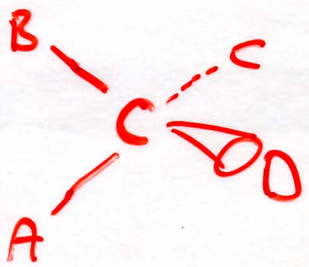
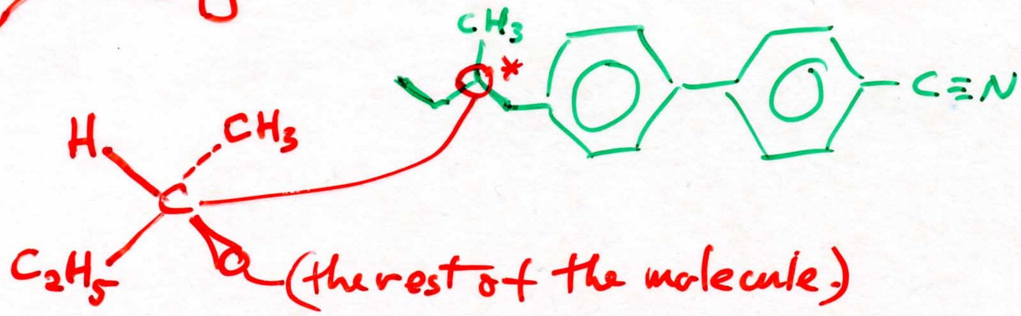
Chirality: lack of mirror symmetry.



Tetrahedral bonding of 4 different "things" to a carbon atom makes it chiral.



Elastic free energy: try adding terms linear in the curvatures, and see that they are eliminated by symmetry. Without mirror symmetry we find a term linear in twist $(n\cdot\nabla\times n)$ is allowed.

nematic: $$f = \frac{1}{2}K_{11}(\nabla\cdot\hat{n})^2 + \underbrace{\frac{1}{2}K_{22}(n\cdot\nabla\times\hat{n}+t_0)^2}+\frac{1}{2}K_{33}(\hat{n}\times\nabla\times\hat{n})^2$$

$$\left.\begin{aligned} n_x &= \cos tz \\ n_y &= \sin tz \\ n_z &= 0 \end{aligned}\right\}$$ Planar system $n\cdot\nabla\times n = -\dfrac{\partial\theta}{\partial z} = -t$

$\frac{1}{2}K_{22}(t_0-t)^2$ minimized when $t=t_0$.

$(\theta = tz)$ ← "cholesteric helix with torsion t, and "Pitch" $P=\dfrac{2\pi}{t}$.

Smectic C: 2 effects on elasticity + polarity

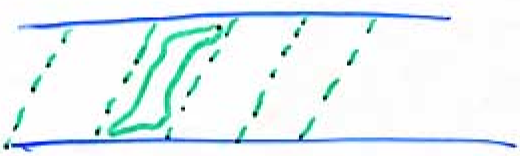
① helix: tilt direction $\hat{c}$ precesses around the layer normal, making a helix, like the cholesteric.

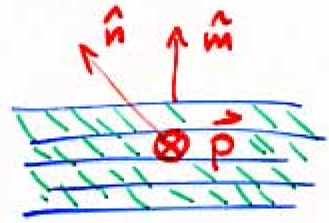
② In a smectic layer: $(\nabla\times\hat{c})_z$ term (spontaneous bend) is allowed in the free energy:

$$f_c = \frac{1}{2}K_s(\nabla\cdot\hat{c})^2 + \frac{1}{2}K_B\left[(\nabla\times\hat{c})_z - b_0\right]^2$$

f_c is minimized for $(\nabla\times C)_z = b_0$. Can this be done uniformly, as the cholesteric helix does for t ?? No! – not in the plane of the smectic layer. The helix (precession of $\hat{c}$ or $\hat{n}$ from layer to layer) involves <u>twist and bend of $\hat{n}$</u>, but the bending of $\hat{c}$ in a single layer is a separate effect!

(3) Chiral smectic C is ferroelectric.

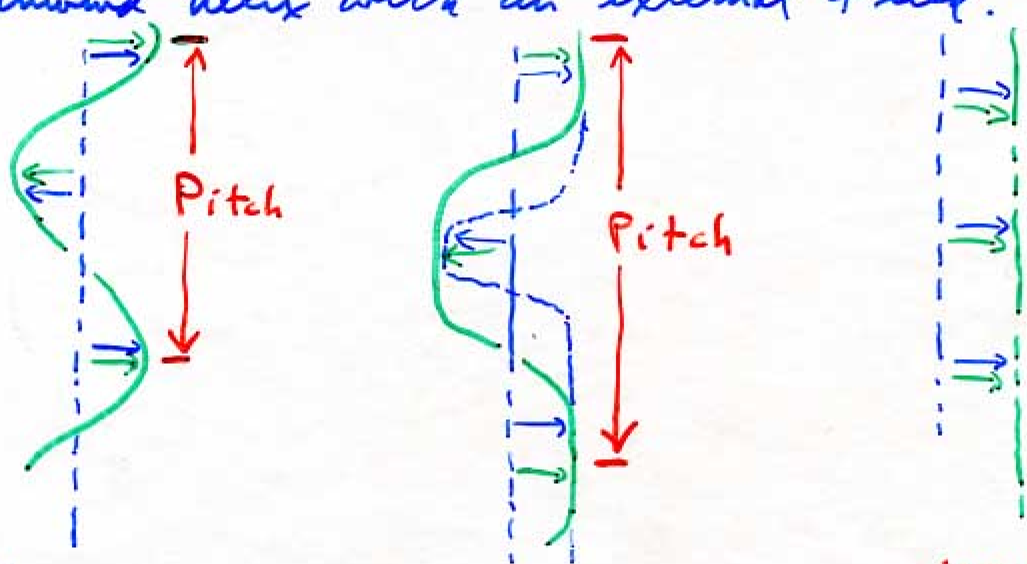




$\vec{P} = P_0\, \hat{m}\times\hat{n}$

Problems involving chirality

(1) Unwind helix with an external field.



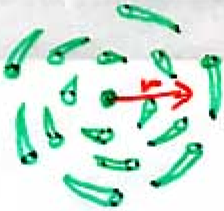
$H = 0$ $\qquad$ $0 < H < H_c$ $\qquad$ $H > H_c$ $\quad \rightarrow = \hat{c}$

$E = 0$ $\qquad$ $0 < E < E_c$ $\qquad$ $E > E_c$ $\quad \rightarrow = \vec{P}$

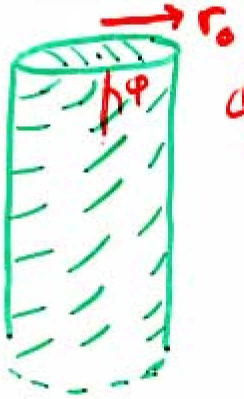
Blue Phases "Modulated Phases"

Double twist cylinders: For the right sign of the saddle-splay elastic energy this cylinder has lower free energy than the simple cholesteric helix. But this is only true within a small radius around the tube axis, up to $\sim \varphi = \pi/4$.

Top



$\varphi = t\, r$

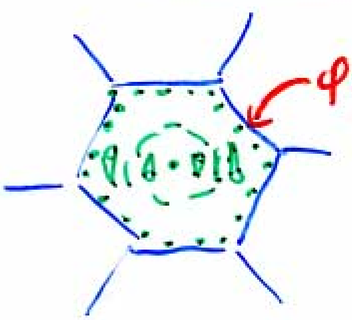


$\varphi(r_0) = \frac{\pi}{4}$

Side

Fill space with tubes packed together.

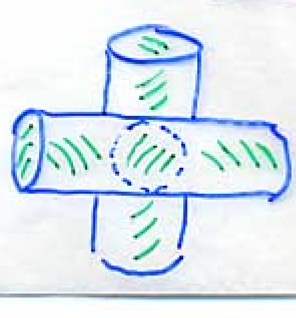
2D hex pattern:



This looks good since tubes join with a continuous vector field (no defects). But for $\frac{\pi}{2} < \varphi < \pi$, saddle splay reverses sign, so $\int dx\,dy\,(\text{tube})\, K_{24}(---) = 0$

Without the negative saddle splay term, all the distortions in this texture just cost energy, compared to the cholesteric helix.

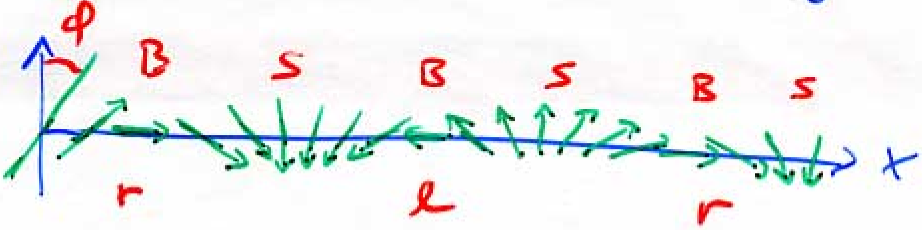
Cubic patterns Pack cylinders with $\varphi = \pi/4$ at r_0, with $\hat{n}$ continuous at contact. Then build 3D network, with arrays of disclination points and/or lines. These internal "surfaces" break the $\iint K_{24}(---)\, d^2r = 0$ rule!!



Sm C* 2D layer: Modulated Phase again!

Try to construct texture with $\nabla \times \hat{c} = b_0$.

try $\phi = \alpha x$



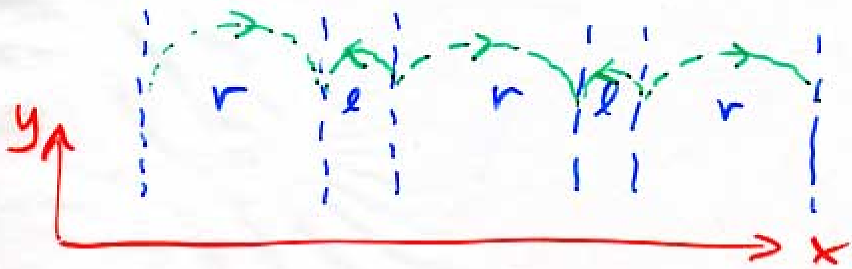
r = right turn in $\hat{c}$ direction

l = left turn in $\hat{c}$ direction

r = good, l = bad } for lowering energy.

$$\int \nabla \times \hat{c}\, dx = 0$$

Next: try to "shrink" bad parts, and expand good parts

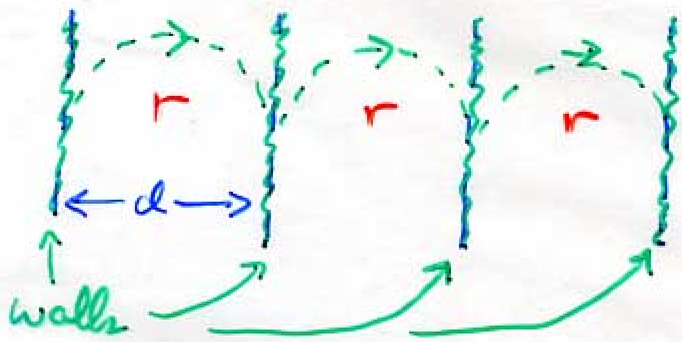


Doesn't help since

$$\int_a^b dx\,(\nabla \times c) = c_y(b) - c_y(a)$$

which is independent of domain width.

Solution: Shrink "bad" parts until they are so small that non-linear terms lower their "bad" energy. Then the "good" parts can win! Final domain width is a balance between negative chiral energy and positive wall energy, and positive (quadratic) curvature energy.



$$f = \underbrace{\frac{-\text{chiral}}{d} + \frac{\text{wall}}{d}}_{\text{if } -,\ \text{then domains}} + \frac{\text{curvature}}{d^2}$$