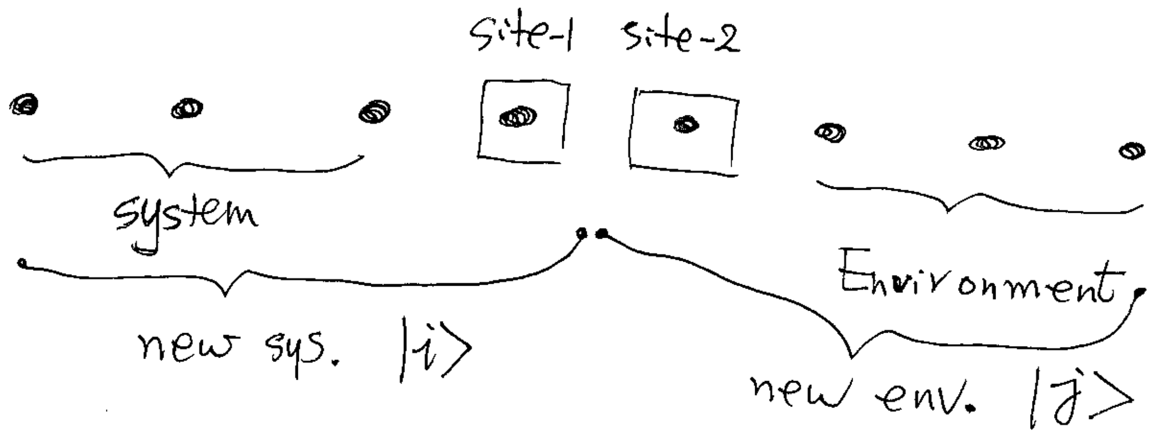
Notes on "DMRG for interacting systems"

I. DMRG set up and symmetries (e.g. SU2)

II. Generalize to fermionic systems

III. Momentum as additional quantum number (or momentum space DMRG) — applying to FQHE

IV. Examples



- use tensor product between system and environment.

$$|\psi\rangle = \sum_{ij} \psi_{ij} \, |i\rangle \otimes |j\rangle$$

① Diagonalize H_{full} to obtain $|\psi\rangle$.

② Construct the reduced density matrix

$$\rho_{ii'} = \sum_k \psi_{ik} \psi^*_{i'k}$$

(matrix representation $\psi \sim M$, $\rho = MM^+$)

diagonalize ρ and keep m eigenvectors corresponding to m largest eigenvalues. $\mathrm{Tr}\,\rho = 1$.

control truncation error $\varepsilon = 1 - \sum_{i=1}^{m} \rho_i$

③ Construct m new basis vectors from $2m$ states

$$|\alpha_l\rangle |S_{l+1}\rangle \Rightarrow |\alpha_{l+1}\rangle$$

$$|\alpha_{l+1}\rangle = \sum_{S_{l+1}, \alpha_l} A^{l+1}_{\alpha_{l+1}, \alpha_l}(S_{l+1}) \, |\alpha_l\rangle \otimes |S_{l+1}\rangle$$

each row is the eigenvector of $\hat{\rho}$.

④ update operators and truncate them, eg.

$$\tilde{H}_{sys} = \underbrace{H'_{sys}}_{\text{previous}} \otimes I_{l+1} + \sum C_{ij} B'_i \otimes B^{l+1}_j$$

$$H_{sys} = OHO^+$$

O is $m \times 2m$ matrix, the rows of O are the kept states.

$$\left(A^{\uparrow}_{\alpha_{\ell+1}, \alpha_\ell} \quad A^{\downarrow}_{\alpha_{\ell+1} \alpha_\ell} \right)$$

* SU2 symmetry

Common eigenstates of $\hat{S}^2$, S_z and H.

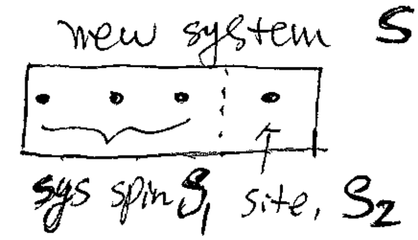
basis states $|S, m, \alpha\rangle$.

The matrix element can be obtained using Wigner-Eckart theorem.

$$\langle S', m', \alpha' | T^{(k)}_{q,i} | S, m, \alpha \rangle = (-1)^{S'-m'} \begin{pmatrix} S' & k & S \\ -m' & q & m \end{pmatrix} \underbrace{\langle S', \alpha' \| T^{(k)}_i \| S, \alpha \rangle}_{\uparrow \text{ reduced element}}$$

$T^{(k)}_i$ the irreducible tensor operator. i- other index, site for example. $T^{(k)}_{q,i}$ it is q component ($q = -k, -k+1, \cdots, k$).

1. Set basis



$$|SM\alpha\rangle = \sum_{m_1 m_2} |S_1 m_1 S_2 m_2, \alpha\rangle \langle S_1 m_1 S_2 m_2 | SM \rangle$$

Clebsch-Gordan coefficients

representing an additional Unitary transformation to basis set of U1.

2. Identifying irreducible tensors

definition $[S^z_i, T^{(k)}_{q,i}] = \hbar q T^{(k)}_{q,i}$

$$[S_{\pm i}, T^{(k)}_{q,i}] = \sqrt{(k\mp q)(k\pm q+1)}\,\hbar\, T^{(k)}_{q\pm 1,i}$$

$$T^{(1)}_1 = -\frac{1}{\sqrt{2}} S^+$$

$$T^{(1)}_0 = S^z$$

$$T^{(1)}_{-1} = \frac{1}{\sqrt{2}} S^-$$

$$\langle S \| T^{(1)} \| S \rangle = \sqrt{(2S+1)(S+1)S}$$

Two irreducible tensor operator $T^{(k_1)}, U^{(k_2)}$ can form a new irreducible operator $X^{(k)}$

$$X^{(k)}_q = \sum_{q_1 q_2} (-1)^{-k_1+k_2+q} \sqrt{2k+1} \begin{pmatrix} k_1 & k_2 & k \\ q_1 & q_2 & -q \end{pmatrix} T^{(k_1)}_{q_1} U^{(k_2)}_{q_2}$$

$$\vec{S}_i \cdot \vec{S}_j = -T^{(1)}_{1i} T^{(1)}_{-1j} - T^{(1)}_{-1i} T^{(1)}_{1j} + T^{(1)}_{0i} T^{(1)}_{0j} \qquad \text{rank-0}$$

similarly, $T^{(1)}$:

$$\begin{cases} T^{(1)}_1 = \dfrac{i}{\sqrt{2}}\left(-\dfrac{1}{\sqrt{2}}\right)\left[(\vec{S}_i \times \vec{S}_j)_x + i(\vec{S}_i \times \vec{S}_j)_y\right] \\ T^{(1)}_0 = \dfrac{i}{\sqrt{2}} (\vec{S}_i \times \vec{S}_j)_z \\ T^{(1)}_{-1} = \dfrac{i}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{2}} \left[(\vec{S}_i \times \vec{S}_j)_x - i(\vec{S}_i \times \vec{S}_j)_y\right] \end{cases}$$

is useful for systems with three vector chiral terms.

3. Beside the transformation to total S basis, and using the reduced matrix elements for irreducible tensor operators, everything else can be carried out similar to $U(1)$-DMRG.

* Example, t-J model

$$H=\sum_{\langle ij\rangle\sigma} -t_{ij}\,(c_{i\sigma}^{+}c_{j\sigma}+h.c.)+\sum_{\langle ij\rangle} J_{ij}\,(\vec{S}_i\cdot\vec{S}_j-n_i n_j/4)$$

Irreducible tensor - rank-$\frac{1}{2}$

$$\begin{cases} T^{(\frac{1}{2})}_{-\frac{1}{2},i}=c_{i\uparrow} \\ T_{\frac{1}{2},i}=-c_{i\downarrow}\end{cases}\qquad \begin{cases} T'^{(\frac{1}{2})}_{-\frac{1}{2}i}=c_{i\downarrow}^{+} \\ T^{(\frac{1}{2})}_{\frac{1}{2}i}=c_{i\uparrow}^{+}\end{cases}$$

Thus that the hopping term H_t can be expressed as

$$H_t=\sum_{\langle ij\rangle}-t_{ij}\left(T'^{(\frac{1}{2})}_{\frac{1}{2}i}T^{(\frac{1}{2})}_{-\frac{1}{2}j}+T'^{(\frac{1}{2})}_{\frac{1}{2}j}T^{(\frac{1}{2})}_{-\frac{1}{2}i}-T'^{(\frac{1}{2})}_{-\frac{1}{2}i}T^{(\frac{1}{2})}_{\frac{1}{2}j}-T^{(\frac{1}{2})}_{-\frac{1}{2}j}T^{(\frac{1}{2})}_{\frac{1}{2}i}\right)$$

Two irreducible $T^{(k_1)}$, $T'^{(k_2)}$ can compose a new tensor $X^{(k)}$:

$$X^{(k)}_q=\sum_{q_1q_2}T^{(k_1)}_{q_1}T'^{(k_2)}_{q_2}\,S^{k_1k_2}_{q_1q_2,k,q}=\sum_{q_1q_2}(-1)^{-k_1+k_2+q}\sqrt{2k+1}\begin{pmatrix}k_1 & k_2 & k\\ q_1 & q_2 & -q\end{pmatrix}\cdot T^{(k_1)}_{q_1}T'^{(k_2)}_{q_2}$$

so
$$X^{(0)}_0=\sum_{q_1q_2}T^{(\frac{1}{2})}_{q_1}T'^{(\frac{1}{2})}_{q_2}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\\ q_1 & q_2 & 0\end{pmatrix}=\frac{1}{\sqrt{2}}\left(T^{(\frac{1}{2})}_{\frac{1}{2}}T'^{(\frac{1}{2})}_{-\frac{1}{2}}-T^{(\frac{1}{2})}_{-\frac{1}{2}}T'^{(\frac{1}{2})}_{\frac{1}{2}}\right)$$

Therefore, we have
$$H_t=\sum_{\{i,j\}}-\sqrt{2}\,t_{ij}\left(X^{(0)}_{0,(ij)}+X^{(0)}_{0,(ji)}\right)$$

II. DMRG for fermionic system

In DMRG, basis states usually form block structure according to their quantum numbers.

basis for system $|qn1, qn2, \alpha \cdots\rangle$

qn1 ... maybe S_z^{tot} of system

qn2 ... maybe electron number N_e^{tot} of system

operators are sparse matrices, which connect certain basis states with fixed change of quantum numbers.

$c_{i\uparrow}^{+}$ gives $\Delta qn1 = \frac{1}{2}$, $\Delta qn2 = 1$. etc.

(In DMRG, you match the quantum number between operators and basis.)

Jordan-Wigner transformation is easy to implement in such a system

$$c_{i\sigma}^{+} = e^{i\pi \sum_{l<i} n_l} b_{i\sigma}^{+}$$

$b_{i\sigma}^{+}$ is the corresponding bosonic creation operator, which adds one particle at site i to the basis state (its a hardcore boson operator).

For $C_{i\sigma}^{+}$ in the system, the phase $e^{i\pi\sum_{l<i} n_l}$ will be combined with $b_{i\sigma}^{+}$ when i-site is being included into system. But for $C_{i\sigma}^{+}$ in the environment, we incorporate part of sign

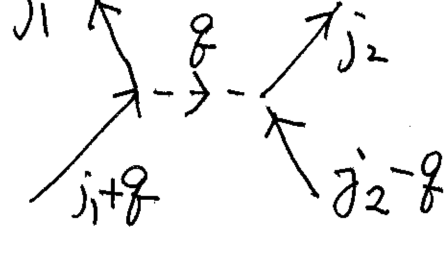
$$C_{i\sigma(env)}^{+} \sim e^{i\pi \sum_{\substack{l<i \\ l\in env.}} n_l}$$

When dealing with H_{full}, all remaining signs for operators from environment or sys/env sites should be taken into account. They are easy to get since each basis has a good quantum number for electron number.

III Momentum space DMRG set up

$$H=\sum_j \xi_j C_j^{+}C_j + \sum_{j_1 j_2 j_3 j_4} A_{j_1 j_2 j_3 j_4} C_{j_1}^{+} C_{j_2}^{+} C_{j_3} C_{j_4}$$

$$=\sum_j \xi_j C_j^{+} C_j + \sum_{j_1, j_2, q} A_{j_1, j_2, j_2-q, j_1+q} C_{j_1}^{+} C_{j_2}^{+} C_{j_2-q} C_{j_1+q}$$



C_j^{+} creates an electron with momentum j. The total momentum is conserved.

* Adding qn3 ~ total momentum to basis quantum numbers.

$$|qn1, qn2, qn3, \cdots \alpha\rangle$$
$$\uparrow \quad \uparrow \quad \uparrow$$
$$S^z \quad N_e^{tot} \quad q^{tot} \quad \cdots$$

* Order the summation to speed up

$$H = \sum_{\substack{j_1, j_2, q \\ \{j_1 < j_2 \\ j_1+q < j_2-q\}}} R_{j_1 j_2, j_2-q, j_1+q}\, C_{j_1}^+ C_{j_2}^+ C_{j_2-q} C_{j_1+q}$$

$$R_{j_1 j_2 j_3 j_4} = A_{j_1 j_2 j_3 j_4} + A_{j_2 j_1 j_4 j_3} - A_{j_2 j_1 j_3 j_4} - A_{j_1 j_2 j_4 j_3}$$

$$A_{j_1 j_2 j_3 j_4} = \int \varphi_{j_1}^*(\vec{r}_1)\, \varphi_{j_2}^*(\vec{r}_2)\, V(\vec{r}_1 - \vec{r}_2)\, \varphi_{j_3}(r_2)\, \varphi_{j_4}(r_1)$$

* Combining similar terms to reduce the number of operators, e.g.

define $\sum_{j_2, q} R_{j_1 j_2 j_2-q\, j_1+q}\, C_2^+ C_{j_2-q} C_{j_1+q} \equiv C_3(j_1)$

$H \to \sum_{j_1} C_{j_1}^+ C3(j_1)$ Only when 3 C operators are all in system or environment!

(keep remaining terms unchanged).

* carefully including all terms to your H_{full} (so many terms ...).

* benefit is that we can target states with different q^{tot}. It works for torus, infinite cylinder (or finite) or sphere.

* Exact diagonalization program for FQHE (torus example)

The single particle wavefunction: (N - Landau level index)

$$\psi_{N,j}(x,y) = \left(\frac{1}{2^N N! \pi^{1/2} L_y l}\right)^{1/2} e^{i\frac{X_j}{l^2}y - \left(\frac{X_j - x}{2l^2}\right)^2} H_N\left(\frac{X_j - x}{l}\right)$$

(infinite system $L_x = \infty$). $X_j = \frac{2\pi l^2}{L_y}$, $j = 0, 1, 2, N_s - 1$.

But, we should use the finite one: $L_x L_y = 2\pi l^2 N_s$ ← number of flux

$$\text{"} \sum_{k=-\infty}^{+\infty} e^{i\frac{X_{jk} y}{l^2}} e^{-(x - X_{jk})^2/2l^2} H_N\left(\frac{X_{jk} - x}{l}\right)$$

$$X_{jk} = \frac{2\pi l^2 j}{L_y} + k L_x \quad \text{(forcing the periodic condition).}$$

$$V_{j_1 j_2 j_3 j_4} = \frac{1}{2}\int d^2\vec{r}_1\, d^2\vec{r}_2\, \psi^*_{Nj_1} \psi^*_{Nj_2} V(\vec{r}_1 - \vec{r}_2) \psi_{N_3 j_3} \psi_{N_4 j_4}$$

set up ED code for $N_s = 6$ (flux number) and $N_e = 2$ (electron number).

totally $\frac{N_s!}{N_e!\,(N_s - N_e)!}$ states $= \frac{6\cdot 5}{2} = 15$

110000 ----
101000

too many, wait, use k-total momentum!

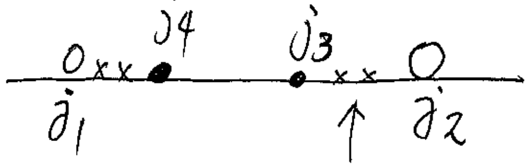
k=0 sector (mod N_s).

$010,\ 001,\quad 1+5=0$

$001,\ 010\quad 2+4=0$

$$\left(\begin{array}{l} R_{j_1j_2j_3j_4} = A_{j_1j_2j_3j_4} + A_{j_2j_1j_4j_3} \\ \quad - A_{j_2j_1j_3j_4} - A_{j_1j_2j_4j_3} \end{array}\right)$$

only 2 states $H = \begin{pmatrix} R_{1551} & f R_{1542} \\ f R^*_{1542} & R_{2442} \end{pmatrix}$ f - fermion sign check (if $Ne > 2$). ($f=1$ here)



are there particles in between to give fermionic signs?

k=1 sector

110, 000

001, 001

000, 110

$$\begin{pmatrix} R_{0110} & f_1 R_{0152} & f_2 R_{0143} \\ * & R_{2552} & f_3 R_{2543} \\ * & * & R_{3443} \end{pmatrix}$$

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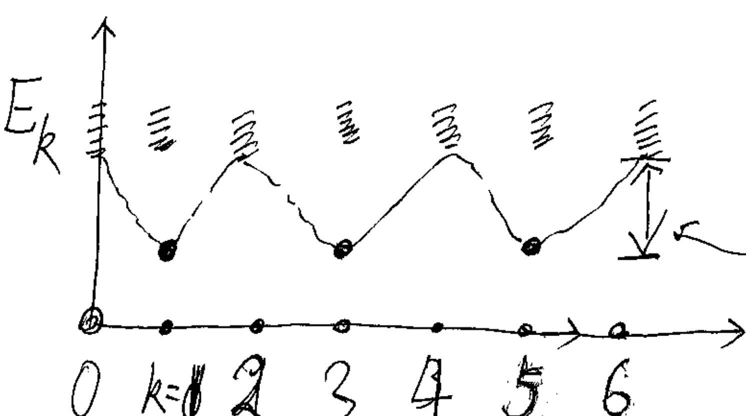
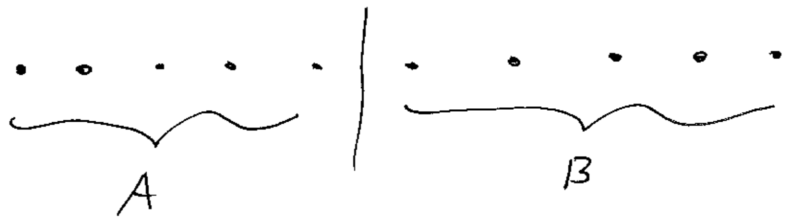


3-fold degenerating Laughlin FQHE

excitation gap is finite and robust

more electrons, separating orbits to A & B



$$|i_A\rangle \equiv |n_0, n_1, n_2, \cdots, n_{N_s/2}\rangle \quad \cdots$$

occupation number for a set of orbitals in A.

a table for finding i_A from $\{n_j\}$.

So once you use $C_{j_1}^{+} C_{j_2}^{+} \cdots |i_A\rangle$ etc

you can get new index $|i_A'\rangle$ quickly.

total basis $|\cancel{k}n\rangle \equiv |i_A\rangle \otimes |j_B\rangle$

new index, adding one basis when

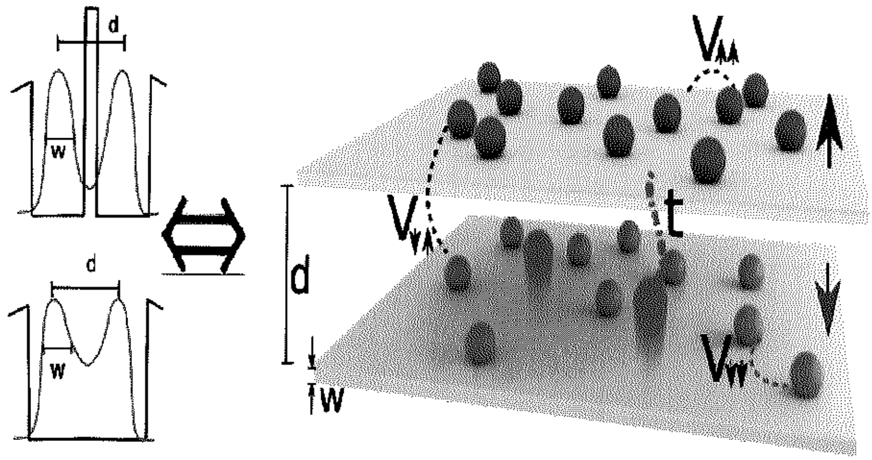
$q_n(iA) + q_n(jB)$ = desired momentum k.

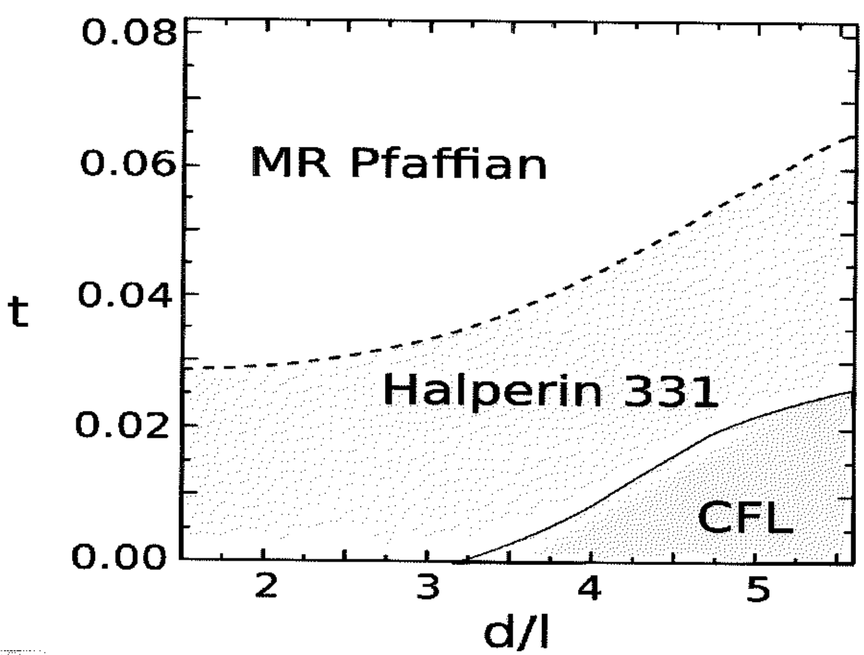
a table for finding i quickly from (i_A, i_B).

Set up matrix elements, then we can diagonalize H for a chosen total momentum k. $|\Psi\rangle = \sum_{ij} \Psi_{ij} |i_A\rangle \otimes |j_B\rangle$

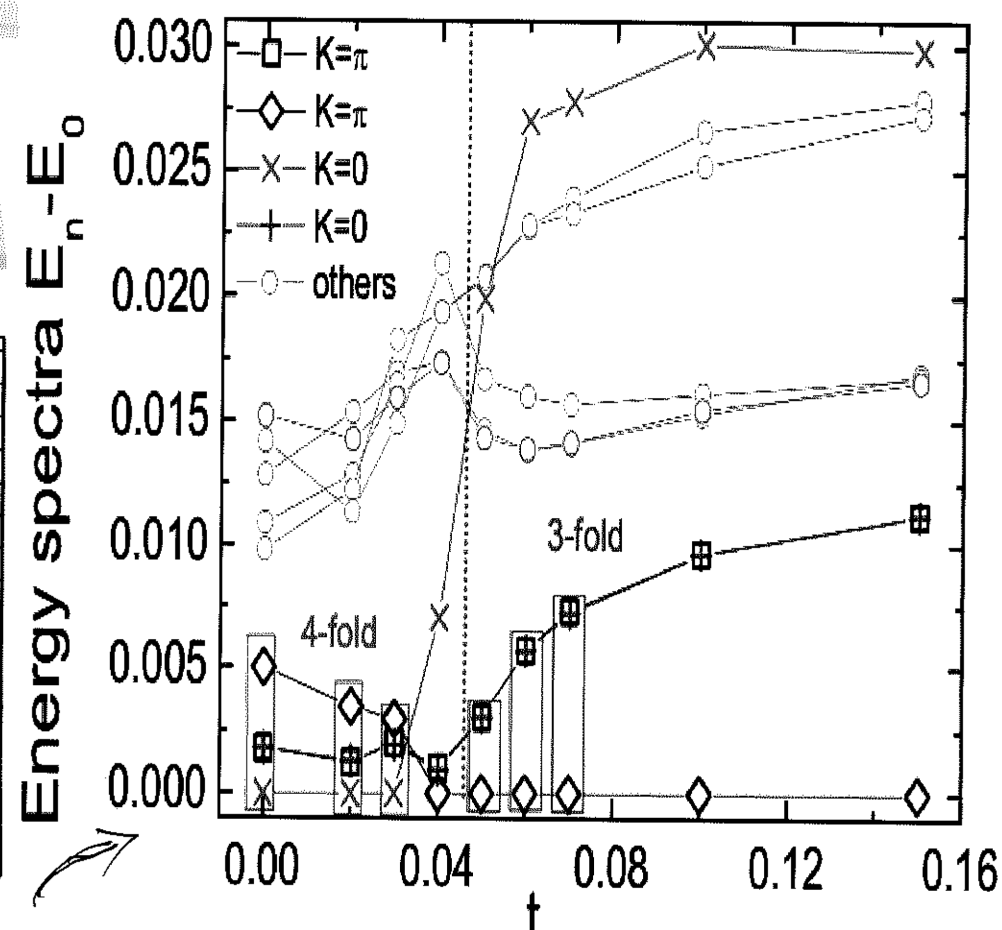
$E(k)$ gives information of topological degeneracy.

Fractional Quantum Hall Bilayers at Half-Filling: Tunneling-driven Non-Abelian Phase





Predicted by Xiao-Gang Wen & other works



Energy spectrum for torus system with varying tunneling $t_\perp$

* Calculating topological Chern number

adding generalized boundary phase θ (α,β)

$$T(L_x\hat{x})\psi_{Nj}(x,y) = e^{i\alpha L_x}\psi_{Nj}(x,y)$$
$$T(L_y\hat{y}) \quad '' \quad = e^{i\beta L_y} \quad ''$$

T is the single-particle magnetic translational operator in the x or y directions.

$$\psi_{N,j}(x,y) = \left(\frac{1}{2^N N! \pi^{1/2} L_y \ell}\right)^{1/2} e^{i\frac{X'_{jk}}{\ell^2} - (x-X'_{jk})^2/2\ell^2} \cdot H_N\left(\frac{X'_{jk}-X}{\ell}\right)$$

N: LL index; j: momentum index in Landau gauge

$$X'_{jk} = \frac{2\pi\ell^2 j}{L_y} + kL_x + \beta\ell^2$$

Nontrivial: changing $\beta = 0$ to $\frac{2\pi}{L_y}$, will increase $j \to j+1$ (representing the center of the orbital).

Obtain many-body wavefunction as before

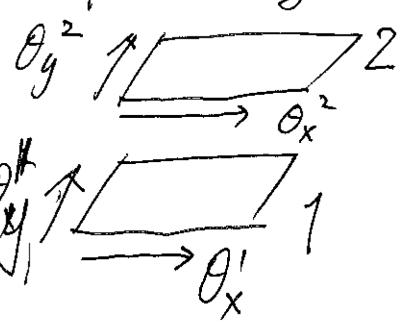
$$|\psi(\alpha,\beta)\rangle = \sum \psi_{ij}(\alpha,\beta)\,|i_A(\alpha,\beta)\rangle \otimes |j_B(\alpha,\beta)\rangle$$

(One should also do unitary transformation, so that
$$\psi \to e^{-i\alpha(x_1+\cdots x_{Ne})} e^{-i\beta(y_1+\cdots+y_N)}\psi$$
Hamiltonian $-i\frac{\partial}{\partial x_i} \to -i\frac{\partial}{\partial x_i}+\alpha$, $-i\frac{\partial}{\partial y} \to -i\frac{\partial}{\partial y}+\beta$)

$$C = \frac{i}{4\pi}\iint d\theta_x d\theta_y \left(\left\langle\frac{\partial\psi}{\partial\theta_x}\middle|\frac{\partial\psi}{\partial\theta_y}\right\rangle - \left\langle\frac{\partial\psi}{\partial\theta_y}\middle|\frac{\partial\psi}{\partial\theta_x}\right\rangle\right) \text{ with } \begin{cases}\theta_x = \alpha L_x \\ \theta_y = \beta L_y\end{cases}$$

For one component FQHE with no disorder, if there is a spectrum gap between ground states and excited states, we have $\Psi_{ij}(\alpha,\beta) \equiv \Psi_{ij}(0,0)$ (interaction does not dependent on boundary phases although $\Psi_{N,j}$ (orbitals) does).
All Berry phase comes from the rotation of the basis vectors, the Chern number $=$ filling number $= N_e/N_s$.

The bilayer case is nontrivial.



if (θ_x^1, θ_y^2) change, but $(\theta_x^2 = \theta_y^1 = 0)$, the interlayer interaction will dependent on boundary phases, we can obtain ~~B~~ drag Hall conductance.

$$C^{\alpha\beta} = \frac{i}{4\pi}\iint d\theta_x^\alpha d\theta_y^\beta \left\{ \left\langle \frac{\partial\Psi}{\partial\theta_x^\alpha} \middle| \frac{\partial\Psi}{\partial\theta_y^\beta} \right\rangle - \left\langle \frac{\partial\Psi}{\partial\theta_y^\beta} \middle| \frac{\partial\Psi}{\partial\theta_x^\alpha} \right\rangle \right\}$$

forms 2×2 Chern number matrix.

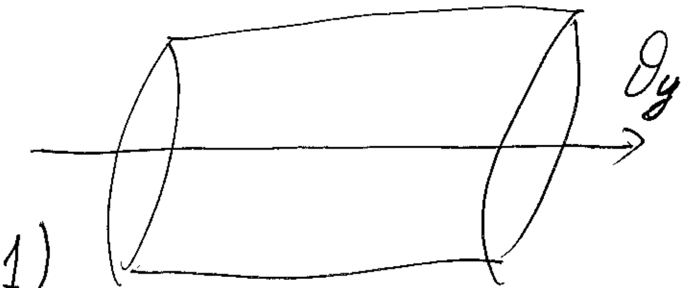
for 331 Halperin state: $C = \frac{1}{8}\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$ inverse of K matrix. $K = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$.

IV Examples

* DMRG inserting flux to detect topological degeneracy

$S_i^+ S_j^- \to e^{i\theta_y} S_i^+ S_j^-$

if the bond crosses the y-boundary (e.g. $i_y = L_y, j_y = 1$)



using $\psi(\theta_y - \Delta\theta)$ as the initial wavefunction for new θ_y. Updating $H(\theta_y)$ locally when we sweep pass different points.

Measuring $\langle S_i^z \rangle$ for different θ_y.

comparing entanglement spectrum as a function of θ_y.

* Spontaneous Quantum Hall Effect

Using complex code to work with minimum entangled state if time-reversal symmetry is spontaneously broken (In real code, ground state would have additional double degeneracy).

* Topological entanglement spectrum and entropy for detecting topological order (FQHE)

root configuration

● ○ ○ , ● ○ ○

1 0 0 , 1 0 0 for 1/3 FQHE

$\left\{\begin{array}{l} 11100, 11100 \text{ for Read-Rezayi } \frac{13}{5} \\ 10101, 10101, 10101 \end{array}\right.$

TABLE I: In this table. we analyze the counting rule of the edge excitations in the "…11100111001110|00 …" sector. which has multiplicities 1, 1, 3, 6, … at $\Delta L = 0, 1, 2, 3, \ldots$.

<table>
<tr><th>$\Delta L = 0$</th><th>$\Delta L = 1$</th><th>$\Delta L = 2$</th><th>$\Delta L = 3$</th><th>$\Delta L = 4$</th></tr>
<tr><td>11100111001110|0000</td><td>11100111001101|0000</td><td>11100111001100|1000</td><td>11100111001100|0100</td><td>11100111001100|0010</td></tr>
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<tr><td></td><td></td><td></td><td>11010110101101|0000</td><td>11100110101010|1000</td></tr>
<tr><td></td><td></td><td></td><td></td><td>11100110100111|0000</td></tr>
<tr><td></td><td></td><td></td><td></td><td>11100101101011|0000</td></tr>
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<tr><td></td><td></td><td></td><td></td><td>11010110101100|1000</td></tr>
<tr><td></td><td></td><td></td><td></td><td>1101011010110101101|0000</td></tr>
</table>

TABLE II: In this table, we analyze the counting rule of the edge excitations in the . . . 111001110011\|00 . . . sector, which has multiplicities $1, 2, 5, 9, \ldots$ at $\Delta L = 0, 1, 2, 3, \ldots$.

$\Delta L = 0$	$\Delta L = 1$	$\Delta L = 2$	$\Delta L = 3$
111001110011|0000	111001110010|1000	111001110010|0100	111001110010|0010
	111001101011|0000	111001110001|1000	111001110001|0100
		111001101010|0000	111001101010|0100
		111001100111|0000	111001101001|1000
		110101101011|0000	111001100110|1000
			111001011010|1000
			110101101010|1000
			110101100111|0000
			1101001101011010110|0000

TABLE III: In this table, we analyze the counting rule of the edge excitations in the . . . 10101101011010l\| . . . sector, which has multiplicities $1, 3, 6, 13, \ldots$ at $\Delta L = 0, 1, 2, 3, \ldots$.

$\Delta L = 0$	$\Delta L = 1$	$\Delta L = 2$	$\Delta L = 3$
101011010110101|0000	101011010110100|1000	101011010110100|0100	101011010110100|0010
	101011010110011|0000	101011010110010|1000	101011010110010|0100
	101011010101101|0000	101011010101100|1000	101011010101100|0100
		101011010101011|0000	101011010110001|1000
		101011001110011|0000	101011010101010|1000
		101010110101101|0000	101011001110010|1000
			101011010011100|1000
			101010110101100|1000
			101011010100111|0000
			101011001101011|0000
			100111001110011|0000
			101010110101011|0000
			011010110101101|0000

TABLE IV: In this table, we analyze the counting rule of the edge excitations in the . . . 1010110101100\|00 . . . sector, which has multiplicities $1, 2, 5, 10, \ldots$ at $\Delta L = 0, 1, 2, \ldots$.

$\Delta L = 0$	$\Delta L = 1$	$\Delta L = 2$	$\Delta L = 3$
1010110100|0000	1010110010|0000	1010110001|0000	1010110000|1000
	1010101100|0000	1010101010|0000	1010101001|0000
		1001110010|0000	1001110001|0000
		1010011100|0000	1010100110|0000
		0110101100|0000	1010011010|0000
			1001101010|0000
			0110101010|0000
			0110011100|0000
			0110101101011 00|0000
			1001110011100 10|0000

III. Results for finite-layer thickness

In a two dimensional system, the ideal Coulomb interaction between electrons has $V(q) = 1/q$. The finite thickness in the normal direction of an experimental quantum Hall system modifies the short-distance part of the ideal 2D interaction, yielding

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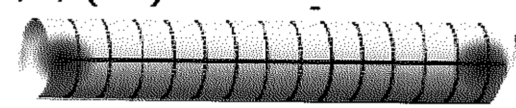
Entanglement Spectrum: 12/5 FQHE as Read-Rezayi state

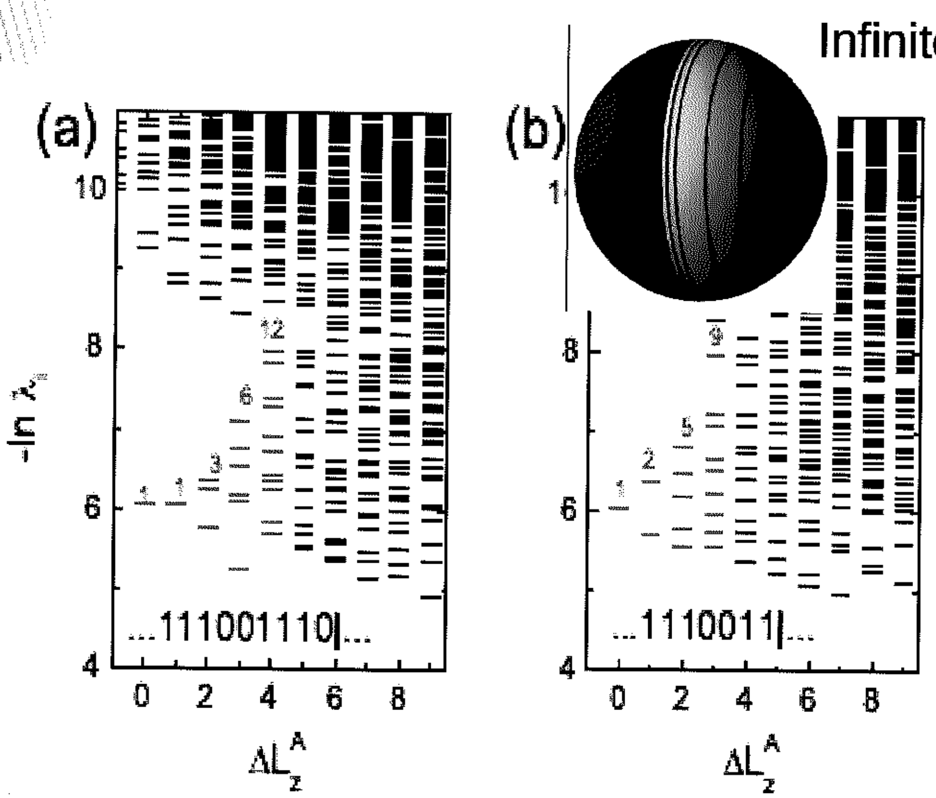
Detected by Entanglement Spectrum:
111001110011100…..
1: 1,1,3,6,12…
111001110011…..
: 1,2,5,9,….

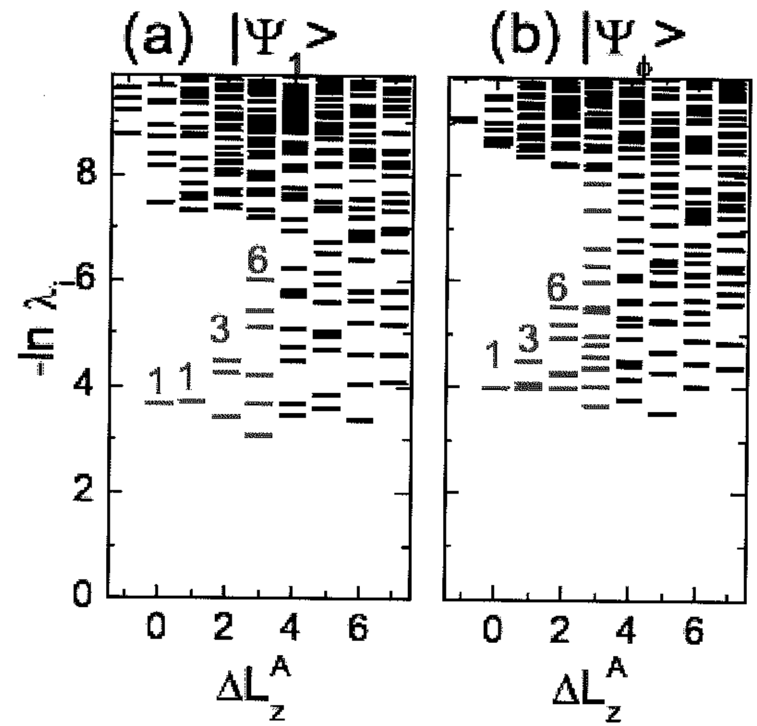
On cylinder, we also find another ground state: 10101 10101 10101

Entanglement spectrum:
1,1,3,6 ... and 1,3,6, (13)



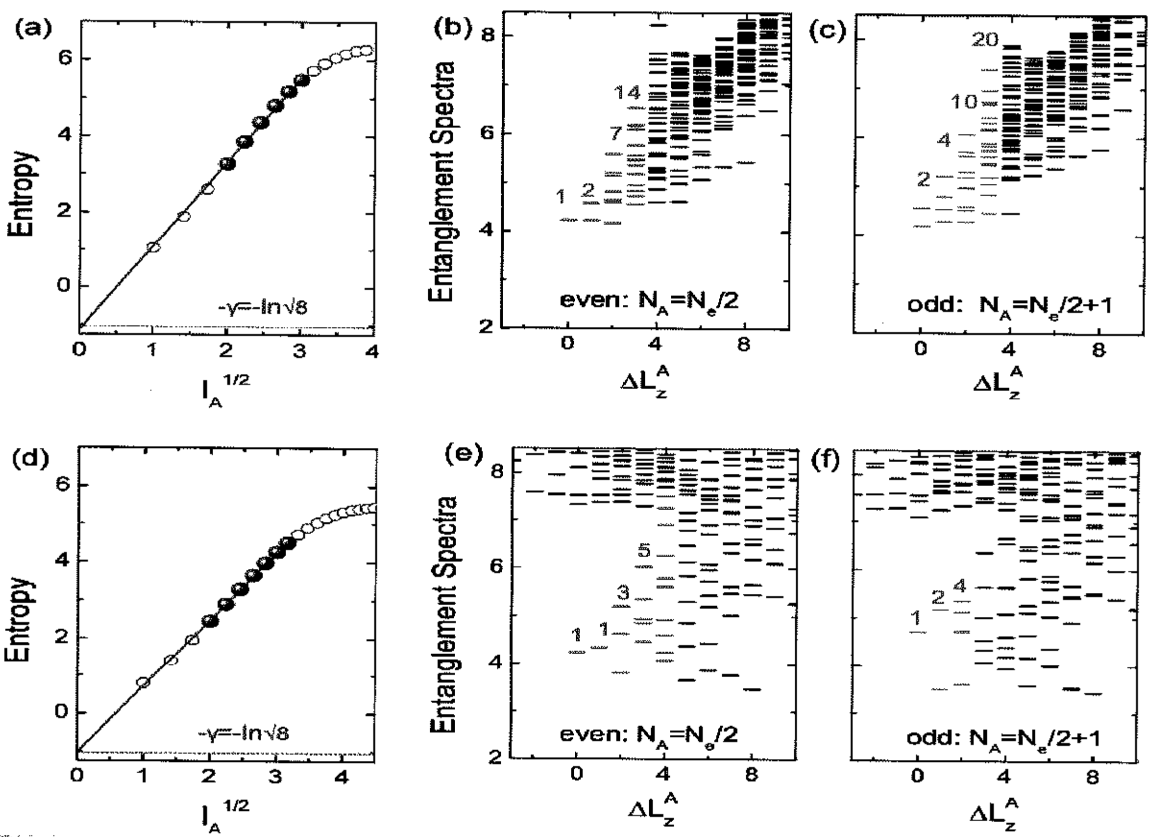
Infinite Cylinder Ly=24

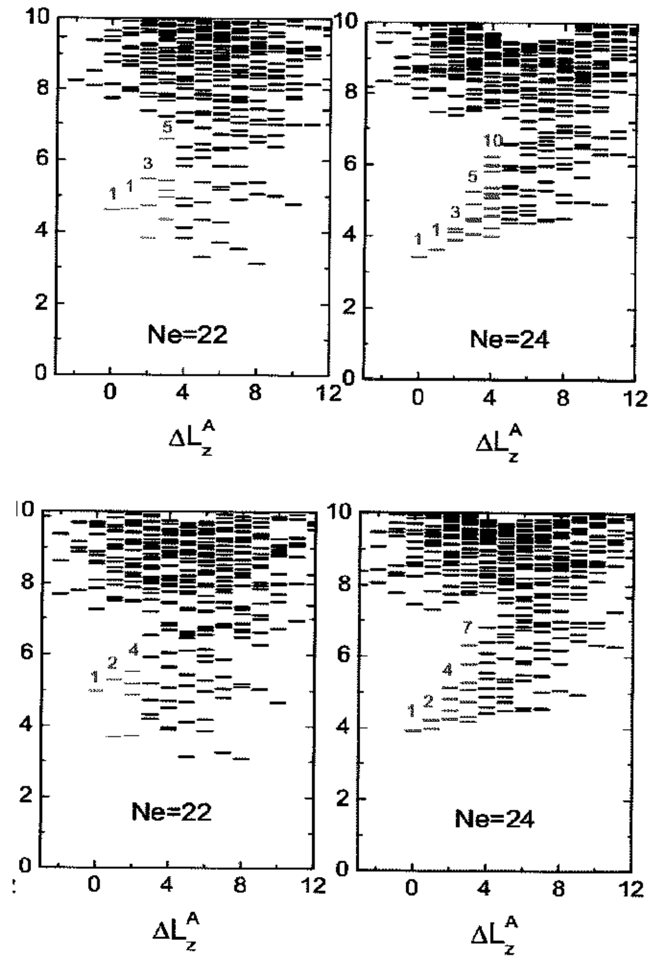




Entanglement spectrum and entropy, Moore-Read vs. Halperin 331

Halperin 331 state





Ne=22 Moore-Read state

Better spectrum for largest system (Ne=24)