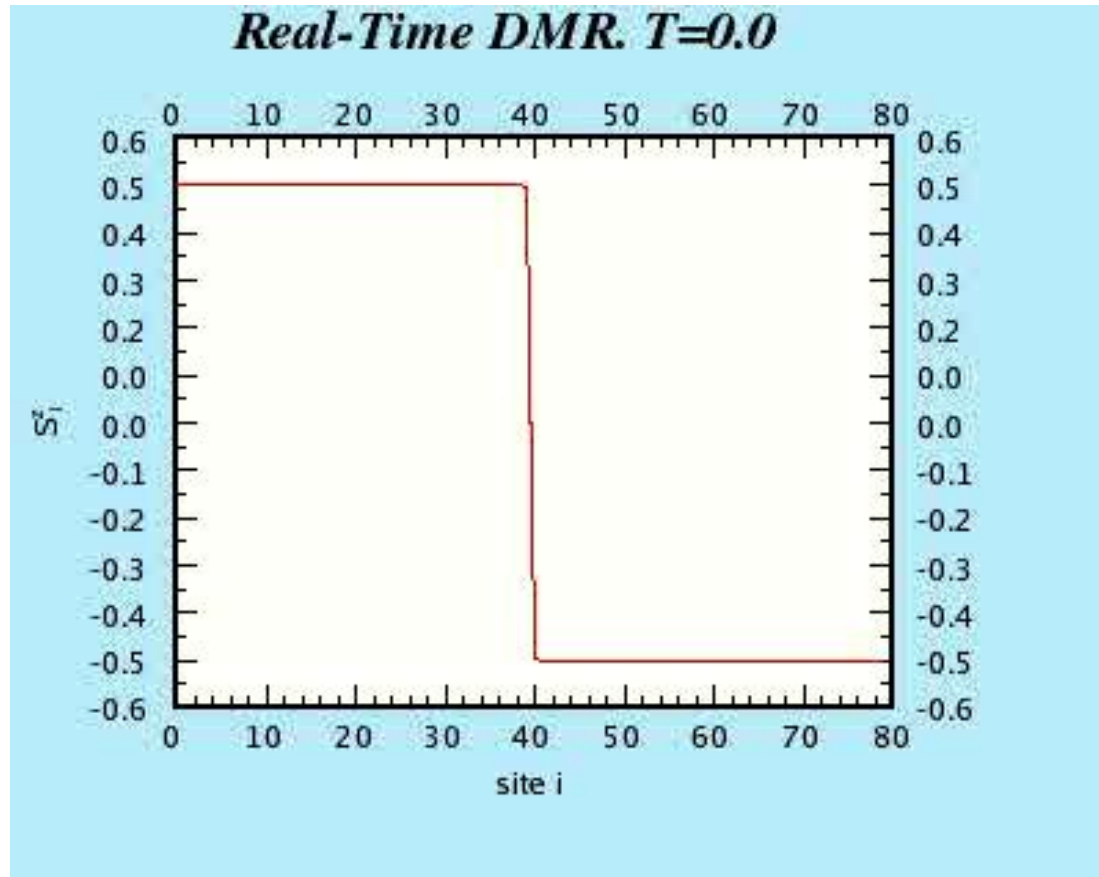


Spin transport

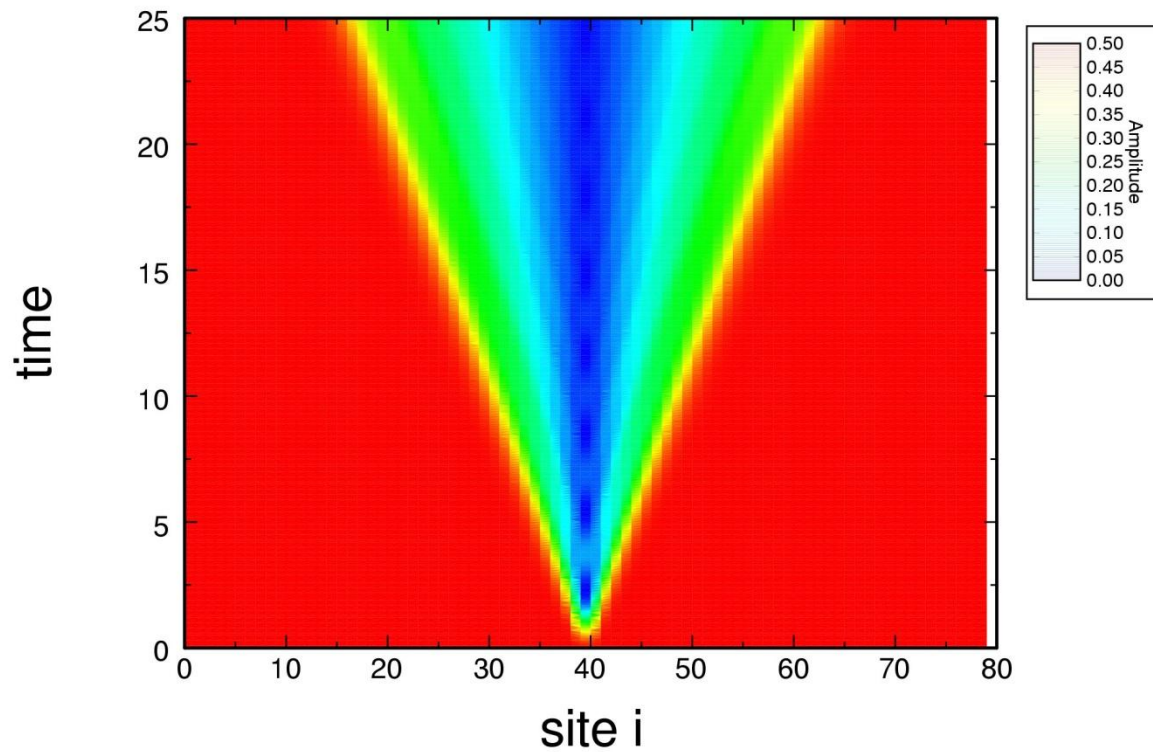
Example: half polarized spin $S=1/2$ chain



Spin transport

Example: half polarized spin $S=1/2$ chain

$$\Delta=0.0$$



The enemy: Entanglement growth

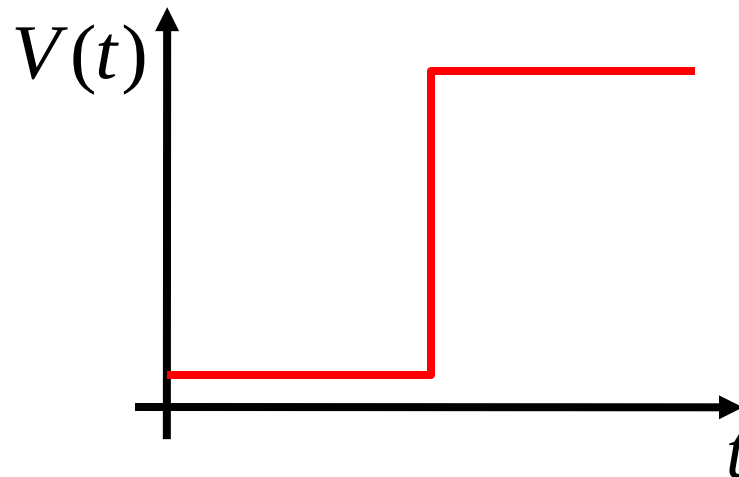
We have seen that the truncation error, or the number of state that we need to keep to control it, depends fundamentally on the entanglement

$$S = S(t)$$

We need to understand this behavior if we want to learn how to fight it!

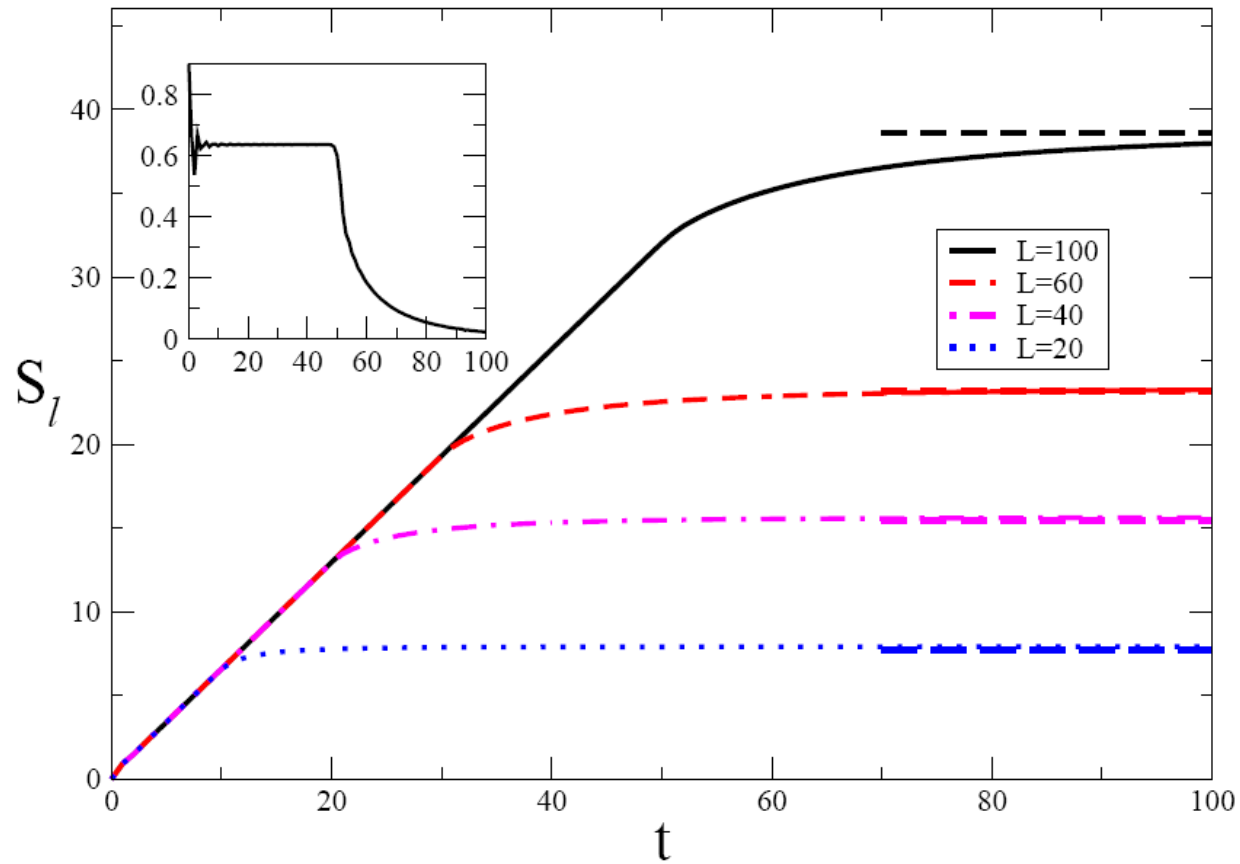
Possible scenarios:

- Global quench
- Local quench
- Periodic quench
- Adiabatic quench
- ...

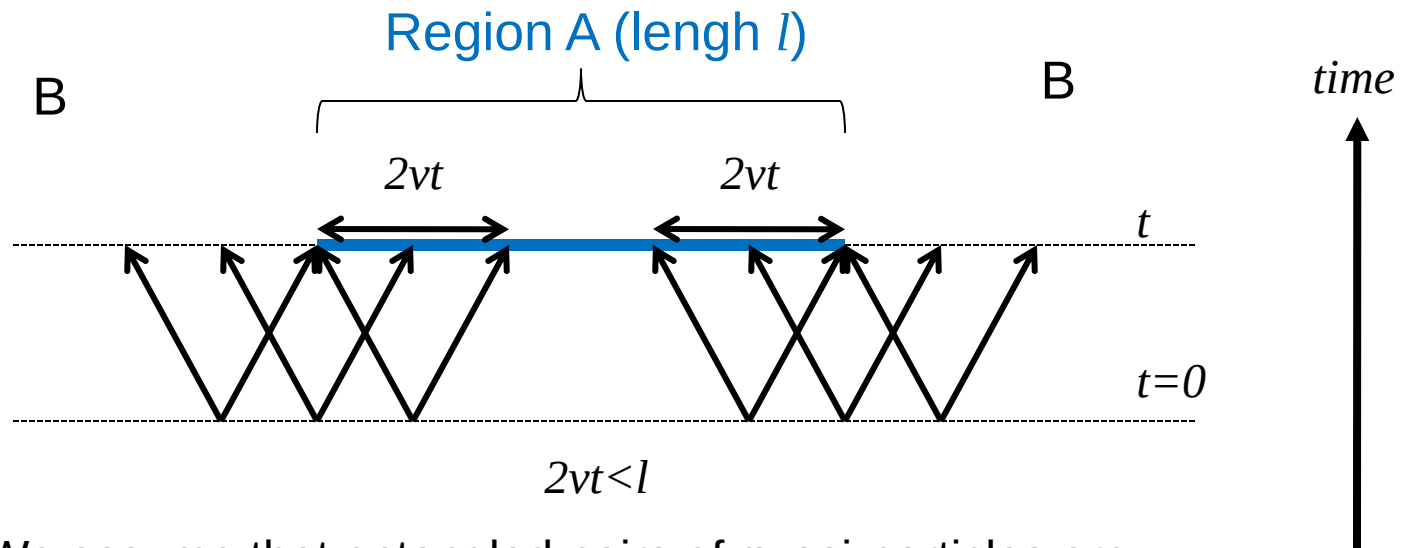


All of a sudden, we are no longer in the ground-state, but some high energy state. Important questions: thermalization vs. integrability

E-growth: global quench



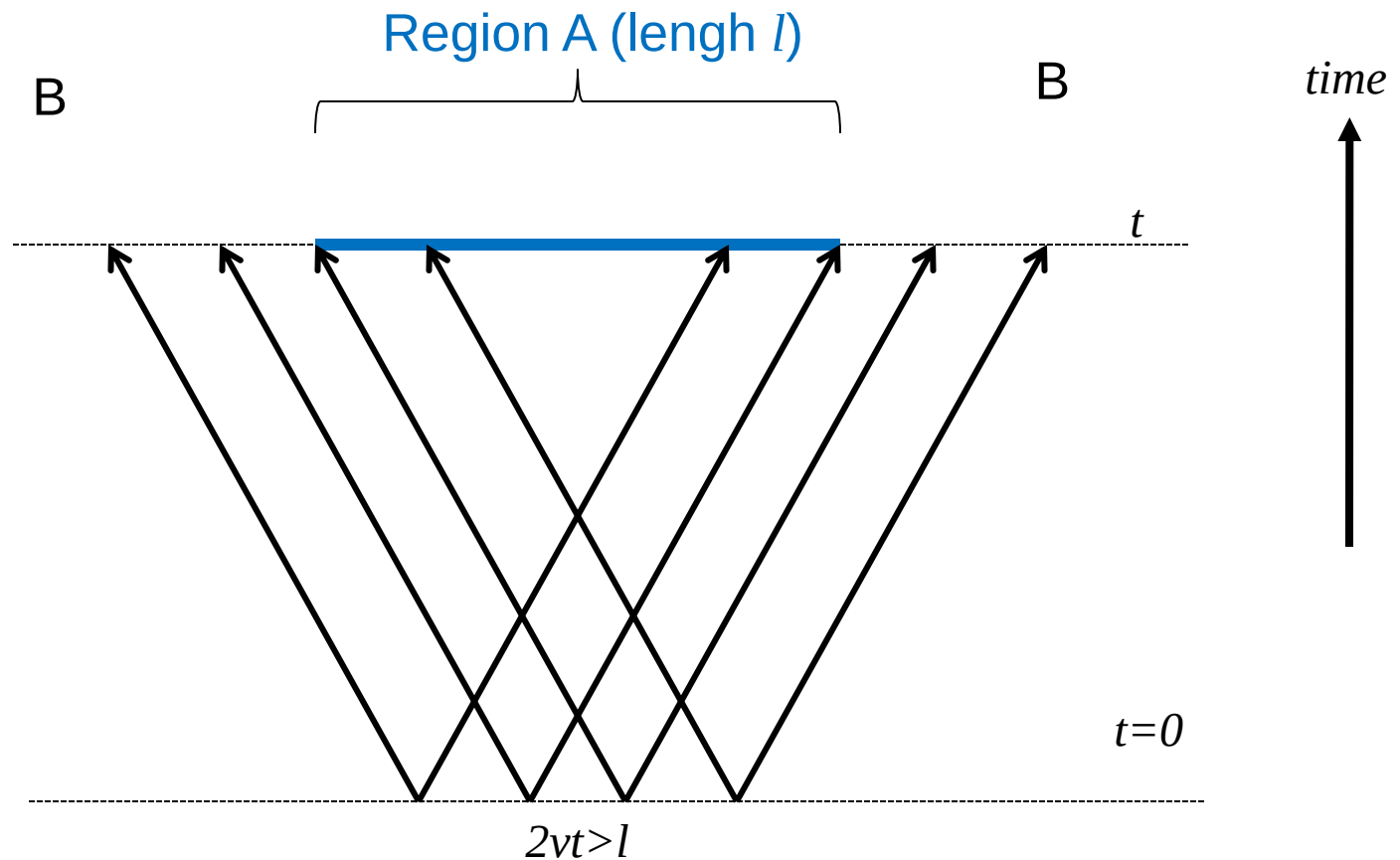
Global quench: qualitative picture



We assume that entangled pairs of quasi-particles are created at $t=0$, and they propagate with maximum velocity

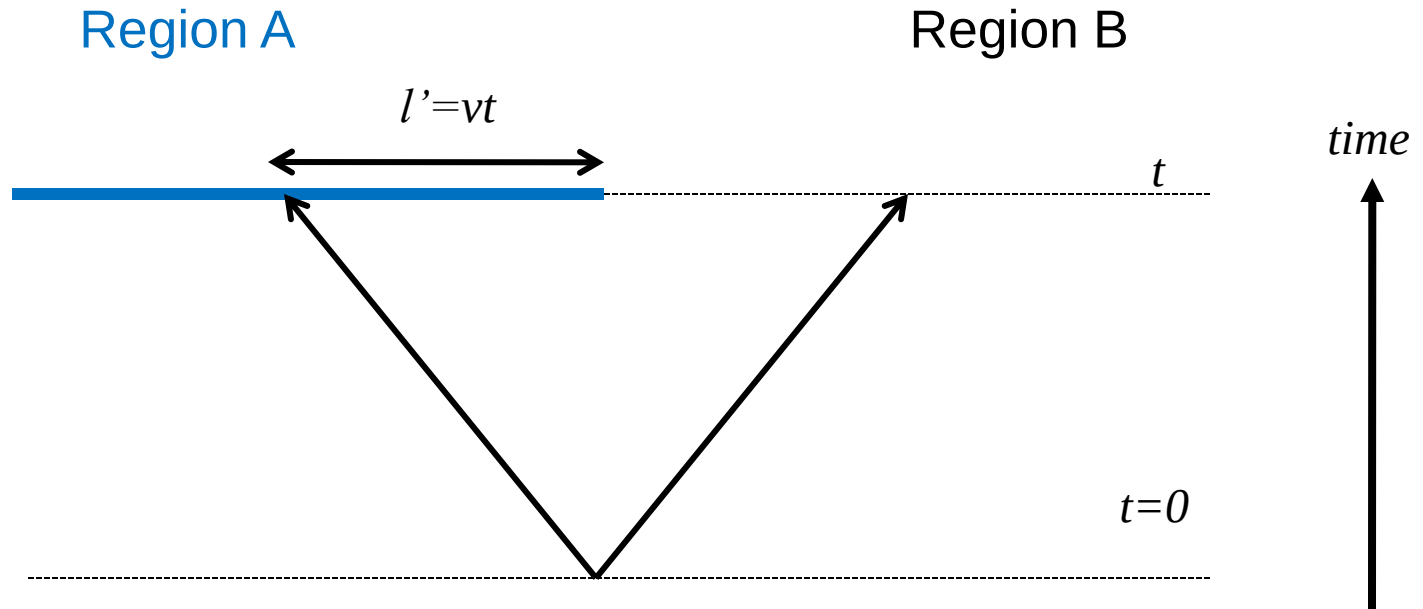
$$\Rightarrow S = S_0 + ct$$

Global quench: qualitative picture



The number of entangled pairs saturates

Local quench: qualitative picture



The perturbation propagates from the center, splitting the system into two pieces, inside and outside of the light-cone

$$\Rightarrow S = S_0 + c' \log(l') = S_0 + c' \log(t)$$

Computational cost

Global quench:

$$S \approx ct \rightarrow m \approx \exp(S) = \exp(ct)$$

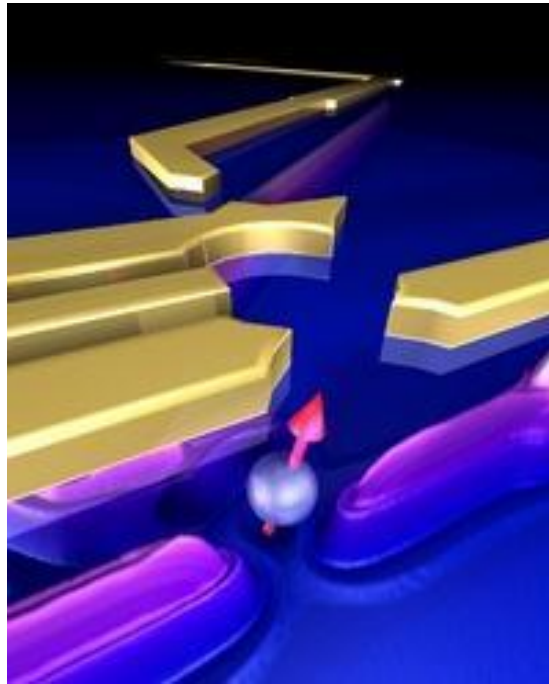
Local quench:

$$S \approx \log(t) \rightarrow m \approx \exp(S) = t^{\text{const.}}$$

Adiabatic quench:

$$S \approx \text{const.} \rightarrow m \approx \text{const.}$$

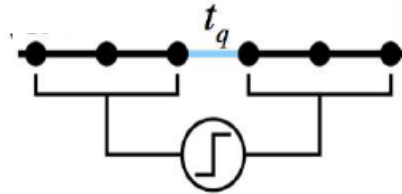
Transport and systems out of equilibrium



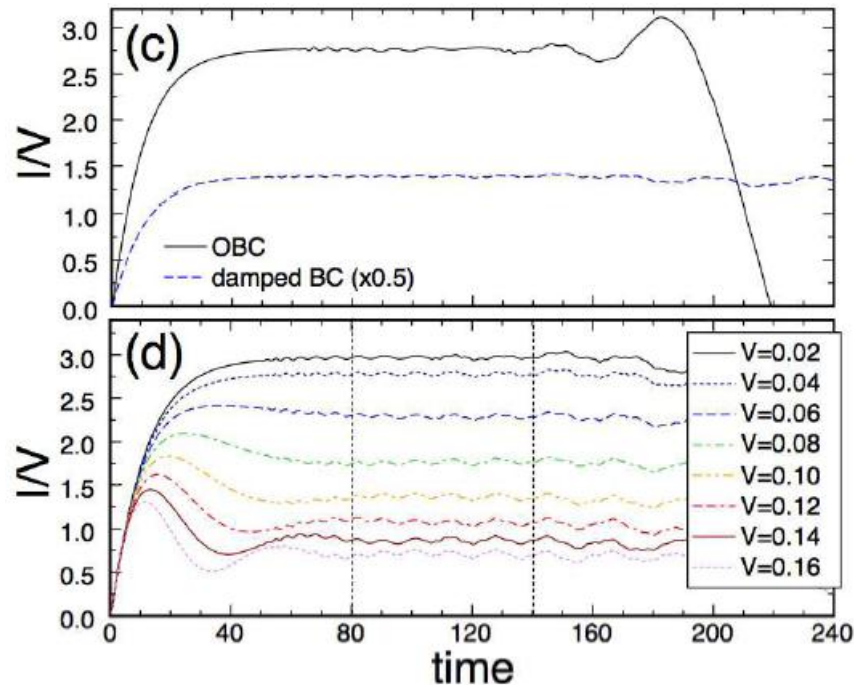
References: PRB **78**, 195317 (2008); PRA **78**, 013620 (2008) ; PRL **100**, 166403 (2008) ; PRB **73**, 195304 (2006); New. J. of Phys (2010)

Thanks to: F. Heidrich-Meisner, K. Al-Hassanieh, C. Busser, G. Martins, E. Dagotto, L. Da Silva, E. Anda

Example: transport in 1d



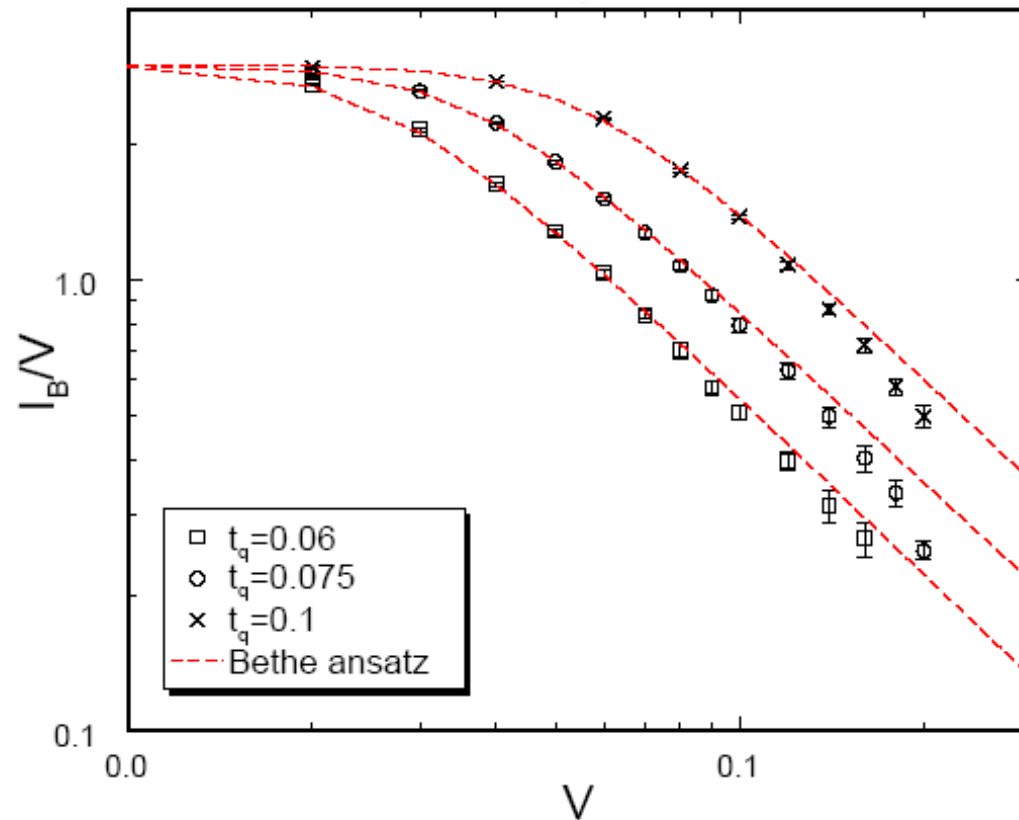
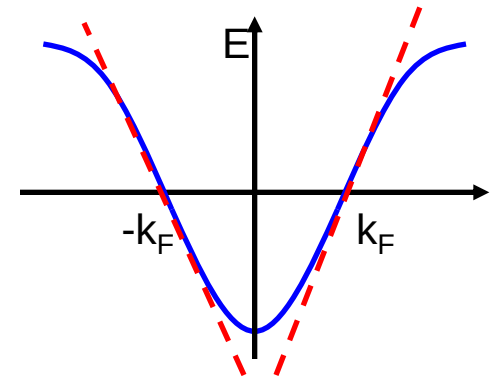
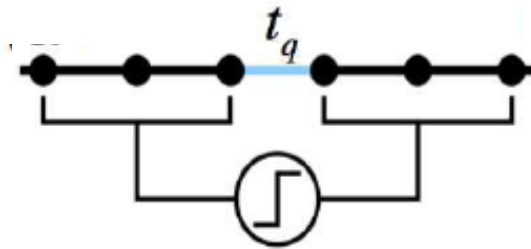
Spinless fermions with interactions.



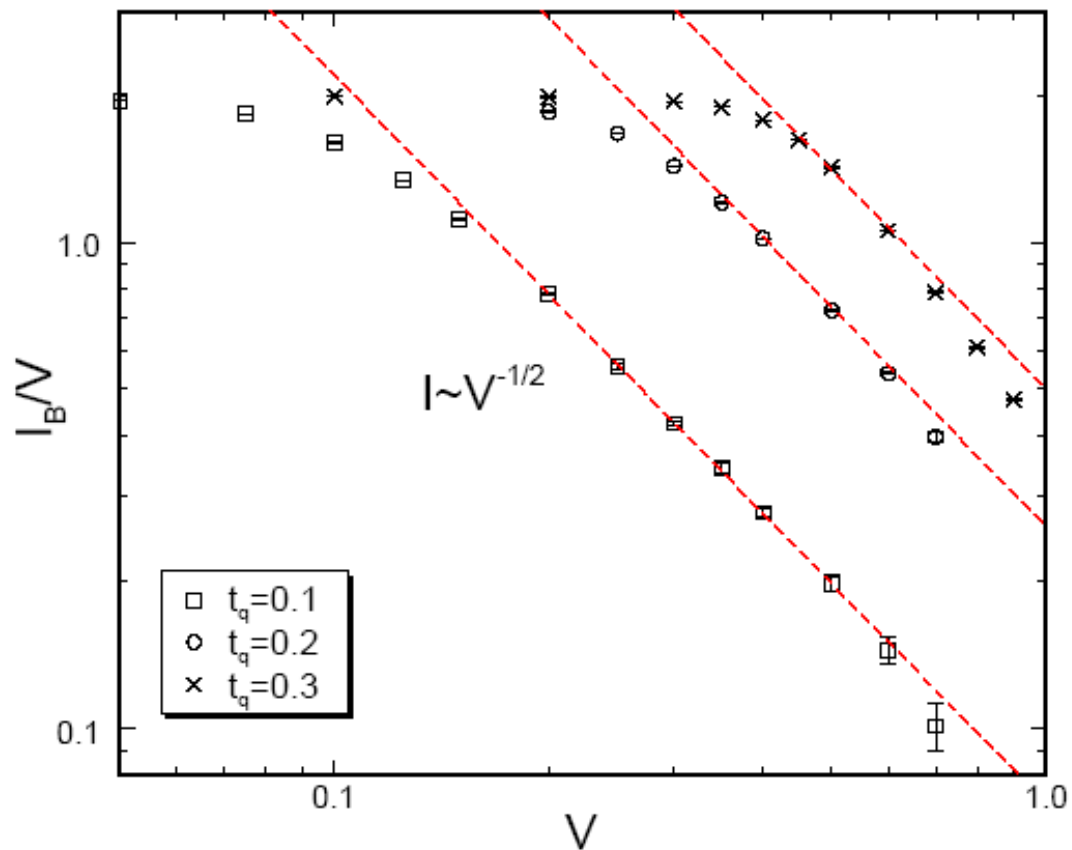
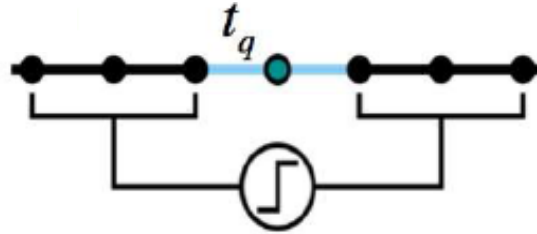
Typical behavior:

- 1) Short time transient
- 2) Plateau (we measure!)
- 3) Reflection at the boundaries. Current changes sign.

Weak link / potential barrier

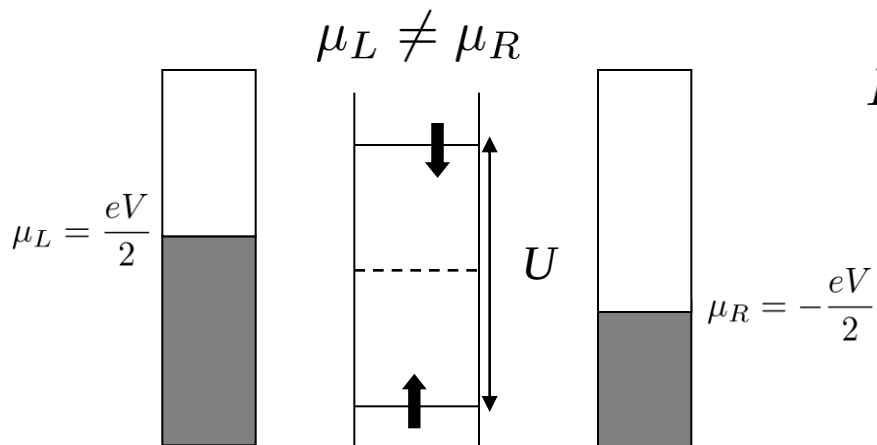
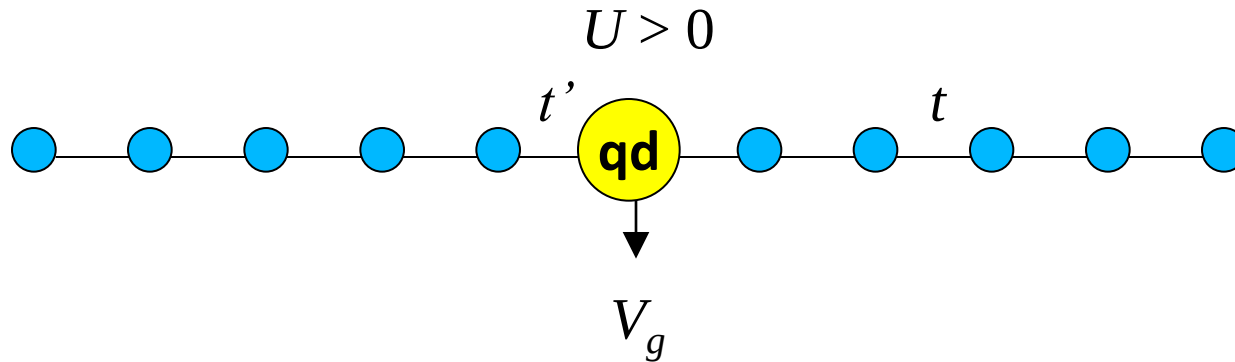


Resonant level / double barrier



Quantum dots

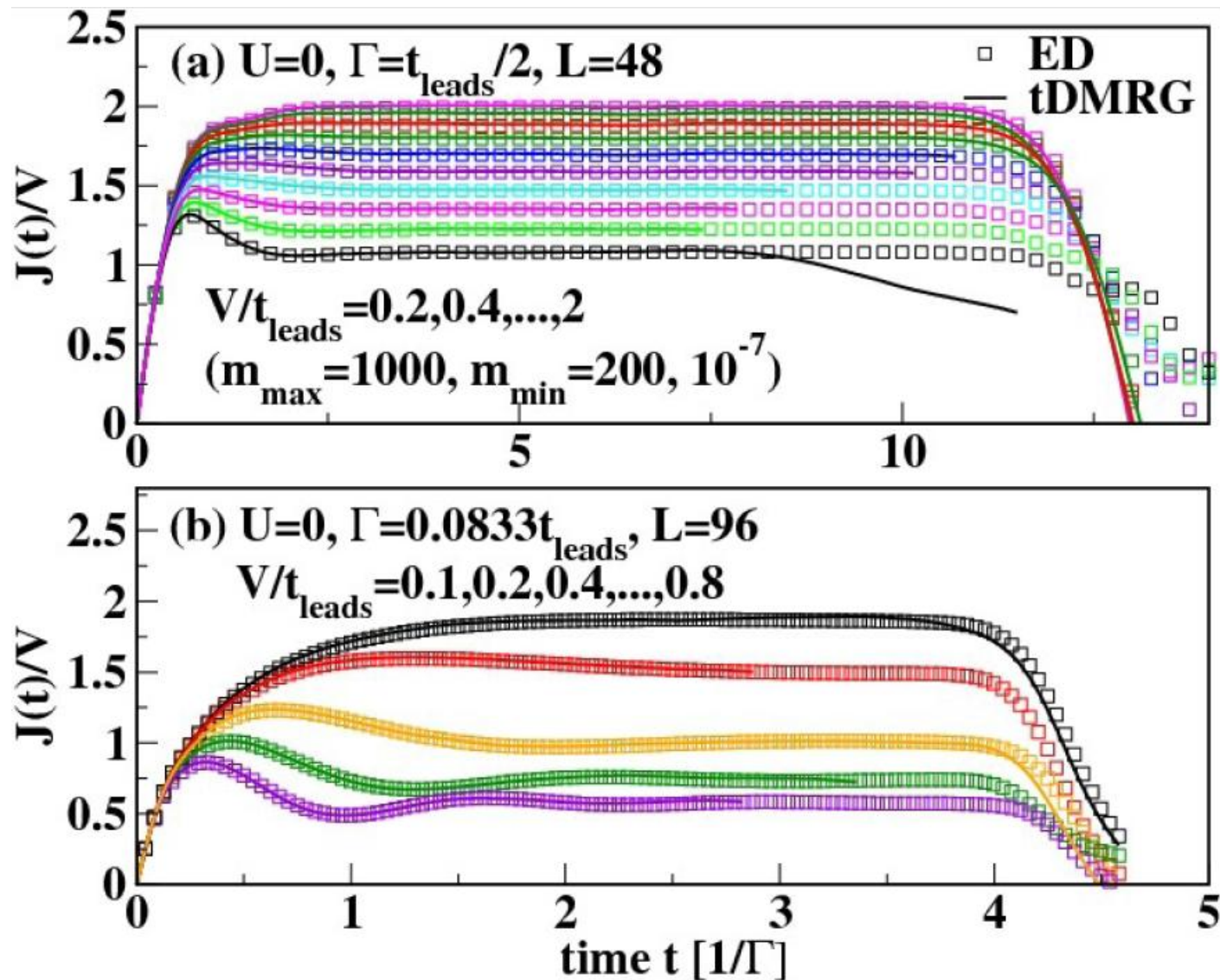
Quantum dot attached to two leads:
single-level Anderson model



$$H = \sum_{k\alpha\sigma} \epsilon_k c_{k\alpha\sigma}^\dagger c_{k\alpha\sigma} + \sum_{\sigma} \epsilon_d n_{\sigma} + U n_{\uparrow} n_{\downarrow} \\ + \sum_{k\alpha\sigma} \left(V_{k\alpha\sigma} c_{k\alpha\sigma}^\dagger d_{\sigma} + \text{h.c.} \right)$$

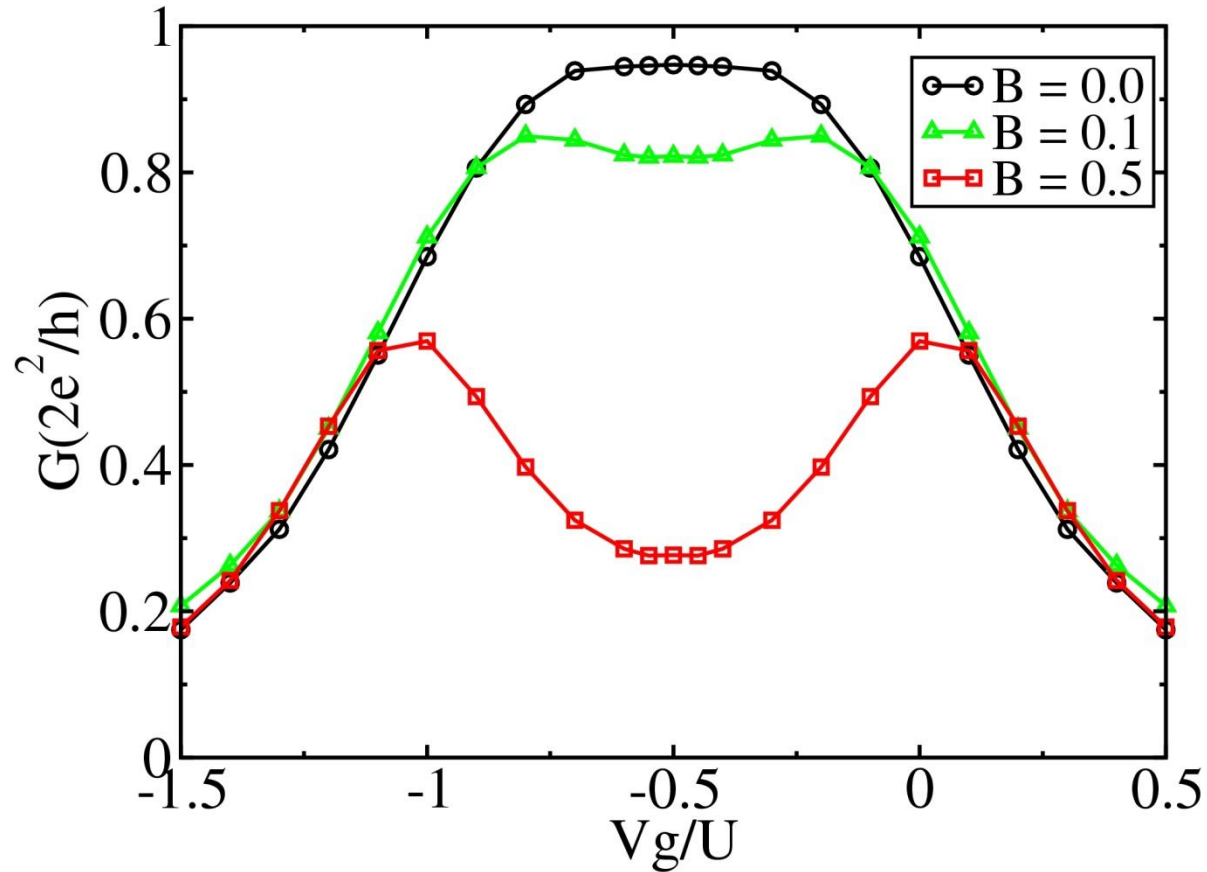
$$\alpha = L, R$$

Non-interacting limit ($U=0$)



tDMRG Results for 1 dot

Kondo Effect and magnetic field

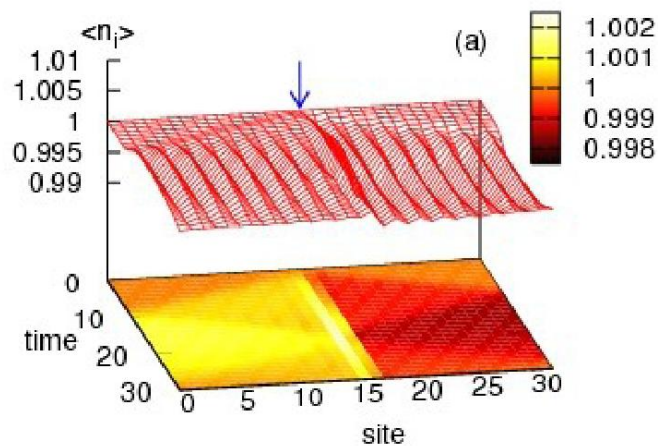
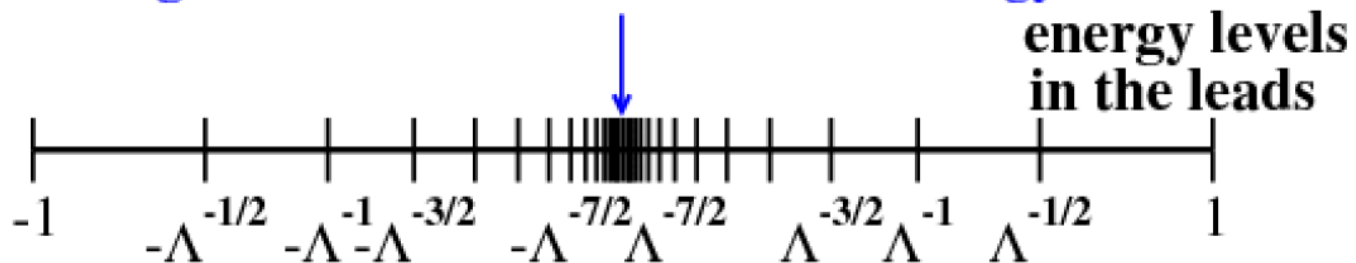


Suppression of Kondo effect: Coulomb Blockade peaks are formed

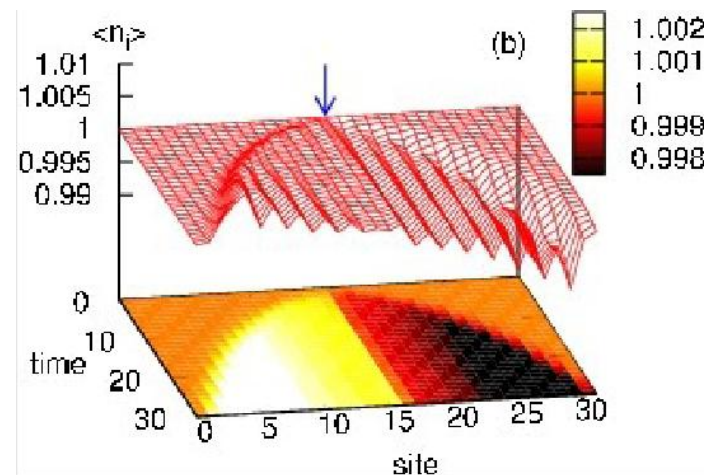
Accessing the Kondo regime

Wilson leads: $t_l = \Lambda^{-1/2}$ ($\Lambda > 1$)

good resolution at the Fermi energy



$\Lambda = 1$

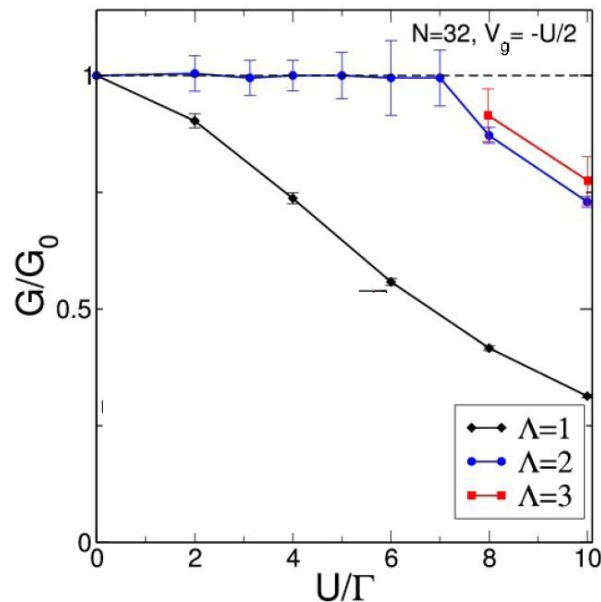
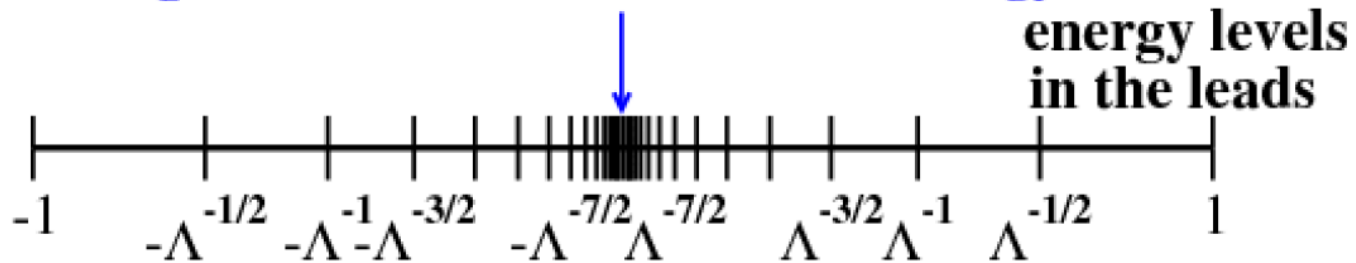


$\Lambda > 1$

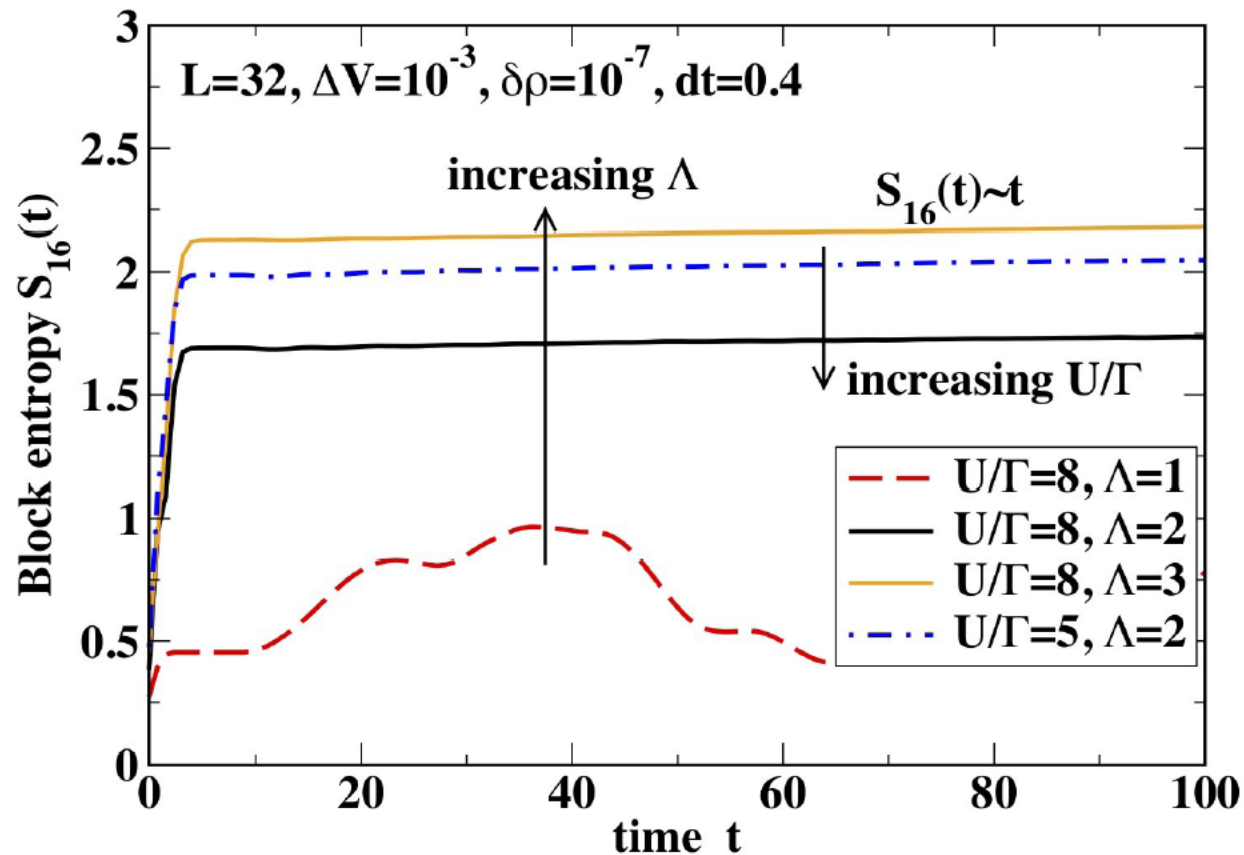
Accessing the Kondo regime

Wilson leads: $t_l = \Lambda^{-1/2}$ ($\Lambda > 1$)

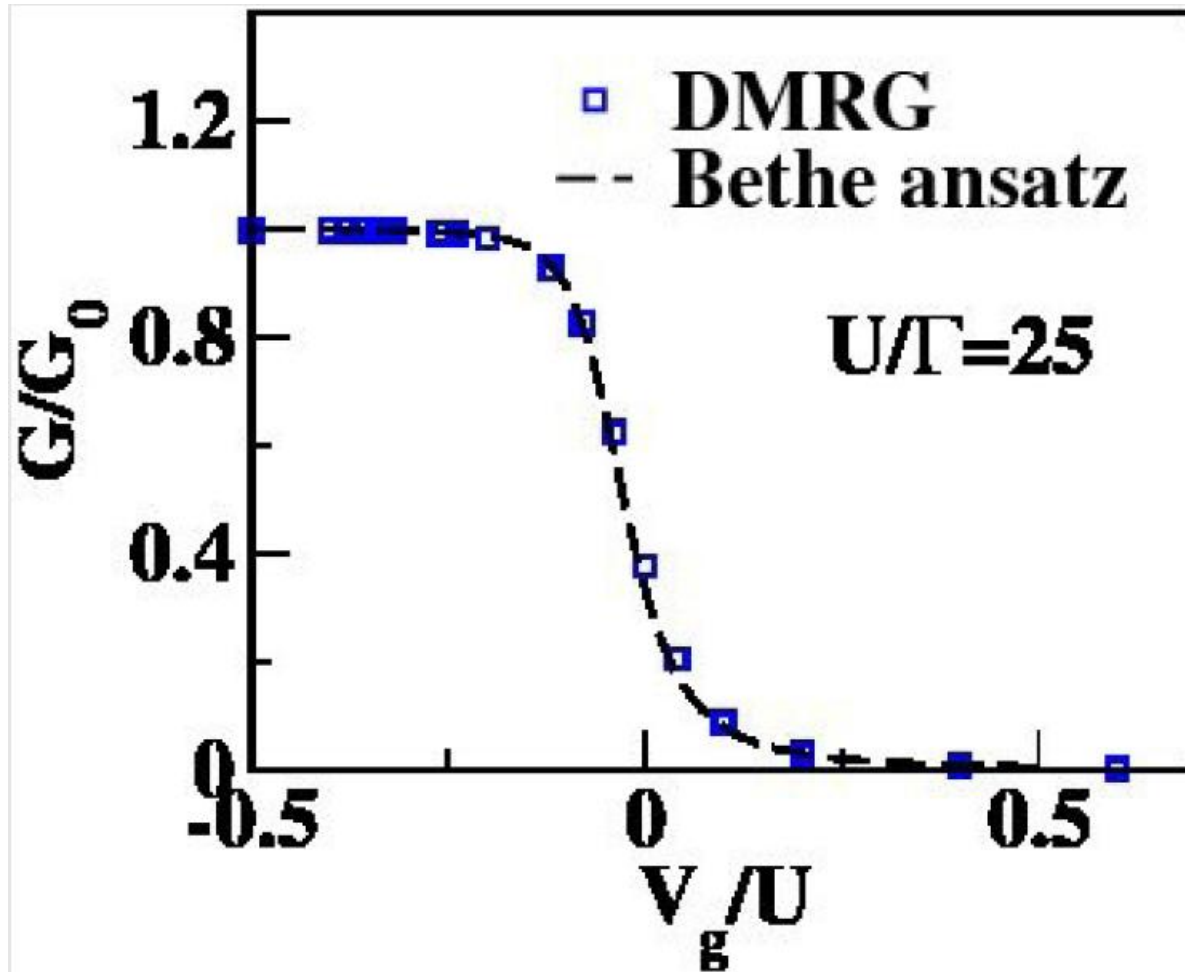
good resolution at the Fermi energy



Entropy growth with Wilson leads

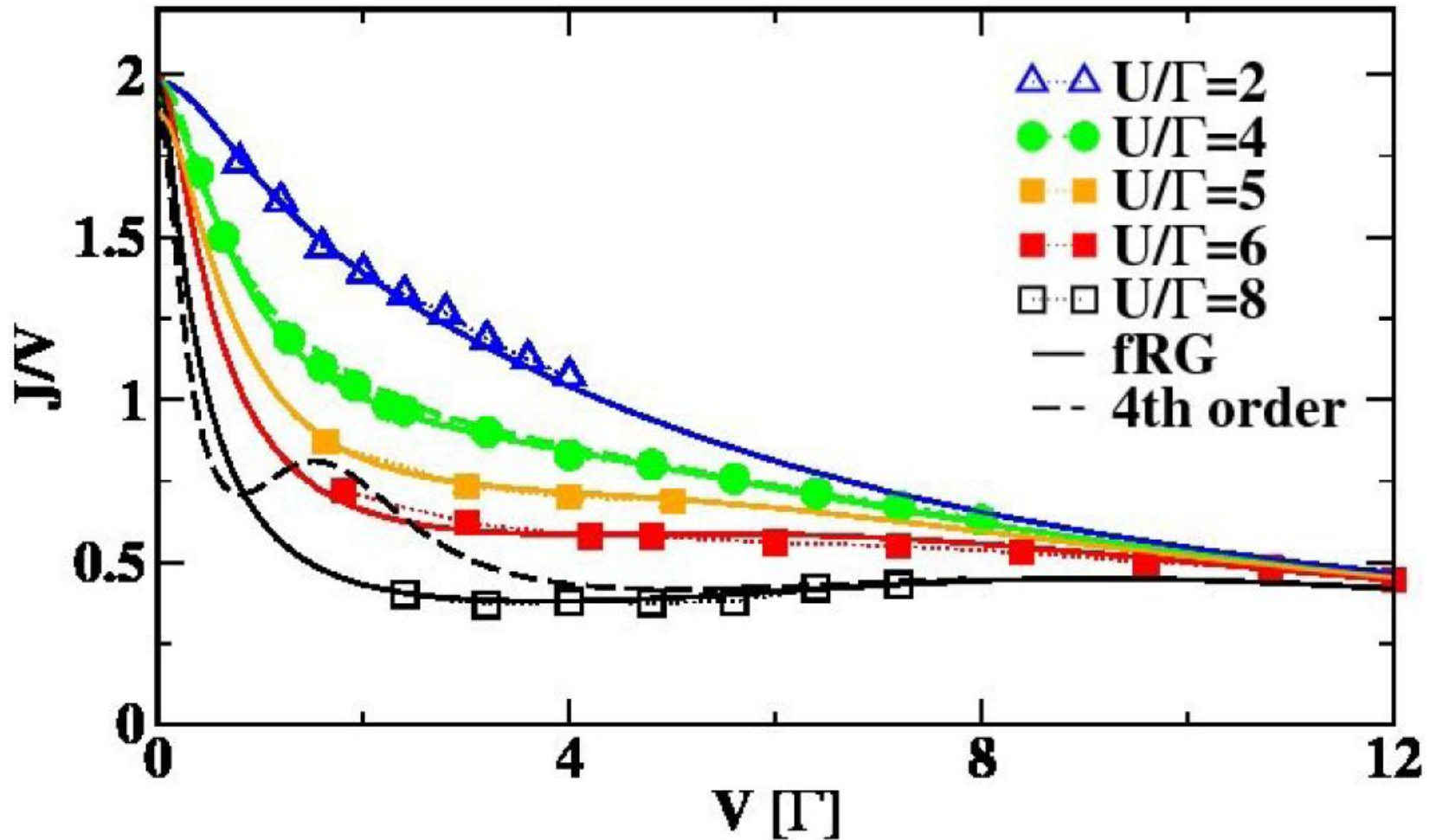


DMRG vs. Bethe Ansatz



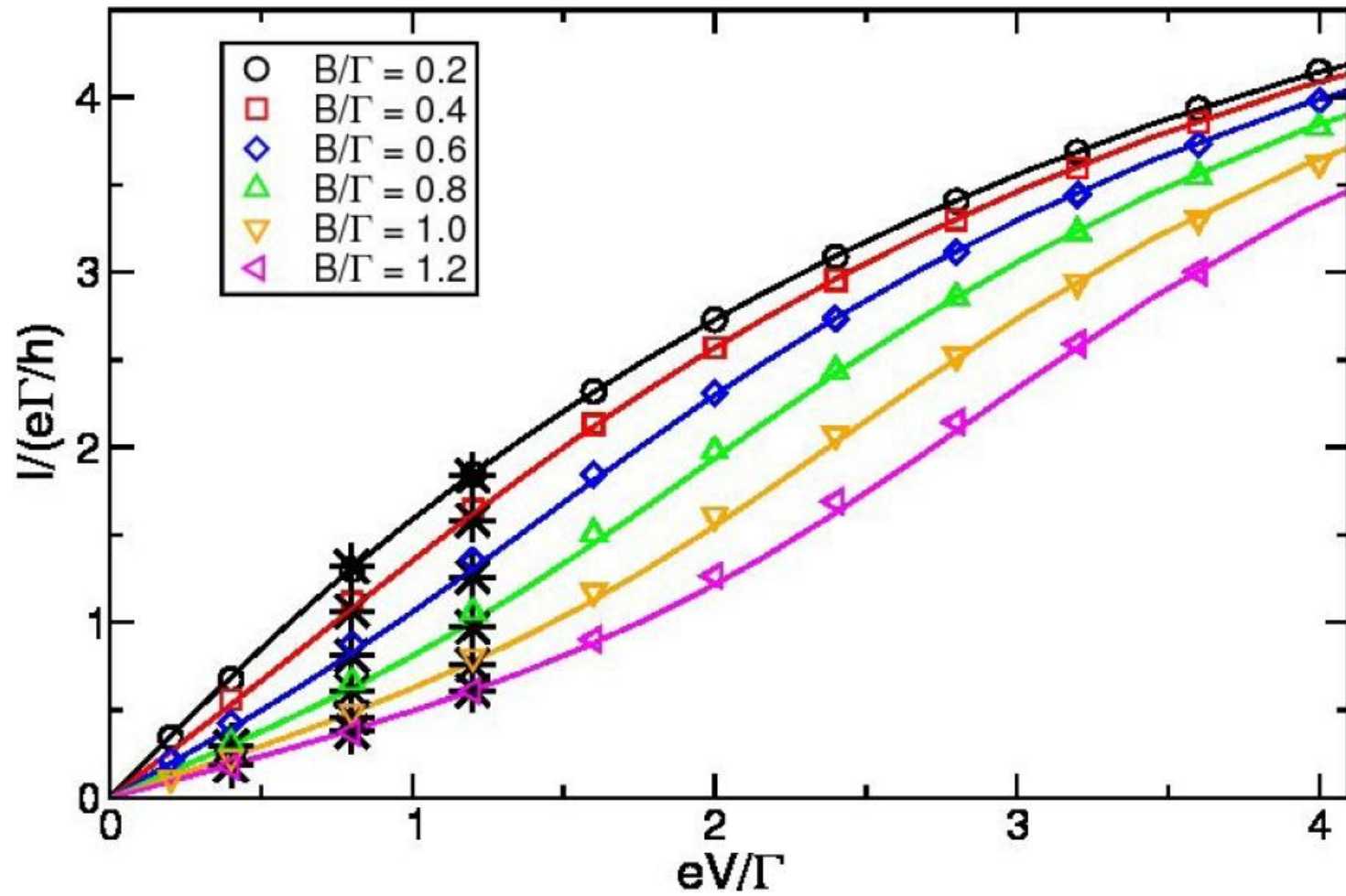
I-V characteristics

Particle-hole symmetric point ($V_g = -U/2$)

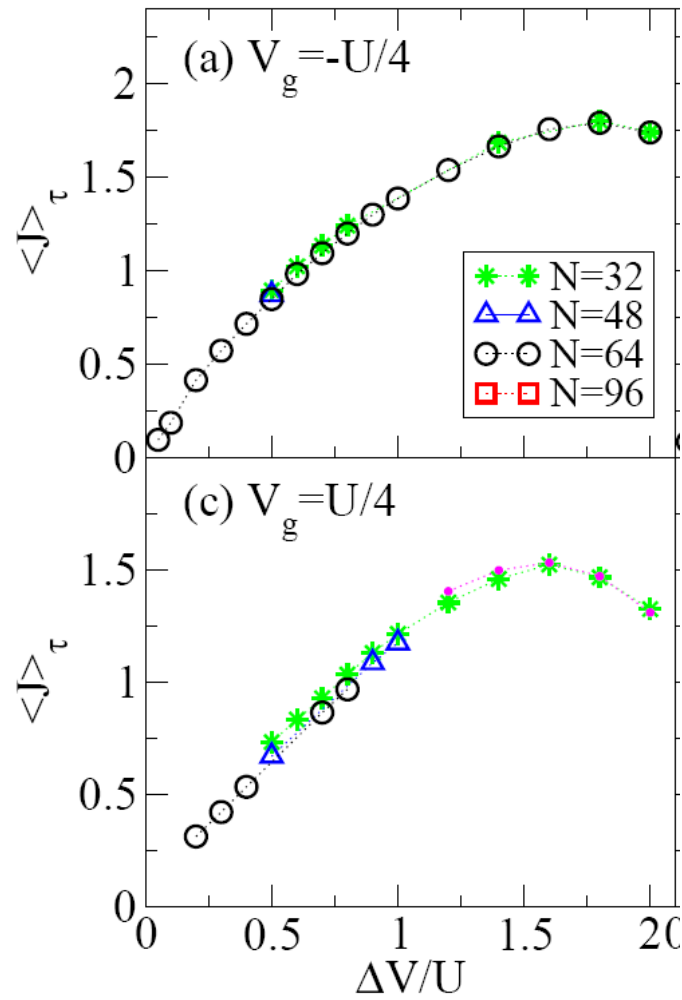


I-V characteristics

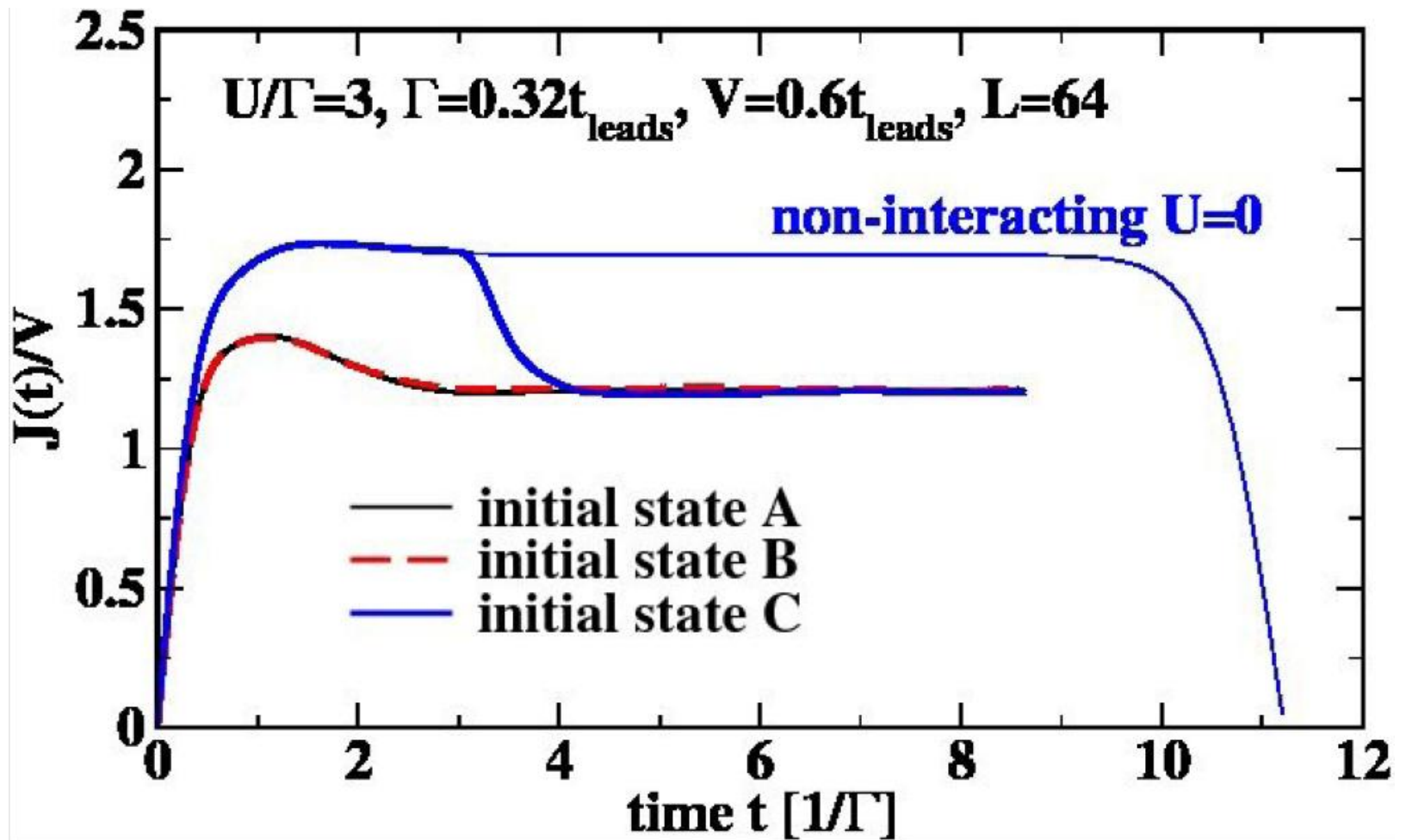
Finite magnetic field



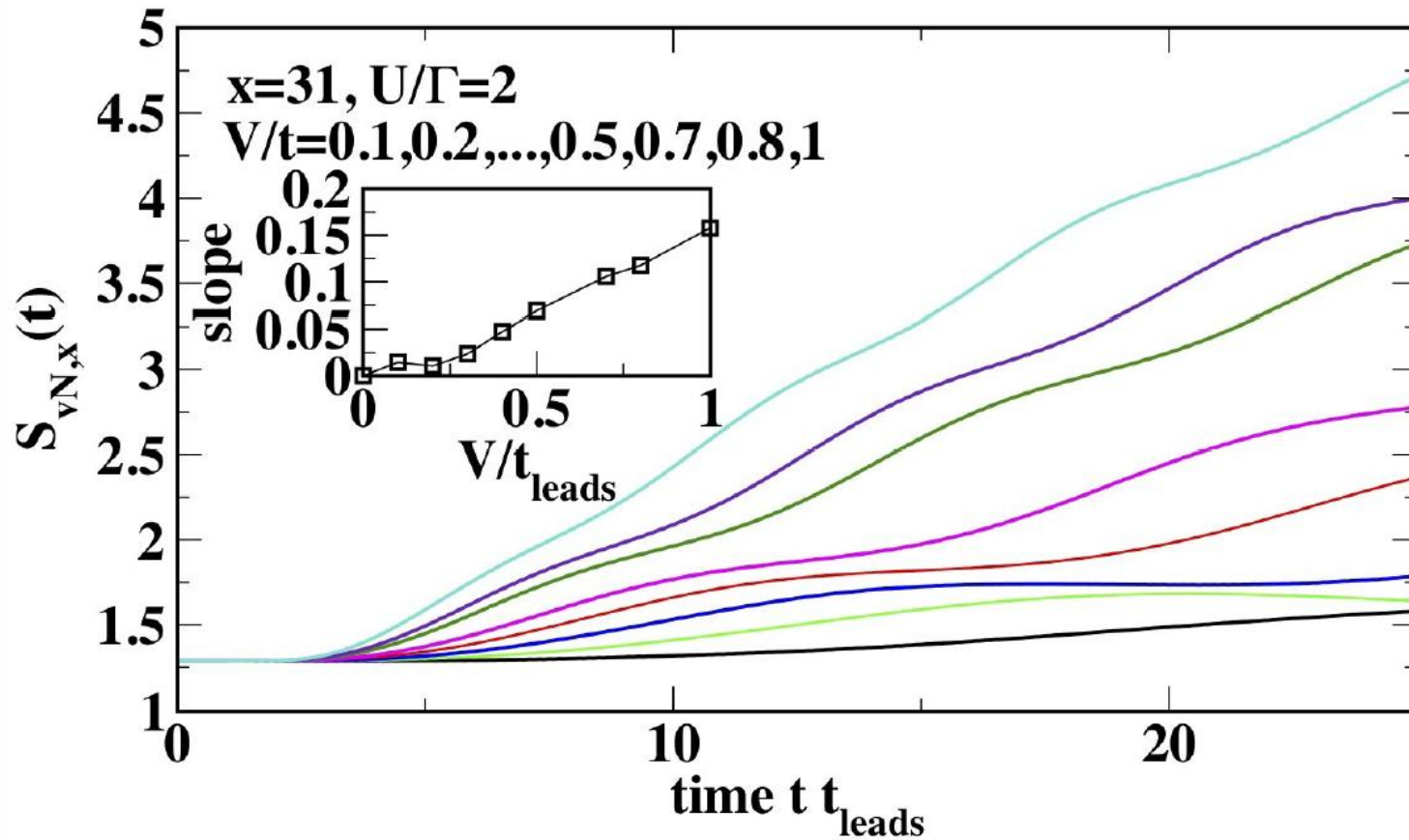
Large bias – out of equilibrium



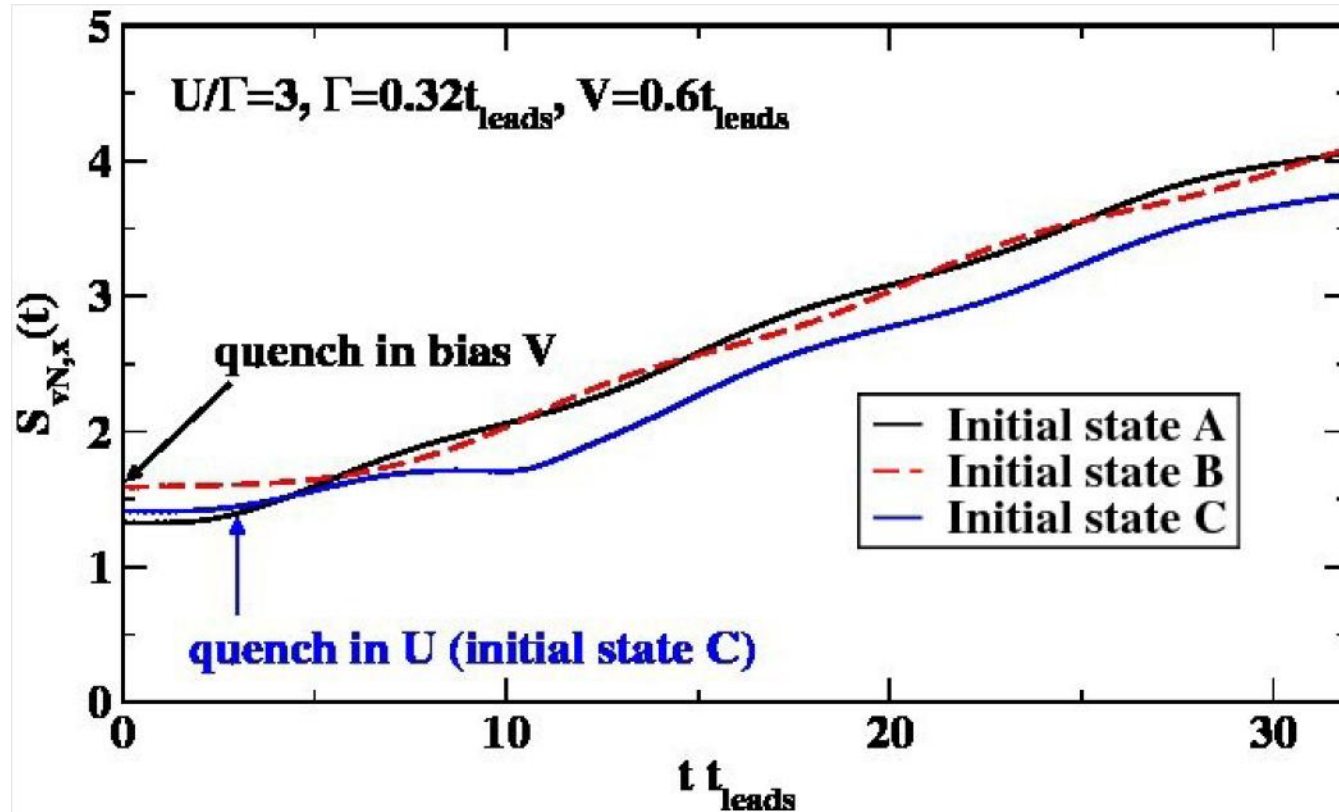
Dependence on the initial state



Computational cost and entropy growth



Computational cost and initial conditions



Entanglement entropy grows linearly in time, once the steady state is reached.