = \vec{x}_i(t) + v_0 \vec{u}_i(t)$

$\vec{u}_i(t+1) = \alpha \vec{u}_i(t) + (1-\alpha) G(\langle u_j(t) \rangle_{n.n.}) + \eta_i(t)$



$x = \sin^{-1}[\sin(\theta - \alpha)]$

$\Phi = \frac{2}{N\pi} \sum_{i=1}^{N} \chi_i$

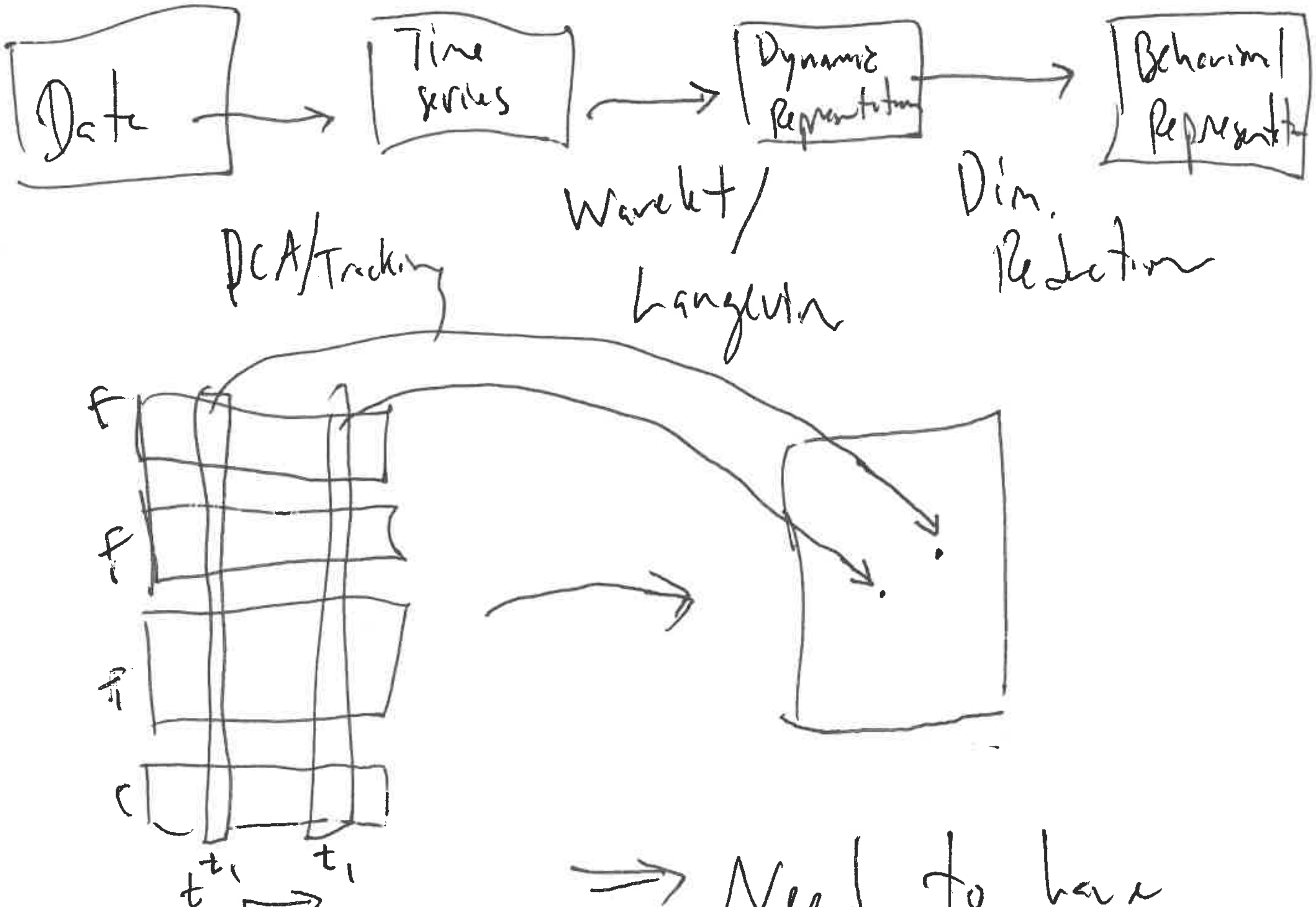
$G(x) = \begin{cases} \frac{(x+1)}{2} & x \geq 0 \\ \frac{(x-1)}{2} & x < 0 \end{cases}$

What makes social behavior difficult?

- → High-dimensional
- → Occurs in space
- → Small, but not large, group of animals
- → Many important states are hidden
- → No good definition of an interaction

First: space

→ Why can't we just include space in our feature vector?



→ Need to have a notion of $D(t_1, t_2)$

→ Wavelets have the same units, so this is somewhat straightforward. But taking space into account is difficult.

→ Units change

Possible solutions:

$$p(\vec{X}_1, \vec{X}_2, \vec{B}) = p(\vec{B} \mid \vec{X}_1, \vec{X}_2)\, p(\vec{X}_1, \vec{X}_2)$$

$$= p(\vec{X}_1, \vec{X}_2 \mid \vec{B})\, p(\vec{B})$$

③ → look at conditionals

Problems: 1) Assumes statistical stationarity
2) Lots of numbers!
3) still just correlative

Next Idea: if we're going to be correlative, own it and do a regression

Typical → 1) Pick a focal event

2) $p(\text{Event} \mid \vec{y}) = g(W\vec{y} + \vec{b})$

($\vec{y}$ ← Environment, other animal, history)

often, $\vec{y}$ is a time trace of many variables

→ Slide

Problems: → Still assumes stationarity
→ Still hard to interpret
→ Requires choice of focal event

What is an interaction?

→ At its core, many of these problems have to do with a lack of understanding of how to quantify the effects of one individual's behavior on another's

→ Example → Jazz Trio

→ There is an understanding of the roles of interaction (sheet music)

→ if they were to play their parts separately, it would inevitably be the same

→ Requires dexterous motor coordination to perform individual roles

→ Not communicated through visual cues

→ Can be measured through collective variable

→ How can you tell that they're interacting?

(Marcus Roberts, Jason Marsalis, Rodney Jordan)

→ Slides (Interaction / Synchronization)

What are some ways to measure an interaction?

1) Synchronization

2) Similarity

3) Emergence

4) Predictability

Takens' Embedding Theorem

Have some dynamical system: $\frac{d\vec{X}}{dt} = f(\vec{X})$

If well-behaved: Take $X_i(t)$ → $\{X_i(t), X_i(t+\tau), \dots, X_i(t+n\tau)\}$

(delay embedding)

↕ 1-1 mapping

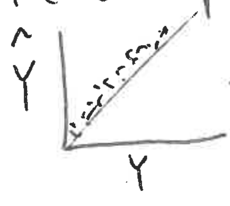
underlying full dynamical system

→ If $\frac{dX_i}{dt}$ depends on X_j → should be able to reconstruct X_j value based on the delay embedding of X_i

In practice: 1) M_X → delay embedding of X   Sugihara, 2012

2) For a point in time → look within a neighborhood in M_X

3) Find the corresponding points in M_Y → Average → $\hat{Y}$



→ correlation coefficient = $C_{X \to Y}$

Idea:

1) Take time series data of two animals (positions / RMW dim red.) X, Y

2) Calculate $C_{X \to Y}$ & $C_{Y \to X}$ for time intervals

3) $X(t) = C_{X \to Y} + C_{Y \to X}$

4) Compare to value for "ghost" animals $X_c(t)$

5) $X(t) - X_c(t) > 0 \Rightarrow$ Interaction