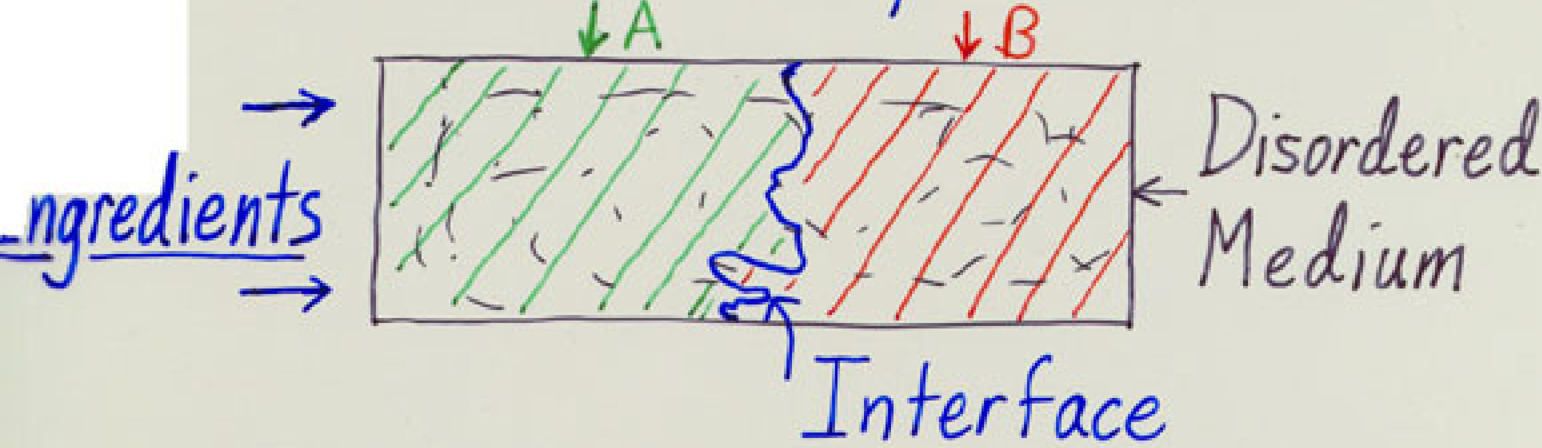
Two different phases (states)



→ Force ⊥ to interface favoring growth of A - external or thermodynamic

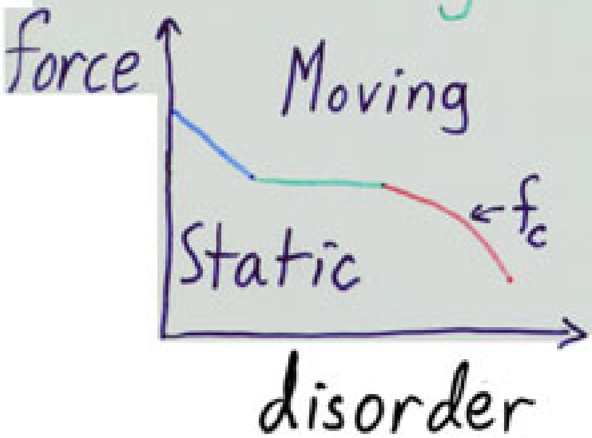
⇒ Ask how force & disorder change interface

Examples	Medium	Phases	Force
Fluid Invasion	Rock	Two fluids	Pressure
" Segregation	Gel	"	Free energy
Domain Growth	Ising	↑ and ↓	Magnetic Field
	Ferromagnet		Free Energy
Spreading	Surface	Two fluids	Gravity, pressure,...
Bacteria -	Petri Dish	Bacteria, Air	Nutrient, light
	Body		

Flux lattices, Charge-density waves, ...

' 1.

Three classes of depinning transition classified by morphology at f_c



— faceted (if on lattice)
— self-affine
— self-similar (percolation)

Large disorder → regions move independently → percolation
Weak disorder → lattice anisotropy wins → "faceted"
Intermediate " → compact growth, self-affine interface
⇒ in some models ⇐

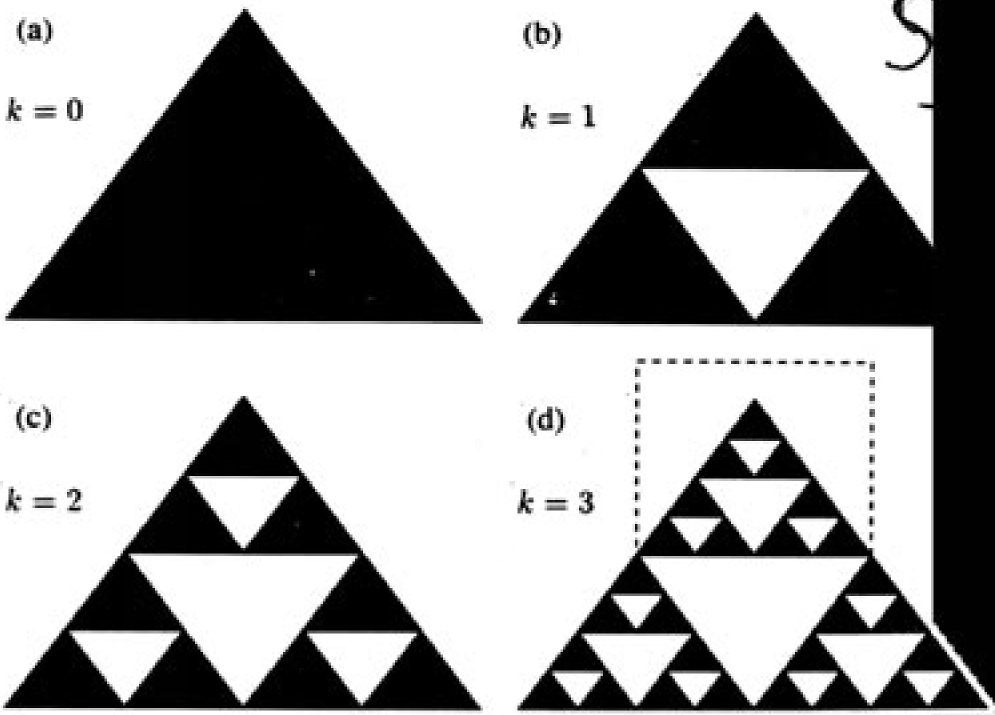
ical transitions between growth morphologies

ical transitions at onset of motion

$f < f_c \Rightarrow V \propto (f_c - f)^{-\gamma}$, power law "avalanche distribution

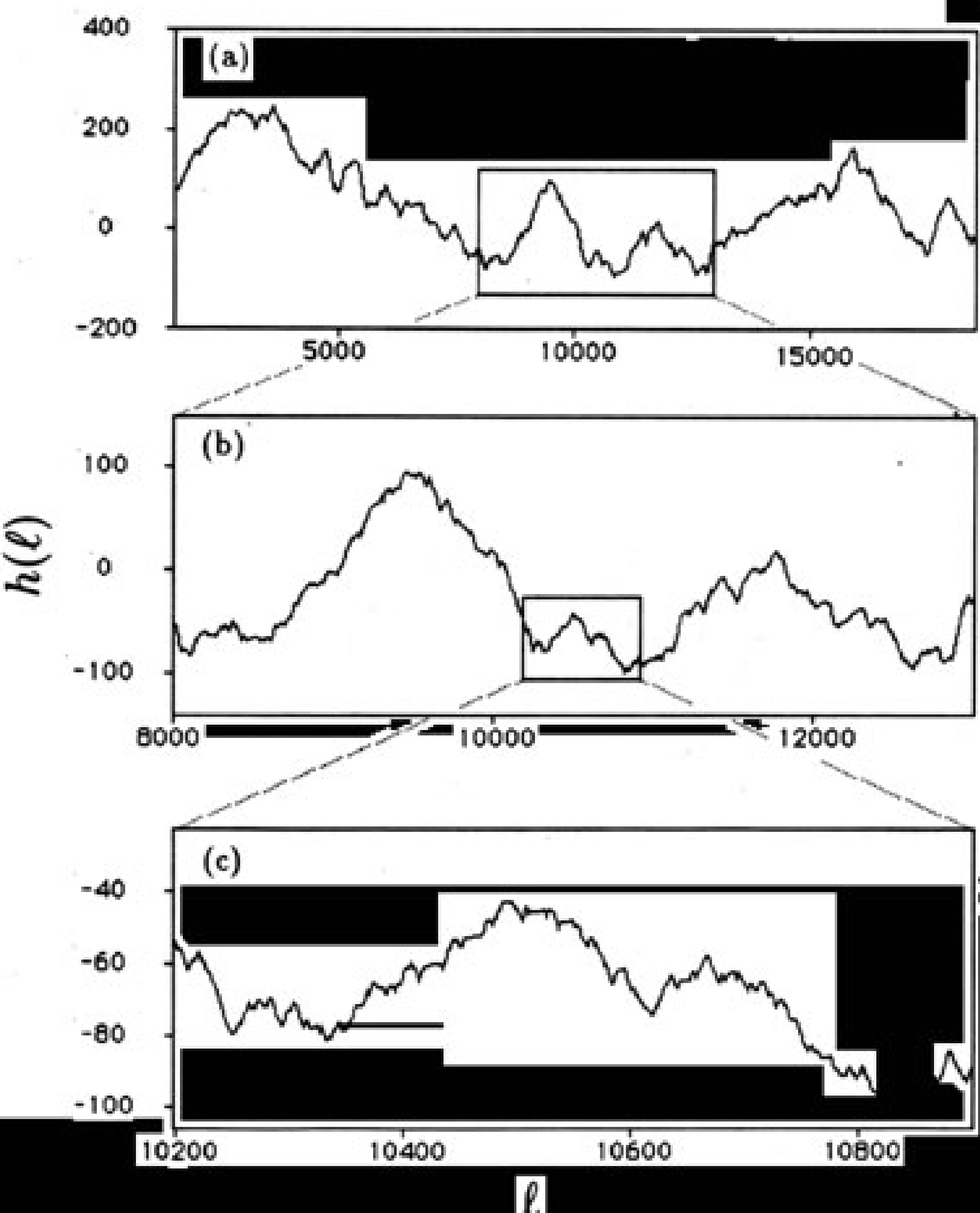
$f > f_c \Rightarrow v \propto (f - f_c)^{\beta}$, power law noise

Figure 3.2 Sierpinski gasket. (a) We start from the filled triangle, (b) remove from the middle a triangle whose area is one fourth of the total. (c) In the next step, we repeat the same procedure for the remaining three filled triangles. (d) This process is iterated indefinitely.

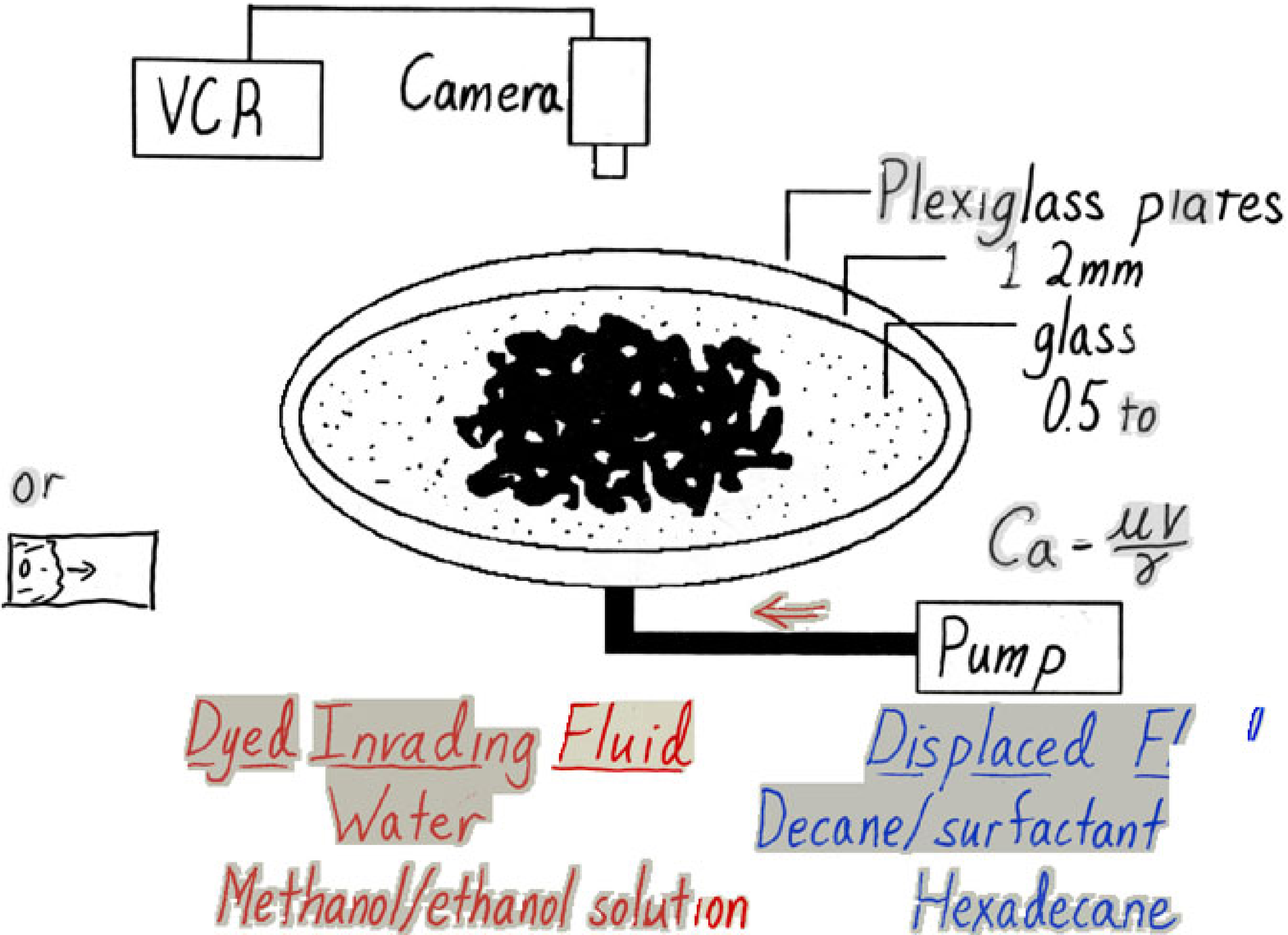


Self -affine

Figure 1.1 Rescaling a self-affine function, in this case the 'DNA walk' introduced in §1.3.2 (cf. Fig. 1.13). Only if the two unequal magnification factors, M_ℓ and M_h, by which the ℓ and h directions are re-scaled, are selected correctly will the enlarged portion have the same statistical properties as the original. (After [416].)



EXPERIMENTAL SET-UP



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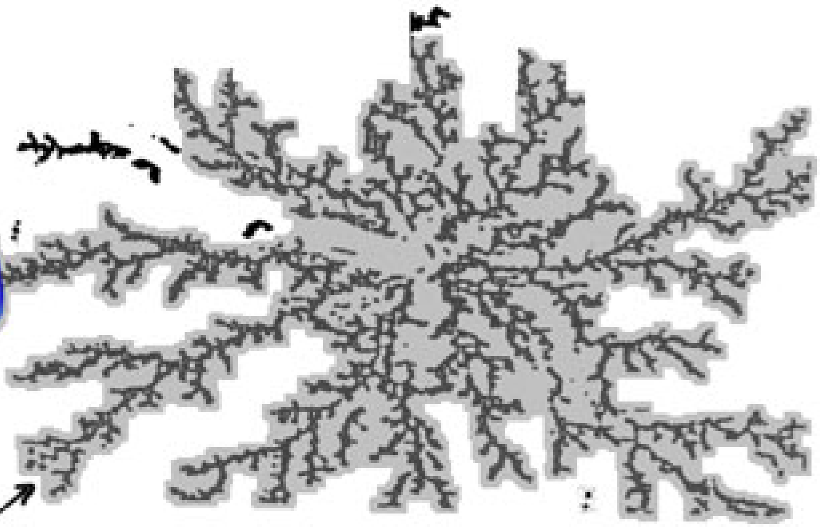
Non-Wetting

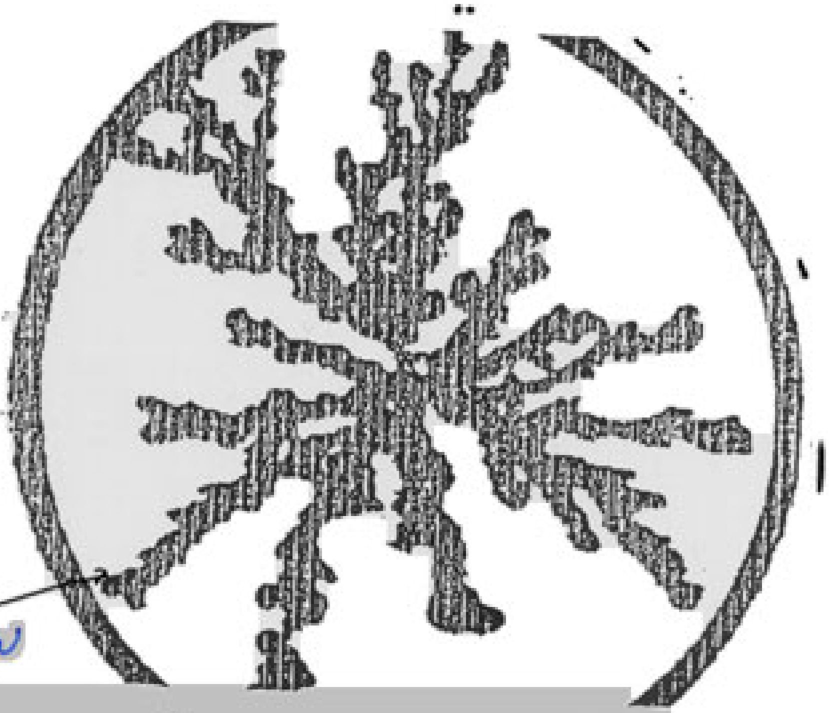


pore-scale fingers

NON-WETTING DISPLACING WETTING

MOBILITY RATIO 200

Wetting



fingerwidth w diverges as velocity decreases $w \propto v^{-1/2}$

WETTING DISPLACING NON-WETTING

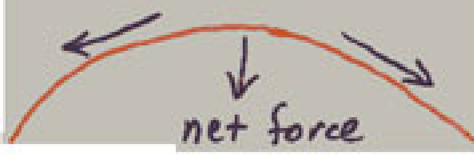
MOBILITY RATIO 200

Stokes et al

Say top view of circular cell

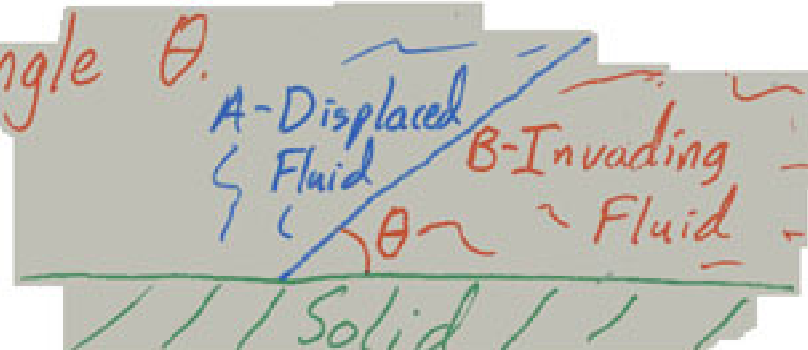
fa

κ_i curvatures in principal directions

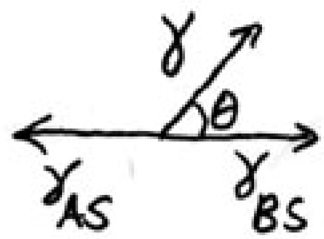


γ = surface tension between fluids

2 Fluid interface intersects solid at fixed contact angle θ.



Force balance:

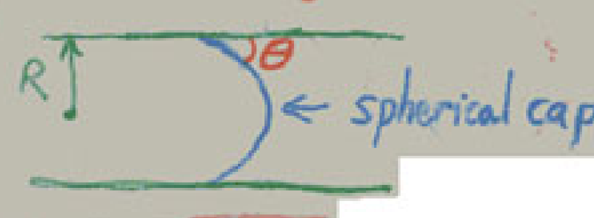


$$\gamma_{AS} = \gamma_{BS} + \gamma\cos\theta$$

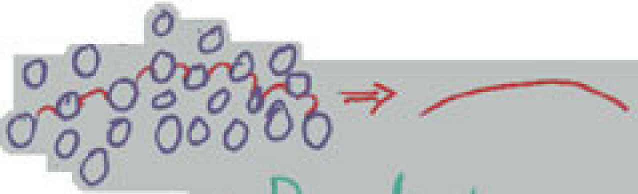
$$\cos\theta = \frac{\gamma_{AS} - \gamma_{BS}}{\gamma}$$

Perfect wetting $\frac{\gamma_{AS} - \gamma_{BS}}{\gamma} > 1$

For cylindrical tube interface only stable at $P = \frac{-2\gamma}{R}\cos\theta$



oni



$$\Delta P_{cap} = \gamma \cdot h_{macroscopic}$$

⇒ Predicts viscous fingering instability

$\lambda \simeq 2\pi \left[3k\gamma/\mu v\right]^{1/2}$ → seen for wetting

Invasion Percolation Model ⇒ Disorder dominates

(Lenormand & Bories '80, Chandler, Koplik, Lerman, Willemsen)

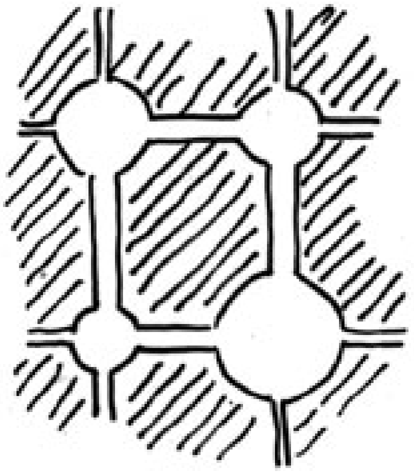
⇒ Pores connected by throats of varying size

At given P some throats are passable

Increase P → more become passable

Problem maps to bond percolation

Consistent with non-wetting invasion



Continuum

- Ignore disorder
- Use Darcy's law $\Rightarrow v = \frac{k}{\mu}(-\vec{\nabla}P)$

k - permeability
μ - viscosity

- Assume surface tension γ at scales $\gg$ pore size

$\Rightarrow$ viscous fingering instability $\lambda \simeq 2\pi\sqrt{\frac{3k}{Ca}}$

$Ca = \frac{\mu U}{\gamma}$

U = velocity

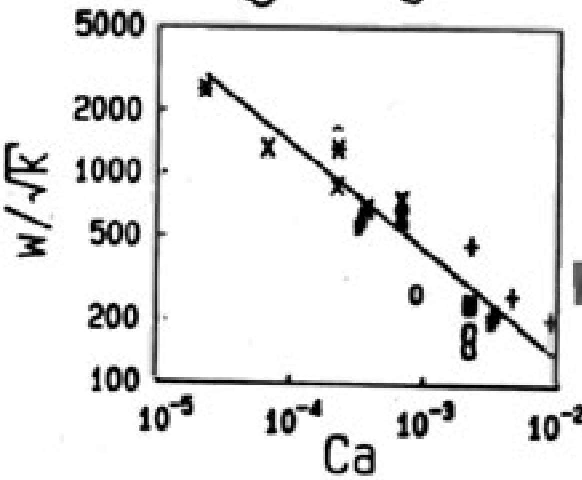


FIG. 2. Normalized finger width ($w/\kappa^{1/2}$) vs N_{Ca} for wetting displacement. The data points refer to asterisks, $d = 0.15$ mm; crosses, $d = 0.5$ mm; circles, $d = 1.7$ mm, all with N_{Ca} varied by changing Q; plusses, $d = 0.5$ mm, $Q = 1.5$ ml/min, with N_{Ca} varied by changing γ; number signs, $d = 0.5$ mm, with N_{Ca} varied by changing μ_2; caret, $d = 0.15$ mm, cell width decreased by a factor of 2. The solid line is a least-squares fit with a slope of -0.51.

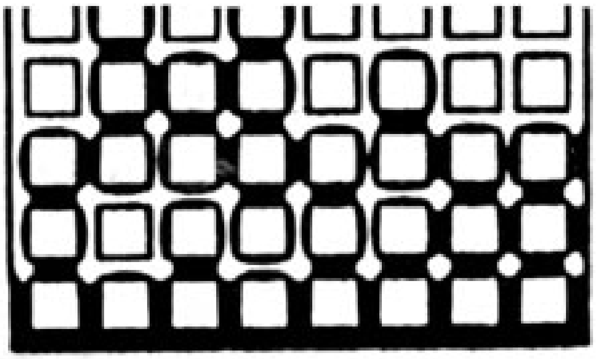
Wetting
Invasion

Non-Wetting
Invasion

PRL, 57, 1718 - Stokes, Weitz, Gollub, Dougherty, Robbins, Chaikin, Lindsay me

ed

Resin Networks (Lenormand + Zarcone 83)



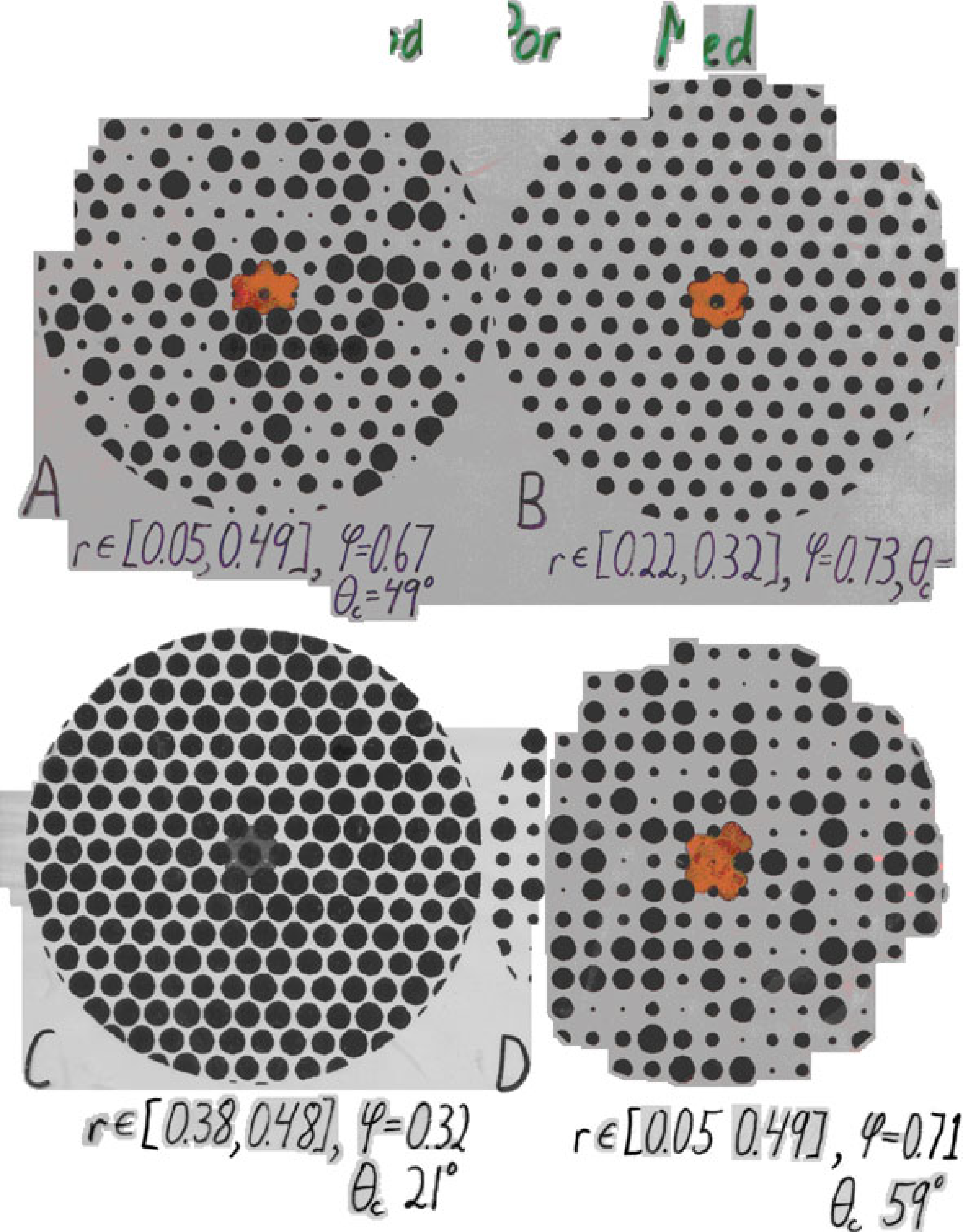
Quasistatic Invasion

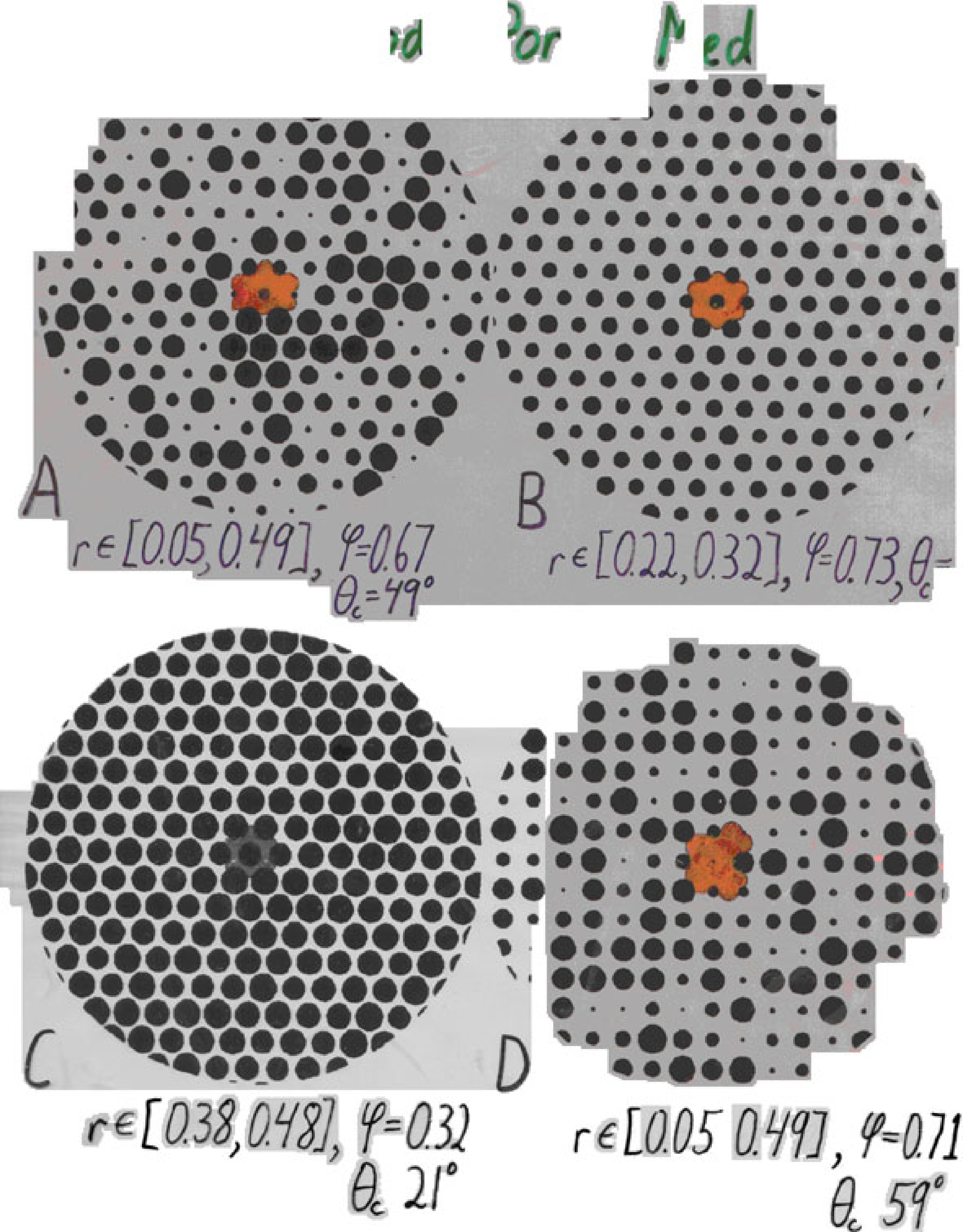
⇓

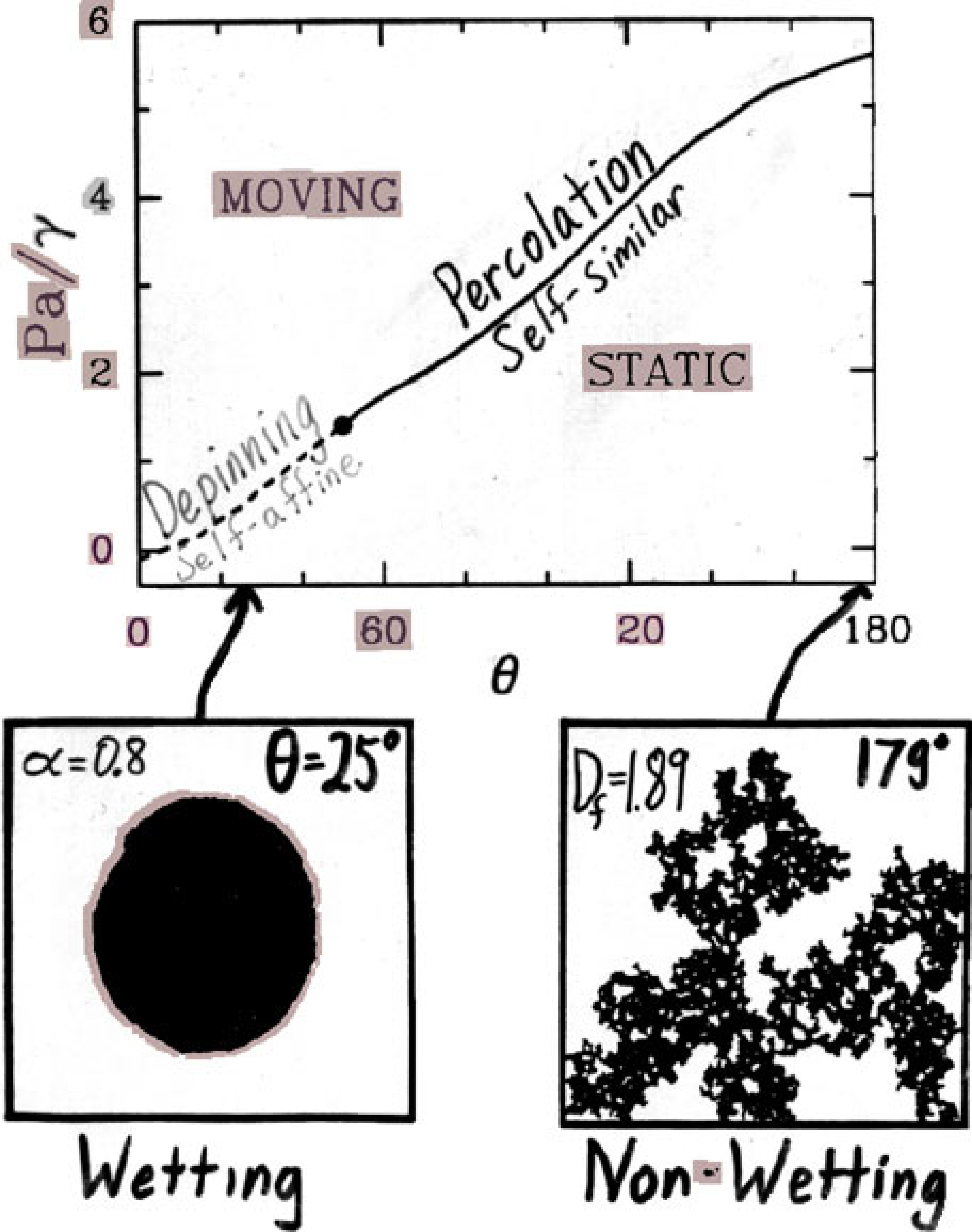
Strong variation with wetting properties

Non-Wetting Invasion

Wetting Invasion


rd
Por
Med
A
r∈[0.05,0.49], φ=0.67
θc=49°
B
r∈[0.22,0.32], φ=0.73, θc
C
r∈[0.38,0.48], φ=0.32
θc 21°
D
r∈[0.05 0.49], φ=0.71
θc 59°

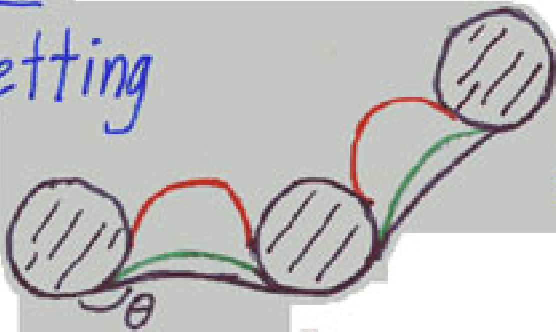

rd Por Med
A
r ∈ [0.05, 0.49], φ=0.67
θc=49°
B
r ∈ [0.22, 0.32], φ=0.73, θc=
C
r ∈ [0.38, 0.48], φ=0.32
θc 21°
D
r ∈ [0.05 0.49], φ=0.71
θc 59°



6
4
2
0
Pa/γ
MOVING
STATIC
Percolation
Self-similar
Depinning
Self-affine
0
60
20
180
θ
α=0.8
θ=25°
D_f=1.89
179°
Wetting
Non-Wetting

$\theta = 180°$

non-wetting

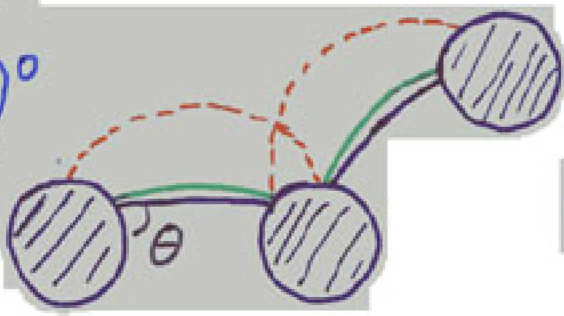


Interfaces in each throat are independent

⇒ invasion percolation

- P_0 - last stable in single throat
- $P_0/2$
- $P_0/4$

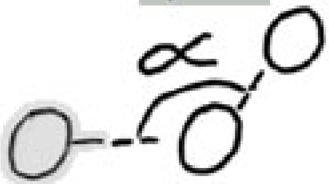
$\theta = 60°$

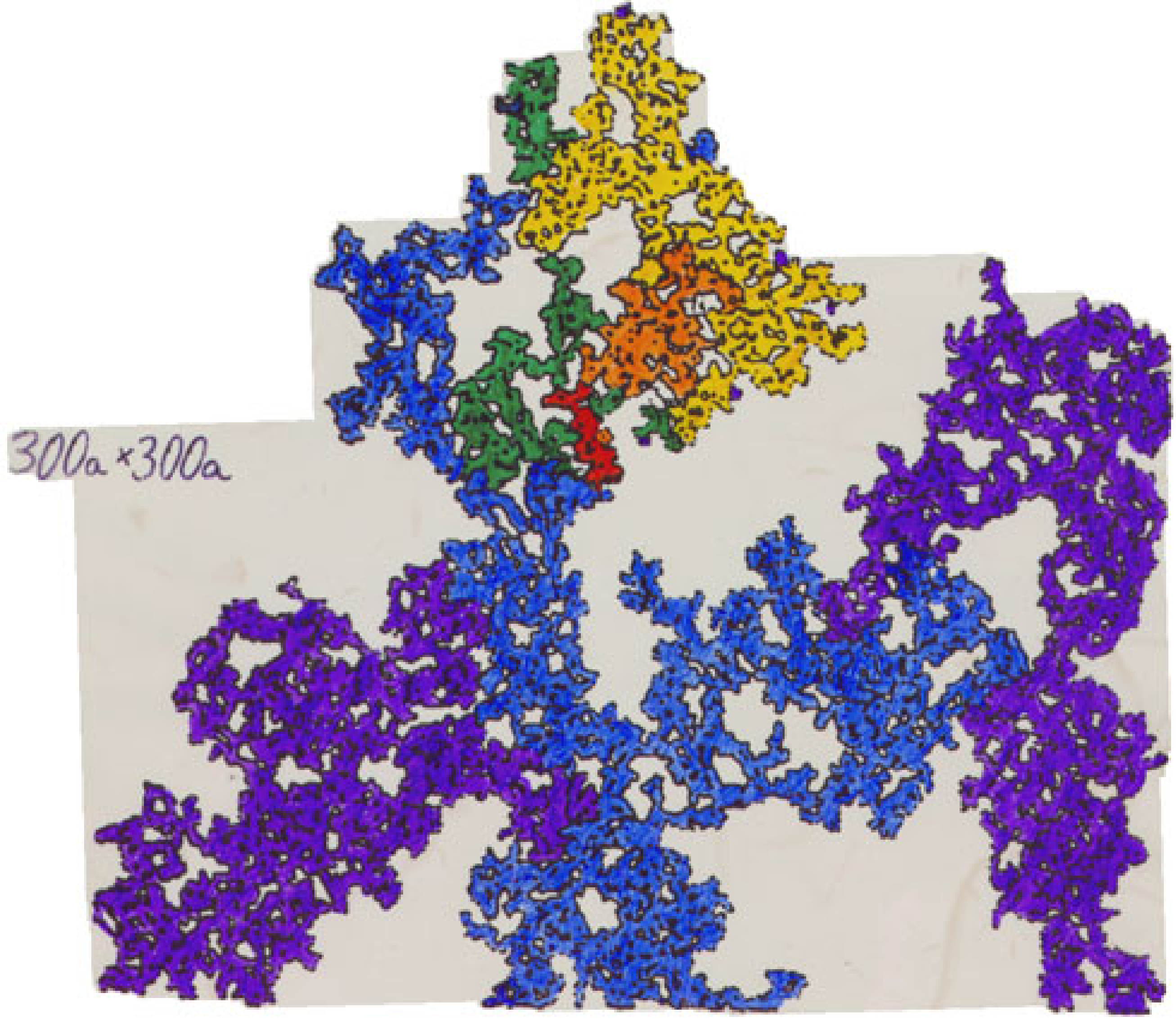


Neighboring interfaces most important factor

⇒ like continuum model growth by "overlaps"

Overlaps eliminate sharpest kinks in interface first – smallest bond angles α least stable





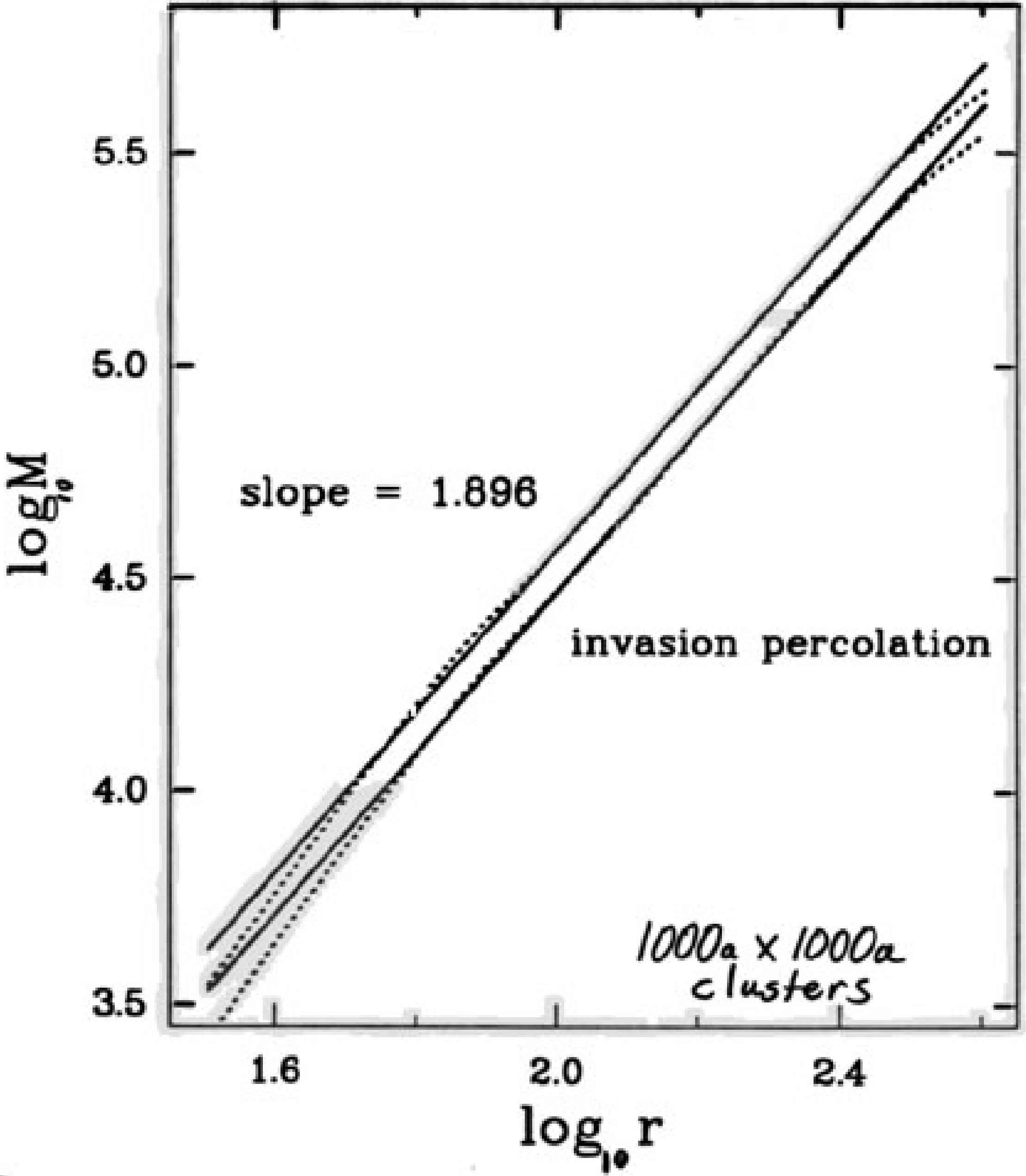
- Bursts dominate growth
- Fractal dimensions

  Bulk 1.89±0.01

  External 1.32±0.03 → consistent with normal percolation

Increasing pressure ↓

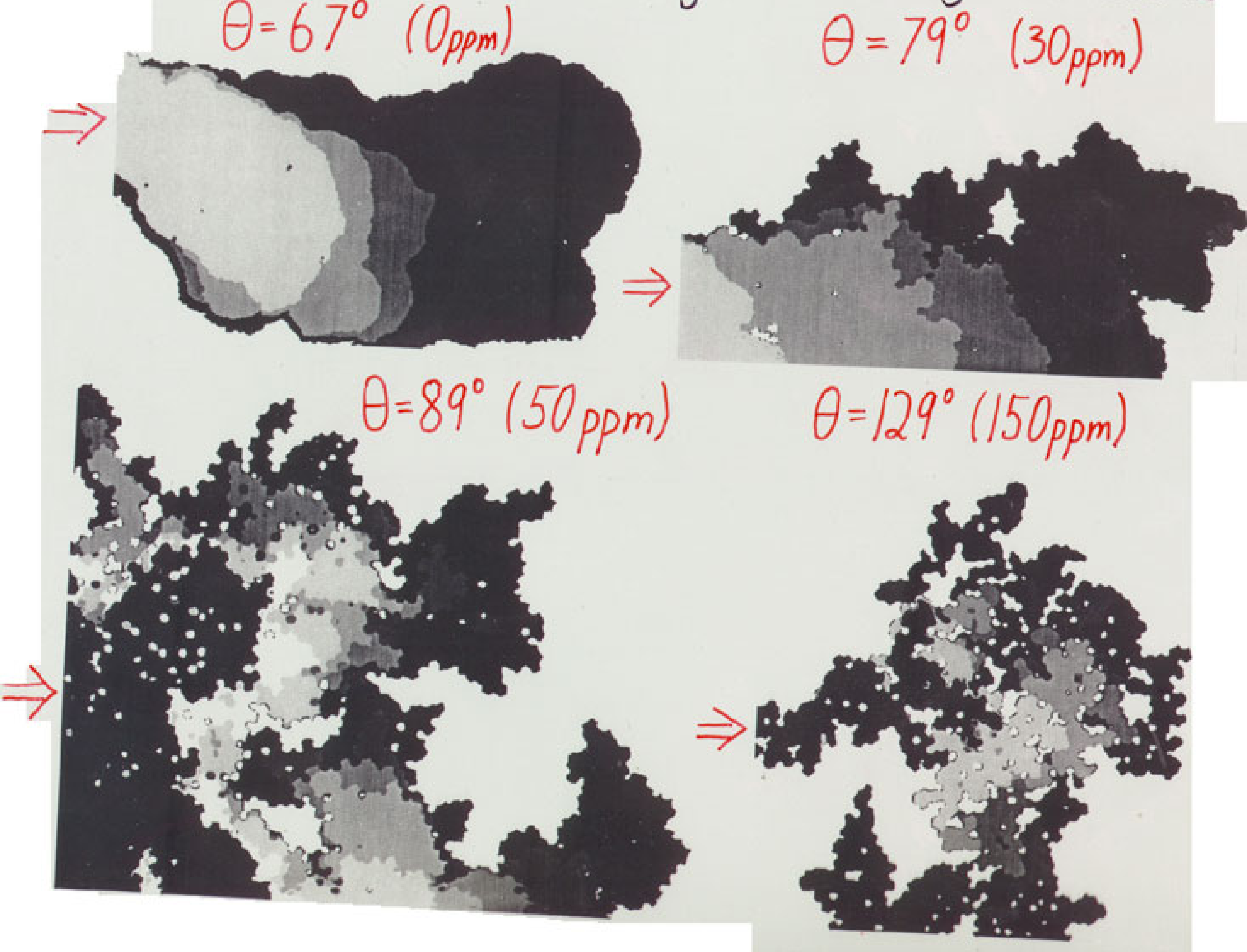
⇒ Value consistent with normal percolation
(from box, mass + interface measures)

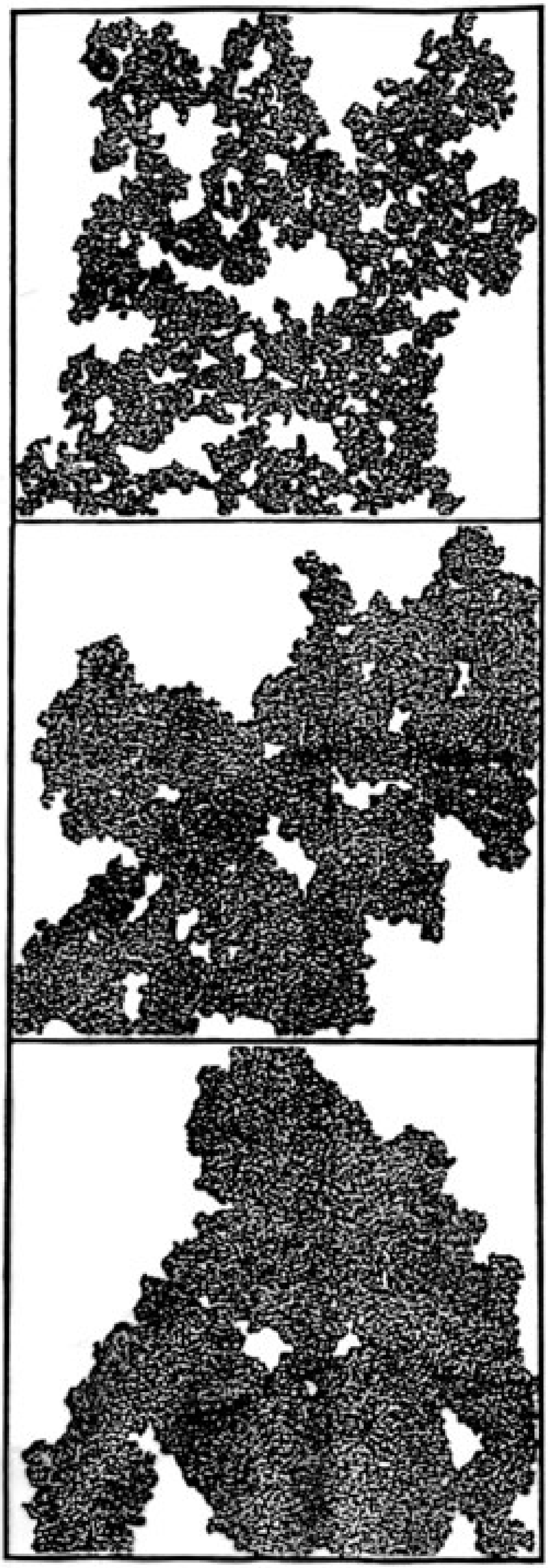


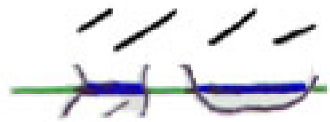
· Previous result – nonuniversal
(Wilkinson + Willemsen)

1.82 – square, honeycomb
1.88 – hexagonal
⇒ site percolatio[redacted]

Snapshots of invaded region - monolayer of beads







finger widt
→ mean width of invaded segments on slices through it

a

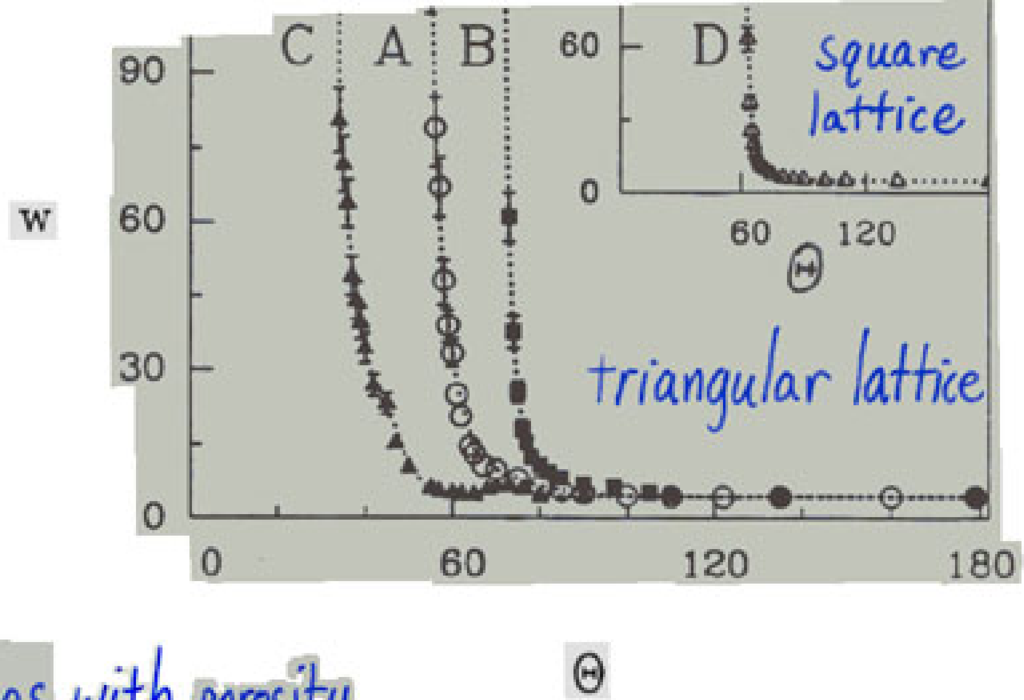
$\theta = 60°$

w diverges as θ decreases

b

$\theta = 58°$

c

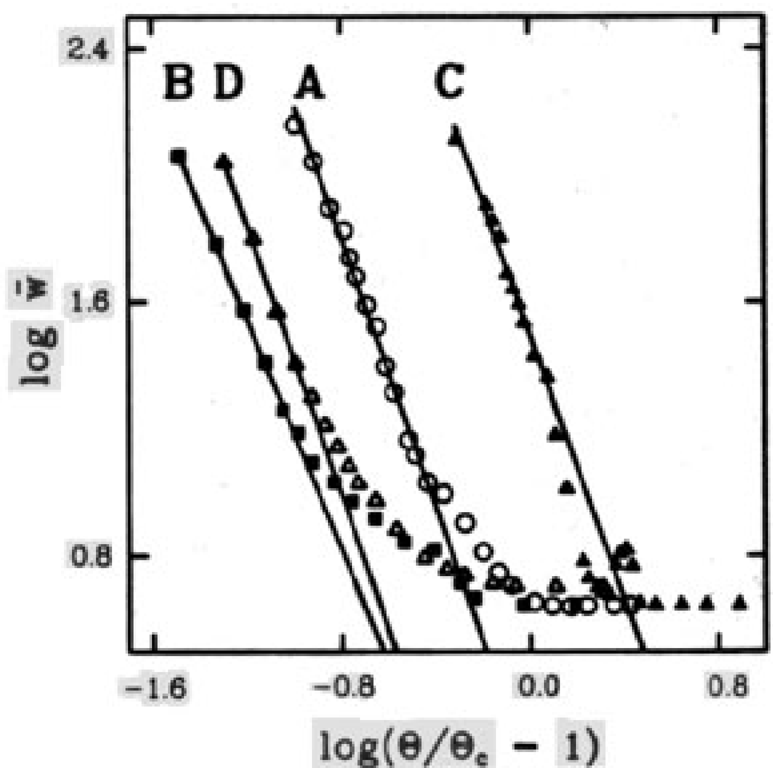


θ_c decreases with porosity

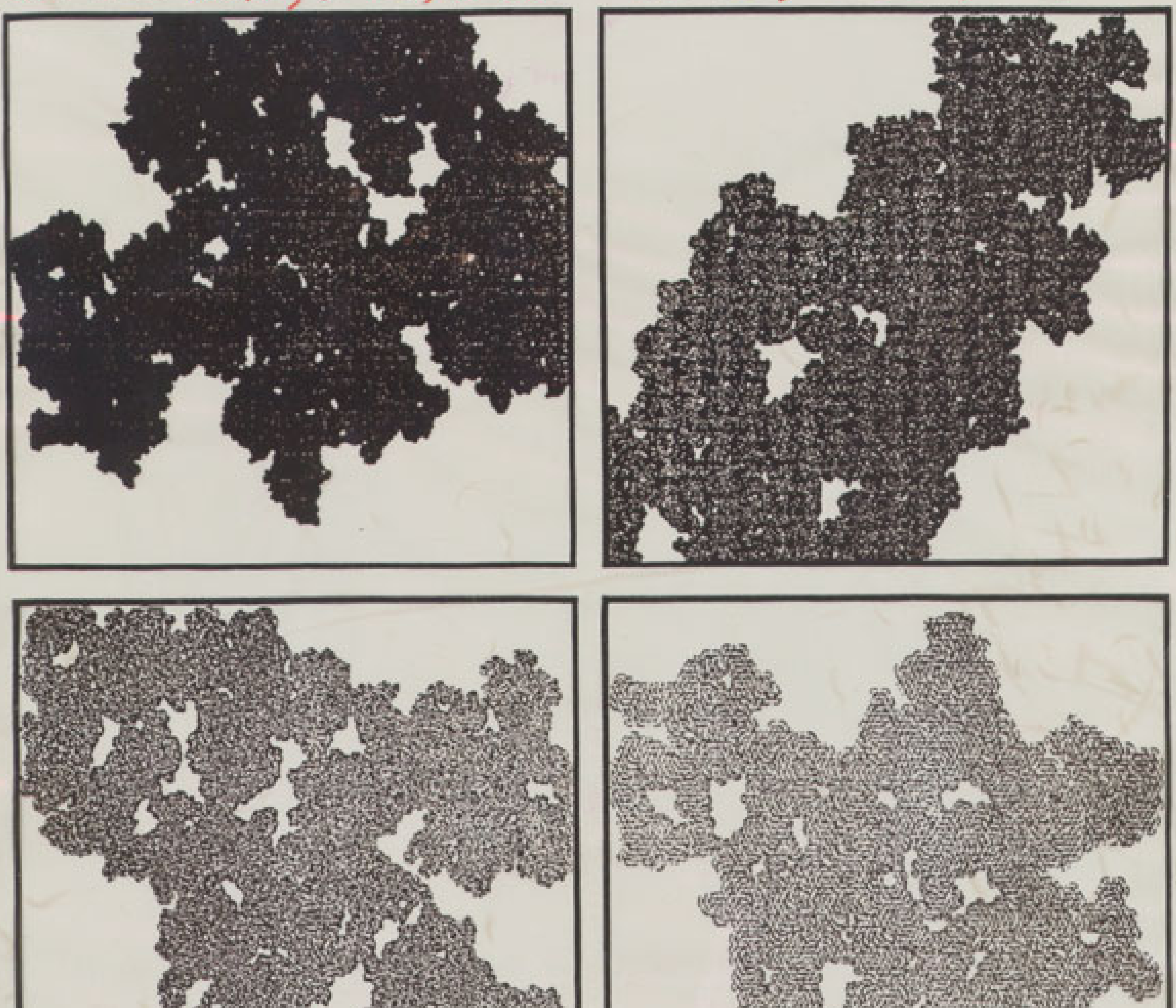
$w \quad (\theta \quad \theta_c)^{-\nu}$

ν - 2. ±0.3 A C, D

ν - 1.8±0.3 B



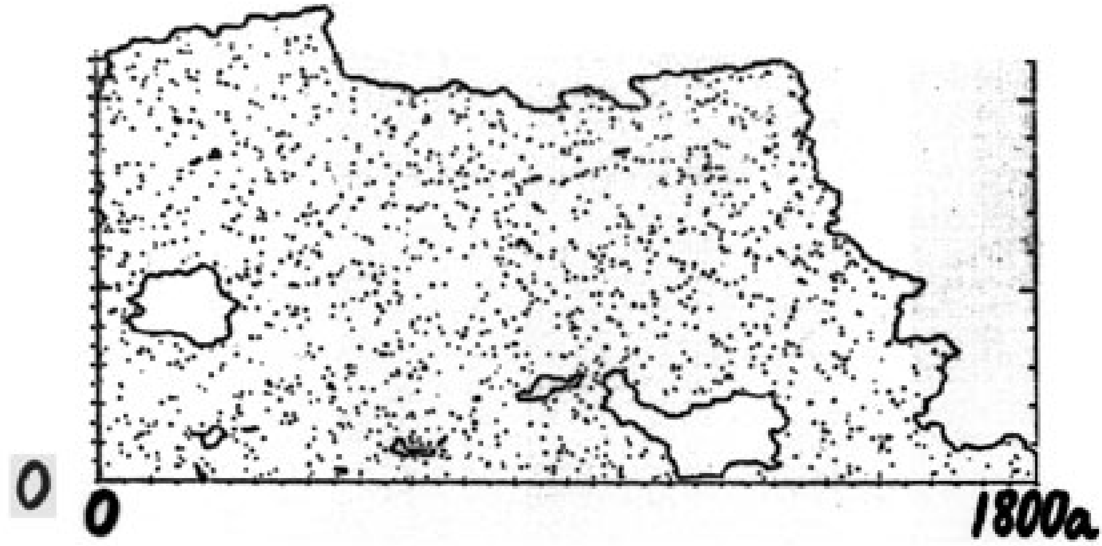
800×800 disks, $\theta = 55°$, $\bar{w} = 86$ 400×400, $\theta = 58.5°$, $\bar{w} = 41$



200×200, $\theta = 62°$, $\bar{w} = 20$ 100×100, $\theta = 70°$, $\bar{w} = 10$

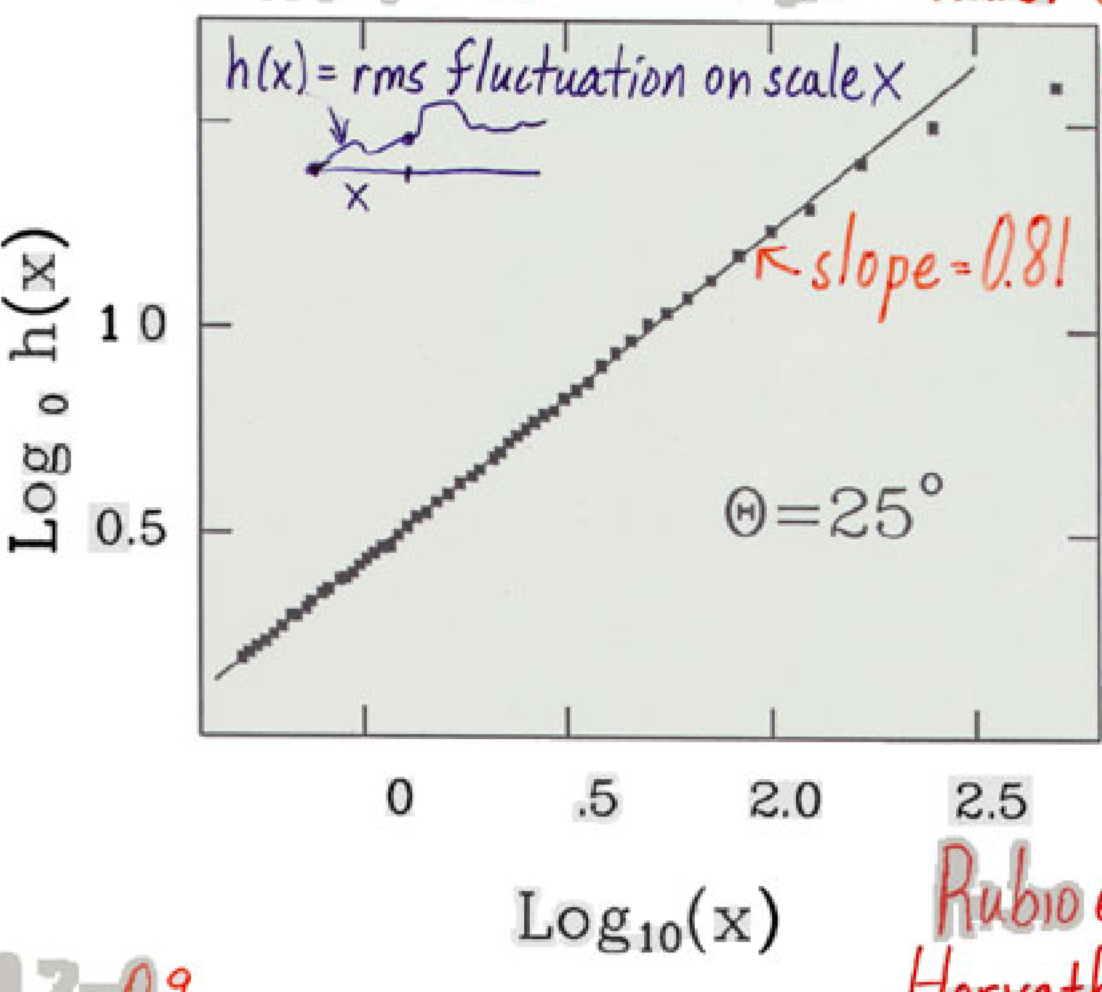
$\Rightarrow$ Only scale of pattern changes as $\theta \to \theta_c$

Pattern at θ 4'



Self Affine structure $\theta < \theta_c$ 49°

$h(x) \propto x^{\alpha}$ $\alpha = .8$ - Hurst exponent or roughness exponent



Rubio et al. 0.73±.03

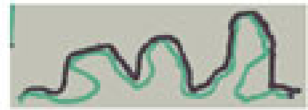
Horvath et al. 0.81

07–09

w = rms variation

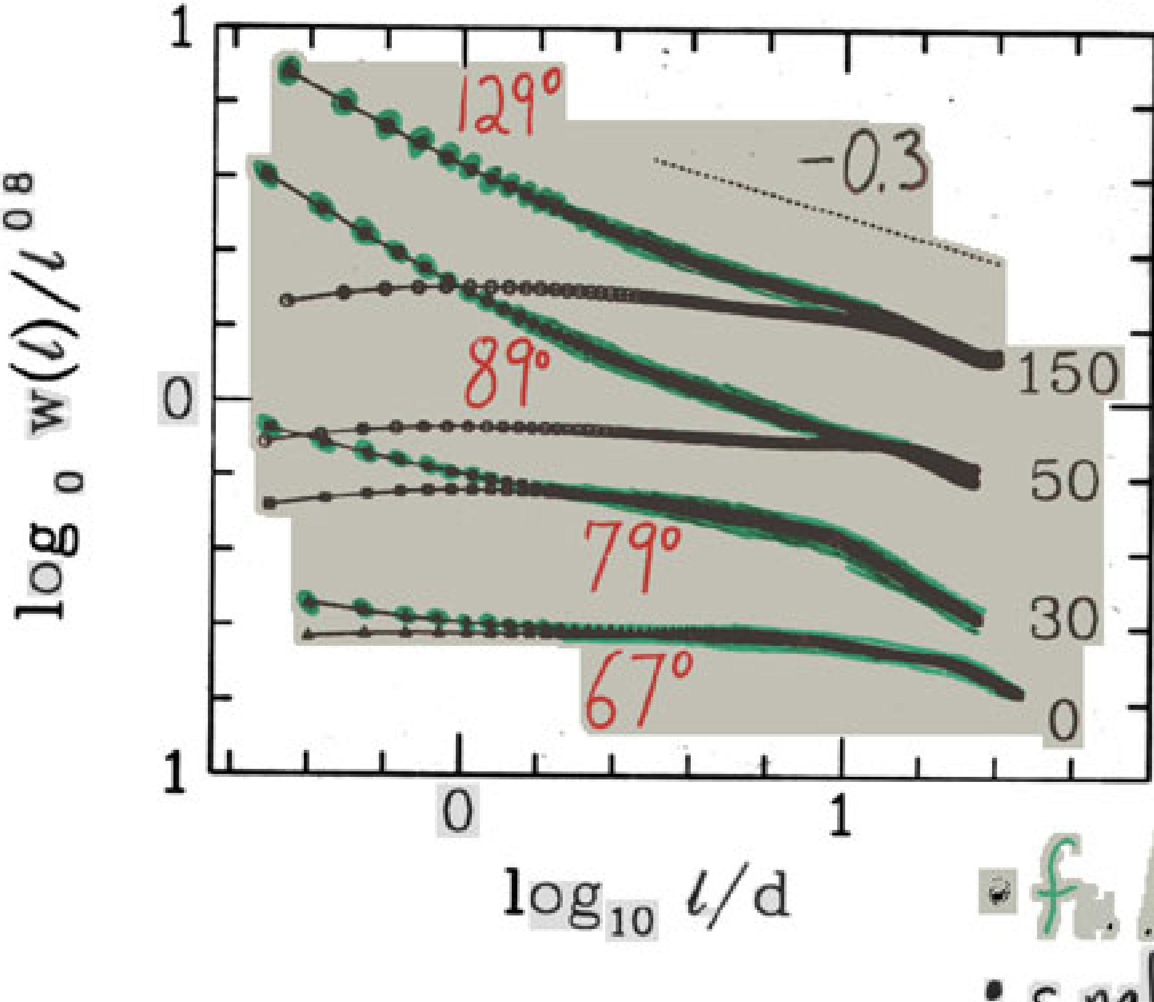
⇒ no overhangs

If self aff ne same result from fu



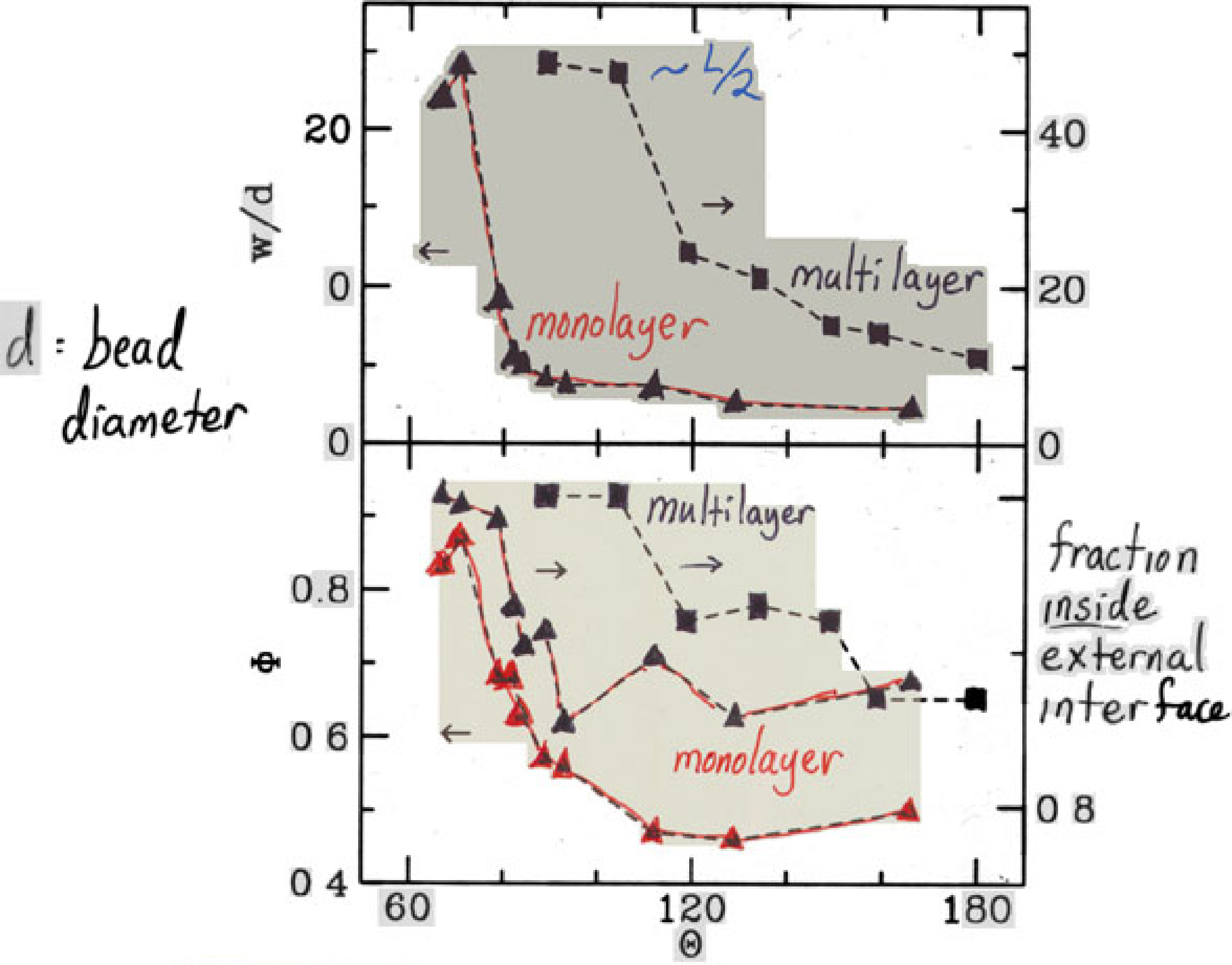
and single valued (maximum interfaces

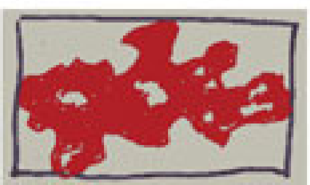
⇒ Self-affine with $\alpha \sim 0.8$ for $\theta < \theta_c$



fu / interface

• s ngle valued


d : bead diameter
w/d
20
0
0
40
20
0
~L/2
multilayer
monolayer
Φ
0.8
0 6
0 4
0 8
fraction inside external interface
multilayer
monolayer
60
120
180
Θ



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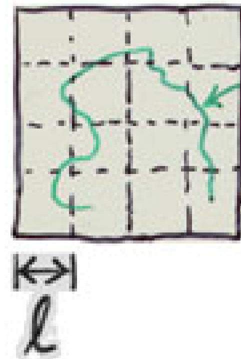


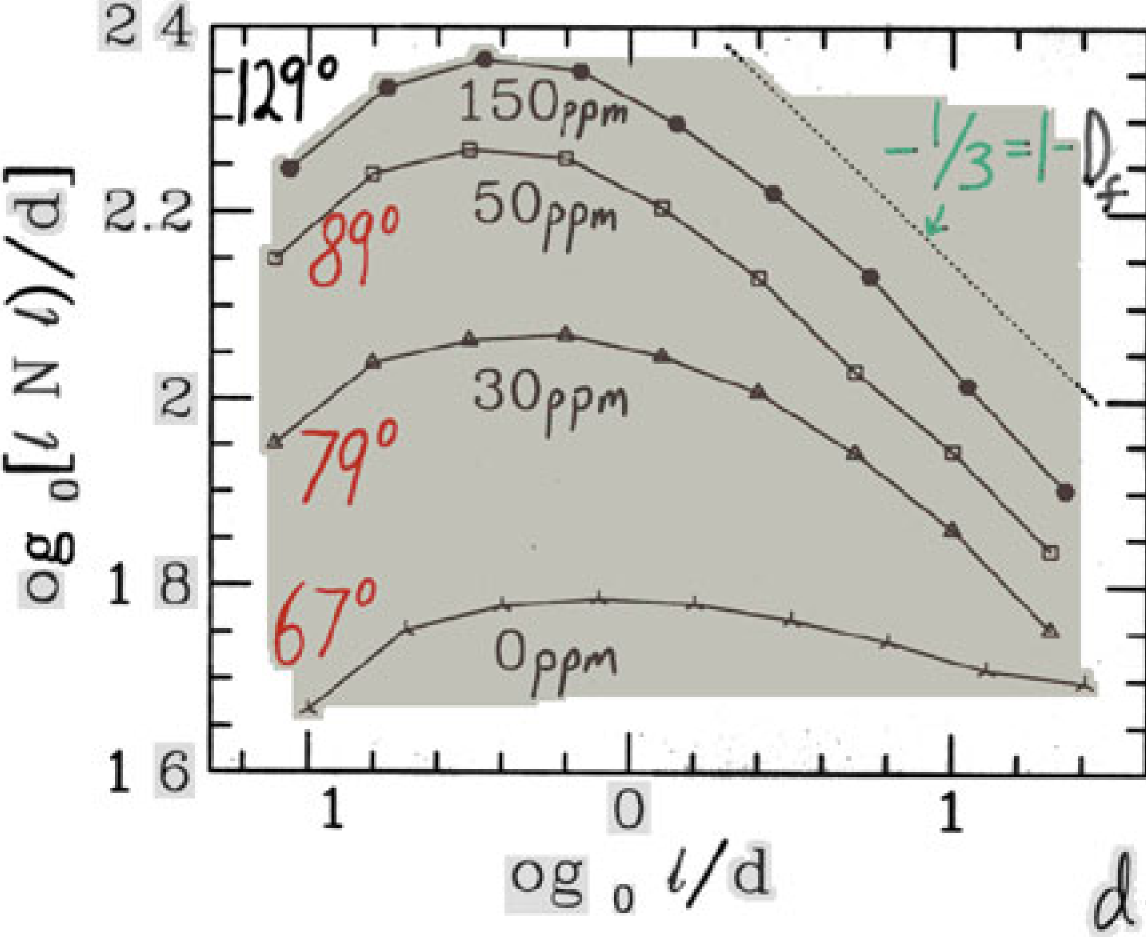
boxes of size ℓ

boxes $N(\ell)$ ℓ

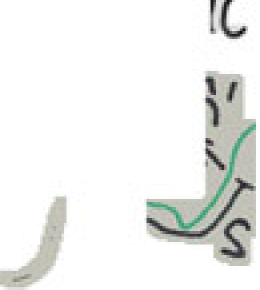
$\ell N(\ell)$ ℓ^{1-D_f}

⇒ 2D percolation D_f 4/3 $\theta > \theta_c$

compact flood D_f 1 $\theta < \theta_c$

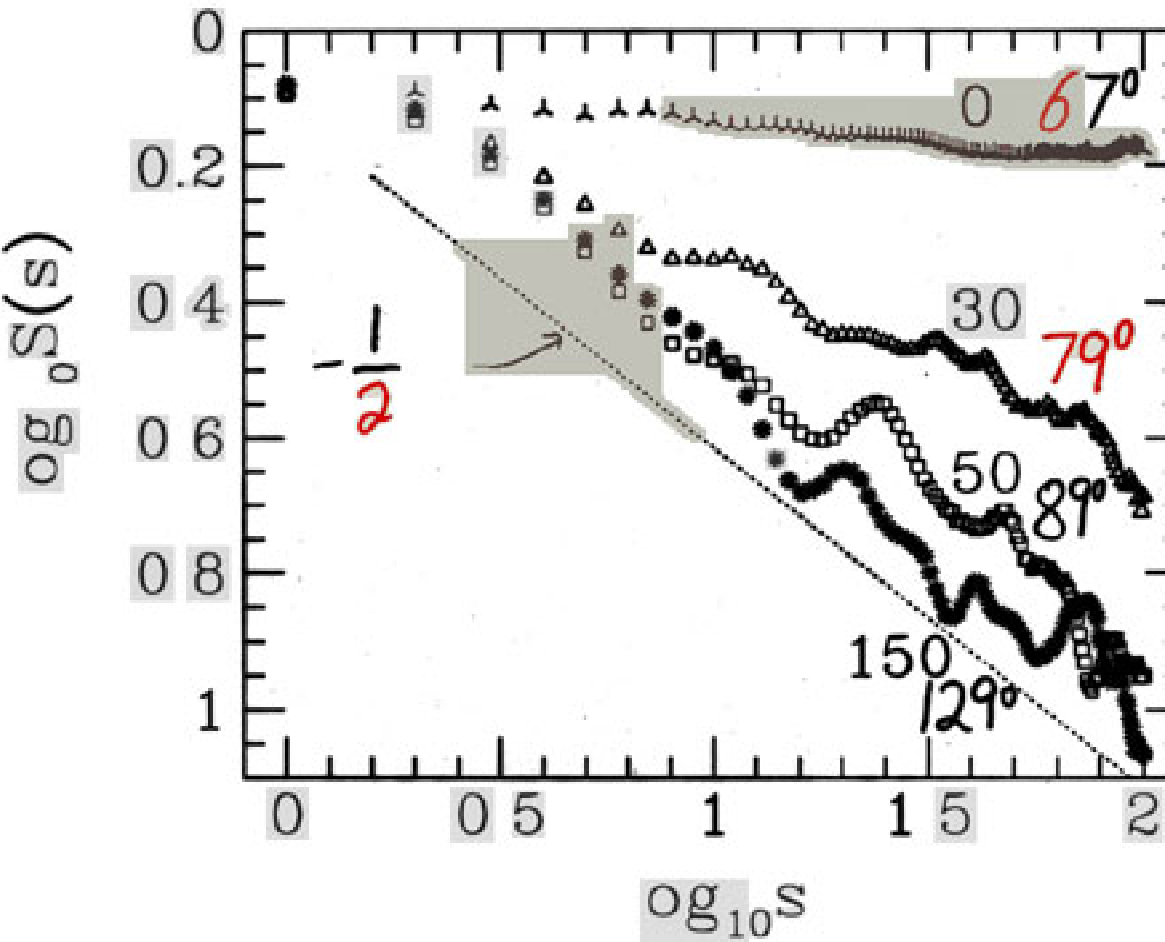


d- bead diameter

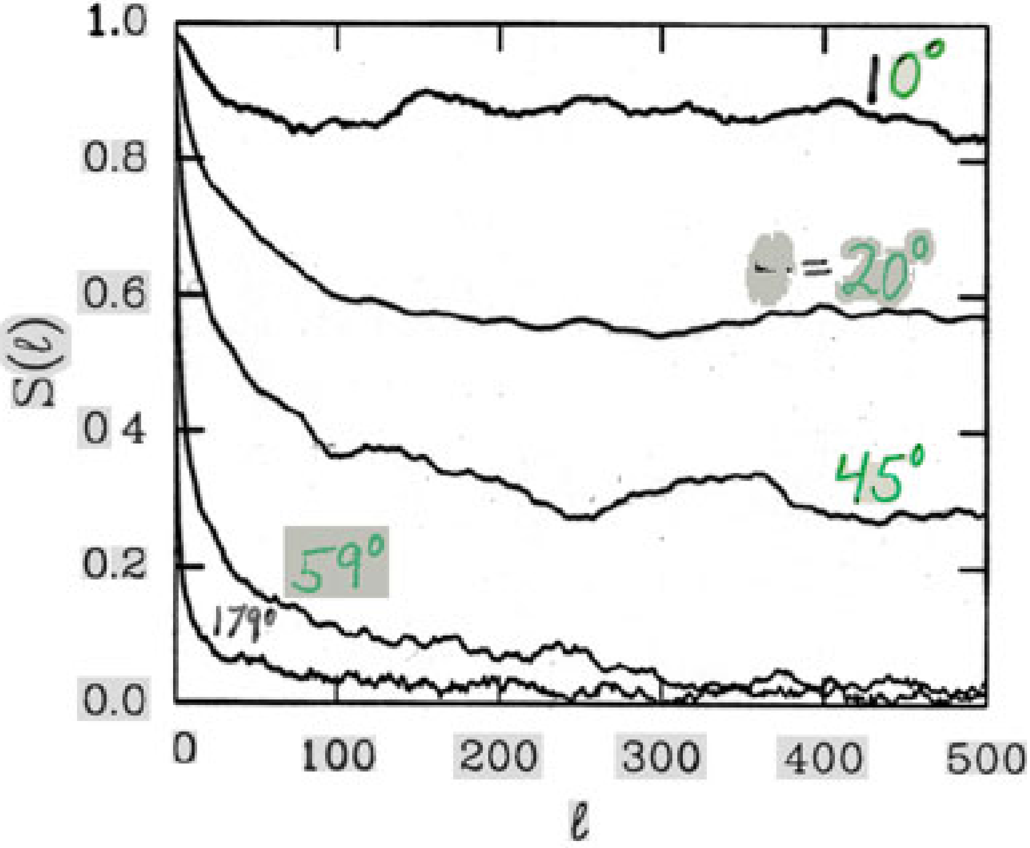


For fracta $S \sim s^{2 \quad D_f})$

$D_f \quad 4/3 \quad S \quad s^{1/2}$



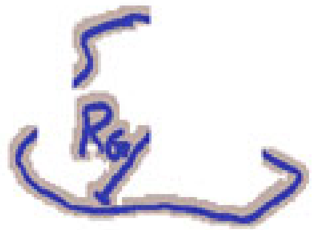
order



$S(\ell)$ surface normal correlation function

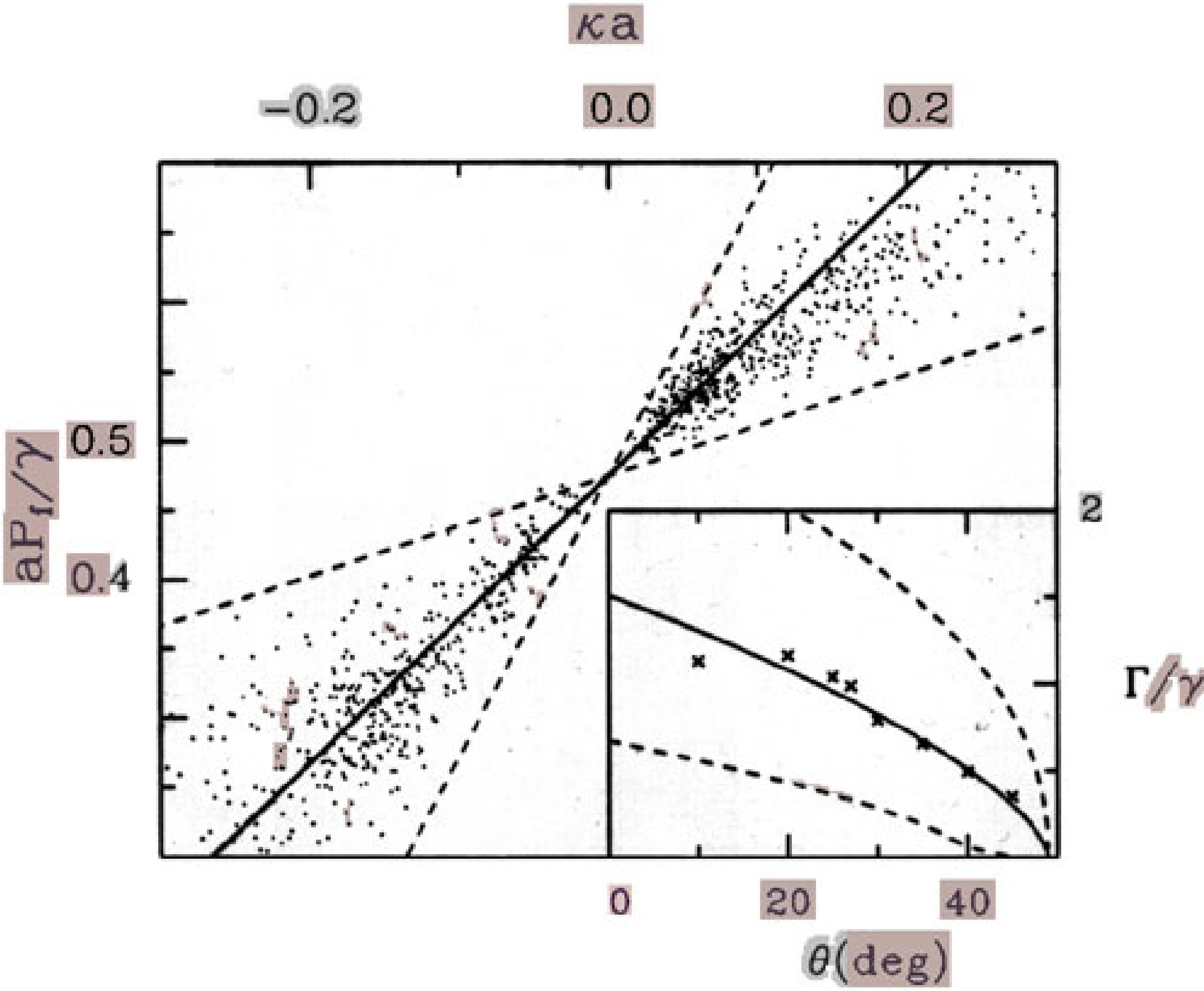
$\langle \hat{n}(\ell'+\ell)\cdot\hat{n}(\ell')\rangle_{\ell'}$, ℓ = arc length along interface

$S(\infty)$ decreases to zero at θ_c



Growth inhibited by Γ Growth aided by Γ

$P_f = P_c + \Gamma/R_G = P_c + \Gamma K$ $P_f = P_c \;\; \Gamma/R_G \;\; P_c + \Gamma K$



M

lom ie

$$H = -J\sum_{\langle ij\rangle} S_i S_j - \sum_i (H + h_i) S_i \qquad S_i = \pm 1$$

exchange coupling — external field — random local field $P(h)$

bounded distribution of random fields $|h_i| < \Delta$

$\frac{1}{J}\Delta \Rightarrow \infty$ – spins flip independently → percolation

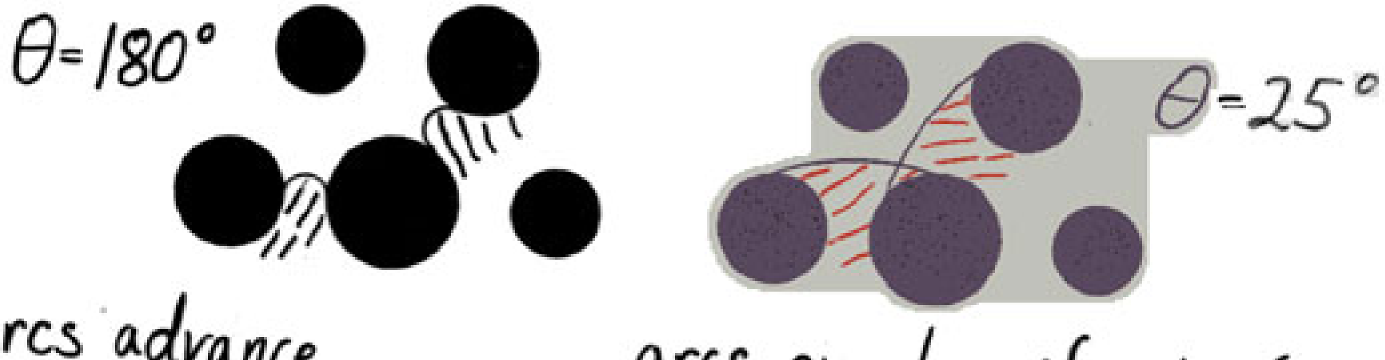
$\frac{1}{J}\Delta$ decreases – exchange ⇒ coherent spin-flips

2D Fluid Invasion – disks of random radius on lattice

Interface – arcs of radius γ/P ← driving pressure

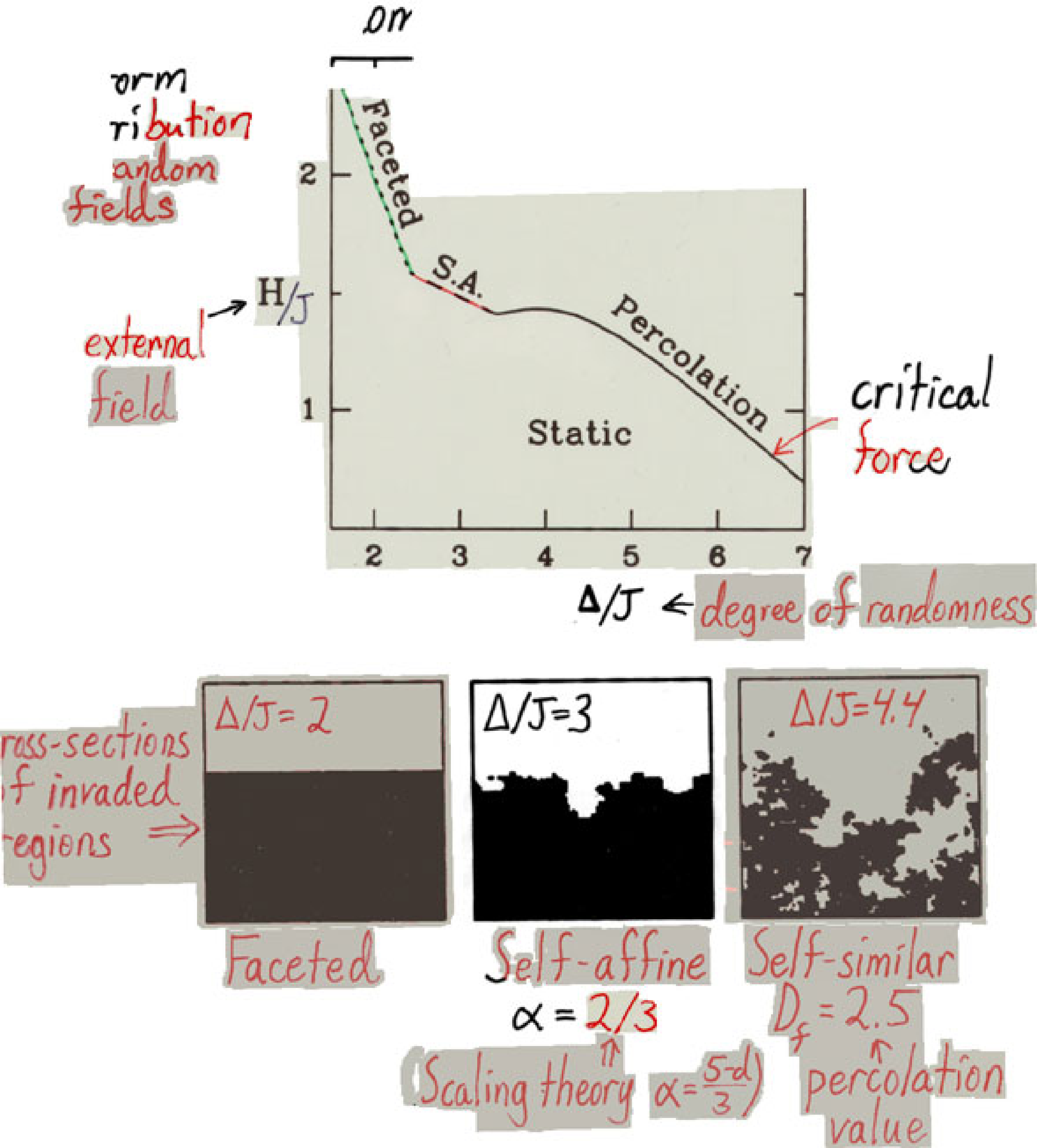
– intersect disks at contact angle θ

Degree of disorder ⇒ range of radii and θ



arcs advance independently

arcs overlap if interface bends sharply ⇒ coherent advance


orm
ribution
andom
fields
Faceted
S.A.
Percolation
Static
H/J
external
field
critical
force
2
1
2
3
4
5
6
7
Δ/J
degree of randomness
Δ/J = 2
Δ/J=3
Δ/J=4.4
ross-sections
f invaded
egions
Faceted
Self-affine
α = 2/3
(Scaling theory α = (5-d)/3)
Self-similar
D_f = 2.5
percolation
value

independent exponents - ν, D_f, D_e

fractal dimension of invaded region & interface

invaded volume	$V \propto (F_c - F)^{-\gamma}$	$\gamma = \nu(D_f + 1 - d)$
interface area	$S \propto (F_c - F)^{-\omega}$	$\omega = \nu(D_e + 1 - d)$
avalanche size	$\langle \delta V \rangle \propto (F_c - F)^{-\phi}$	$\phi = \nu(D_f - D_e) + 1$
" distribution	$\rho(\delta V) \propto \delta V^{-\tau'}$	$\tau' = 1 + (D_e - \nu^{-1})/D_f$

	2D	3D
percolation	ν 1 32(5)	ν - 0 9(1)
	D_f 1.88(4) D_e 1 32(2)	$D_f - D_e = 2.48(3)$
affine	ν 1 30(5)	$\nu = 0.75(5)$
$= d$ D_e +	$\alpha = 0\ 81(5)$	$\alpha - 0\ 67(3)$

$$\langle A \rangle \propto \frac{1}{S}\frac{dA_t}{dP} \propto (P_c - P)^{-\nu(D_f - D_e) -}$$

$\uparrow$ interface length

$S \propto L \xi^{D_e - 1}$

D_e external dimension

Scaling relation

$$\rho(A) \propto A^{-\alpha} e^{-A/\xi^{D_f}} \Rightarrow \langle A \rangle \propto \int dA\, A^{(1-\alpha)} e^{-A/\xi^{D_f}} \propto \xi^{D_f(2-\alpha)}$$

$$D_f(2-\alpha) = D_f - D_e + 1/\nu$$

$$D_f(1-\alpha) = 1/\nu - D_e$$

Finite size scaling functions

$$A_t = L^{D_f} f[(P - P_c) L^{1/\nu}]$$

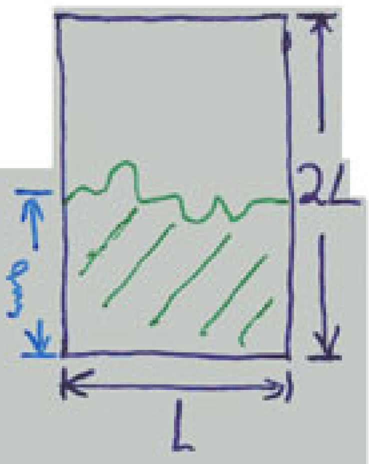
$$\langle A \rangle = L^{D_f - D_e + 1/\nu} g[(P-P_c) L^{1/\nu}]$$

$$B_A = D_f - D_e + 1/\nu$$

Percolation $D_f = \frac{91}{48}$, $\nu = 4/3$, $D_e = 4/3$

$\Rightarrow B = 2/16 \approx 13 \quad \alpha = \frac{119}{91} \approx 3$

ff



compact $\Rightarrow \xi \propto \frac{1}{L} A_{tot} \propto (P_c - P)^{-\nu}$

$$\chi \equiv \langle A \rangle \propto \frac{1}{L} \frac{d}{dP} A_{tot} \propto (P_c - P)^{-(1+\nu)}$$

since interface length, instability probability } ~ constant

also compact $\Rightarrow \langle \lambda^2 \rangle \propto \langle A \rangle \propto (P_c - P)^{-(1+\nu)}$

Ansatz: power law distributions of λ^2, A

$$\rho(A) \propto A^{-\tau} e^{-A/\xi^2}$$

$$\langle A \rangle \propto \int dA\, A^{1-\tau} e^{-A/\xi^2} \propto (\xi^2)^{2-\tau} \propto (P_c - P)^{-2\nu(2-\tau)}$$

$$2\nu(2-\tau) = 1+\nu \Rightarrow 3 - 2\tau = 1/\nu$$

one exponent determines all others

ξ L $\Rightarrow$ Finite size scaling $\Rightarrow$ only L/ξ matters

$$\frac{1}{L} A_{tot}(P,L) = L\, f((P-P_c) L^{1/\nu})$$

$$\chi(\ ,L) \quad L^{1+1/\nu} f((P-P_c) L^{1/\nu})$$

$$\rho(A) = L^{-2\tau} g(A/L^2)$$

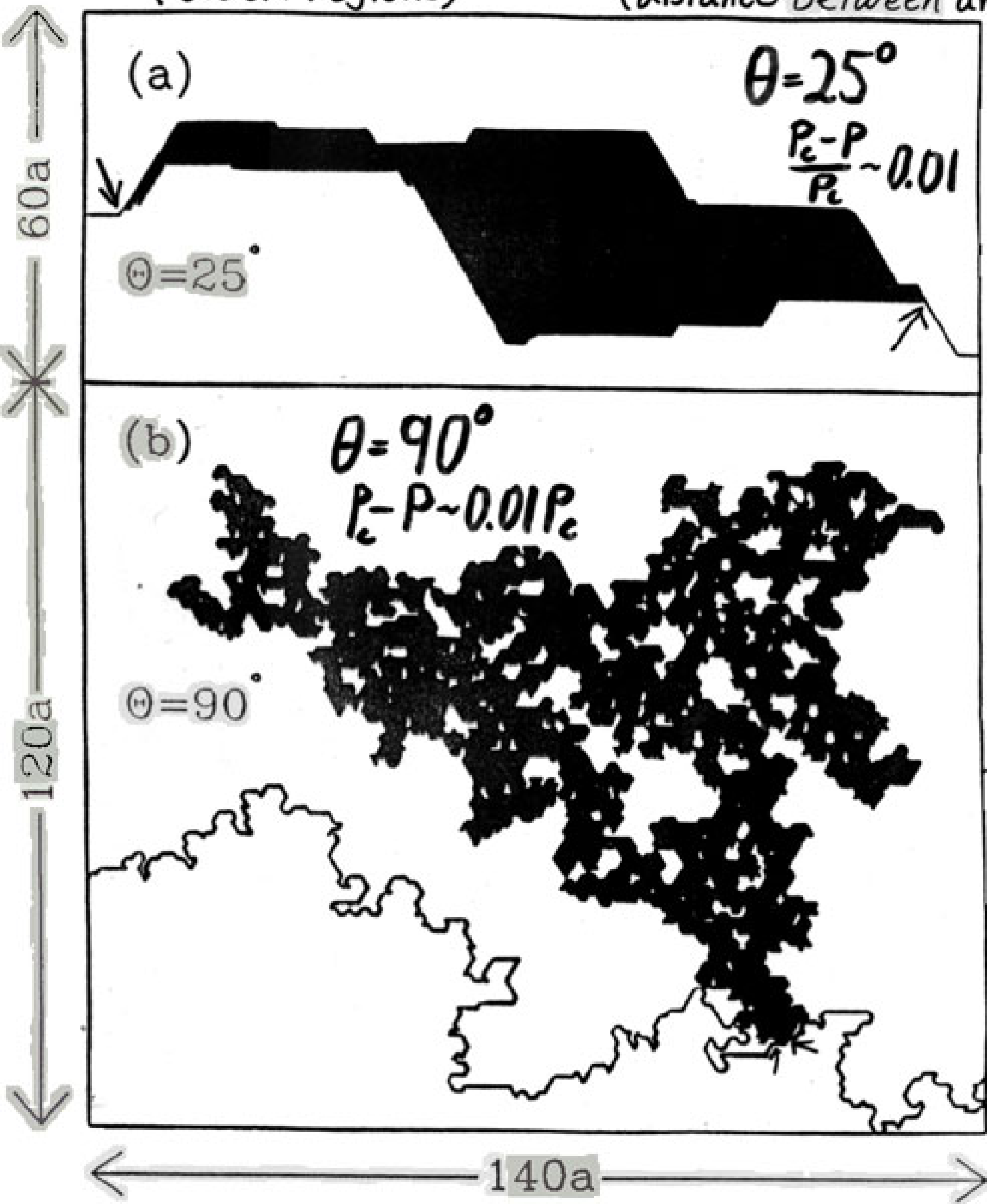
Find $\nu = 1.3 \pm 0.$ $\tau = 1.25 \pm 0.025$

wths with P

Increase P + // single arc unstable

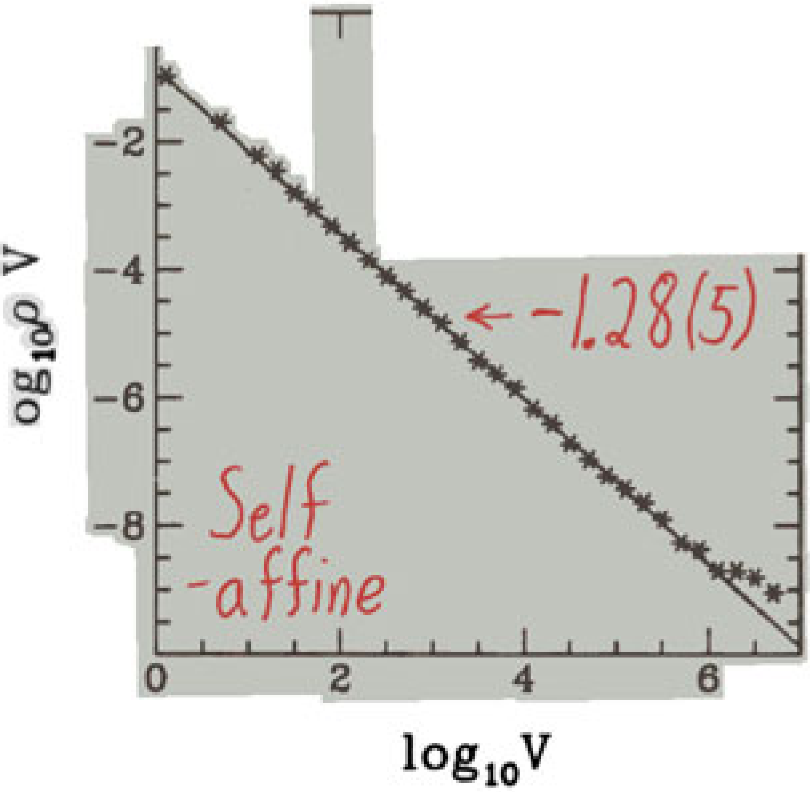
A = new area invaded (black regions)

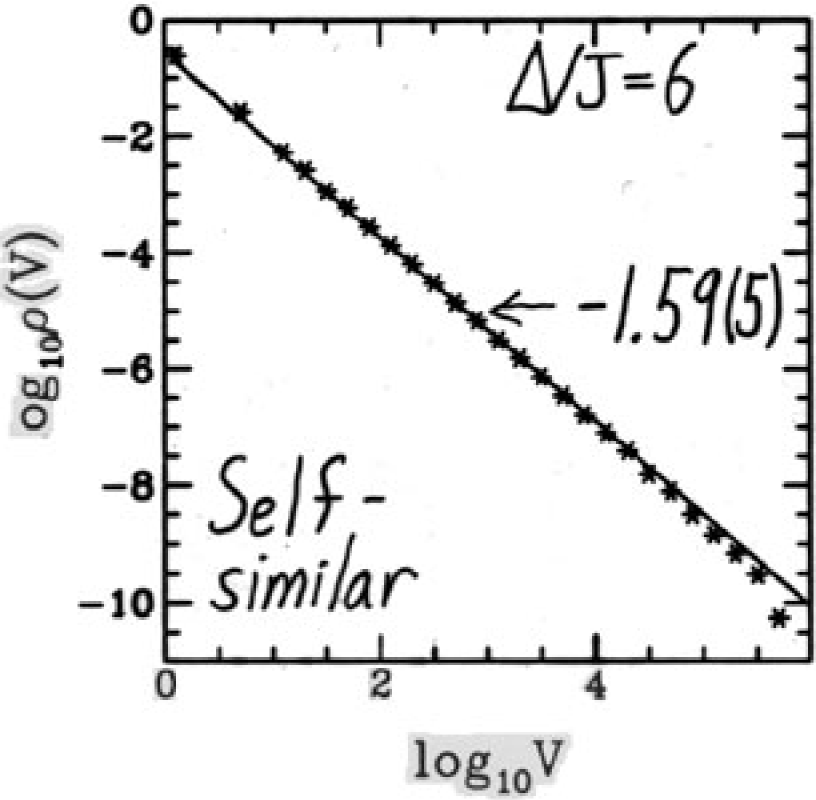
λ ≡ width of interface advance (distance between arrows)



2 Fluid Invasion

on





Data averaged over $L^{1/\nu}(H_c-H)/H_c \,|<1$

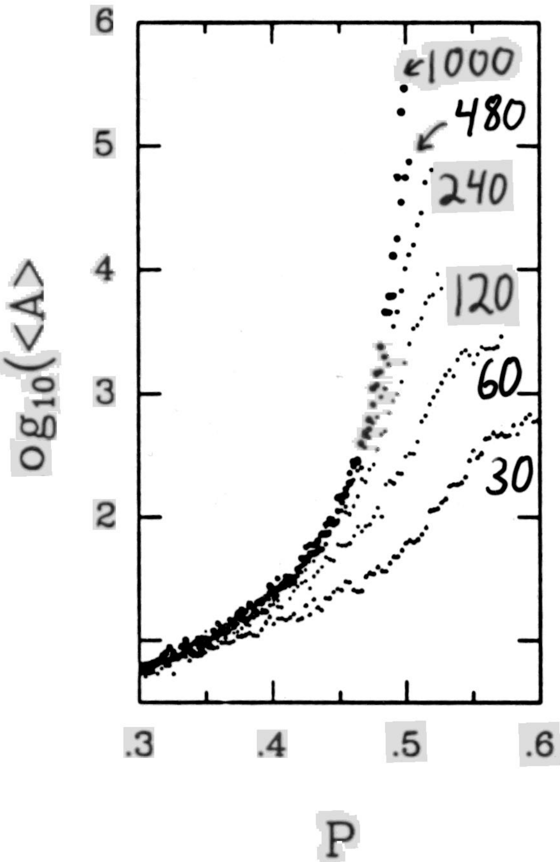
$\rightarrow \xi > L$

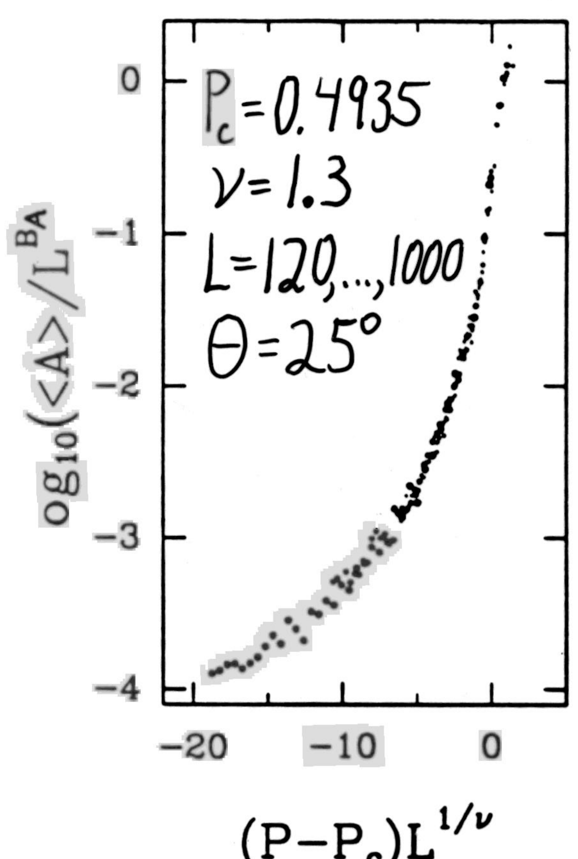
aling of Susceptibility

$$\frac{1}{S}\frac{dA_t}{dP} = L^{B_A} f'[(P-P_c)L^{1/\nu}]$$

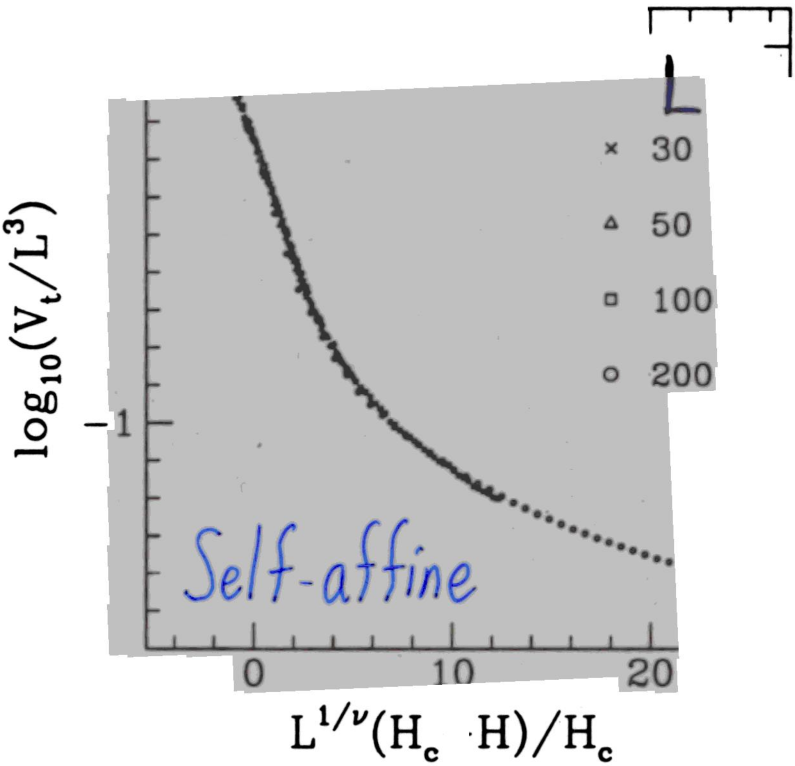
S- interface length

$$B_A = 1 + 1/\nu$$





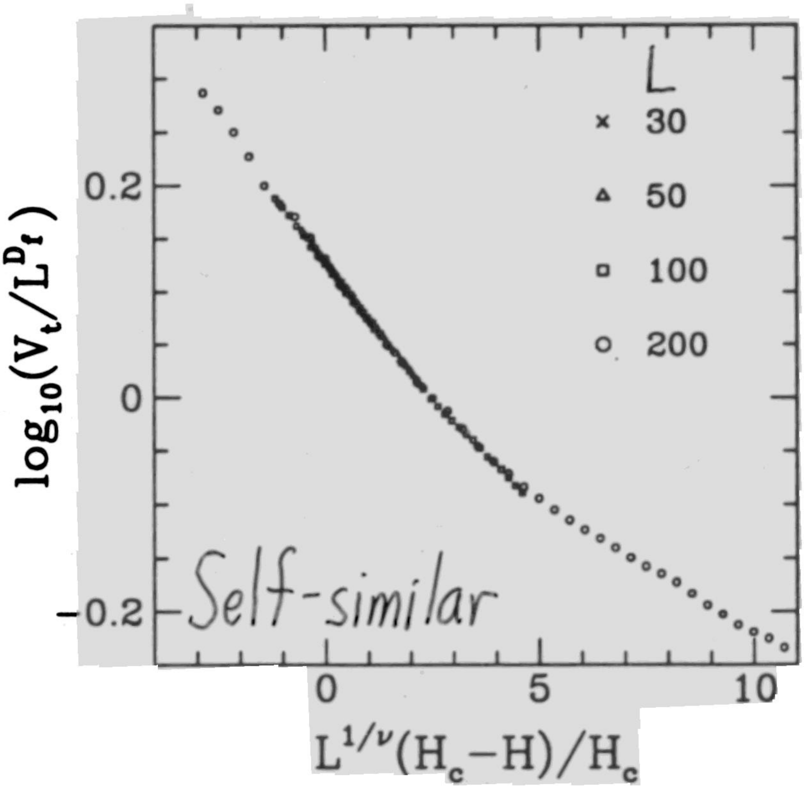
q of invaded volume - 3D random Isin



Δ/J

$\nu = 0.75(5)$

$\alpha = 0.67(3)$



$\Delta/J=6$

$D_f = 2.48(3)$

$\nu = 0.9(1)$

$H_c = 1.011(2)$

flow velocity $v \equiv \frac{1}{L}\frac{dV}{dt} \propto (F-F_c)$

- fluctuations $S_v(\omega) \propto \omega^{-\rho}$ at F_c
- morphology $w(x,t) \propto x^{\alpha} f[t^{\beta}/x^{\alpha}]$

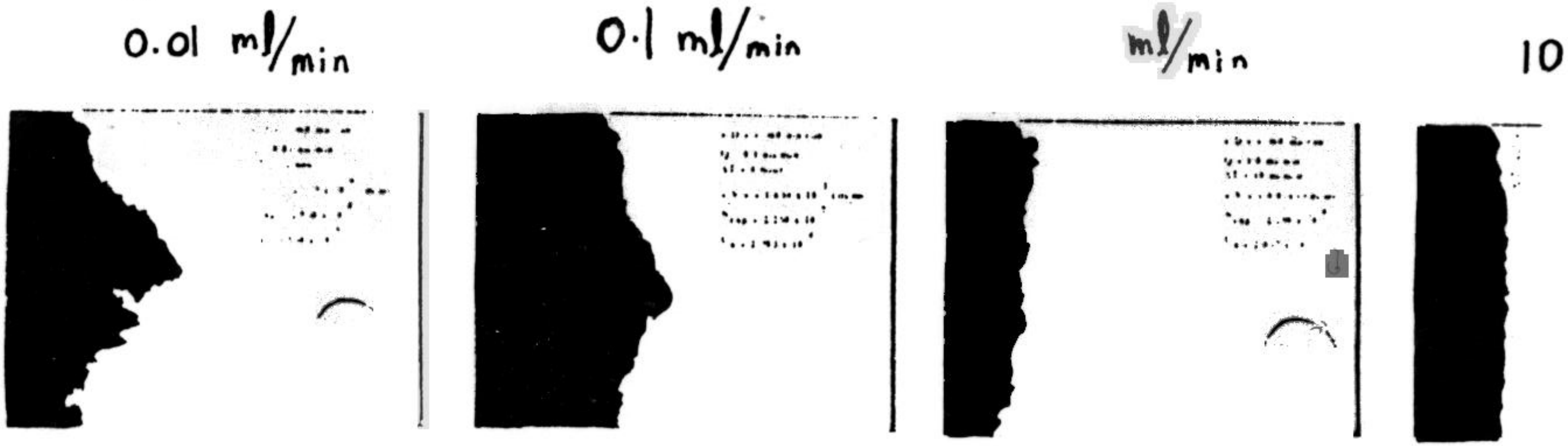
$x < \xi$ α, β characteristic of quenched disorder

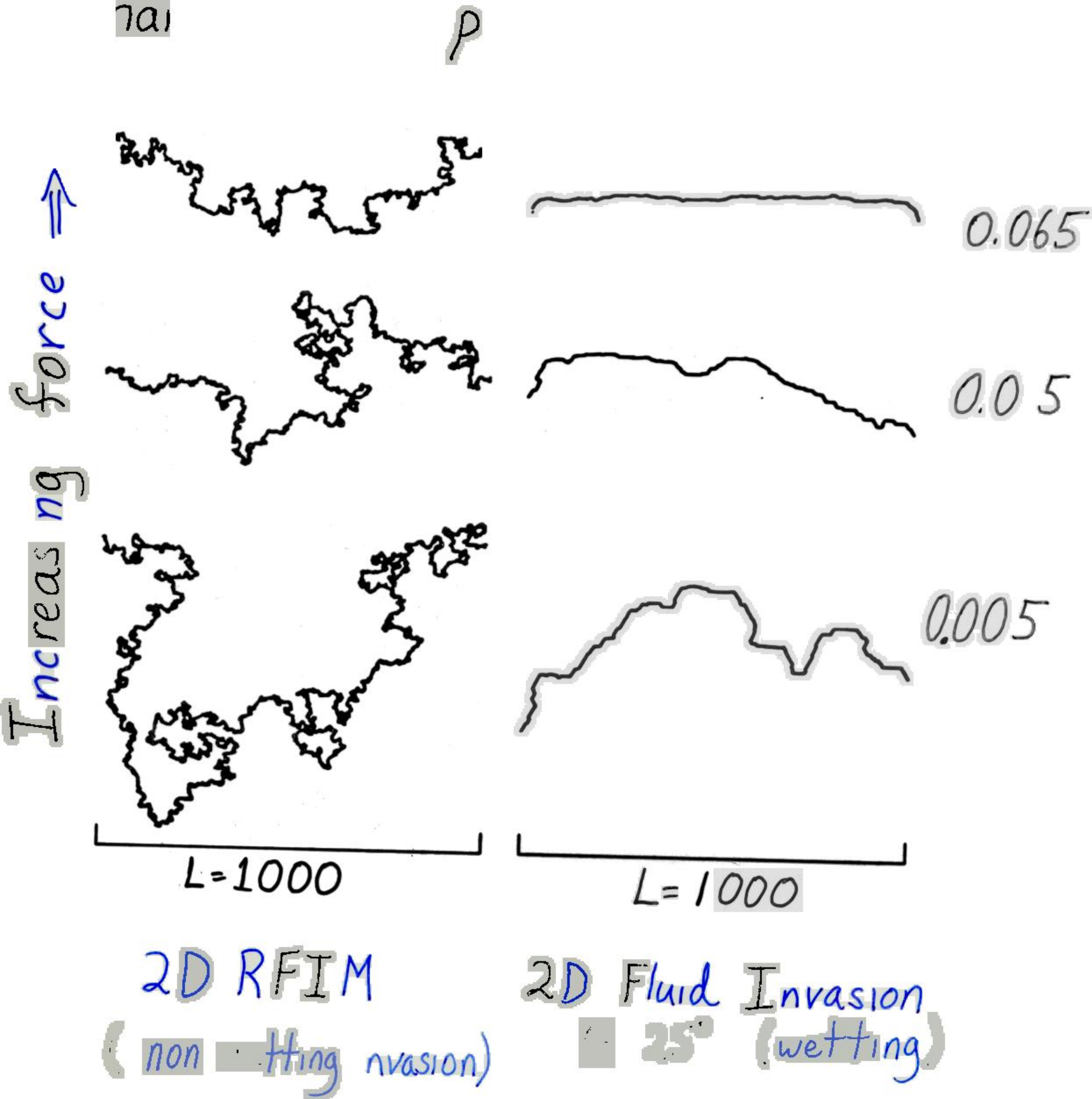
$x > \xi$ α', β' " " annealed

	2D percolation	3D self-affine	
ν	1.33(2)	0.75(5)	3/4
ς	0.42(5)	0.65(5)	2/3 ?
ρ	1.55(5)	1.5(1)	
α	1	0.64(3)	2/3
β		0.45(5)	3/7 ?
α'	0.505(10)	0.25(1)	
β'	0.33(1)	0.14(1)	

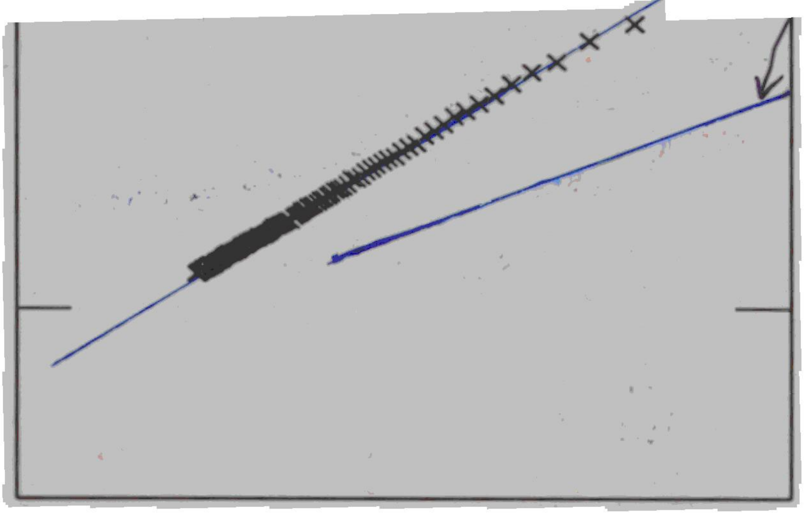
RG $\Rightarrow$ $\nu = \frac{1}{2-\alpha}$, $\varsigma = \nu(\alpha/\beta - \alpha)$, $\alpha = \frac{\varepsilon}{3}$, $\varepsilon = 5-d$

Natterman, Stepanow, Tang, Leschhorn Narayan, Fisher


0.01 ml/min
0·1 ml/min
ml/min
10



nai
p
Increas ng force ⇒
0.065
0.05
0.005
L=1000
L=1000
2D RFIM
(non tting nvasion)
2D Fluid Invasion
25° (wetting)



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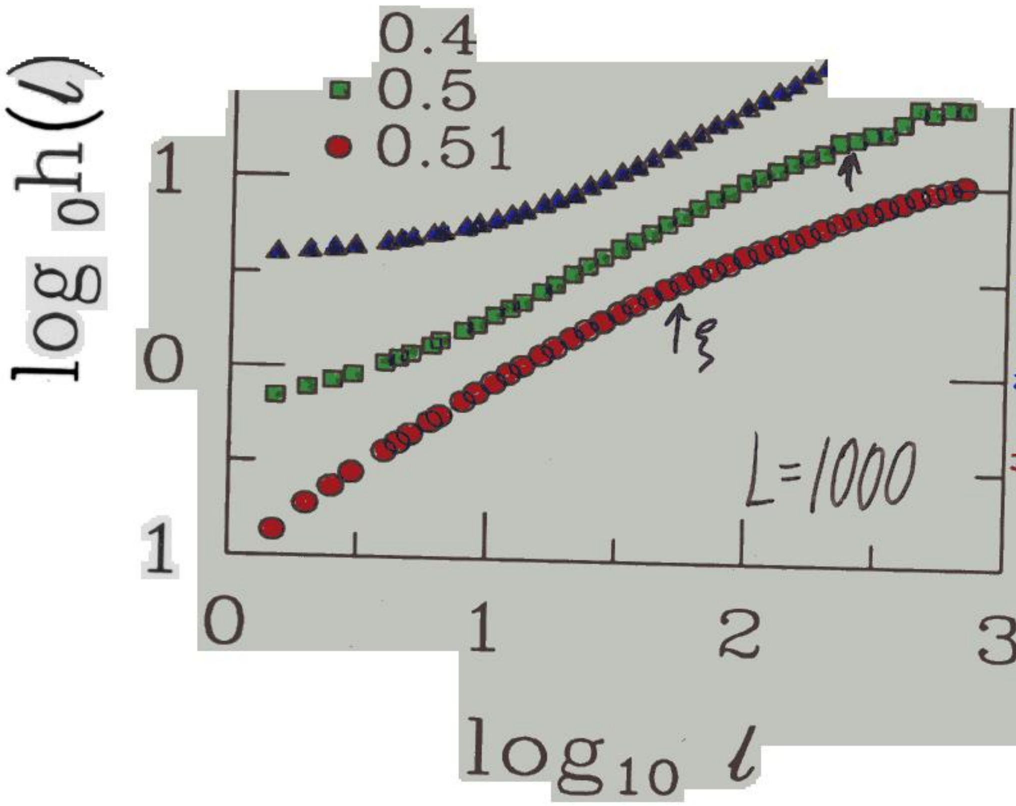


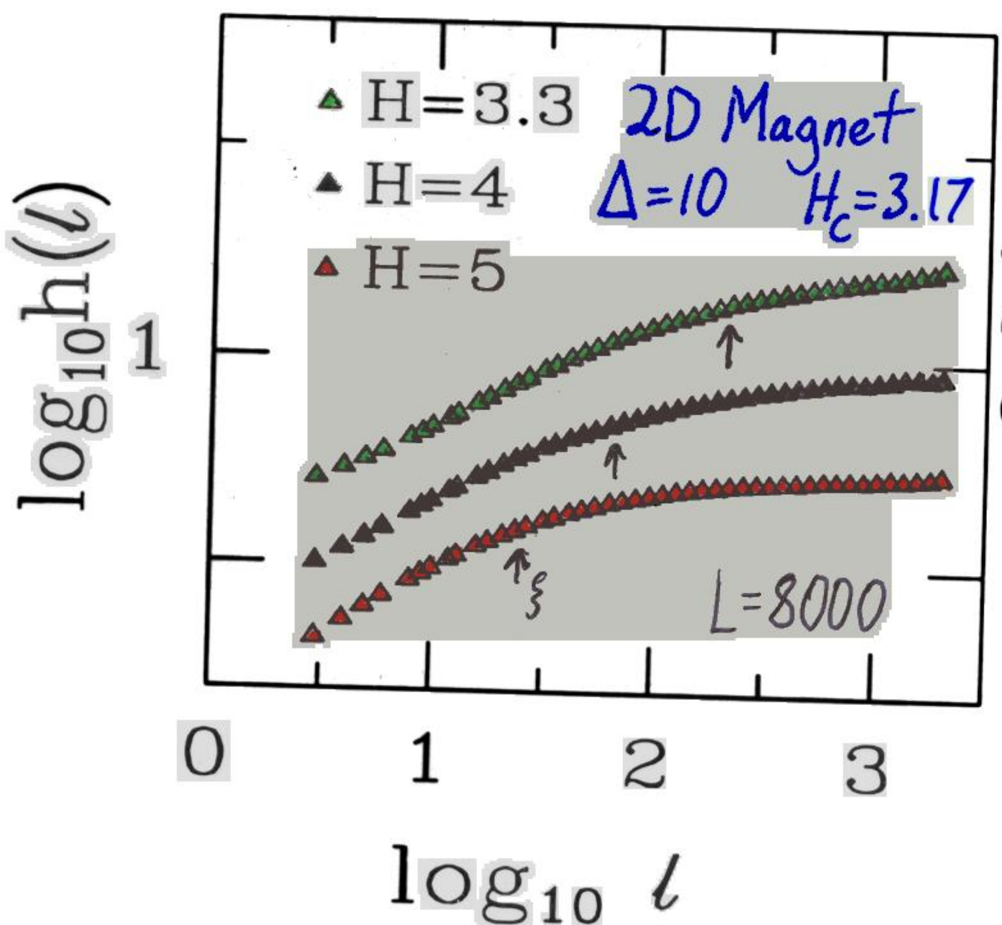
Increase P

⇒ smoother

⇒ fewer overhangs

⇒ new large scale behavior

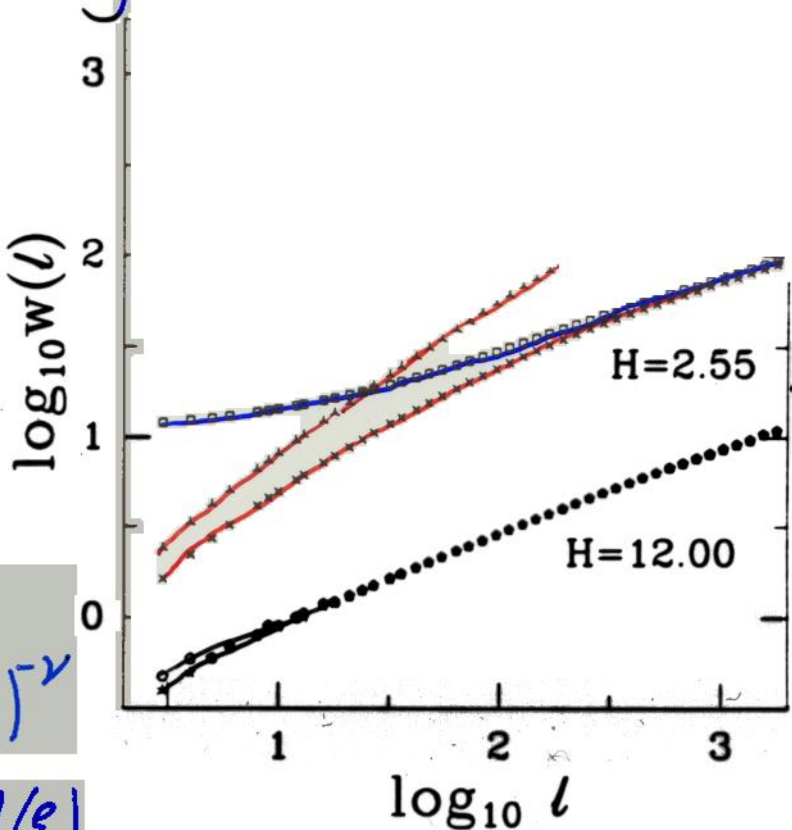
⇒ crossover scale ξ decreases



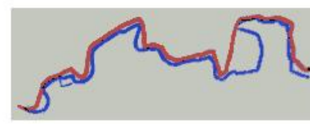
Same trends in self similar regime of magnets

(sequential

ng



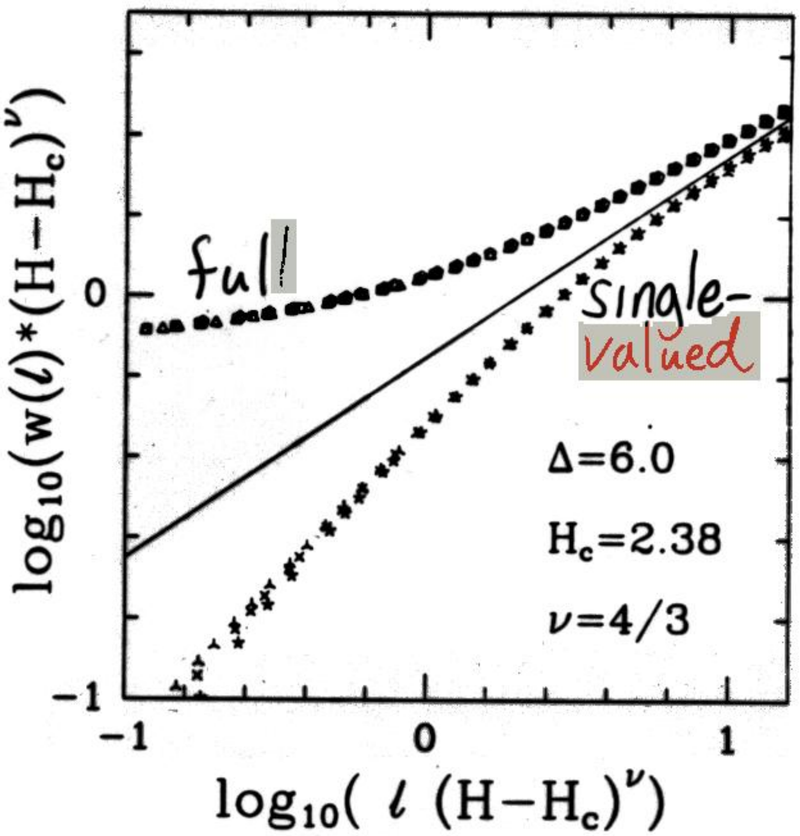
valued
— full
interface



Assume single scale $\xi \propto (H-H_c)^{-\nu}$

Then $w/\xi = f(\ell/\xi)$

Data fall on universal curve



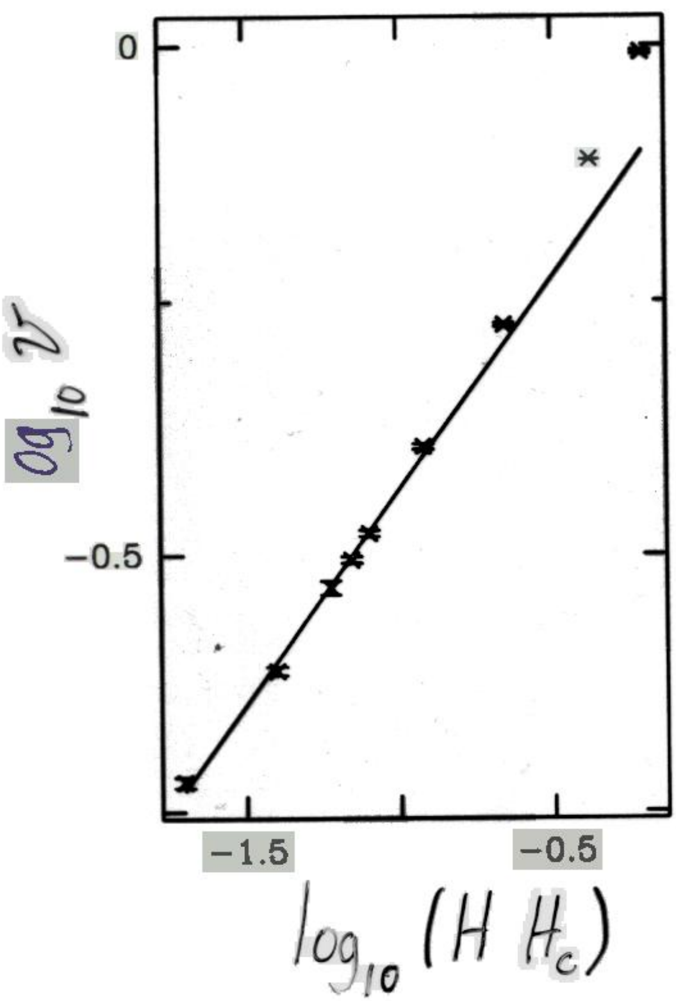
Sc f -1 0/d

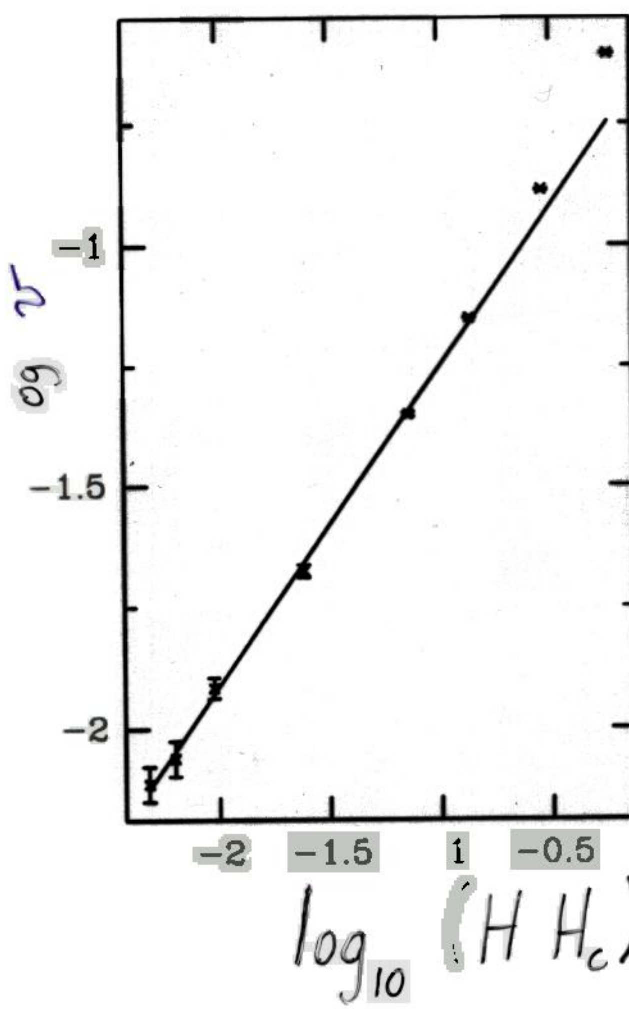
2D percolation

ς 0 43(2)

2D se affine

$\varsigma = 0\ 65(2)$





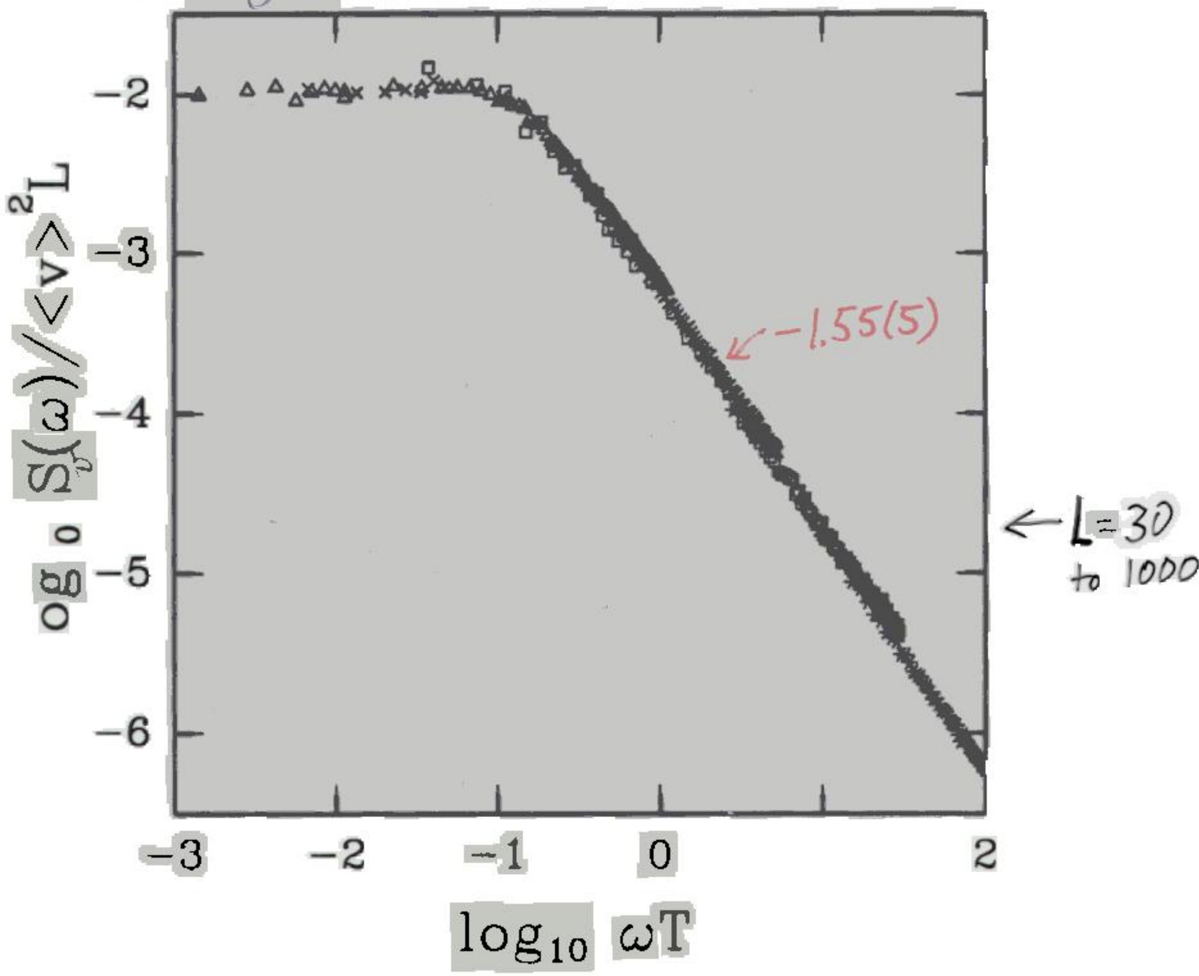
$\varsigma = \nu\,(D_f \quad D_{min})$

↑ dimension of minimal path

r law noise spectrum $S_v(\omega$

clusters of a scales up

ime to fill largest → $T \propto \xi^{D_f - 1}/\langle v \rangle$



$S(\omega) \propto \langle v^2 \rangle L$ → adds incoherently

→ constant fraction of v^2 at low ω

3D self-affine $T \; (F \; F_c)^{-1.3} \sim \xi/\langle v \rangle$

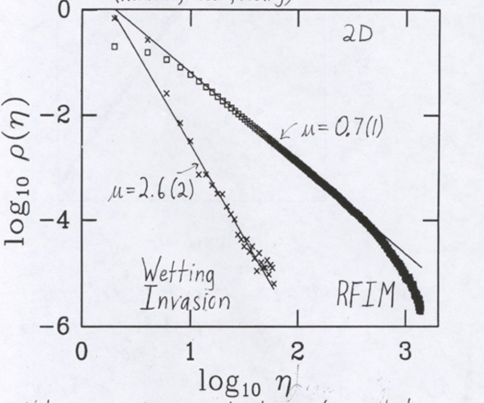
Power law distribution of noise amplitudes can increase α above ½ (Zhang, 1990)

$$\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + f + \eta(x,t) \qquad \rho(\eta) \sim 1/\eta^{1+\mu}$$

$\mu < 2 \Rightarrow$ self-similar, $\mu = 2.6 \Rightarrow \alpha \simeq 0.8$

Take surfaces separated by $\Delta t \Rightarrow \eta \simeq \frac{h(x,t+\Delta t) - h(x,t) - \Delta h}{\Delta h}$

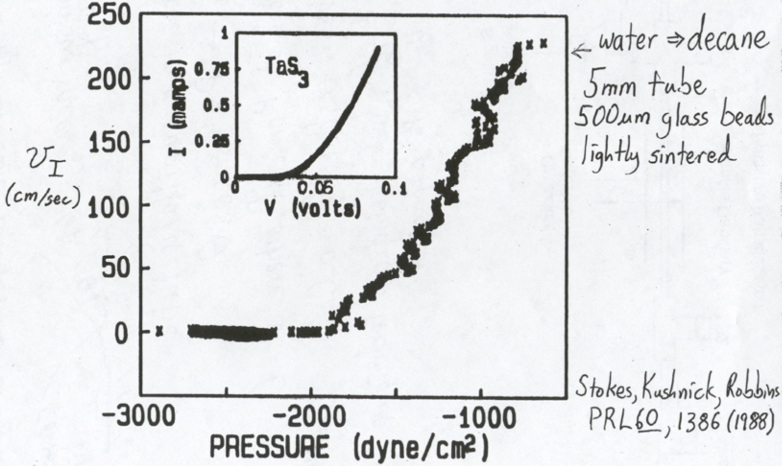
(Horvath, Vicsek, Family)



$\Rightarrow$ Values consistent with observed morphology and measurements on wetting invasions

Horvath Vicsek Family $2.7 = \mu$

Interface Velocity vs. Interfacial Pressure Drop
Compared to Charge-Density Wave Response

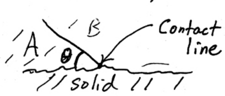


Stokes, Kushnick, Robbins
PRL 60, 1386 (1988)

Experimental Studies of Contact Line Motion

Suman Kumar, Dan Reich (Stokes, Kushnick, Bhattacharya

Disorder on surface pins contact line



For interface in a capillary tube

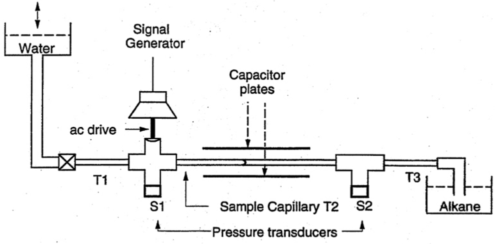
Force → $P = \frac{-2\gamma}{R}\cos\theta$

Pinning ⇒ range of P or $\cos\theta$ where $v = 0$

When moves $\cos\theta_a - \cos\theta = \alpha\, Ca^x$ $\quad Ca = \frac{\mu v}{\gamma}$

Measure $\frac{d\cos\theta}{dCa} \sim Ca^{x-1}$ using ac technique

v_{dc} with capacitor



Pure water with decane or hexadecane
Etching increases disorder ⇒ range of pinning
BUT see same exponent $x = 0.21 \pm 0.02$ in all cases
RG ⇒ $x = \frac{11}{9}$?