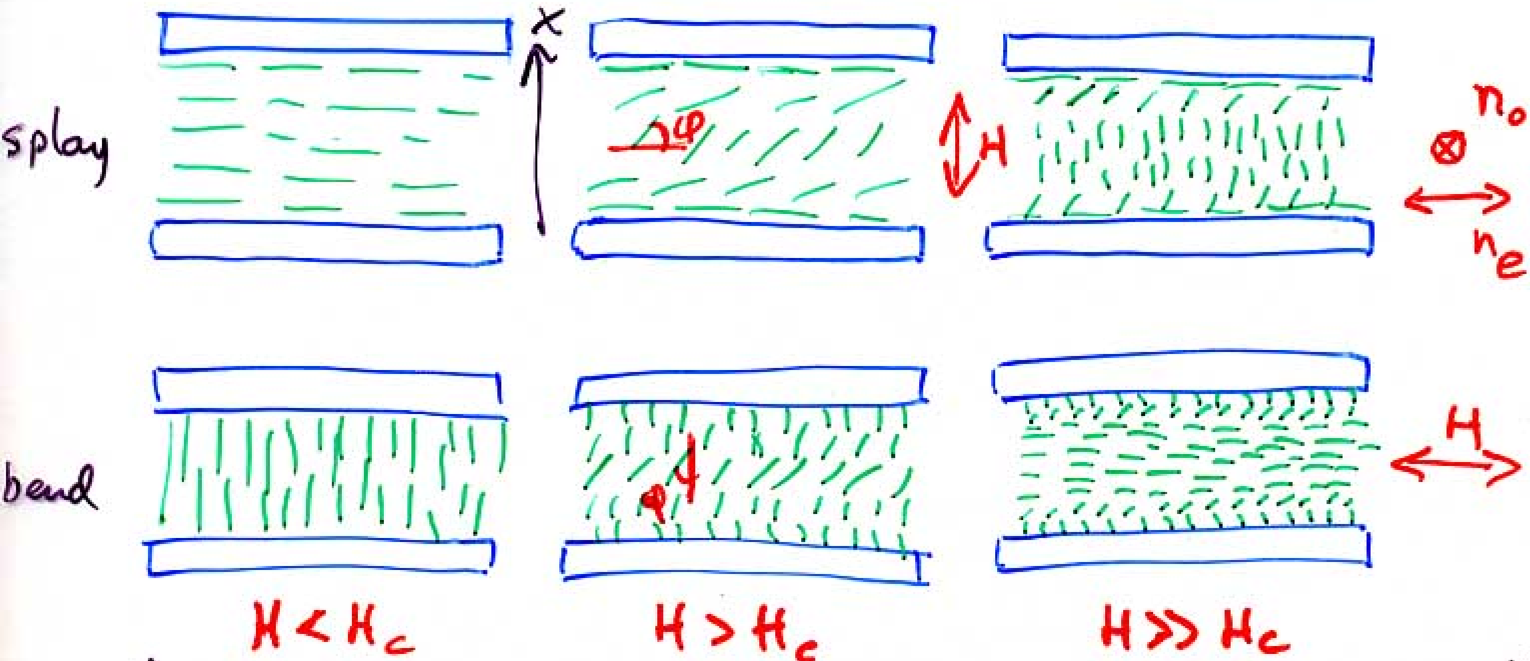
Optics of thin layers of liquid crystal.

① Splay-bend Fredericks transition



Light polarized $\perp$ to plane of the page is "ordinary", with index of refraction n_o - independent of tilt angle φ.

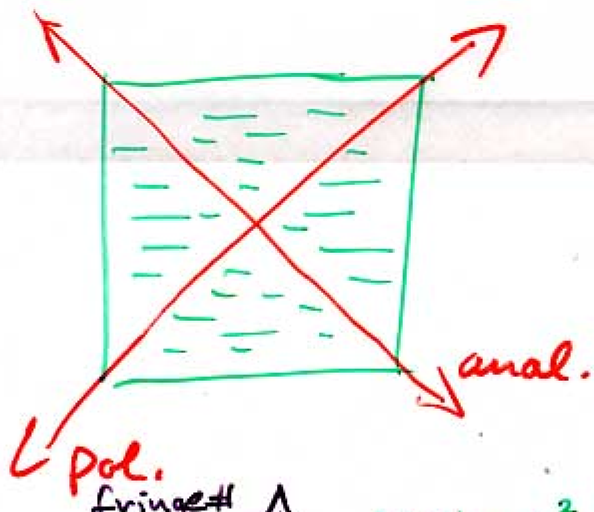
Light polarized parallel to the plane of the page is "extraordinary", with $n_e(\varphi)$.

Phase shift between ord. and extraord. waves in crossing the sample is

$$\Delta\psi = \frac{2\pi}{\lambda}\int_0^d (n_e(x) - n_o)\,dx = \frac{4\pi}{\lambda}\int_0^{\varphi_{max}} \frac{n_e(\varphi) - n_o}{\left(\frac{d\varphi}{dx}\right)}\,d\varphi$$

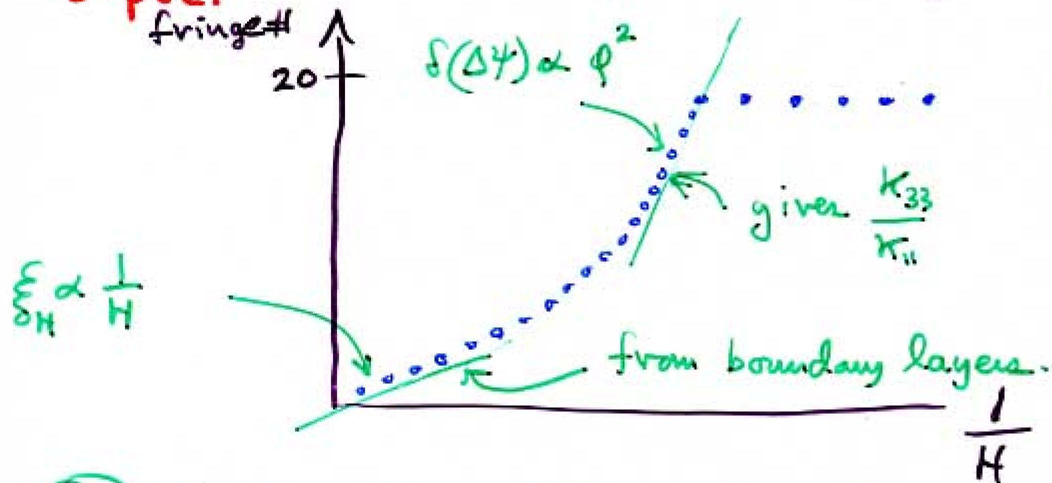
This is the integrated birefringence of the layer. Each change of $\Delta\psi$ by 2π produces an "interference fringe".

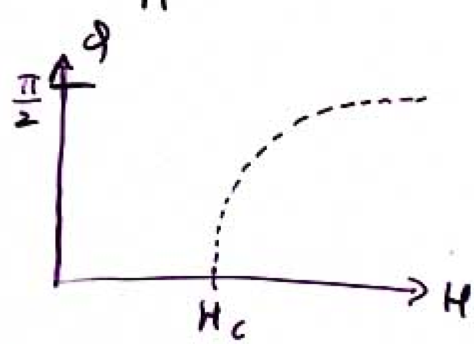
Using crossed polarizers with the sample between them:



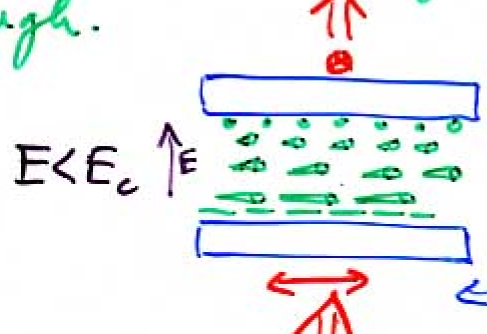
For the splay geometry, start with a sample with $\Delta\psi = 2\pi m$, with $m = 20$. For $H < H_c$ the sample is dark – no transmitted light.

Plot fringe # vs $\frac{1}{H}$





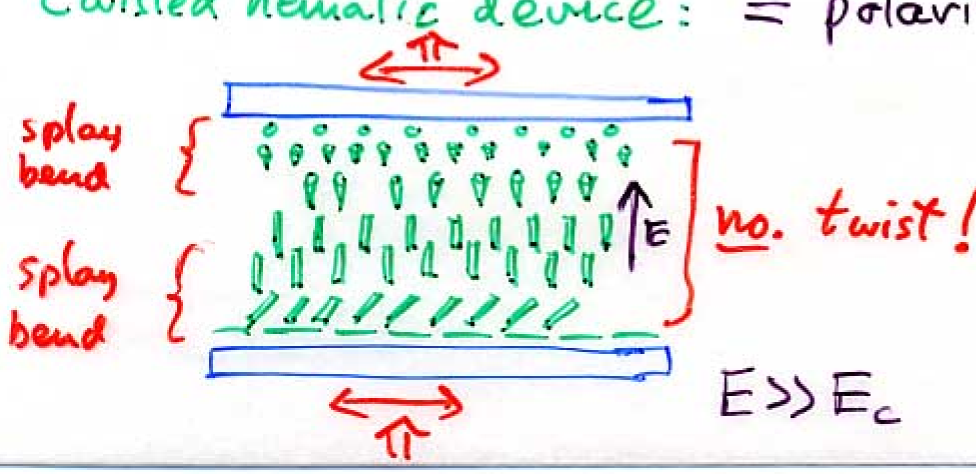
(2) twist effect: Polarization follows twist, if gradual enough.



rotates to exit parallel to n.

Polarization parallel to $\hat{n}$ entering sample

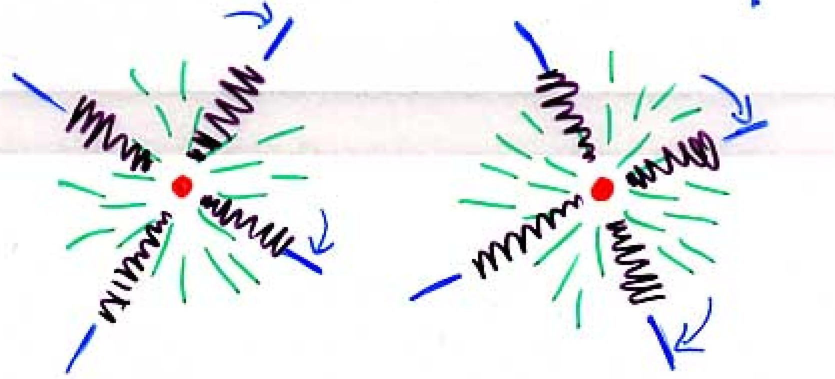
"twisted nematic device" = polarization rotation switch.



Polarization does not rotate while passing through sample.

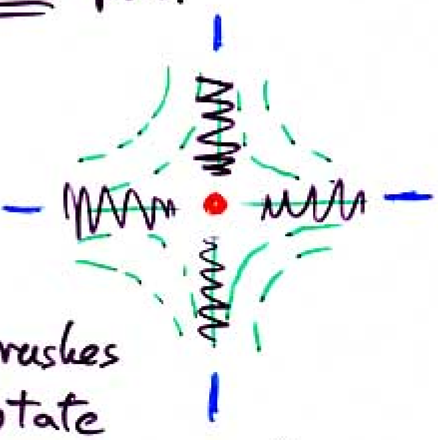
③ Mapping the director with crossed polarizers:

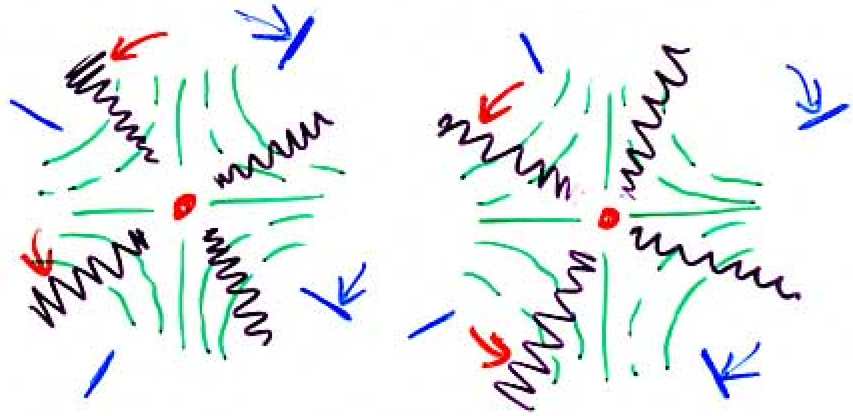
+1



brushes rotate **with** pols.

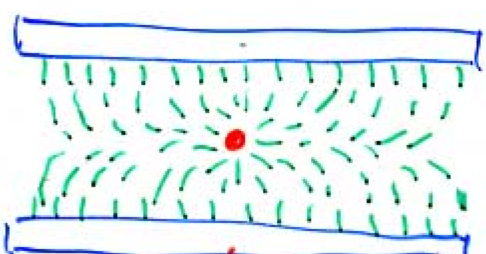
−1





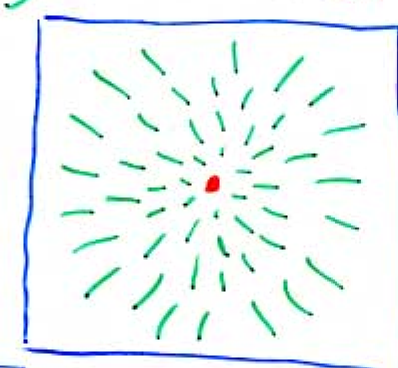
brushes rotate **opposite** to pols.

Combining ① and ③: Point disclination in a flat layer with normal boundary conditions:



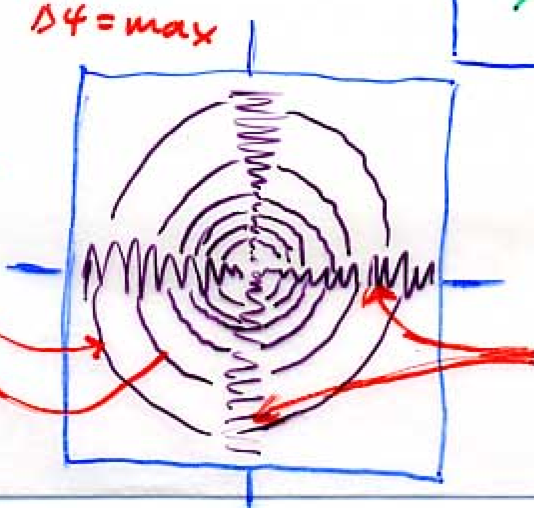
$\Delta\psi = 0$ $\Delta\psi = max$

side view



top view

$\Delta\psi = 2\pi n$ rings



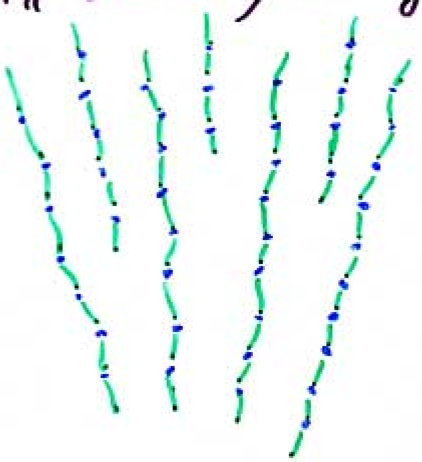
Between crossed polarizers

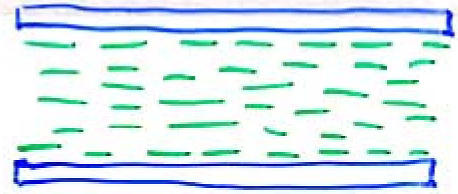
extinction brushes

Splay avoidance in the Frederiks transition

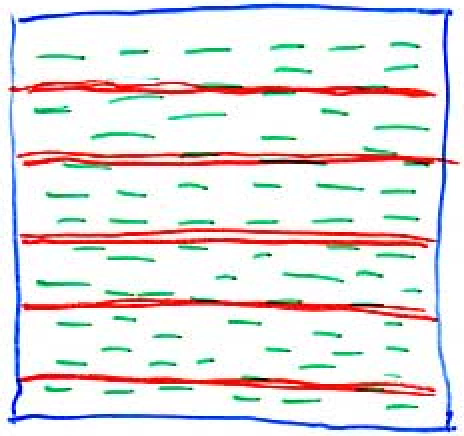
In nematics based on very long chain molecules,

K_{11} is very large:

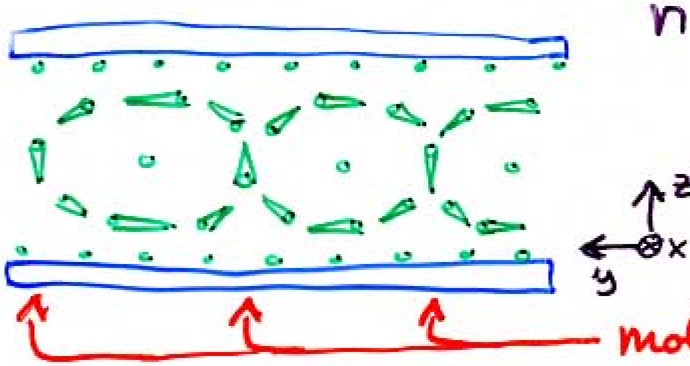




$$H_c = \frac{\pi}{d}\sqrt{\frac{K_{11}}{\chi_a}}$$



Above some H_0, see stripes <u>parallel</u> to the initial director



$\hat{n} = (1, n_y, n_z)$ <u>linearized free energy</u>

$$\frac{F}{\ell} = \frac{1}{2}\iint dy\, dz \Big[K_{11}\Big(\frac{\partial n_y}{\partial y} + \frac{\partial n_z}{\partial z}\Big)^2 + K_{22}\Big(\frac{\partial n_y}{\partial z} - \frac{\partial n_z}{\partial y}\Big)^2 - \chi_a H^2 n_z^2 \Big]$$

Note: set $n_y = 0$, E.L. eqn is $\frac{\partial^2 n_z}{\partial z^2} = -\frac{\chi_a H^2}{K_{11}} n_z$, giving

$n_z = A\cos\left(\frac{\pi z}{d}\right)$, $H_c = \frac{\pi}{d}\sqrt{\frac{K_{11}}{\chi_a}}$, as before.

From translational symmetry, we can say that:

$n_z = f(z)\cos(qy) \quad n_y = g(z)\sin(qy)$ integrate over y:

$$\frac{F}{\ell} = \frac{1}{4}\int_{-\frac{d}{2}}^{+\frac{d}{2}} dz \Big[K_{11}(gq + \underbrace{f_z}_{\frac{\partial f}{\partial z}})^2 + K_{22}(\underbrace{g_z}_{\frac{\partial g}{\partial z}} + qf)^2 - \chi_a H^2 f^2 \Big]$$

E.L. equations: $K_{11}(g q^2 + q f_z) - K_{22}(g_{zz} + q f_z) = 0$

$$-K_{11}(q g_z + f_{zz}) + K_{22}(q g_z + q^2 f) - \chi_a H^2 f = 0$$

general solution: $f = A_1 \cosh(q_1 z) + A_2 \cos(q_2 z)$

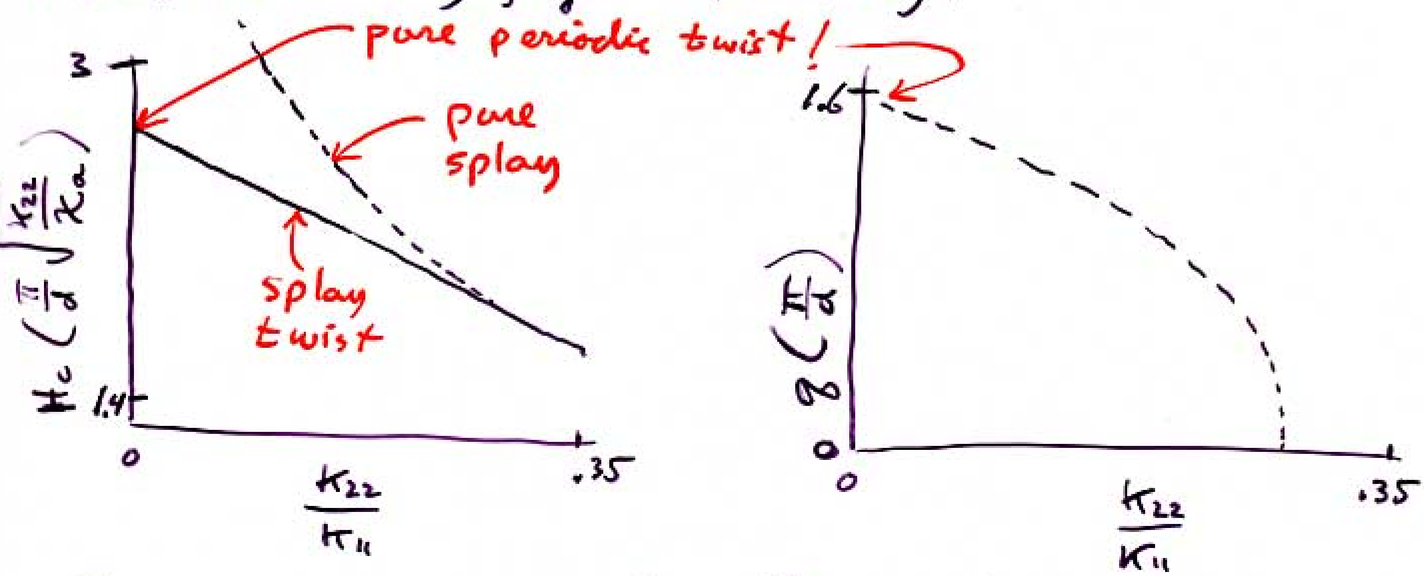
$$g = B_1 \sinh(q_1 z) + B_2 \sin(q_2 z)$$

Get q_1 and q_2 from the E.L. equations

Boundary conditions: $f = g = 0$ at $z = \pm \frac{d}{2}$:

$$\left(\frac{B_1}{A_1}\right)\left(\frac{A_2}{B_2}\right) \tanh\left(\frac{q_1 d}{2}\right) = \tan\left(\frac{q_2 d}{2}\right)$$

Solve numerically, get following:



For $K_{11} \to \infty$, no splay allowed $(gq + f_z = 0)$

$$n_z = f_q \cos qy \quad n_y = -f_z \sin qy$$

$$q = 1.51 \frac{\pi}{d} \quad H_c = 2.64 \frac{\pi}{d}\sqrt{\frac{K_2}{\chi_a}}$$

Important concept: Given freedom, system will respond in the lowest free energy mode possible.

* At higher H, there is a transition back to the pure splay mode as it becomes impossible to avoid splay.