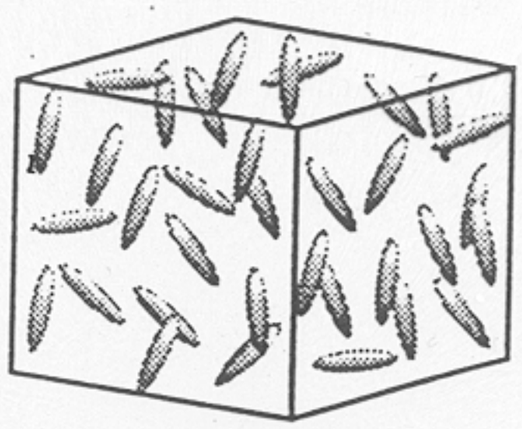
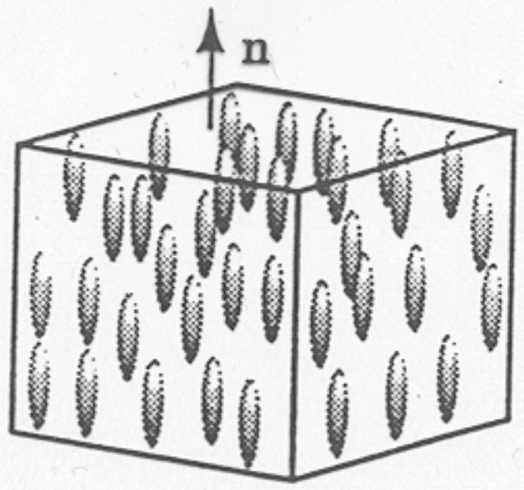
Isotropic



(a)

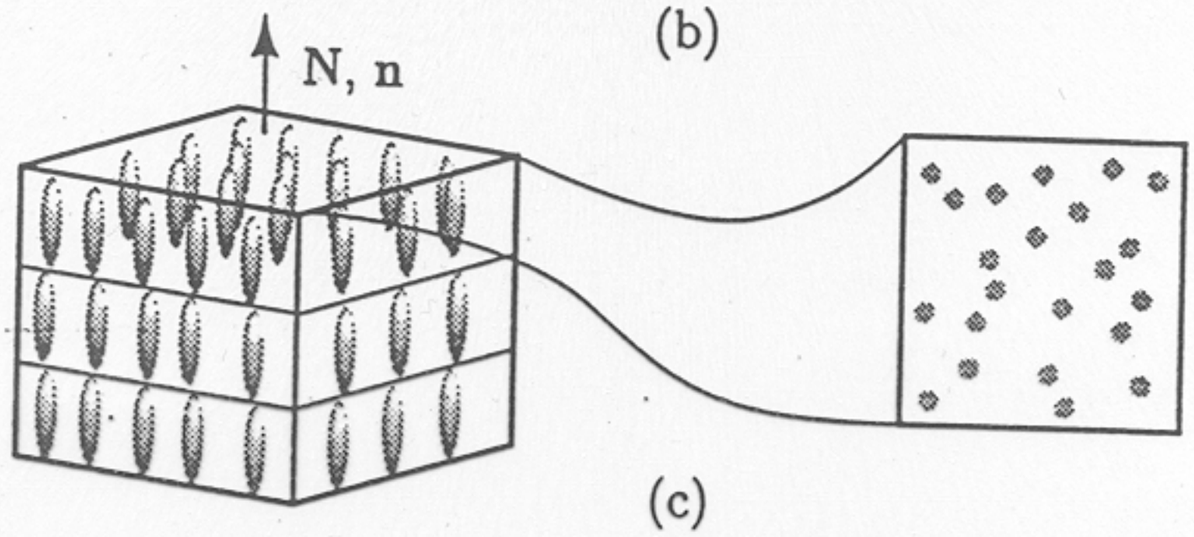
Nematic



(b)

molecular orientational order (MOO)

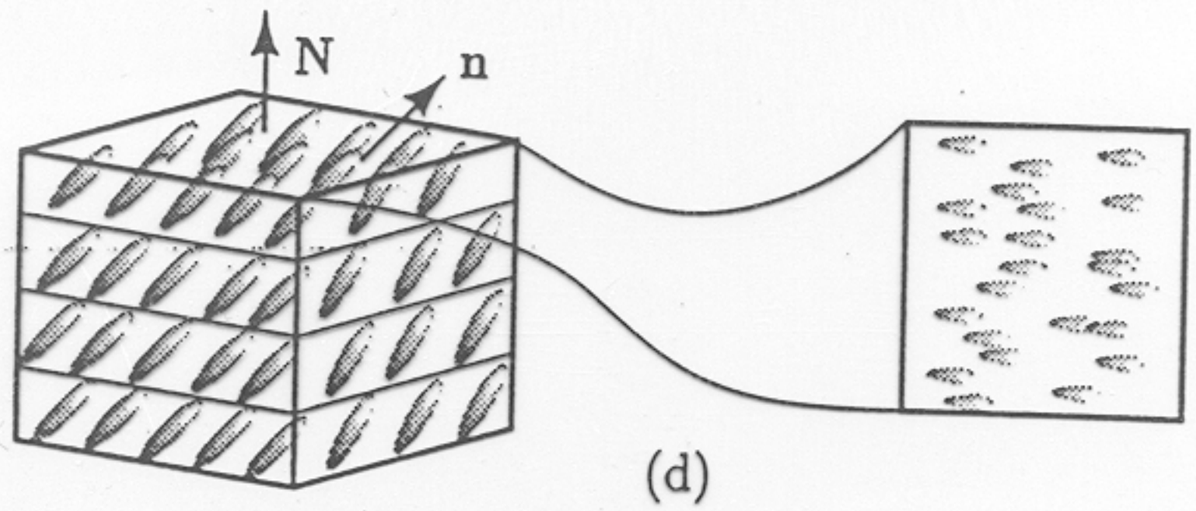
Smectic A



(c)

MOO + 1d positional order

Smectic C

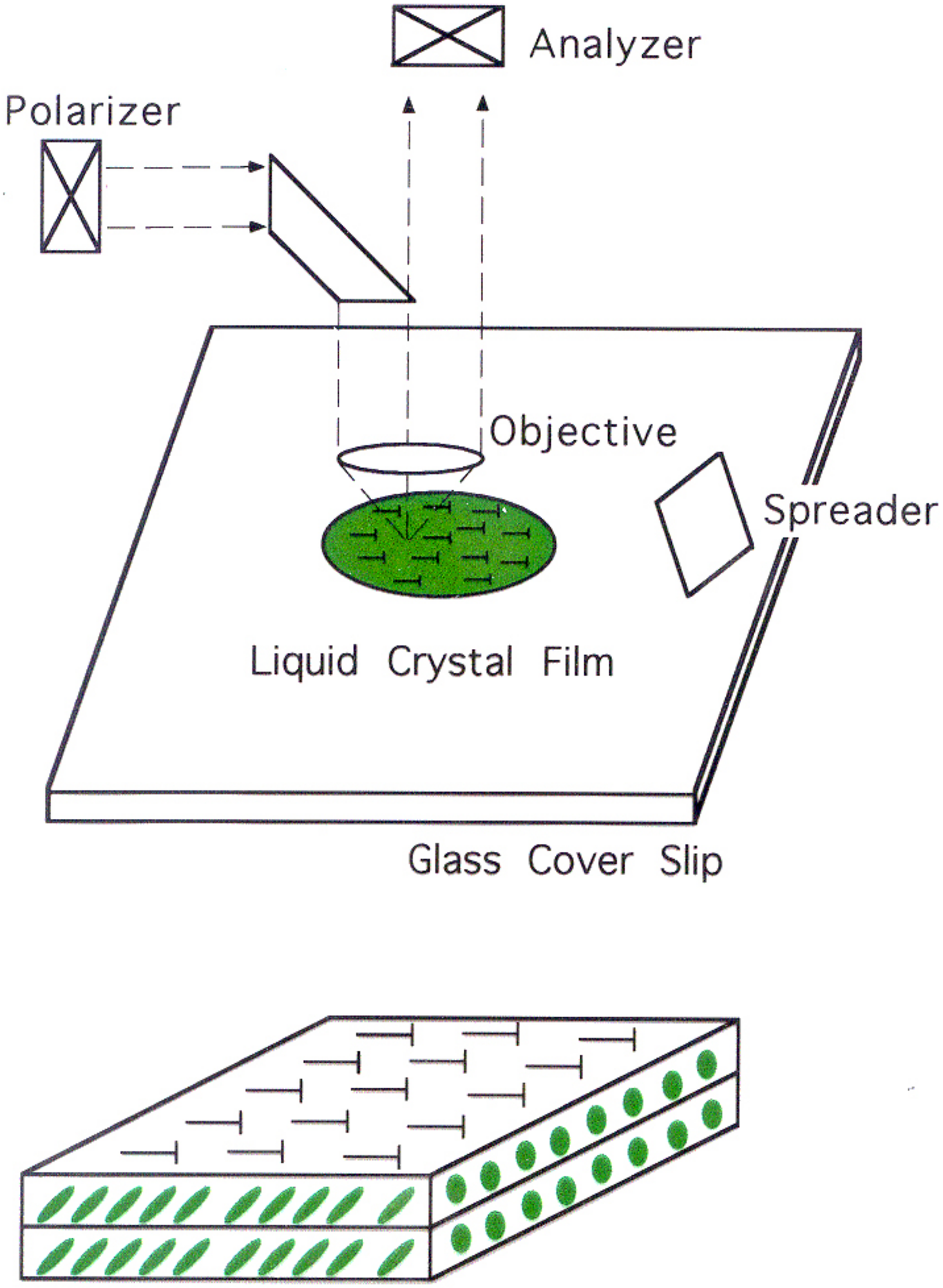


(d)

MOO + 1d positional order + tilt order

Fig. 2.7.3. Schematic representation of the position and orientation of anisotropic molecules in (a) the isotropic, (b) the nematic, (c) the smectic-*A*, and (d) the smectic-*C* phases. The direction of average molecular alignment in all but the isotropic phase is specified by a unit vector **n**. The layer normal in the smectic phases is indicated by the unit vector **N**. In the smectic-*A* phase, **n** is parallel to **N**, whereas in the smectic-*C* phase, it is not. In the text $\mathbf{N} = \mathbf{e}_z$ is parallel to the *z*-axis. (c) and (d) also show the arrangement of molecules in the smectic planes in the smectic-*A* and -*C* phases. In the smectic-*C* phase, the projections of molecular axes onto the plane perpendicular to **N** align on average along the **c**-director.

Freely Suspended Films

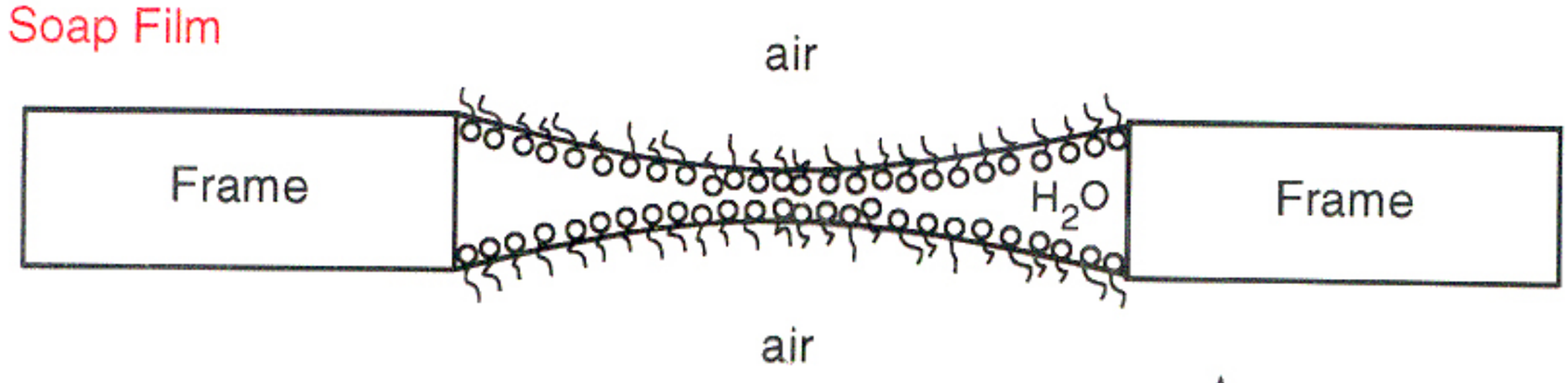


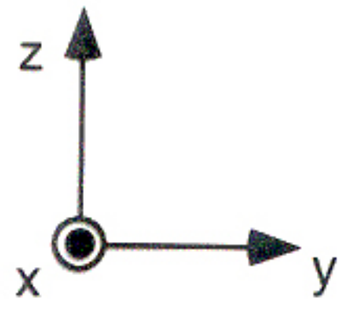
c-director

Freely Suspended Films

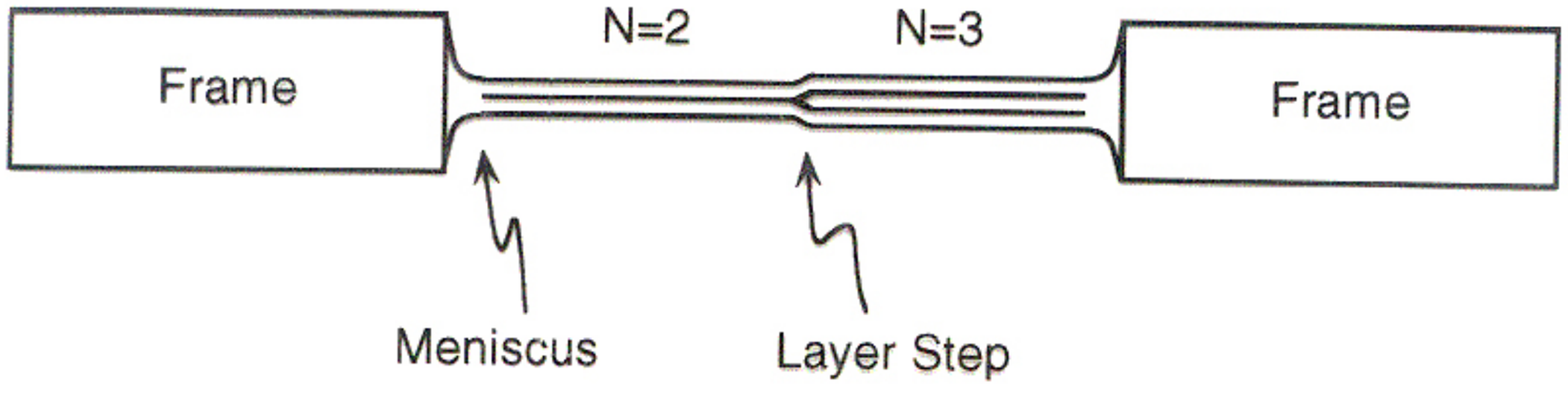
Cross-Section

Soap Film

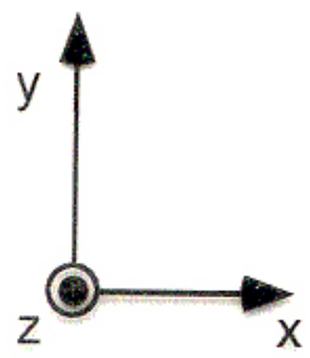


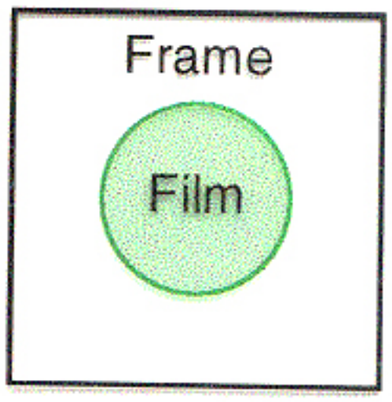


Liquid Crystal Film



Top View





FRANCK ELASTICITY

SPLAY →

$$F_S = \frac{1}{2} K_S \left(\frac{\partial \phi}{\partial x}\right)^2$$

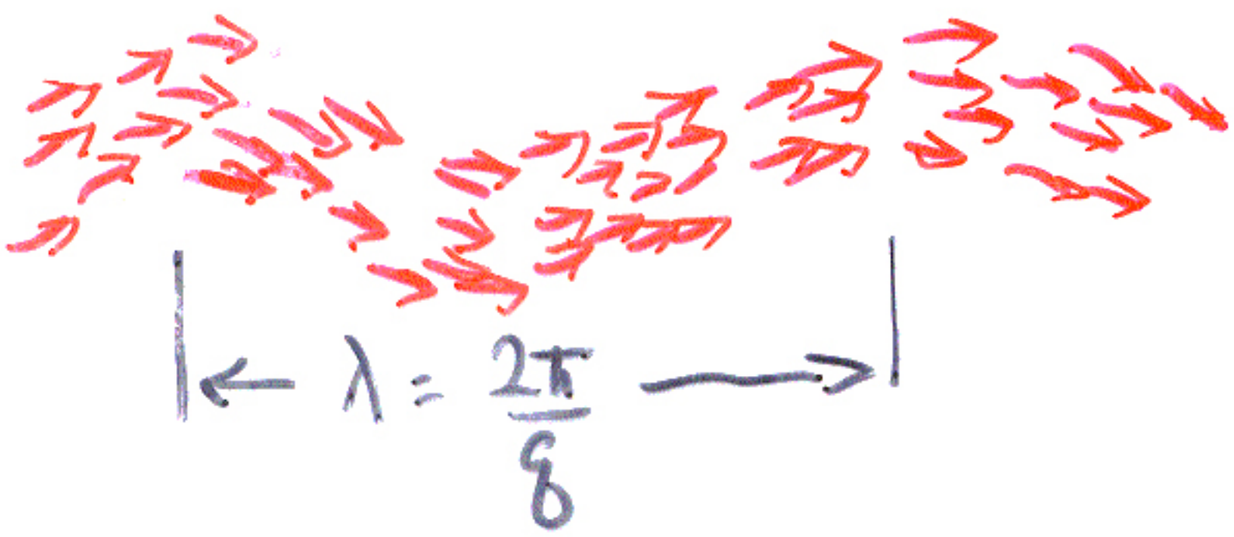
BEND →

$$F_B = \frac{1}{2} K_B \left(\frac{\partial \phi}{\partial y}\right)^2$$

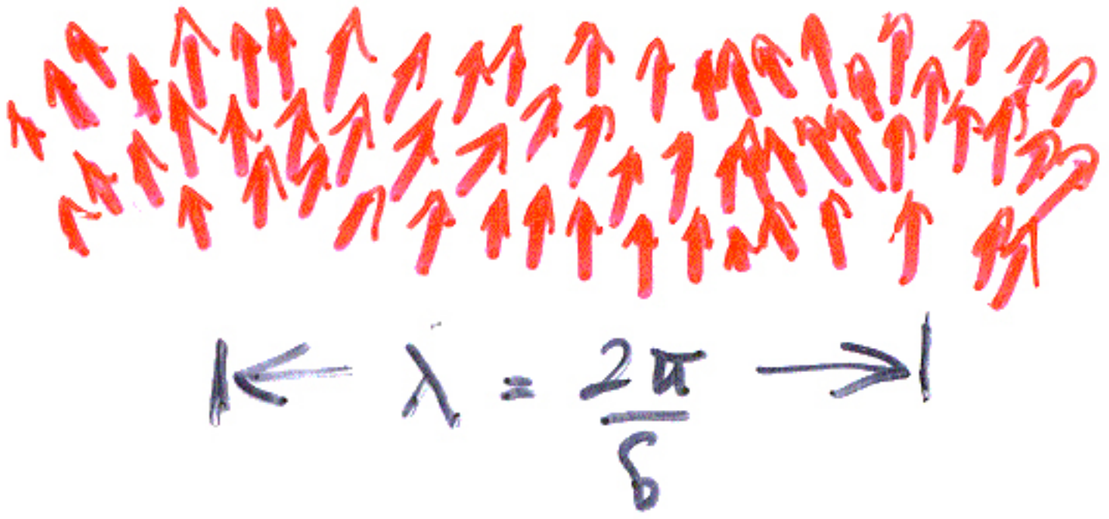
if $K_S = K_B = K$:

$$F = \frac{1}{2} K \left\{ \left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 \right\}$$

BEND WAVE



SPLAY WAVE



$$\phi_q = \frac{1}{L} \int\int d^2r \, \phi(r) e^{i\vec{q}\cdot\vec{r}}$$

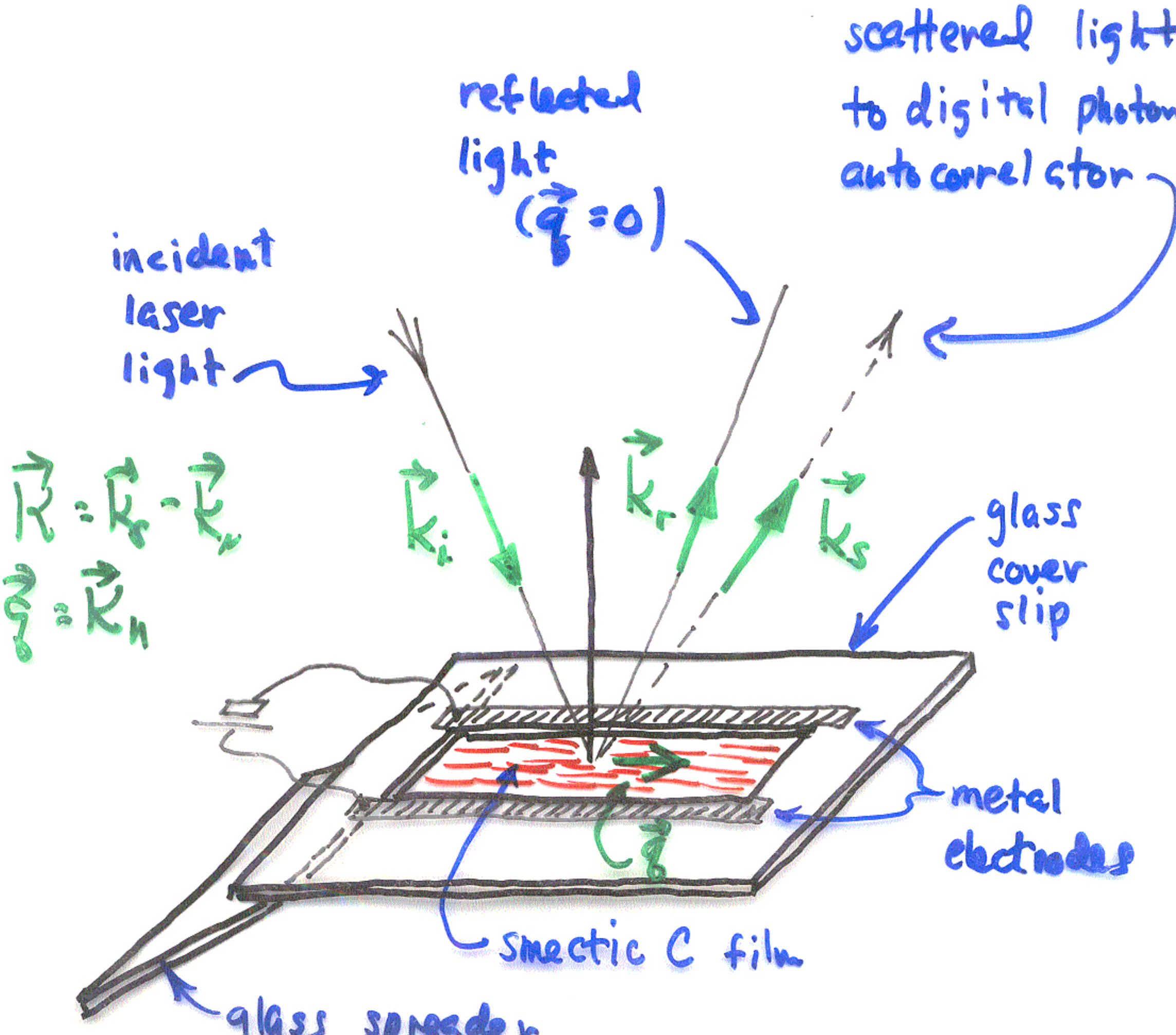
$$F = \frac{1}{L^2} \sum_q \frac{Kq^2}{2} |\phi_q|^2$$

$$\langle |\phi(q)|^2 \rangle = \frac{k_B T}{K q^2}$$

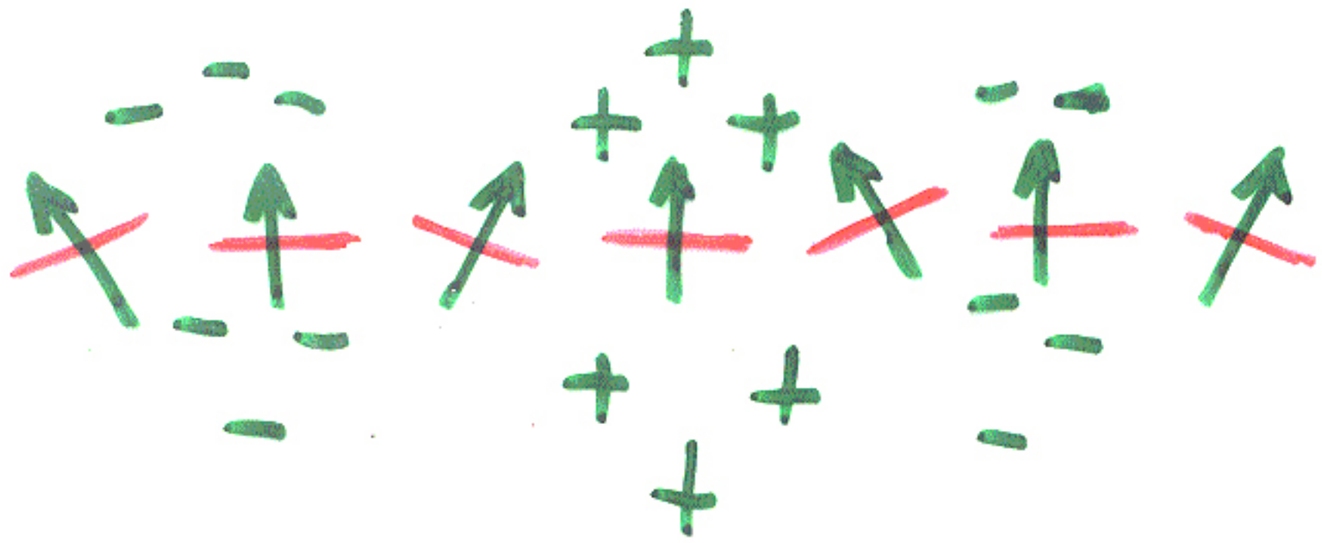
k space - depolarized light scattering spectroscopy (inelastic & elastic)

$$I(\vec{q}) \propto \langle |\delta n(\vec{q})|^2 \rangle$$

$$\langle I(\vec{q},0)\, I(\vec{q},\tau) \rangle \Rightarrow \langle \delta n(\vec{q},0)\, \delta n^*(\vec{q},\tau) \rangle$$



splay $\vec{q} \perp \hat{n}$



bend $\vec{q} \parallel \hat{n}$

$$\mathcal{F} = \int\left[\frac{1}{2}K_s(\nabla\cdot\hat{n})^2 + \frac{1}{2}K_b(\nabla\times\hat{n})^2 - \vec{P}_0\cdot\vec{E} + \int\frac{(\nabla\cdot\vec{P}_0)(\nabla\cdot\vec{P}_0')}{|\vec{r}-\vec{r}'|}d^2r\right]d^2r'$$

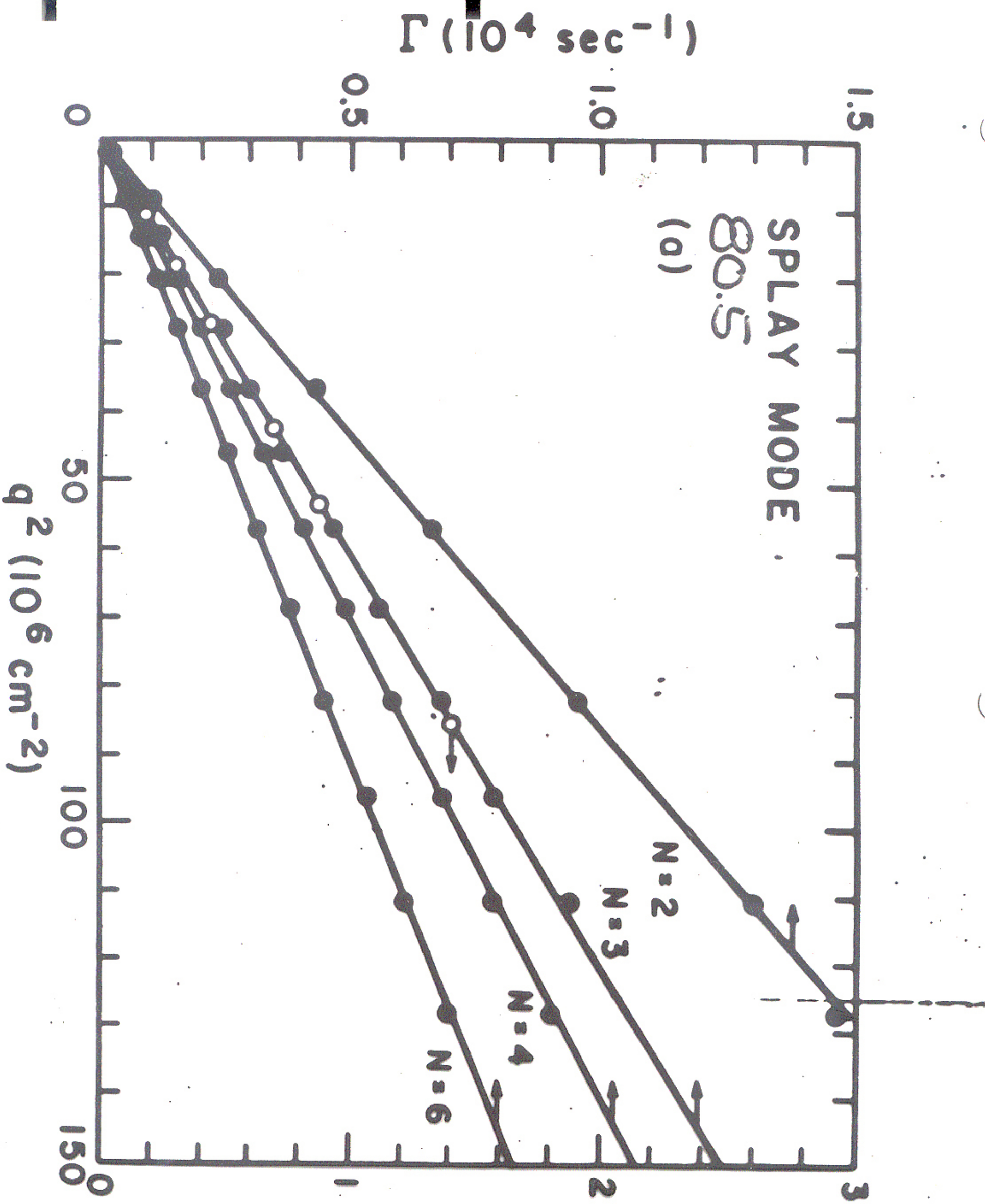
$$= \frac{1}{2}\left[K_s q^2 + P_0 E\right]|\delta n_s|^2 + \left[K_b q^2 + 4\pi P_0^2 + P_0 E\right]|\delta n_b|^2$$

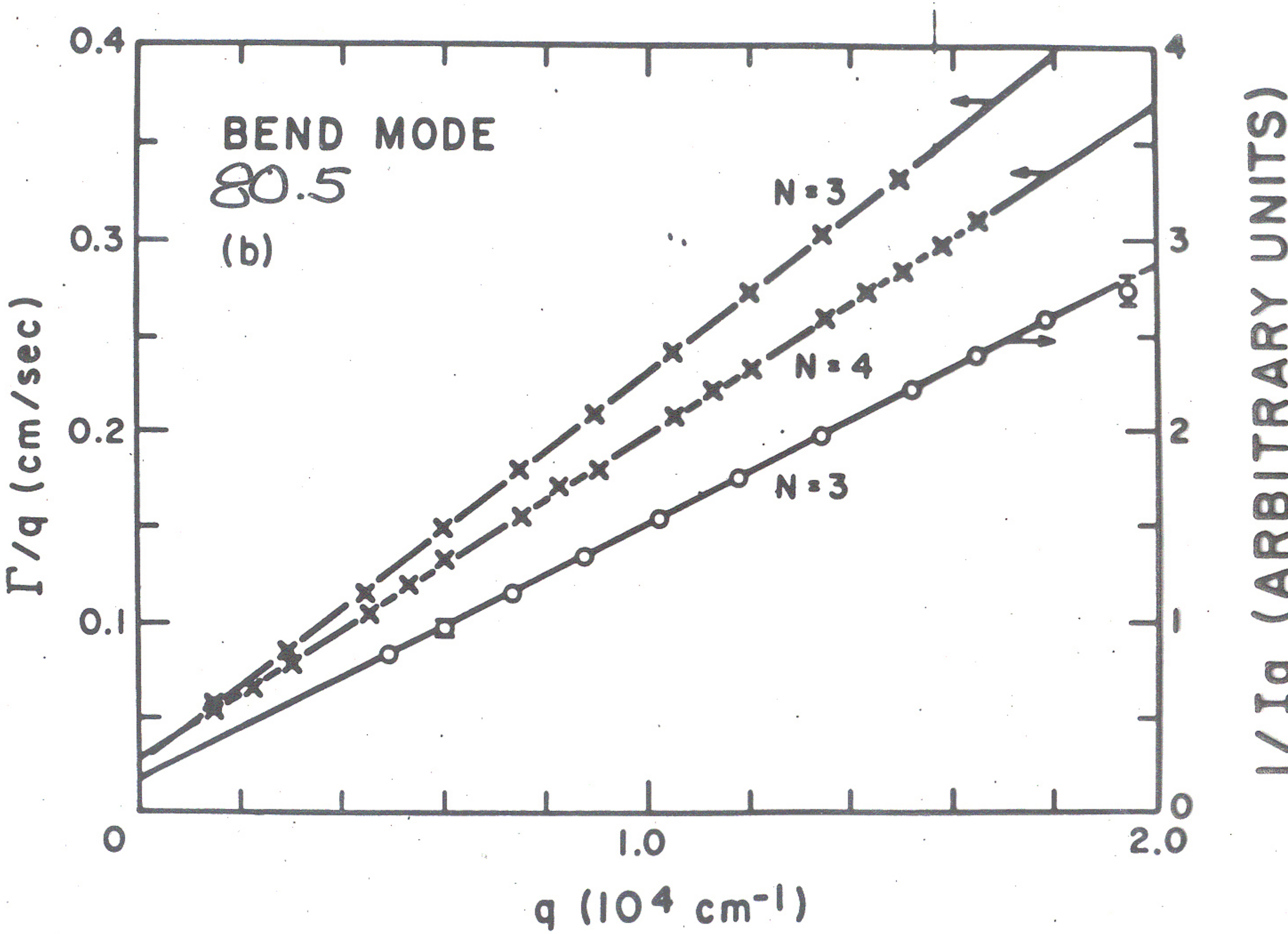
$$\Gamma_b(q) = \frac{k_B T}{K_b q^2 + 4\pi P_0^2 |q| + P_0 E} \quad (\vec{q} \parallel \hat{n})$$

$$\Gamma_s(q) = \frac{k_B T}{K_s q^2 + P_0 E} \quad (\vec{q} \perp \hat{n})$$

$$\Gamma_b = \frac{K_b q^2 + 4\pi P_0^2 |q| + P_0 E}{\gamma_b}$$

$$\Gamma_s = \frac{K_s q^2 + P_0 E}{\gamma_s}$$


Γ (10⁴ sec⁻¹)
0
0.5
1.0
1.5
SPLAY MODE
80.5
(a)
N=2
N=3
N=4
N=6
q² (10⁶ cm⁻²)
50
100
150
0
1
2
3
I/I (ARBITRARY UNITS)


BEND MODE
80.5
(b)
N = 3
N = 4
N = 3
0.4
0.3
0.2
0.1
0
1.0
2.0
4
3
2
1
0
Γ/q (cm/sec)
I/Iq (ARBITRARY UNITS)
q (10⁴ cm⁻¹)

Direct Measurement of Orientational Correlations in a 2 Dimensional Liquid Crystal System.

D. Van Winkle

2d nematic (de Gennes, 1974)

$$F = \frac{1}{2}\left\{K_B(\sigma \times \hat{c})^2 + K_S(\sigma \cdot \hat{c})^2\right\}$$

$$\rightarrow \frac{1}{2} K(\nabla\phi)^2$$

$$\sigma^2(\vec{\rho})_\tau = \langle |\varphi(0) - \phi(\vec{\rho})_\tau|^2 \rangle$$

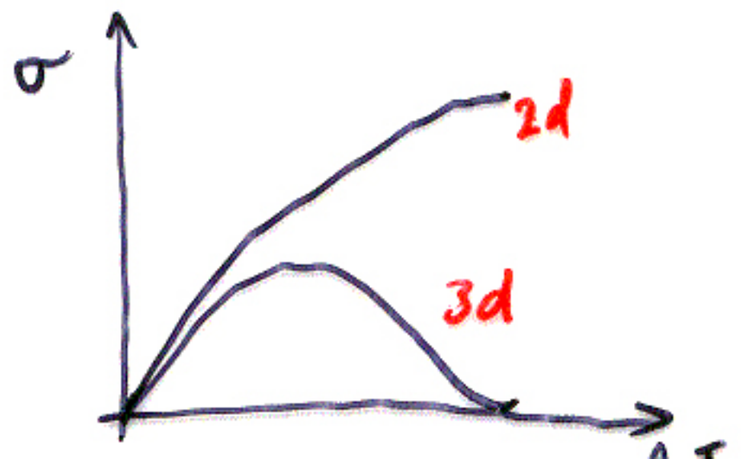
$$= \frac{2k_B T}{KL} \int \frac{d^2q}{4\pi^2 q^2} \left\{1 - e^{i\vec{q}\cdot\vec{\rho}}\, e^{-Dq^2\tau}\right\}$$

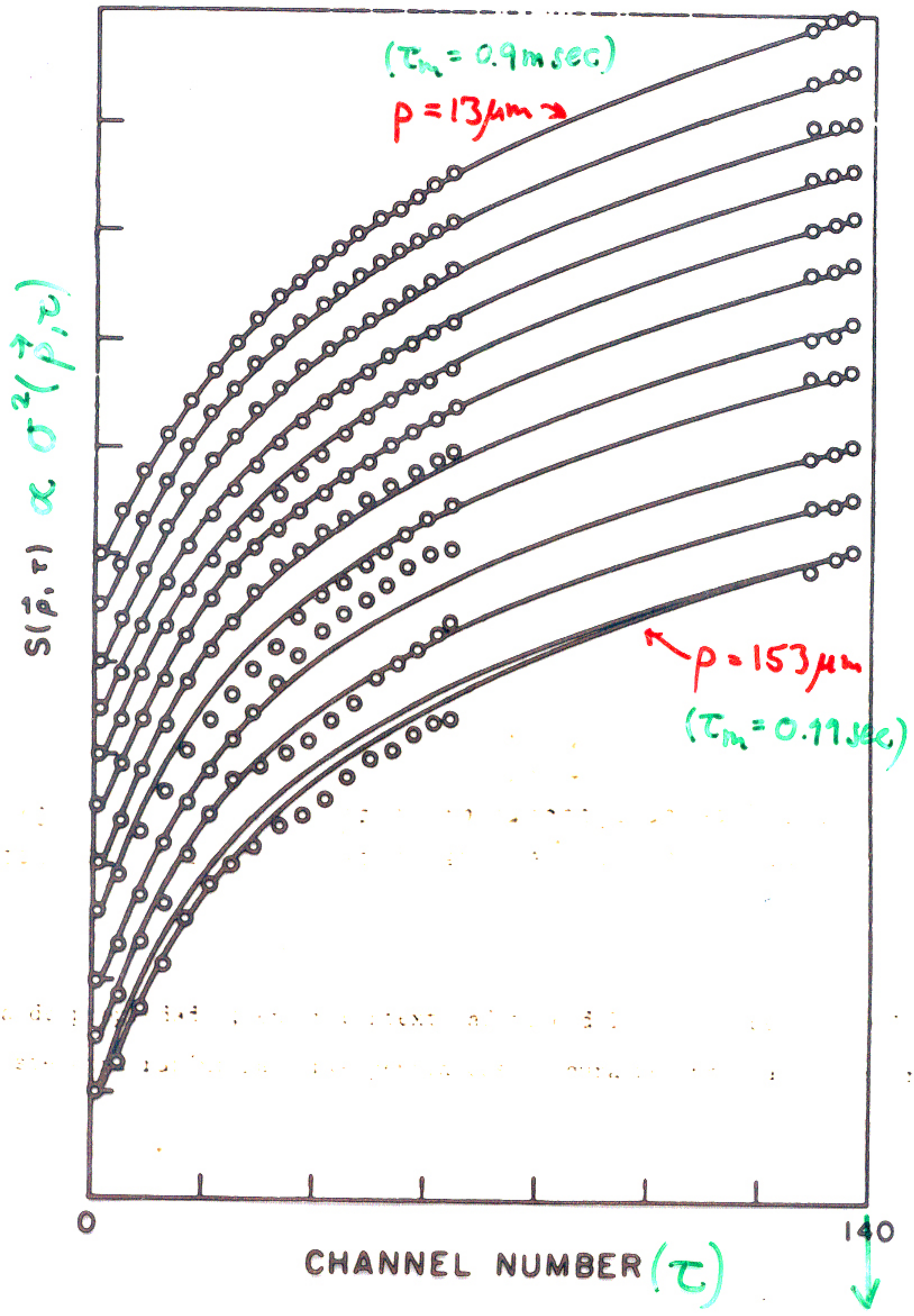
$$\dot{\phi} = D\nabla^2\phi$$

$$\langle \phi_q(0)\, \phi_q(\tau) \rangle = \langle \phi_q(0)^2 \rangle e^{-Dq^2\tau}$$

$$\sim \ln(\rho/a) + \ln(\tau/\tau_c) \qquad \tau_c = \rho^2/AD$$

L-P divergence { Landau, Peierls




$(\tau_m = 0.9 m sec)$
$p = 13 \mu m$
$S(\vec{p},\tau) \propto \sigma^2(\vec{p},\tau)$
$p = 153 \mu m$
$(\tau_m = 0.11 sec)$
0
140
CHANNEL NUMBER (τ)
$\tau_m \sim p^2$

DoBAMBC
N = 3

T_c

$\kappa \cdot 10^{-13}$ ergs/

0 1 2 3 4

92 96 100 104 108 112

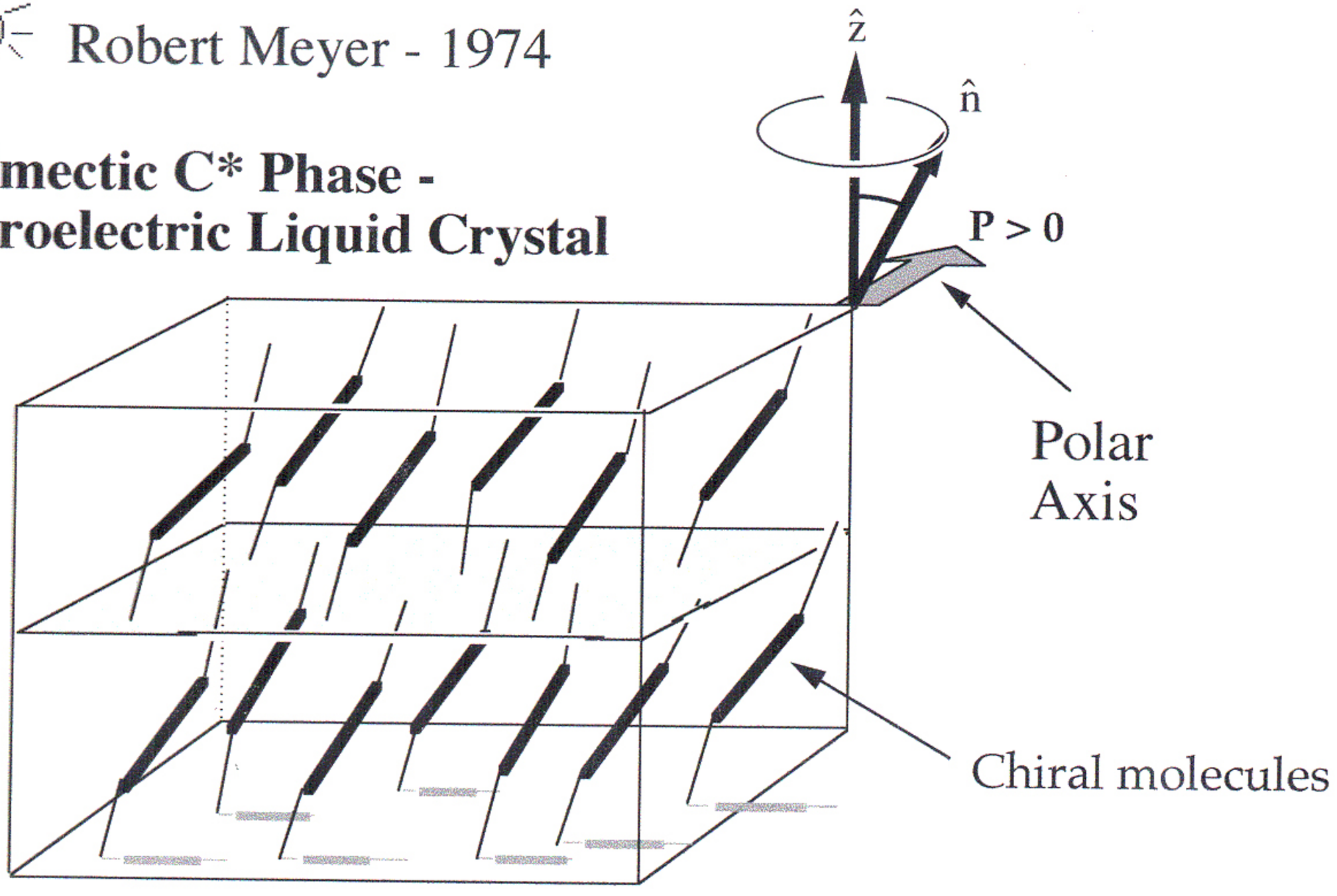
TEMPERATURE (°C)

Nelson - Kosterlitz $k_B T_c = \frac{\pi K}{2}$

Can We Make a Polar Liquid?

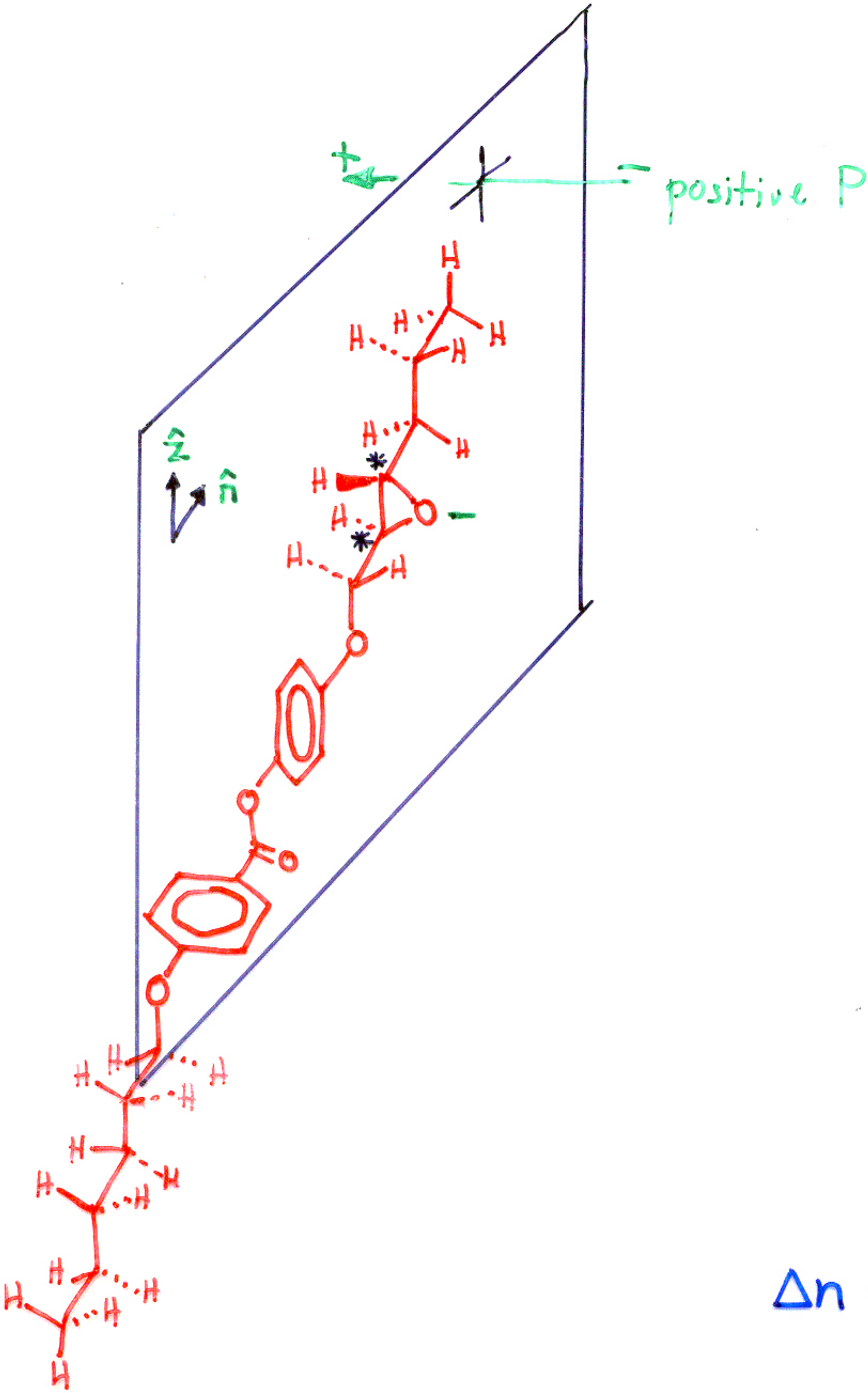
Robert Meyer - 1974

The Smectic C* Phase -
A Ferroelectric Liquid Crystal



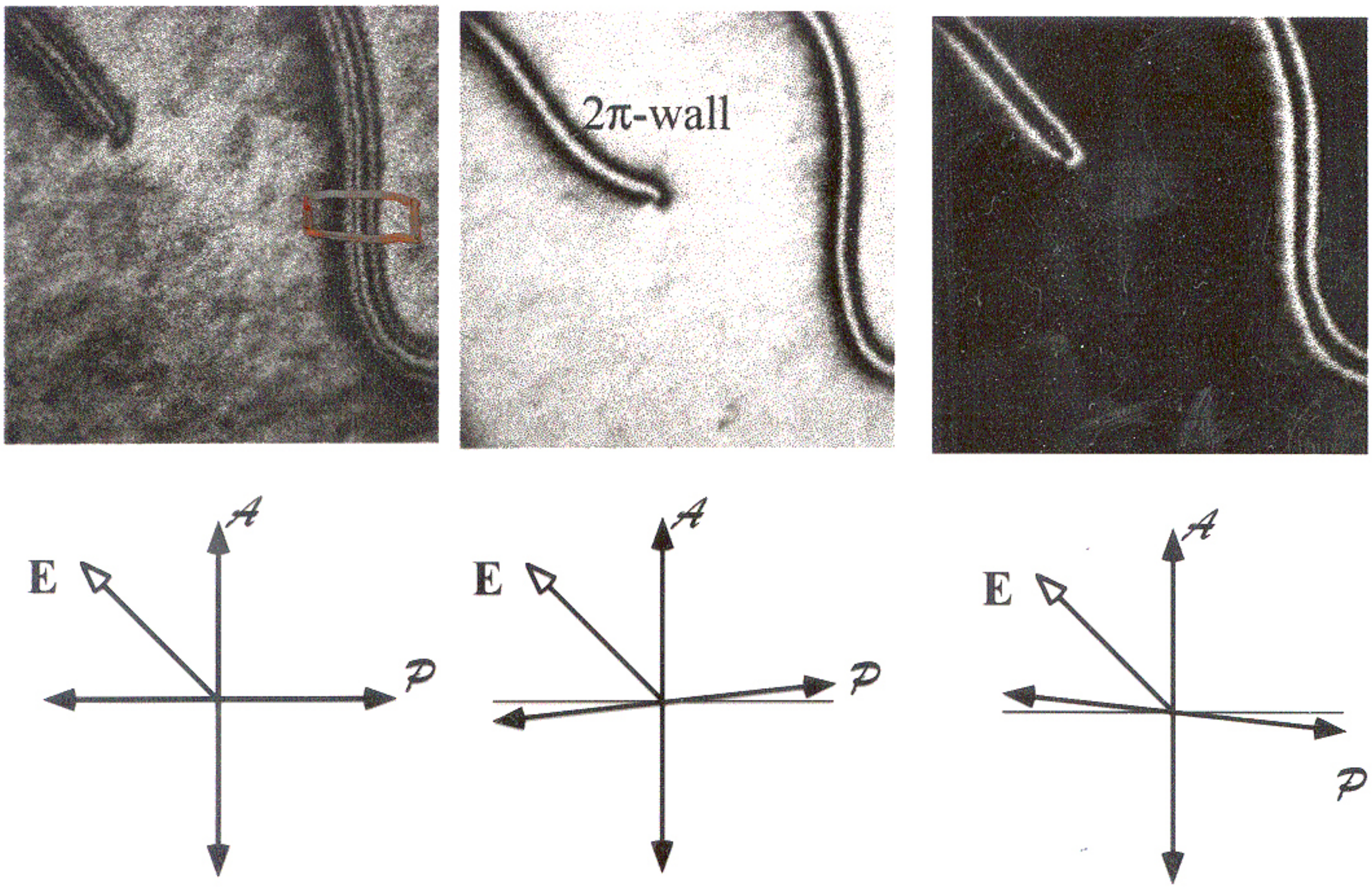
P has equal magnitude but opposite sign for enantiomers

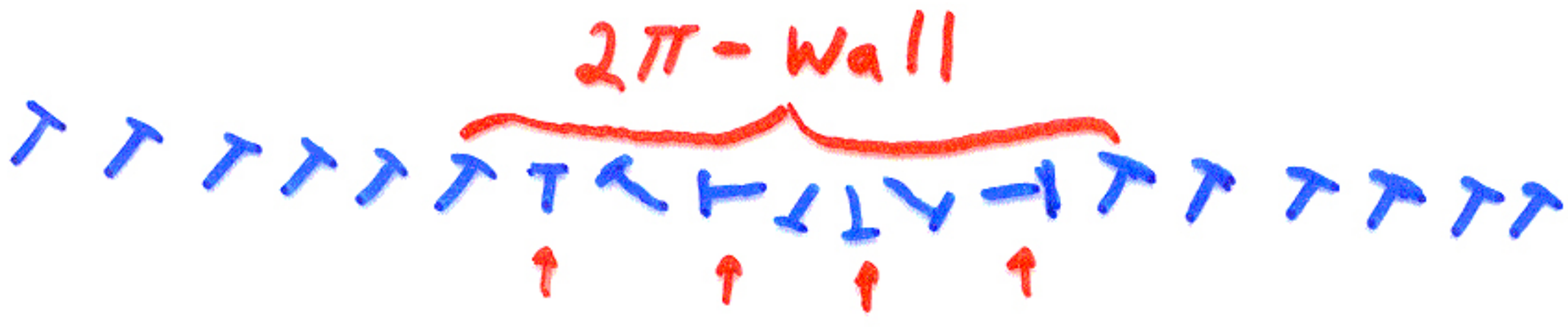
P along $\hat{z} \times \hat{n} \equiv$ positive

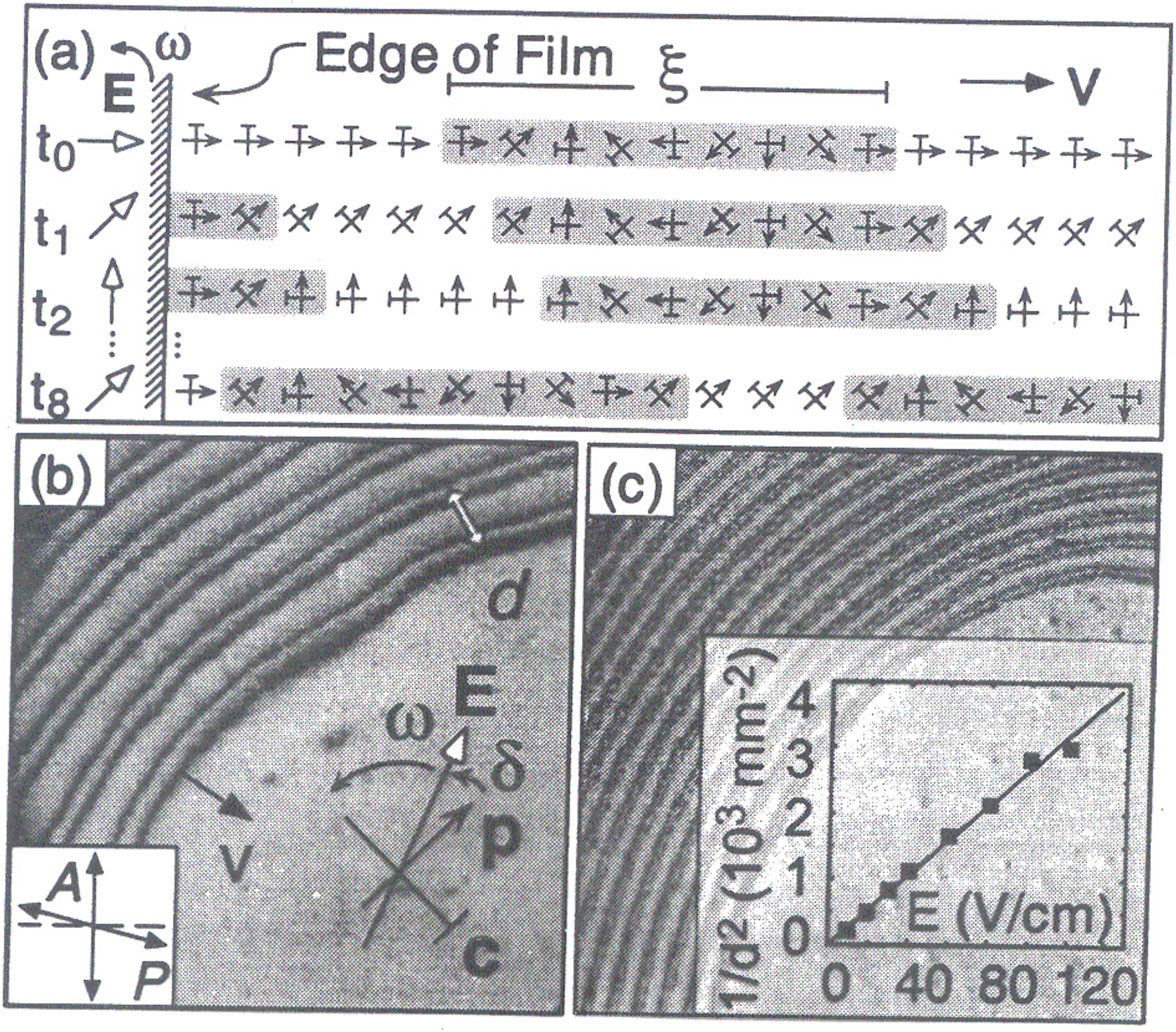


$\Delta n \sim 0.1 \rightarrow 0.2$

Point Defect in Sm C* Film






(a)
ω
E
Edge of Film
ξ
V
t_0
t_1
t_2
t_8
(b)
d
E
ω
δ
p
V
A
P
c
(c)
$1/d^2$ (10^3 mm^{-2})
4
3
2
1
0
E (V/cm)
0
40
80
120

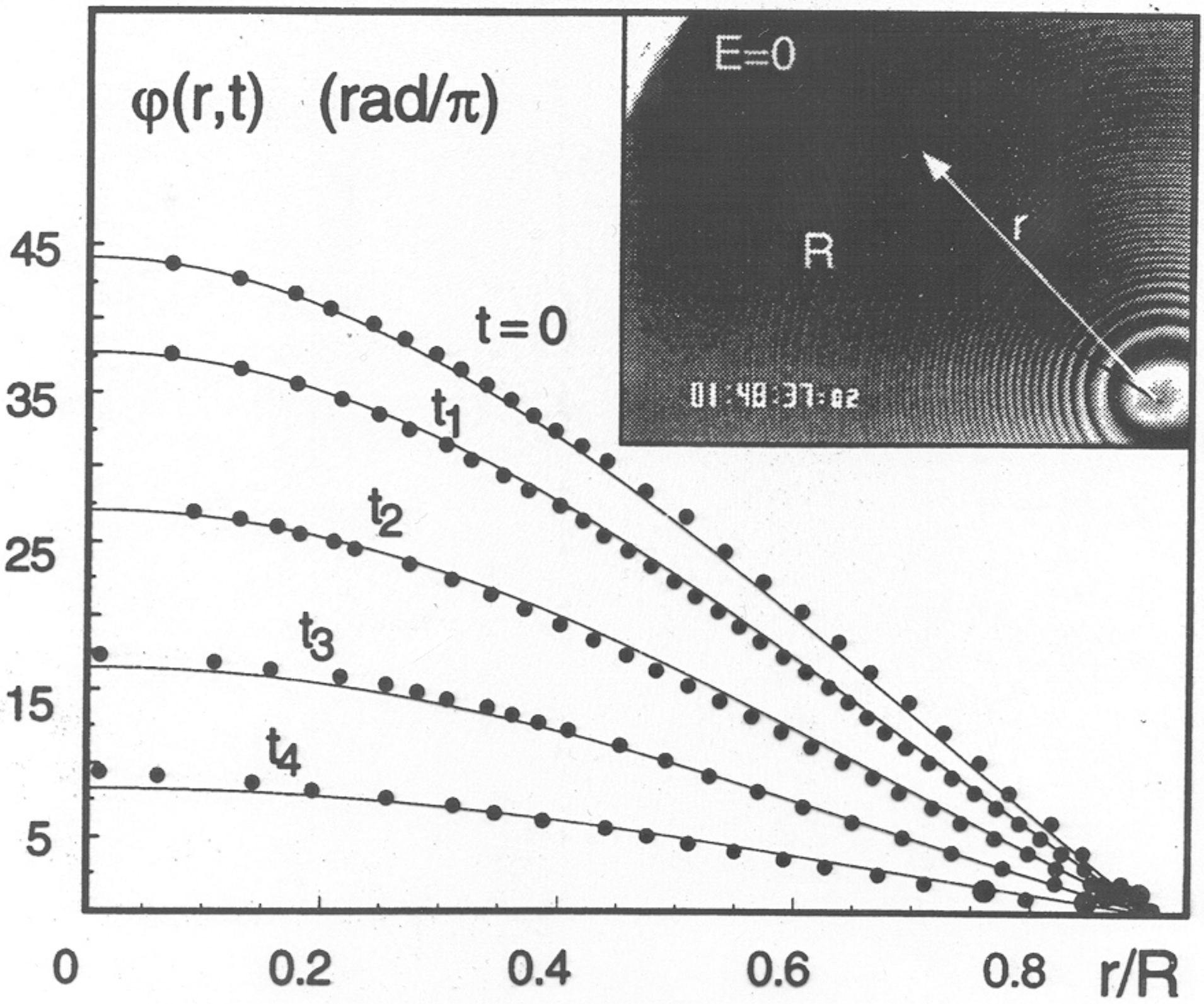
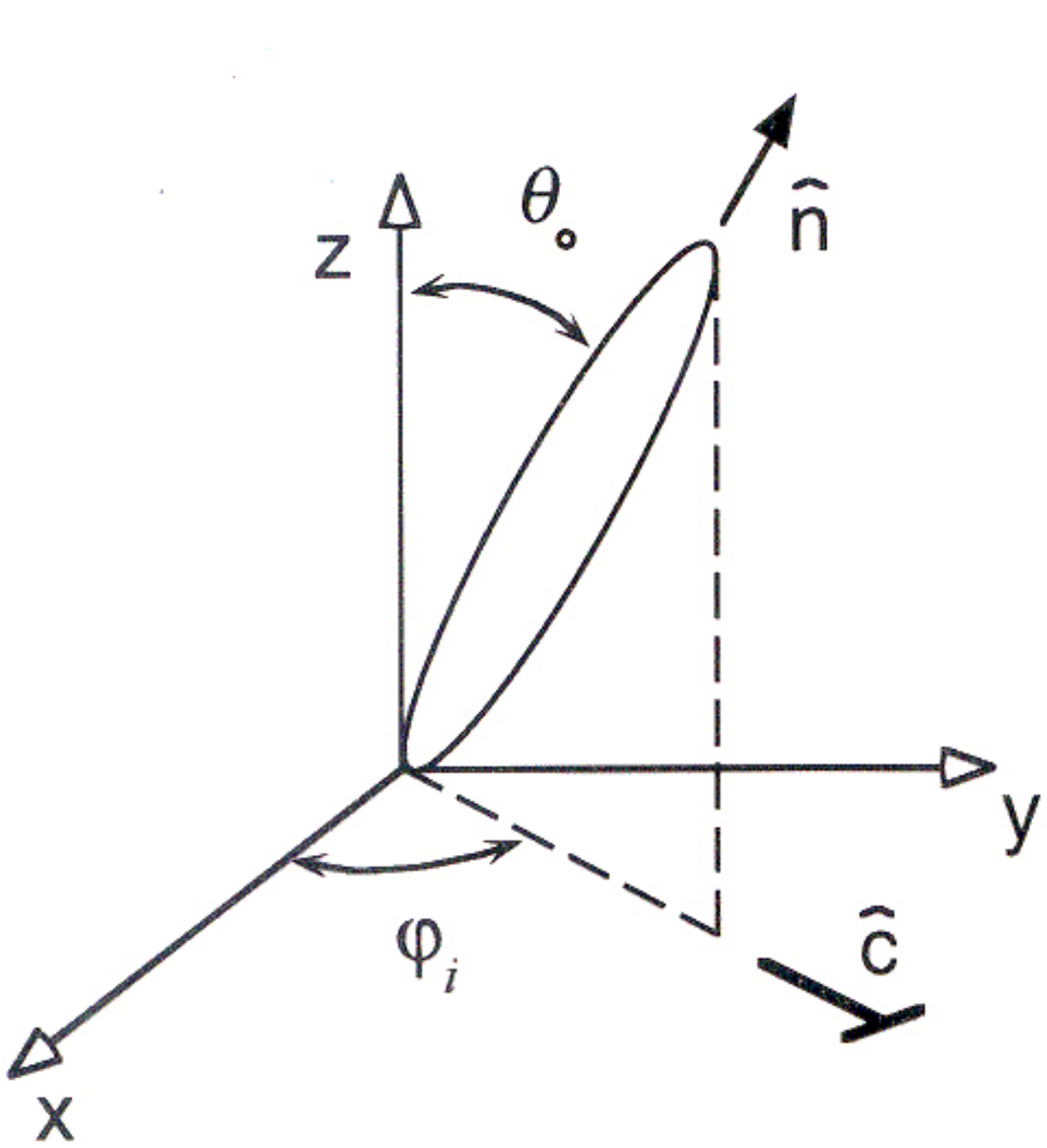
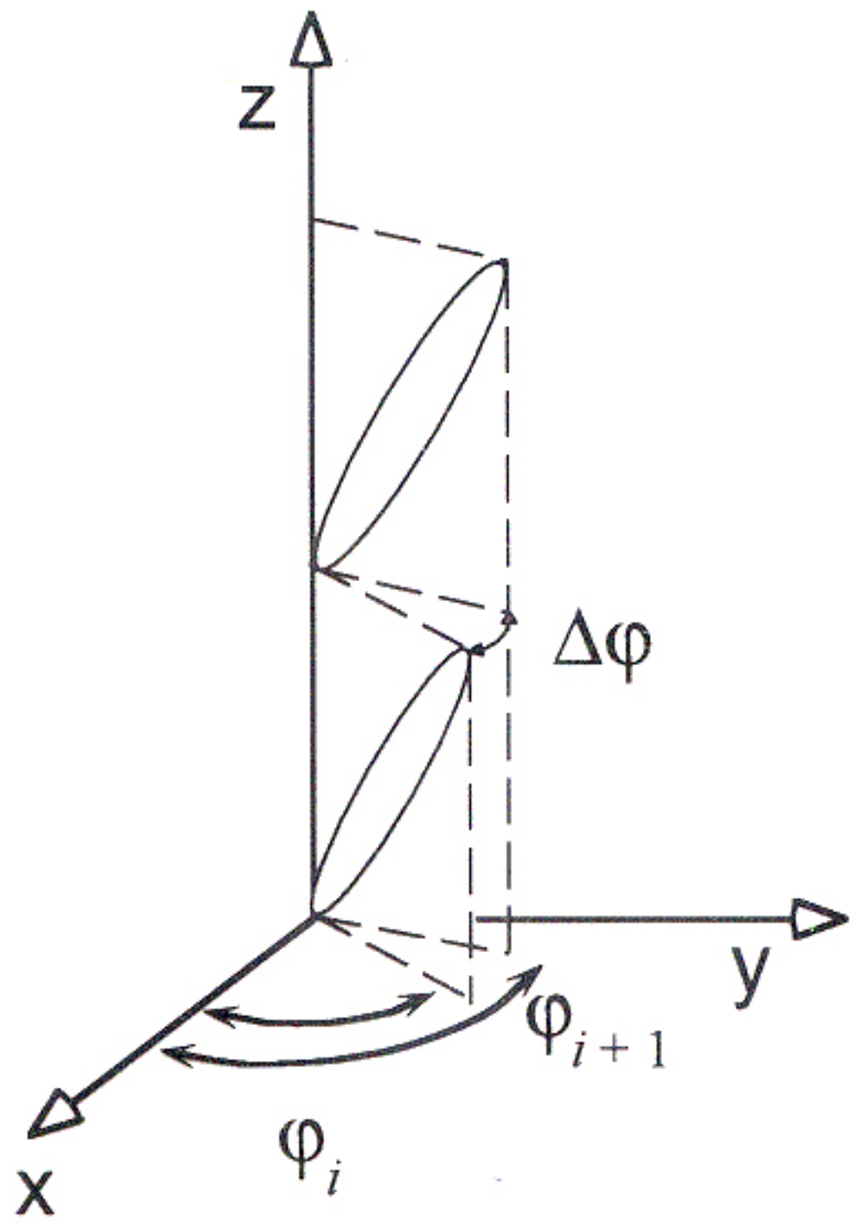
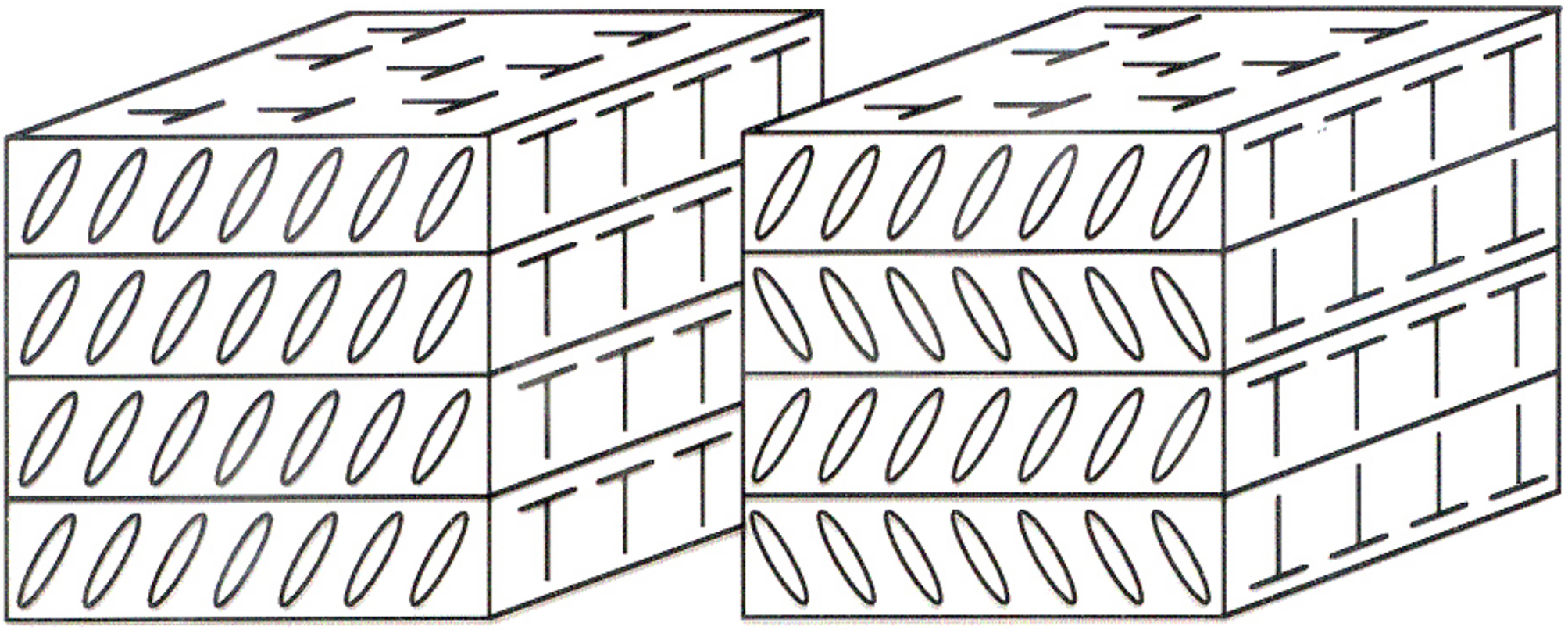


FIG. 3. Ring relaxation. The phase of **c**, $\varphi(r,t)$, is shown (●) for a five-layer film of DOBAMBC relaxing without a field. The curves are fits to the first three terms of Eq. (4) using $A_1 = 134.9$, $A_2 = 0.29$, $A_3 = 1.15$, and $\gamma/K = 2.0 \times 10^5$ s/cm^2 at times $t_1 = 41.8$, $t_2 = 134.7$, $t_3 = 275.1$, and $t_4 = 467.4$ s. The inset is a typical video image of a film of radius R during relaxation.

Smectic C and Smectic C_A Phases

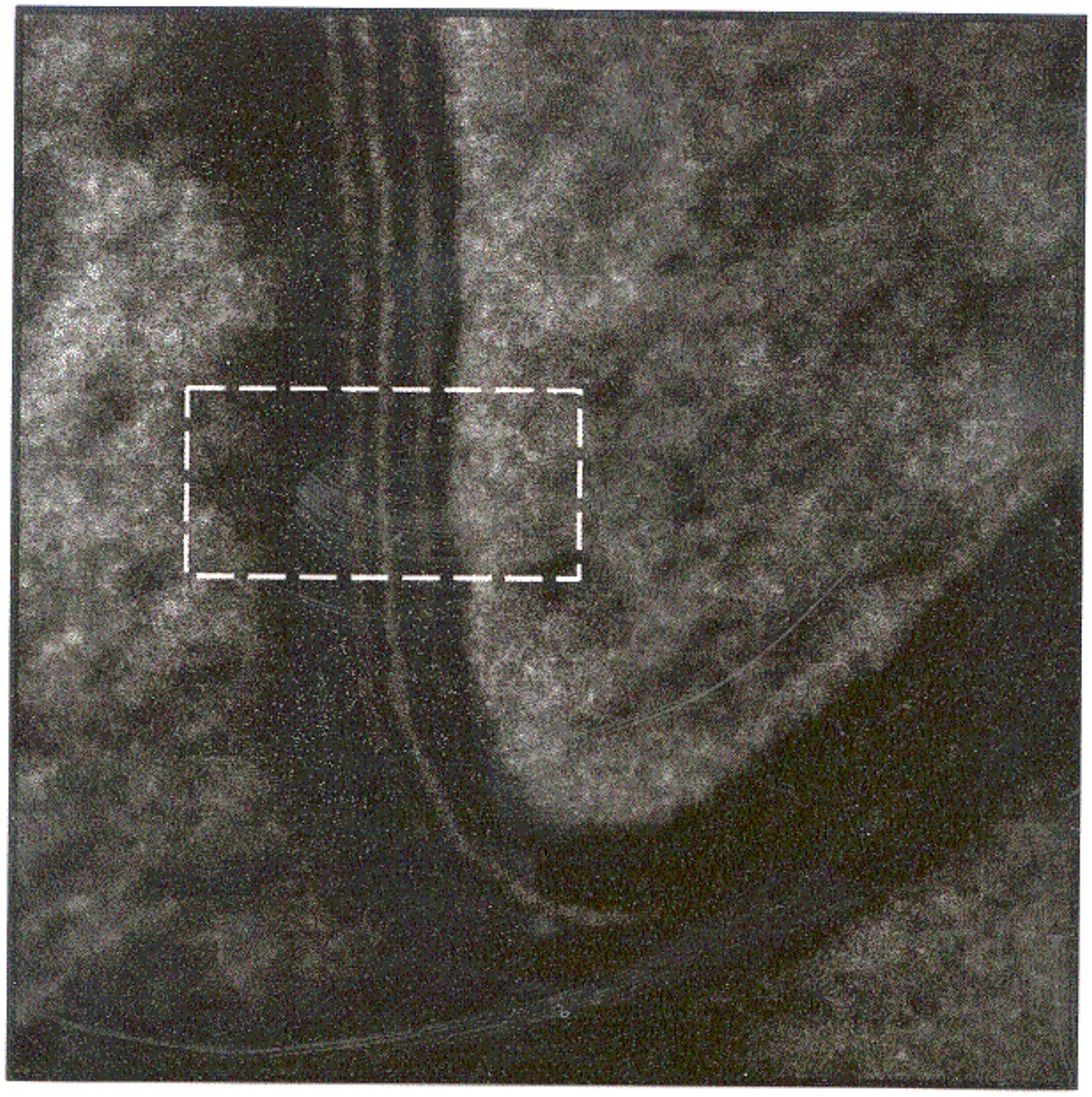


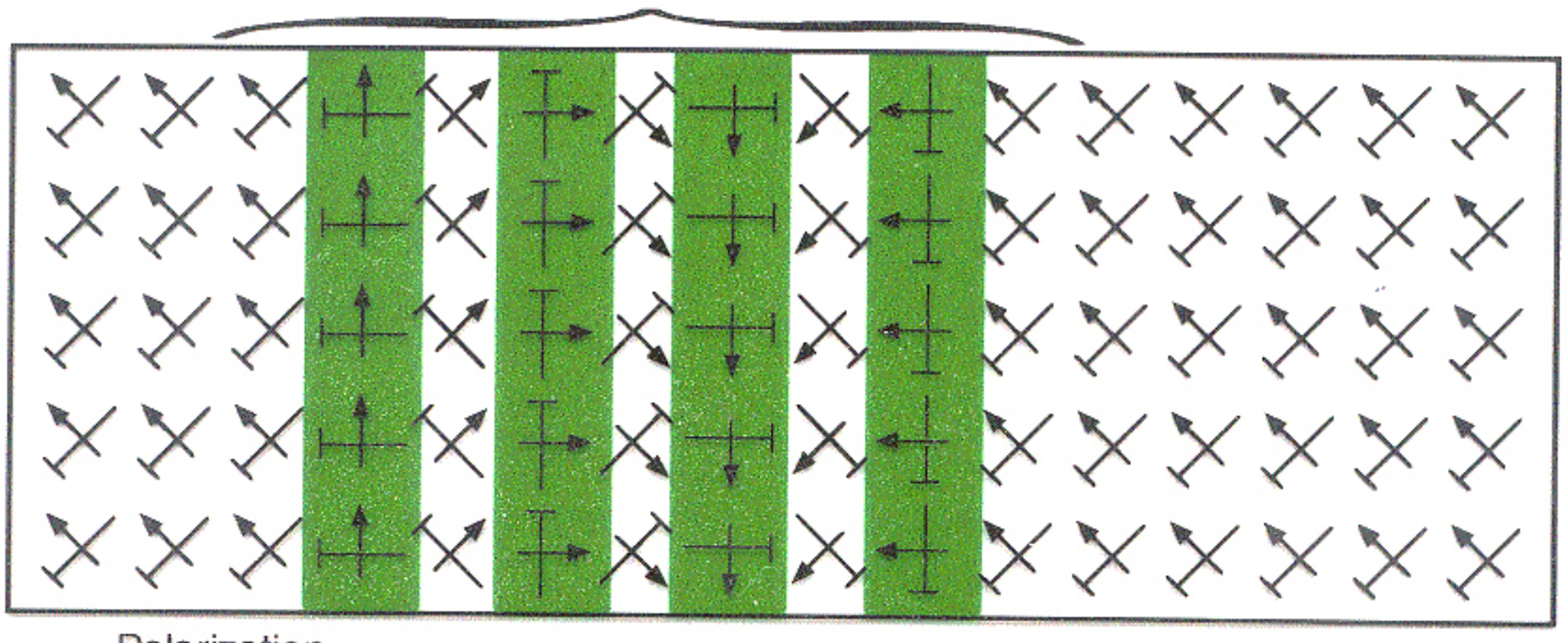




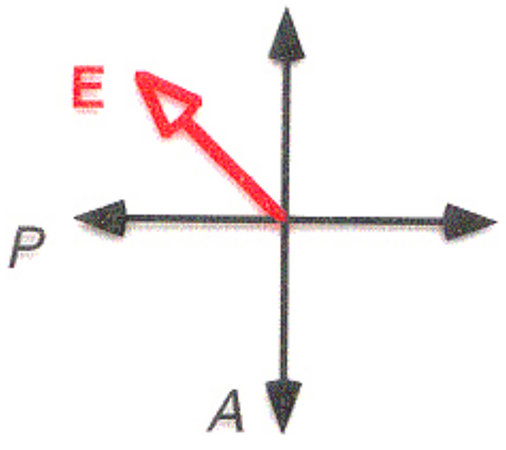
synclinic $\Delta\varphi = 0$ anticlinic $\Delta\varphi = \pi$

2π-Wall





$$\xi = \sqrt{\frac{K}{PE}}$$



TILT

PLANE

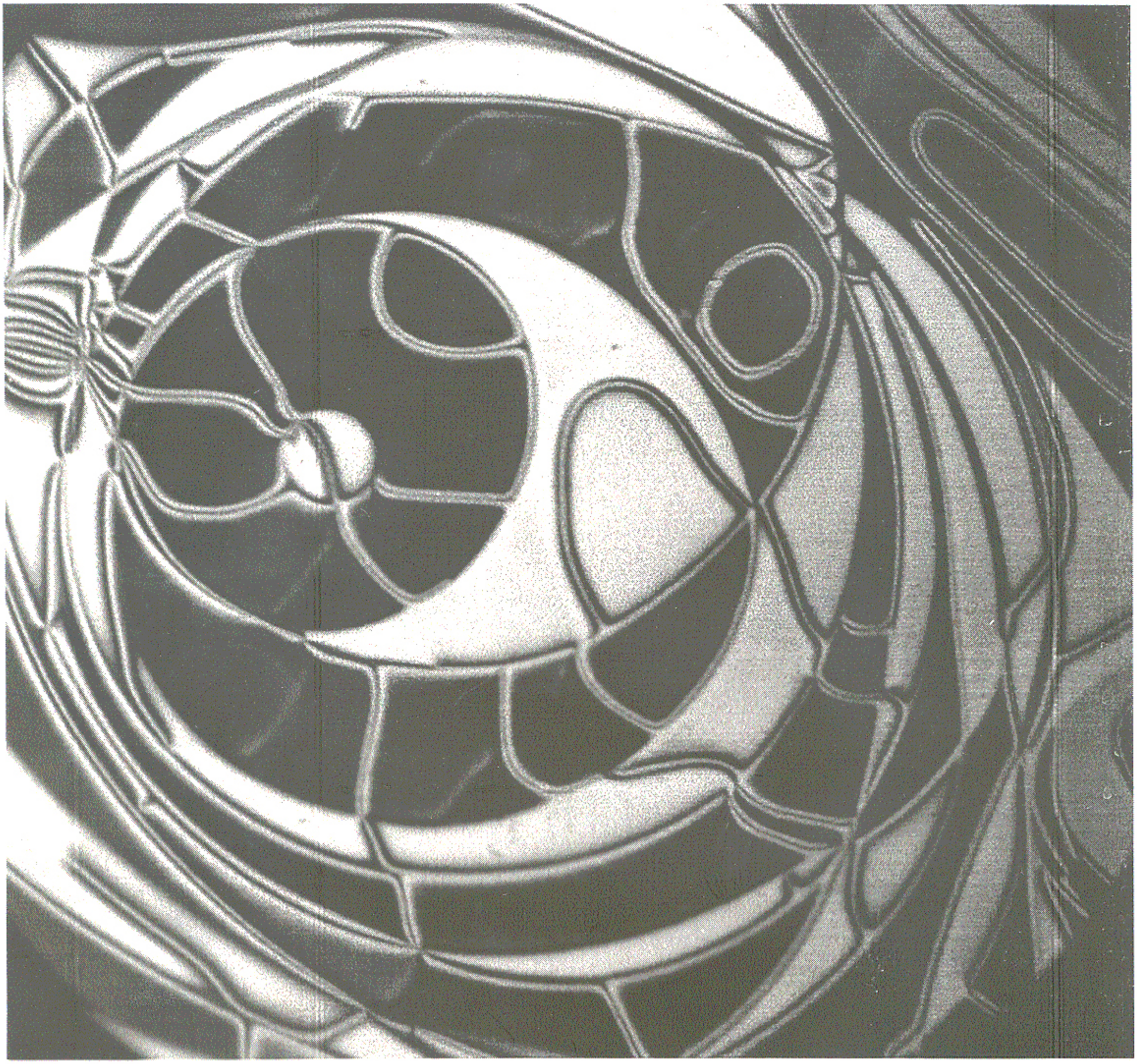
N=2

$\vec{P}$

$\parallel \vec{E}$

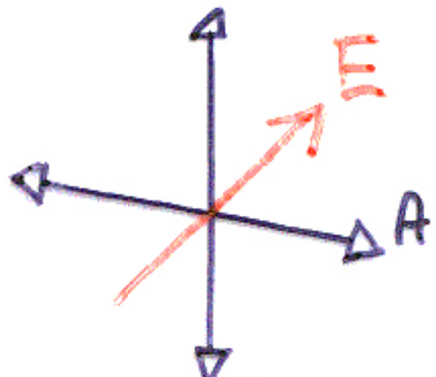
N=3

$\perp \vec{E}$



antiferroelectric texture in freely-suspended film

МНРОВС



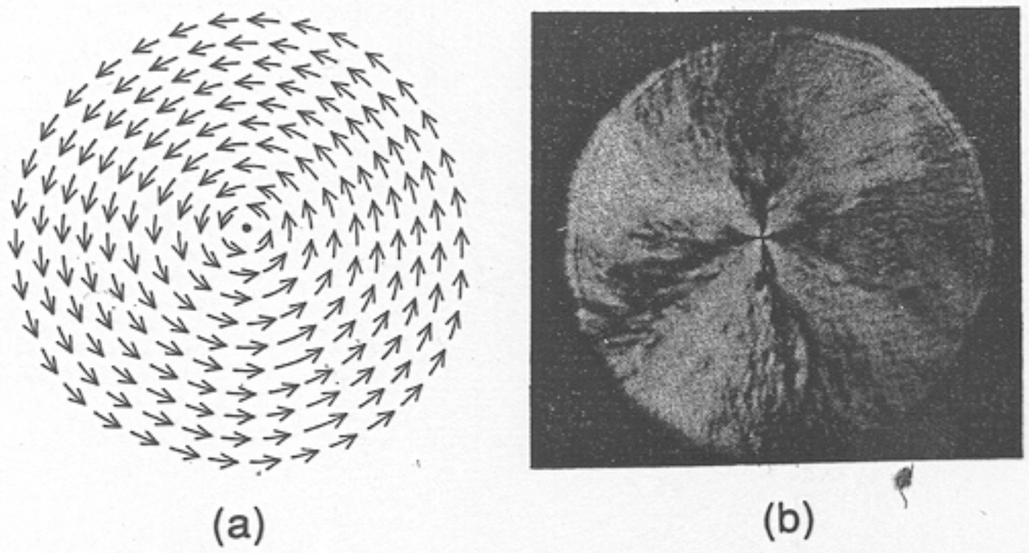


FIG. 1. (a) The topological defect studied in these experiments: a 2π ($s=+1$) point bend disclination in $\mathbf{c}(\mathbf{r})$, the 2D nematic director field of a freely suspended smectic-C film. (b) Appearance of the vortex of (a) in depolarized reflected light microscopy (DRLM) [9]. The diameter of the illuminated area is 1.8 mm. The film diameter is 3.7 mm.

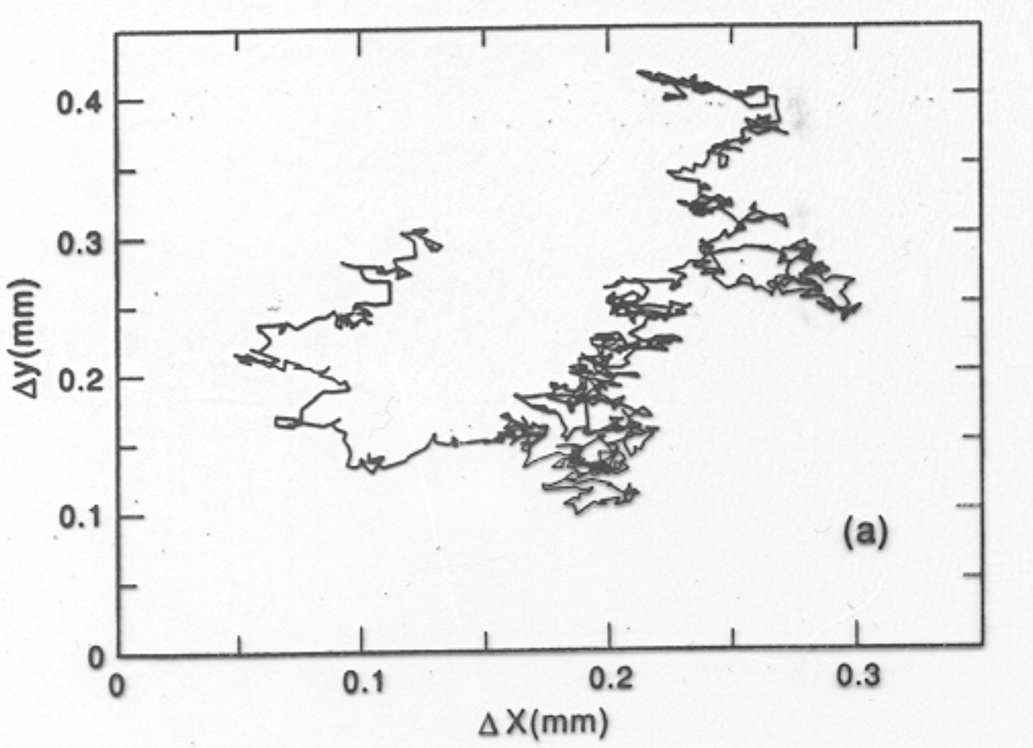


FIG. 2. (a) The trajectory of a defect over a 2-h time period at 55.1 °C. (b) Mean-square displacement vs time for the data in (a), giving a diffusion constant $D=0.8\times10^{-7}$ cm^2/sec.

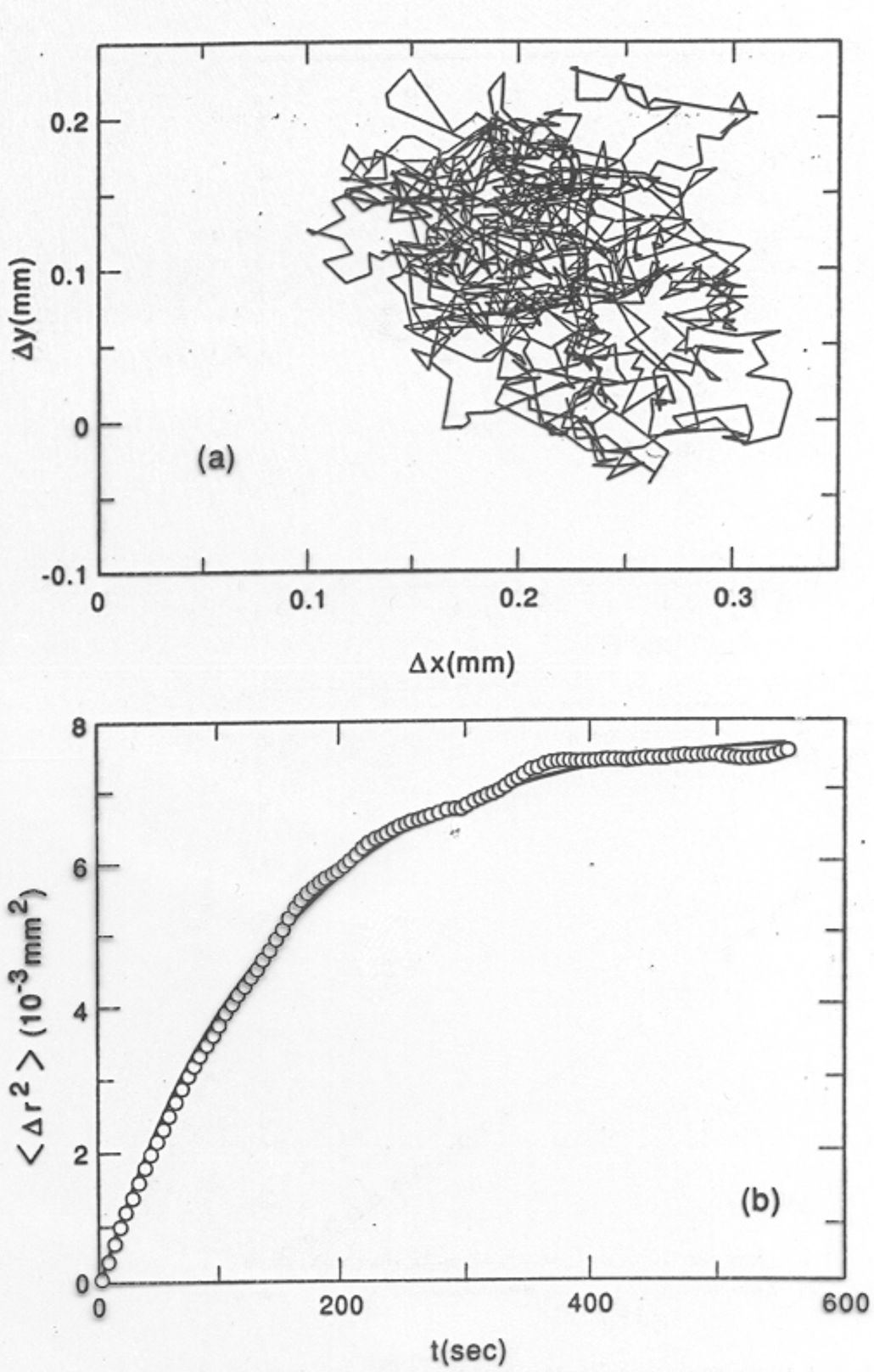
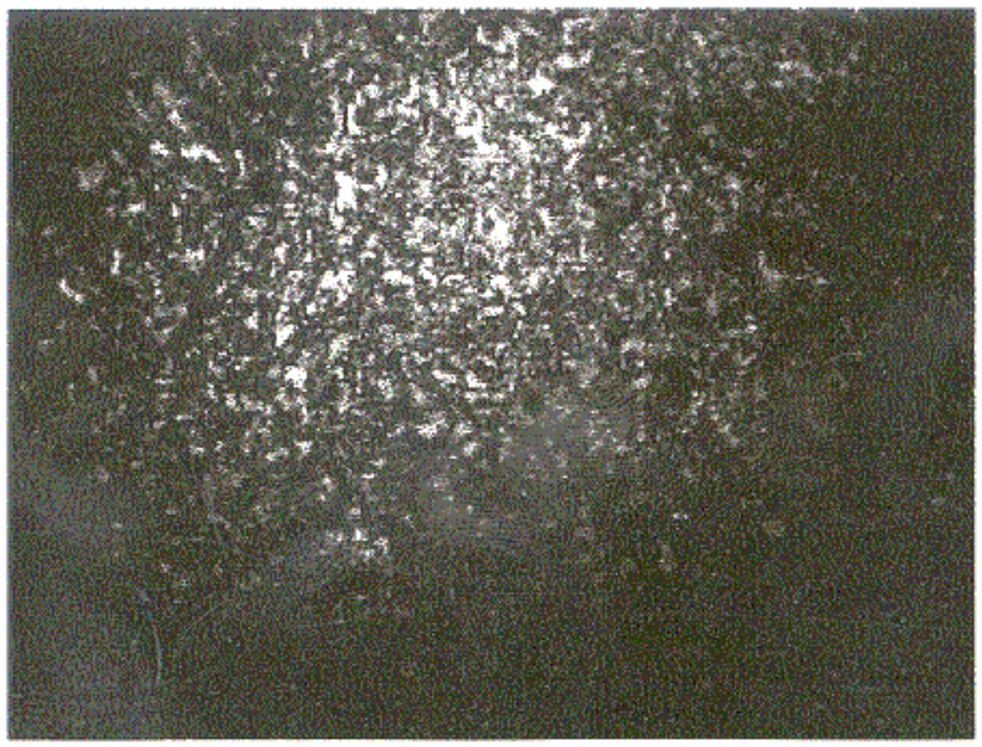
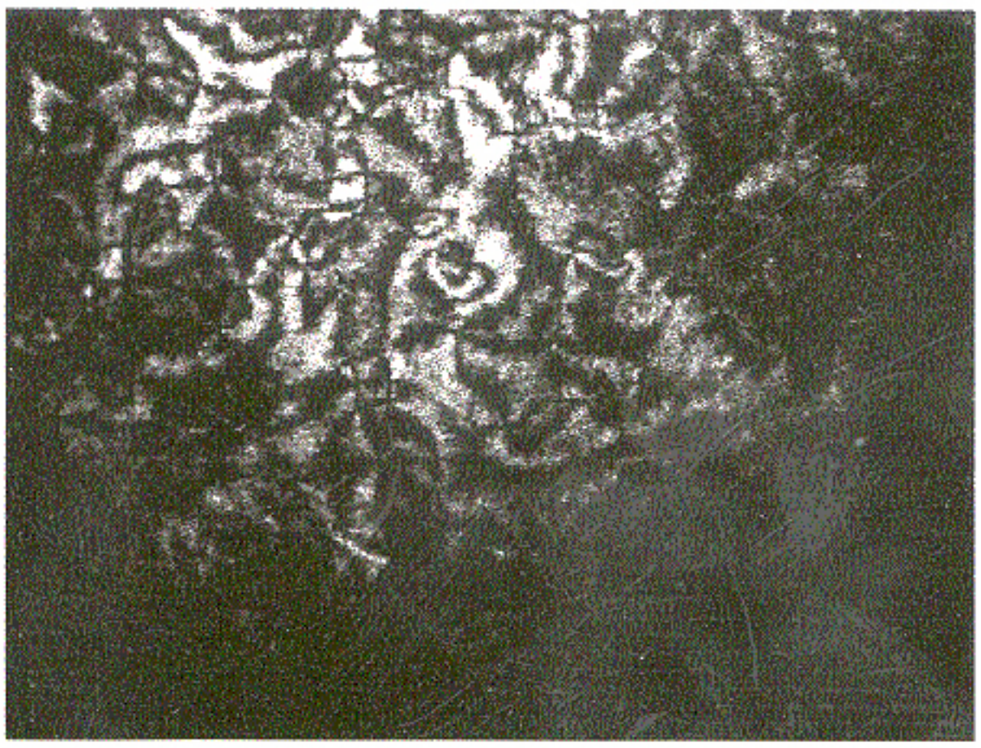
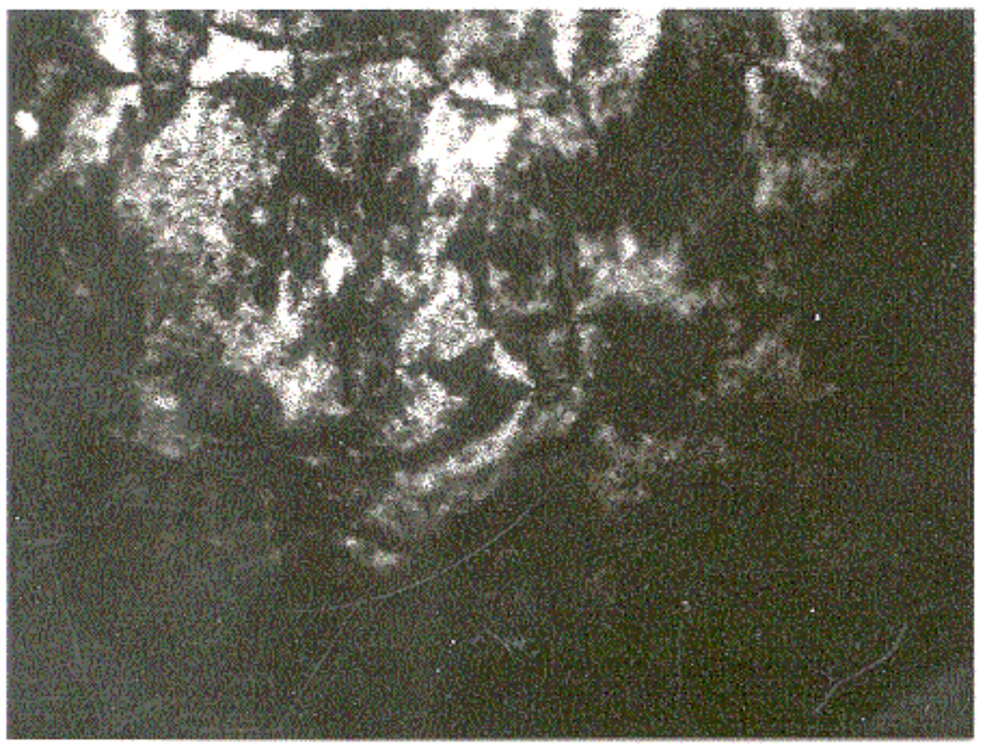


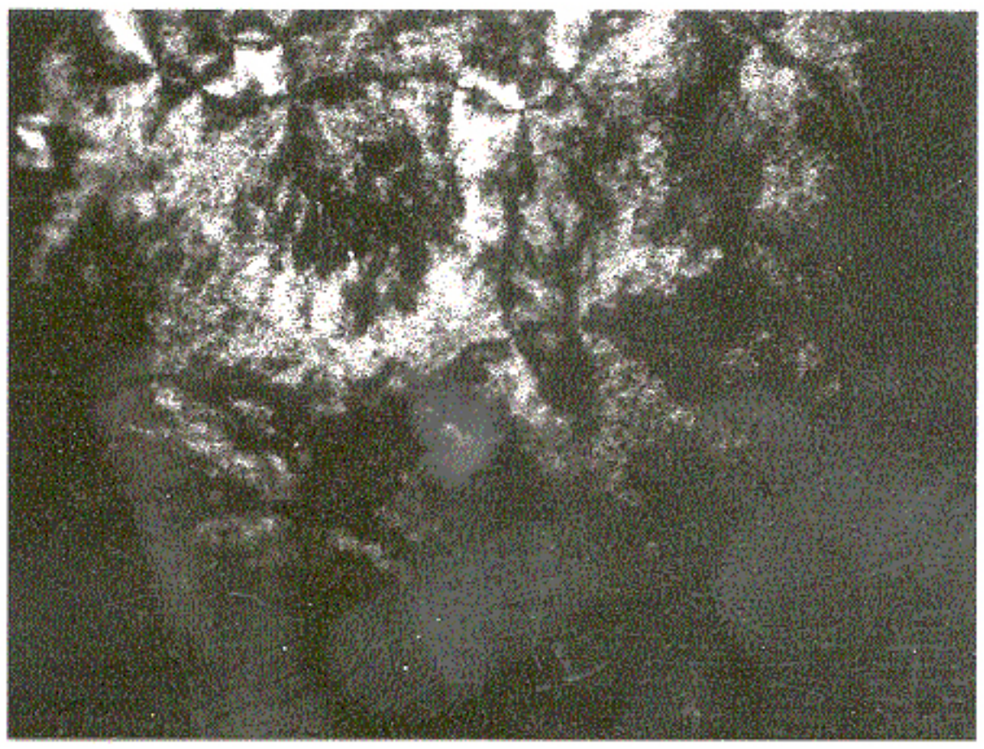
FIG. 3. (a) The trajectory of a defect over a 2-h time period at 74.6 °C. (b) Mean-square displacement vs time for the data in (a). The saturation of $\langle\Delta r^2\rangle$ at long times shows the effect of the binding potential on the motion.

Time Sequence of Defect Coursening



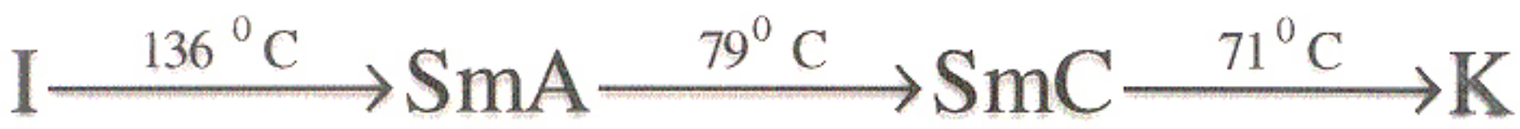




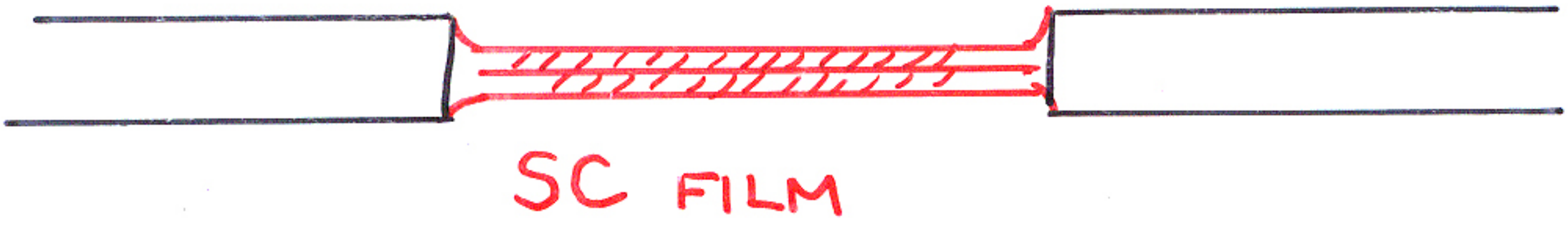


2 layer films at T= 80°C

3M EX - 900084

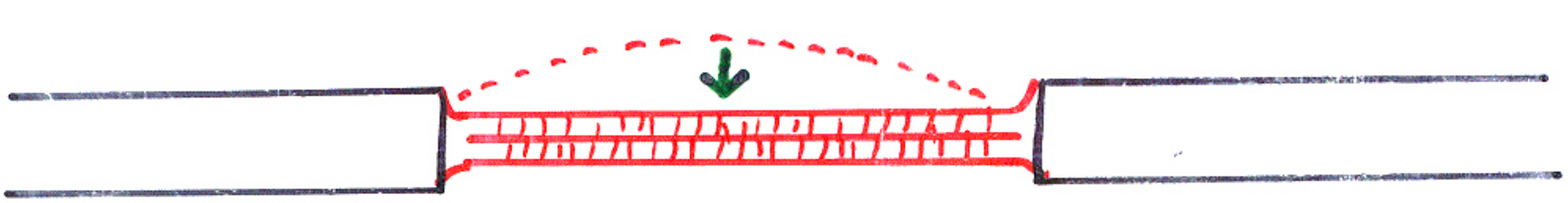


QUENCHING THE A → C TRANSITION



SLOWLY BLOW A BUBBLE

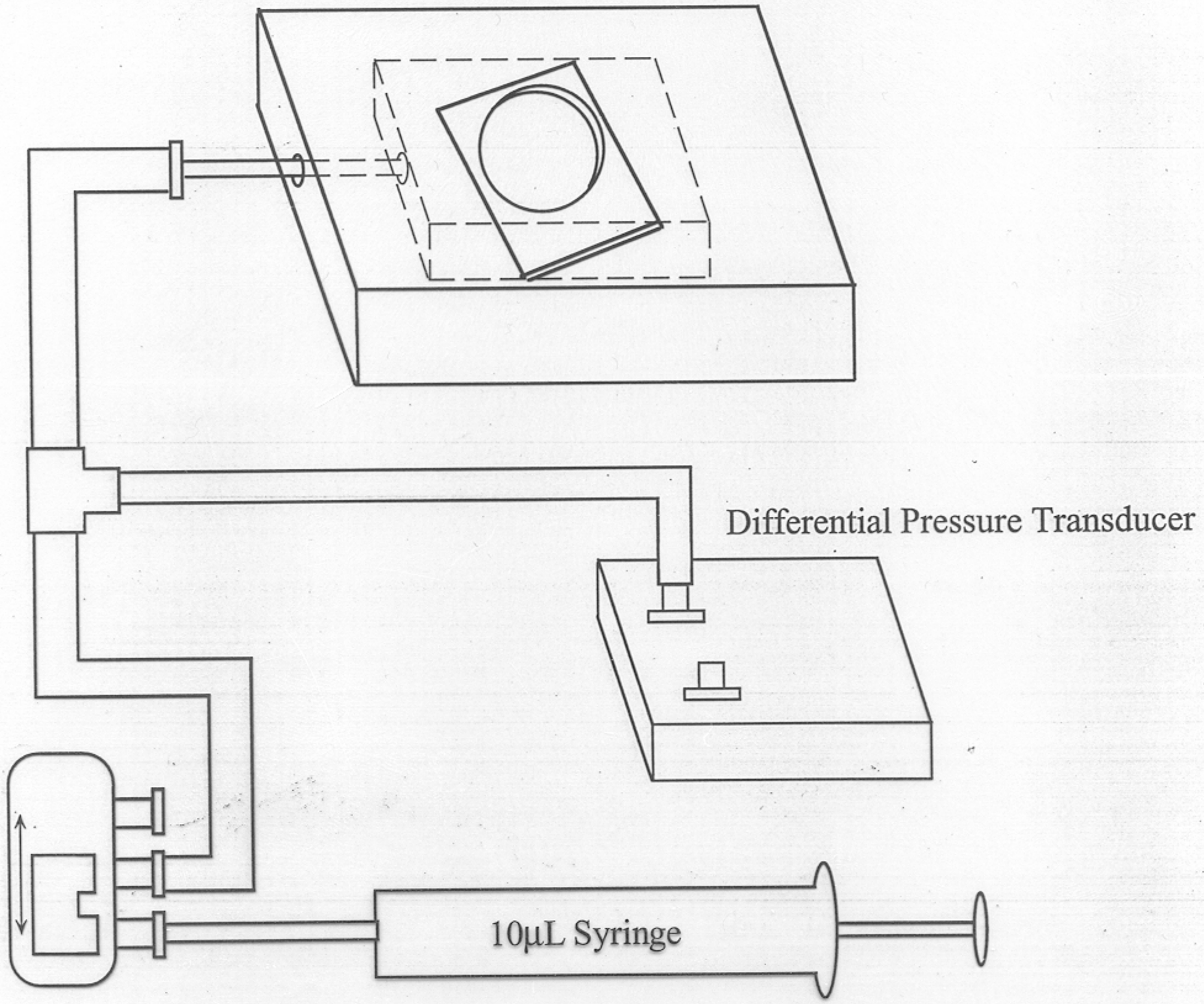
FILM REMAINS SC

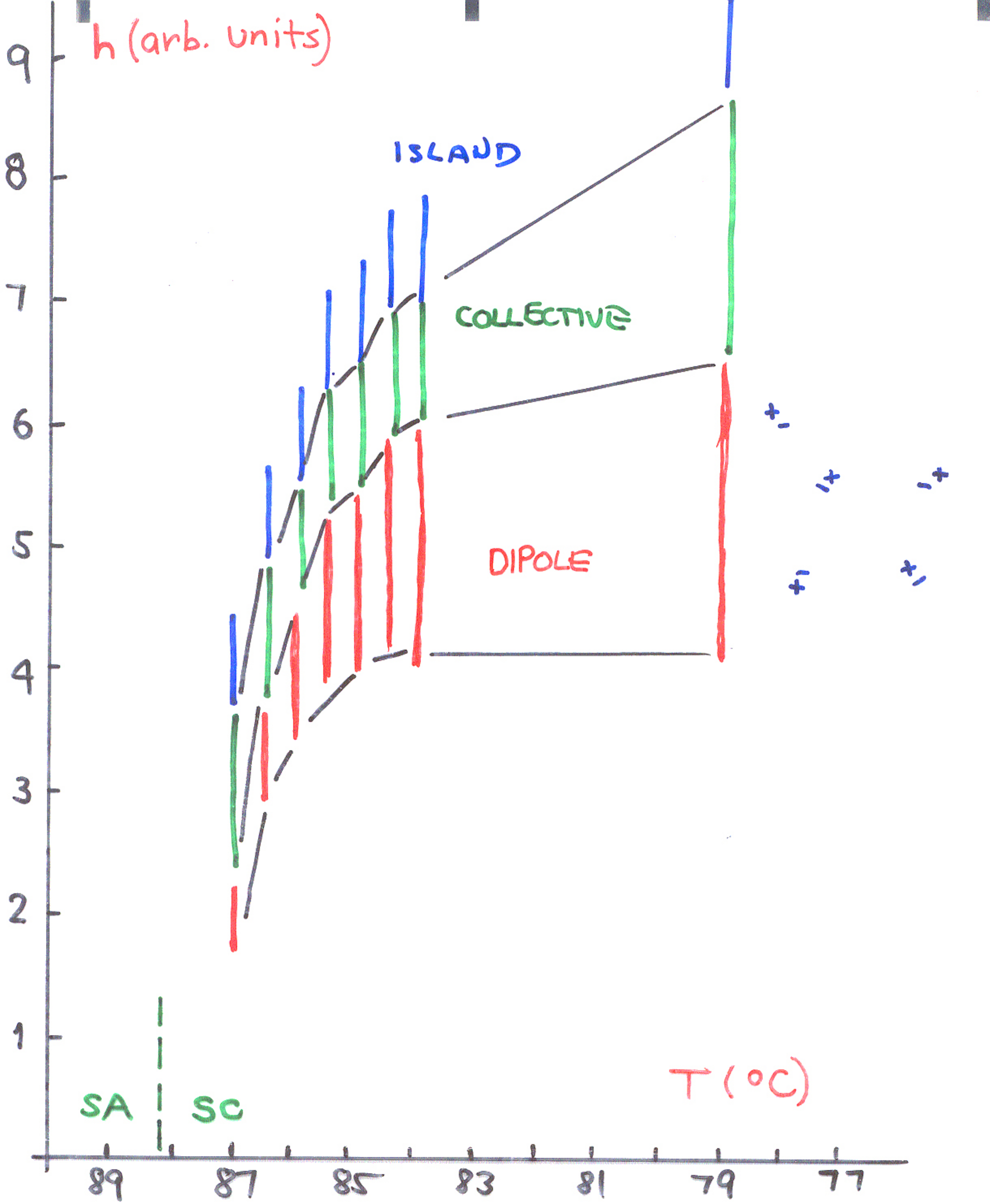


QUICKLY RELEASE THE AIR PRESSURE TO GENERATE TRANSIENT IN-PLANE STRESS $\sigma \sim \Delta A/A$.

→ SC → SA → SC QUENCH CYCLE

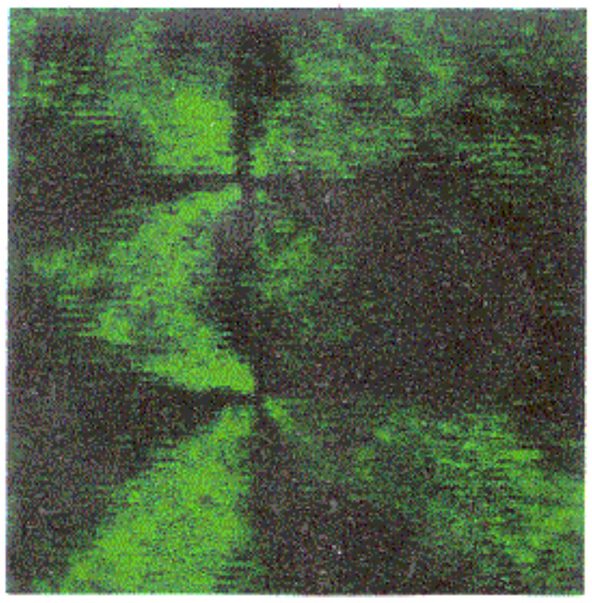
Stress Induced Transitions for the Smectic C Phase to the Smectic A Phase



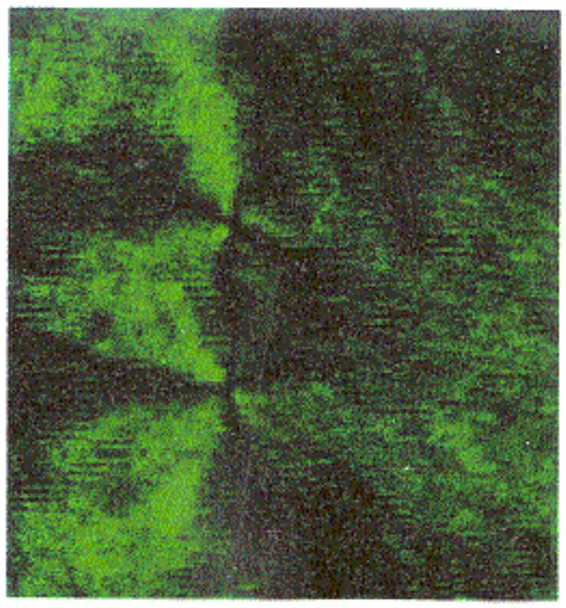

h (arb. units)
9
8
7
6
5
4
3
2
1
ISLAND
COLLECTIVE
DIPOLE
+-
-+
+-
-+
SA
SC
89
87
85
83
81
79
77
T (°C)

Annihilation of Oppositely Charged Topological Defects

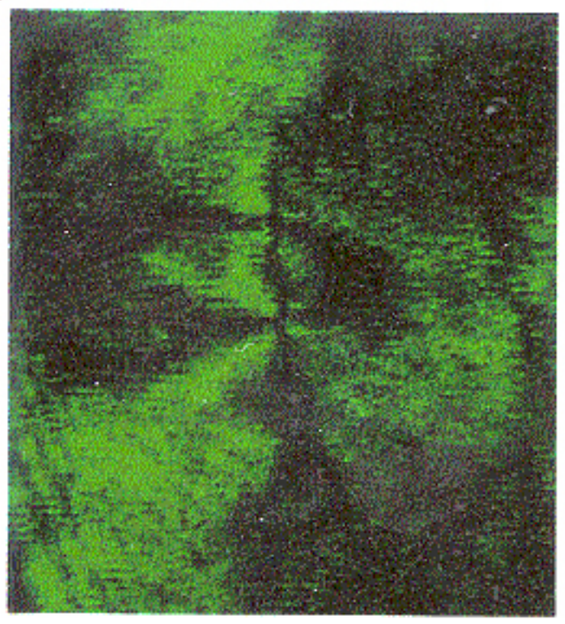
t=13.37



t=32.00

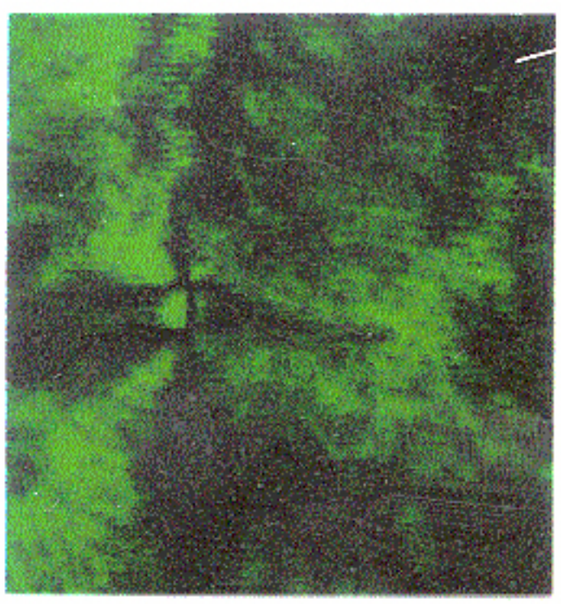


t=34.66

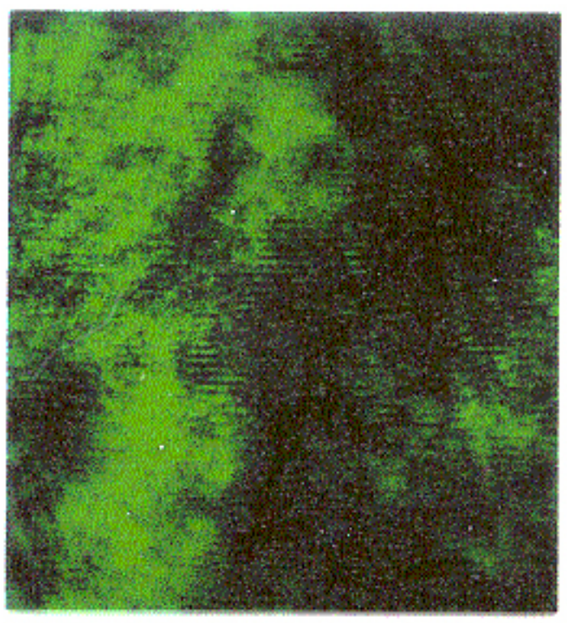


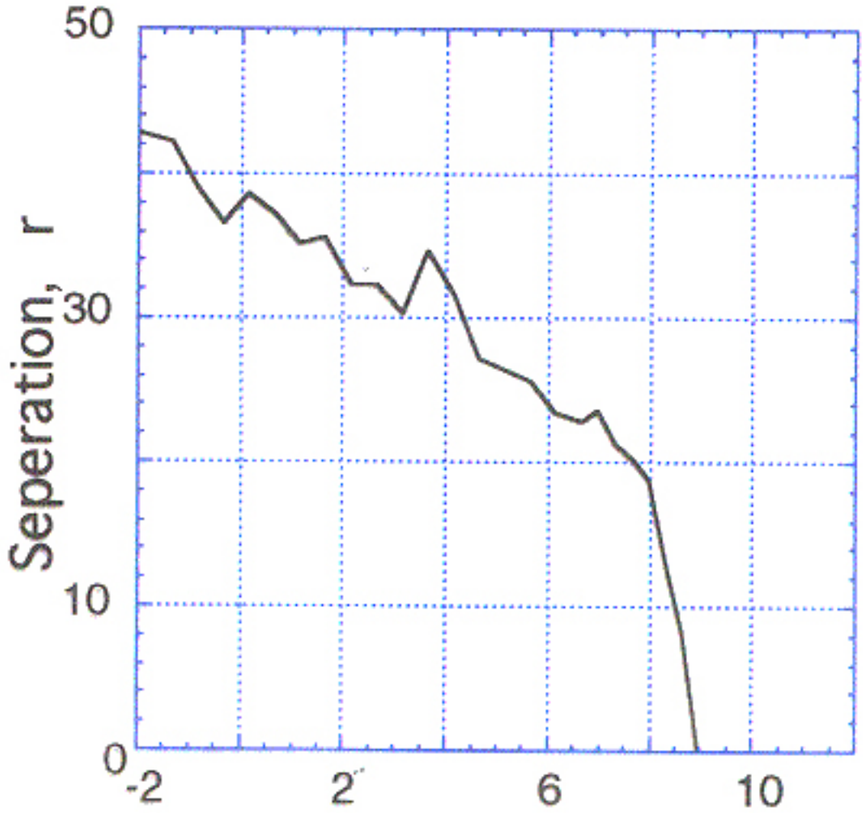
←~100 μm→

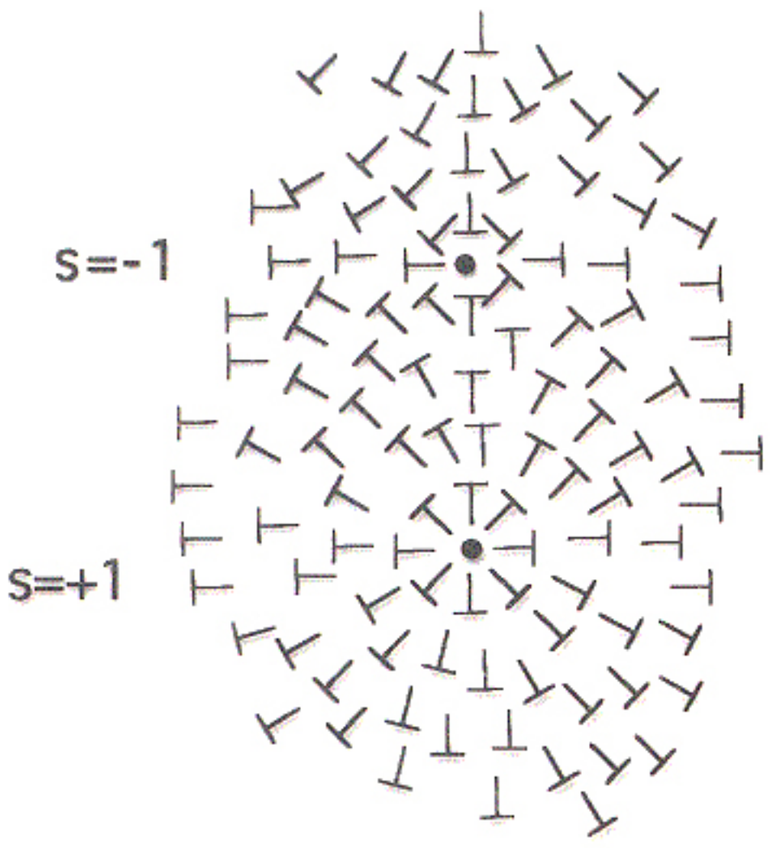
t=38.73

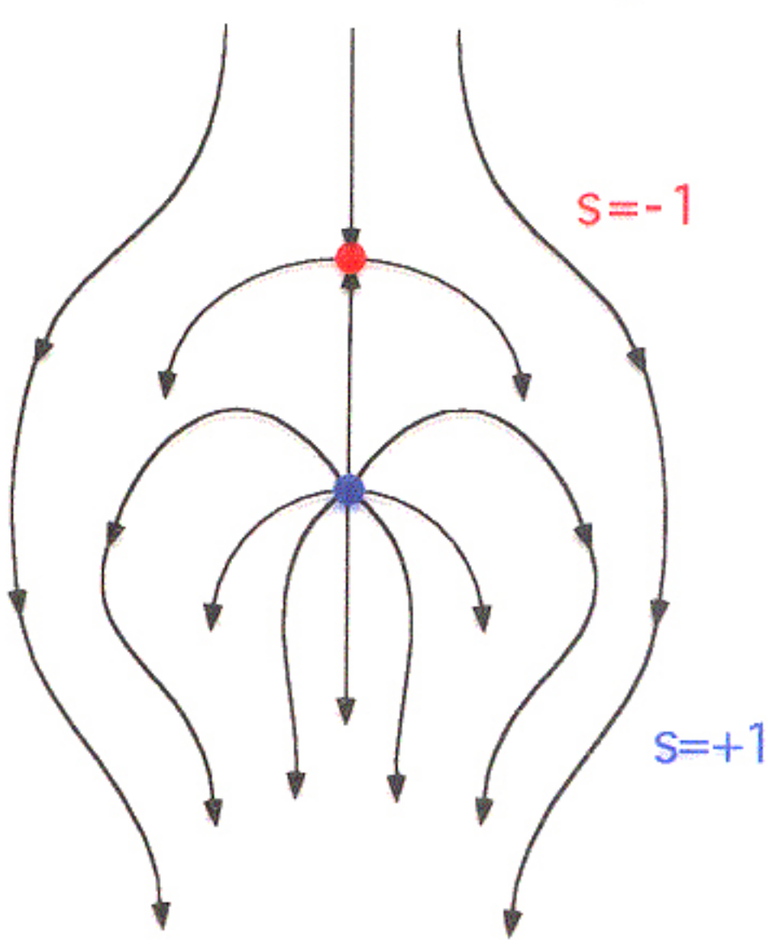


t=40.00









Defect pairs have an attractive force between them:

$$F = -\nabla U = -\frac{\pi k s^2}{r}$$

$$\pi k \rightarrow k$$

$$s^2 = 1$$

$$F = -\nabla U = -\frac{k}{r}$$

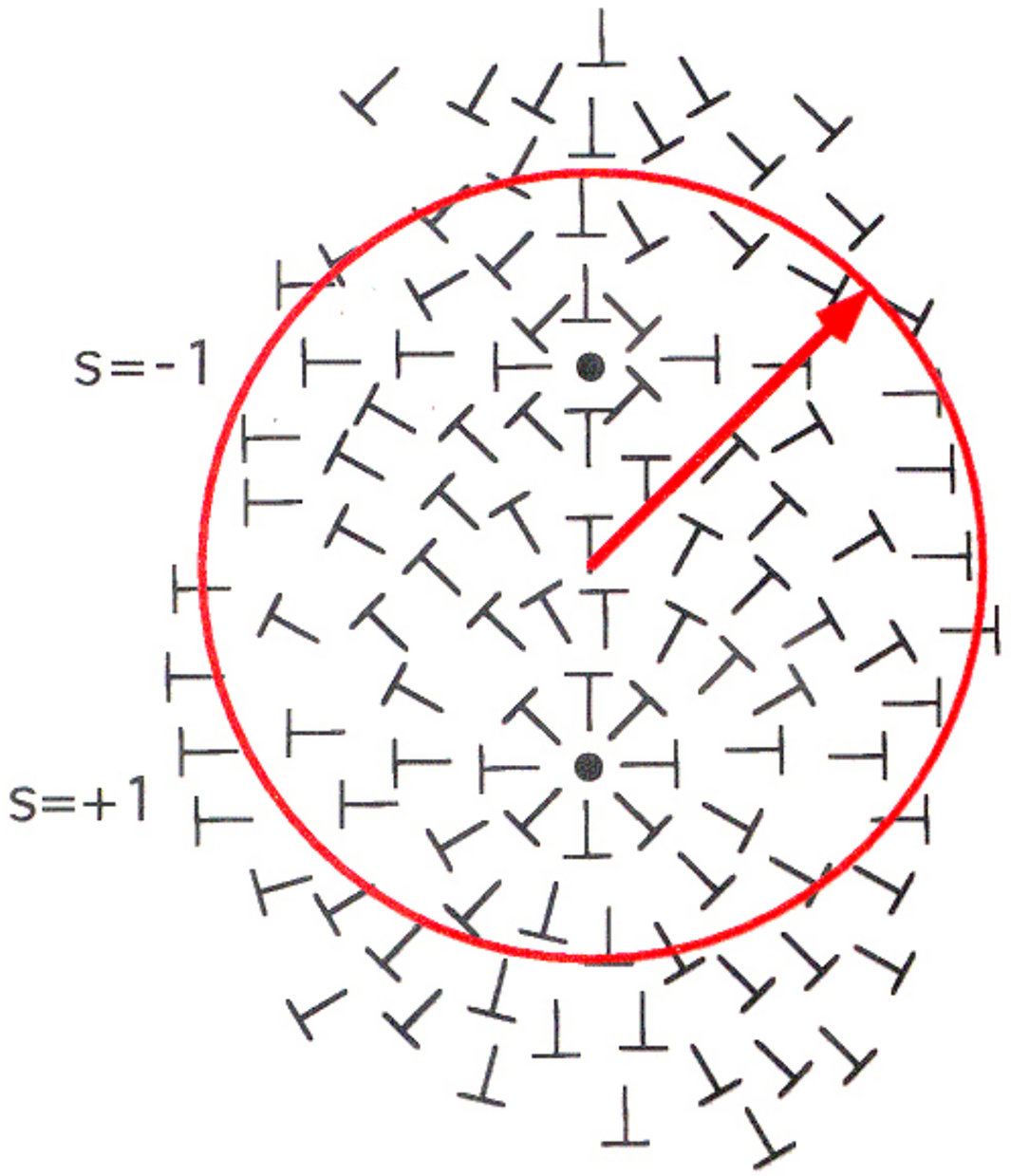
Defects can move through the films with some mobility, μ.

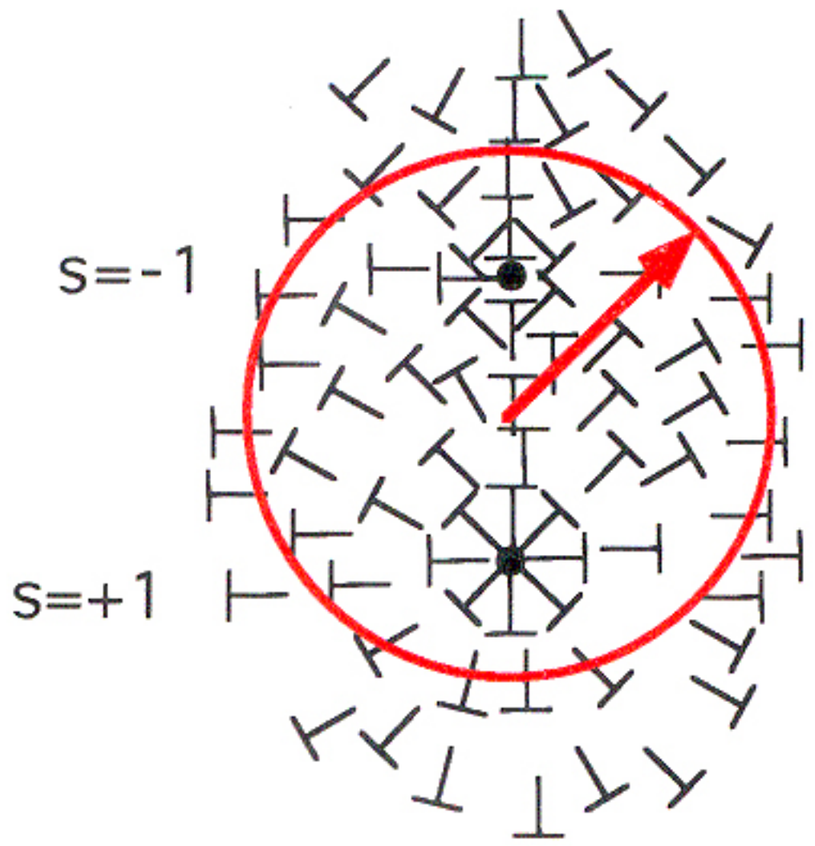
$$\dot{r} = \frac{dr}{dt} = \mu F$$

$$\dot{r} = -\frac{\mu k}{r}$$

$$\dot{r} = f(r)$$

What is the nature of the movement of a point defect?





There is no mass transport, only the reorientation the c-director.

Rotational viscosity, $\eta=1/\mu$, is the source of the overdamping of the force. The region in which rotation of the c-director occurs is proportional to *r* resulting in an *r* dependence of μ:

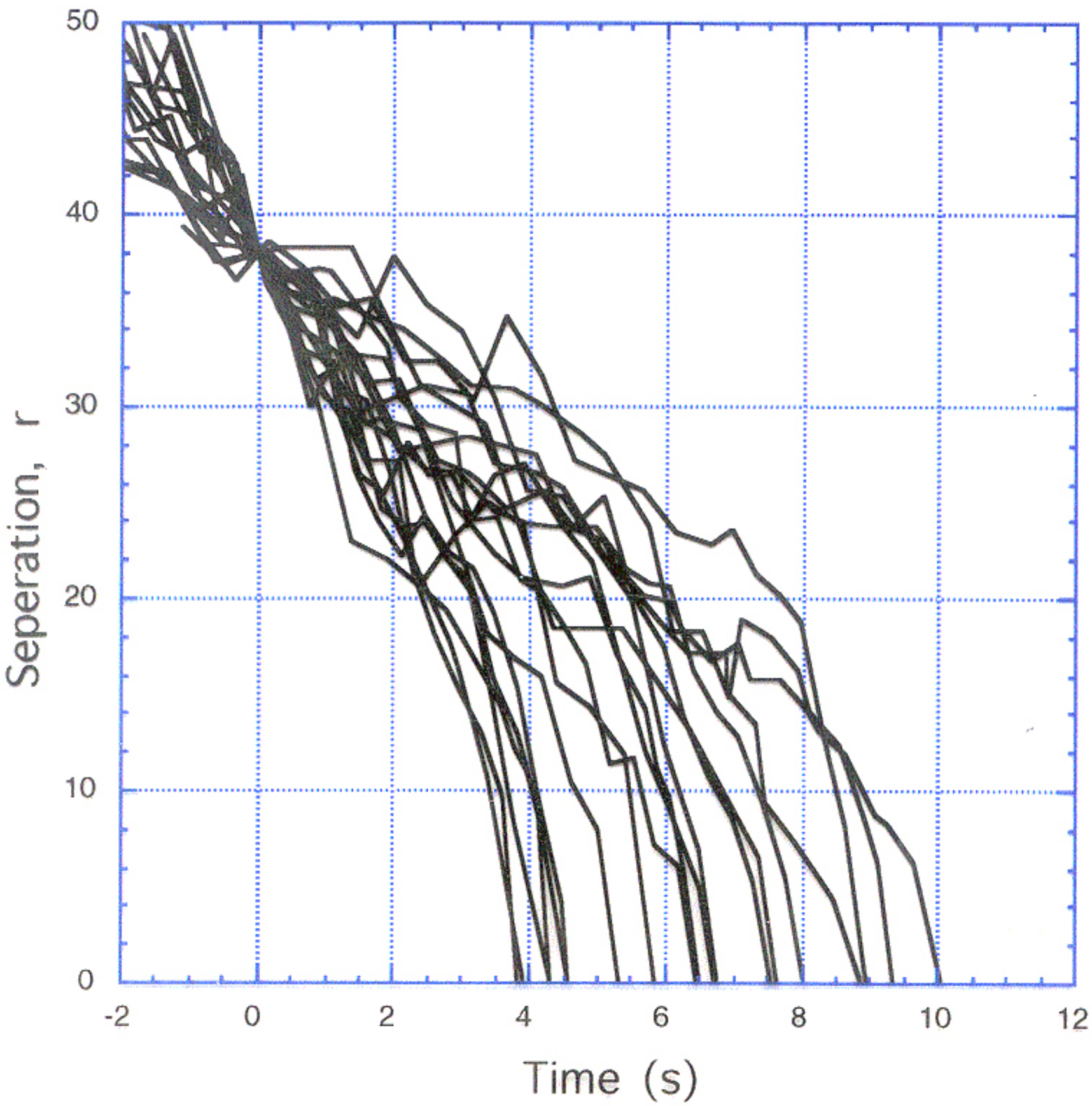
$$\mu = \mu_0 \left(\ln \frac{r}{r_{core}} \right)^{-1}$$

H. Pleiner, Phys. Rev A 37, 3986 (1988).

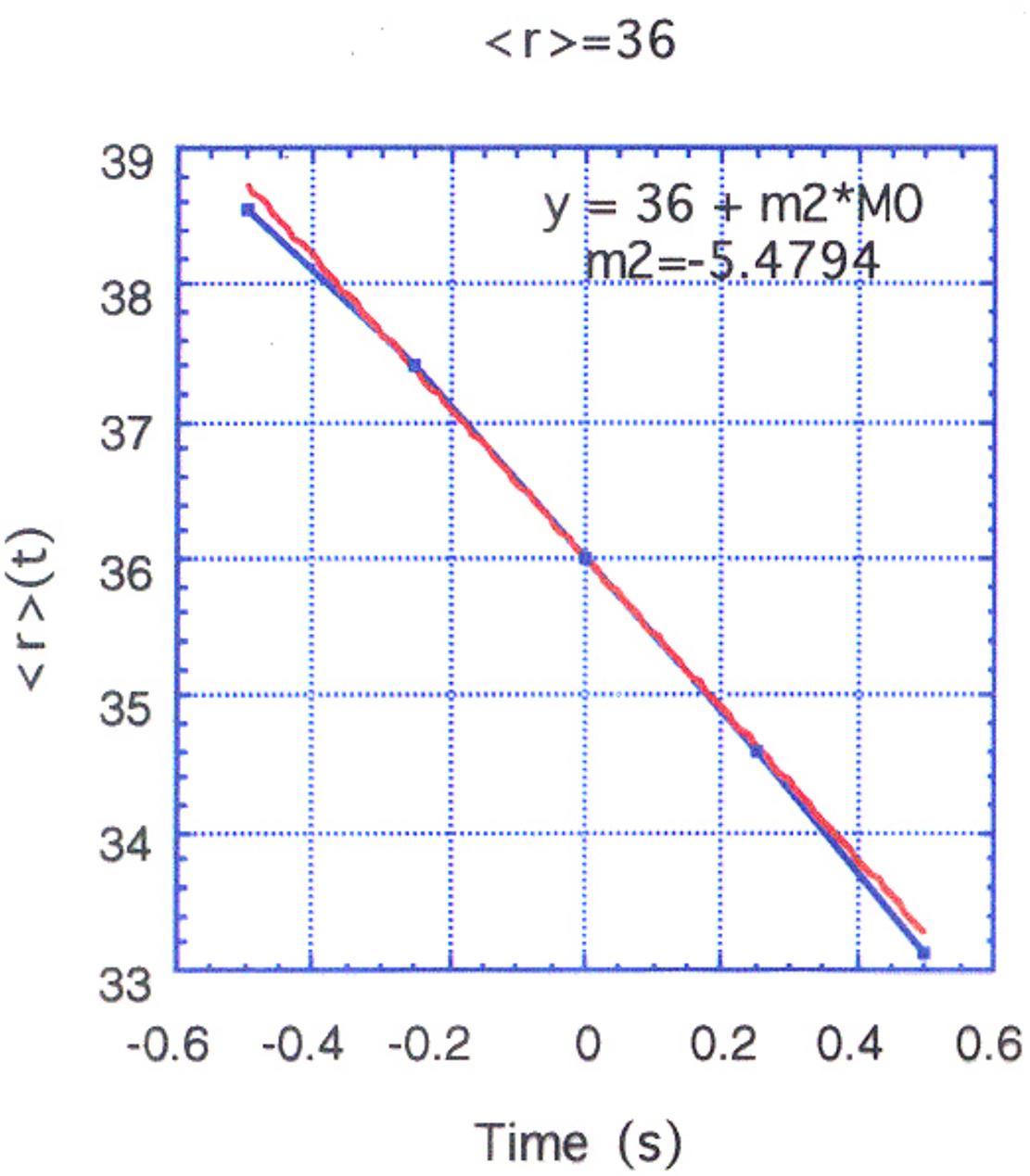
B. Yurke, A. N. Pargellis, T. Kovacs, and D. A. Huse, PRE **47**, 1525 (1993).

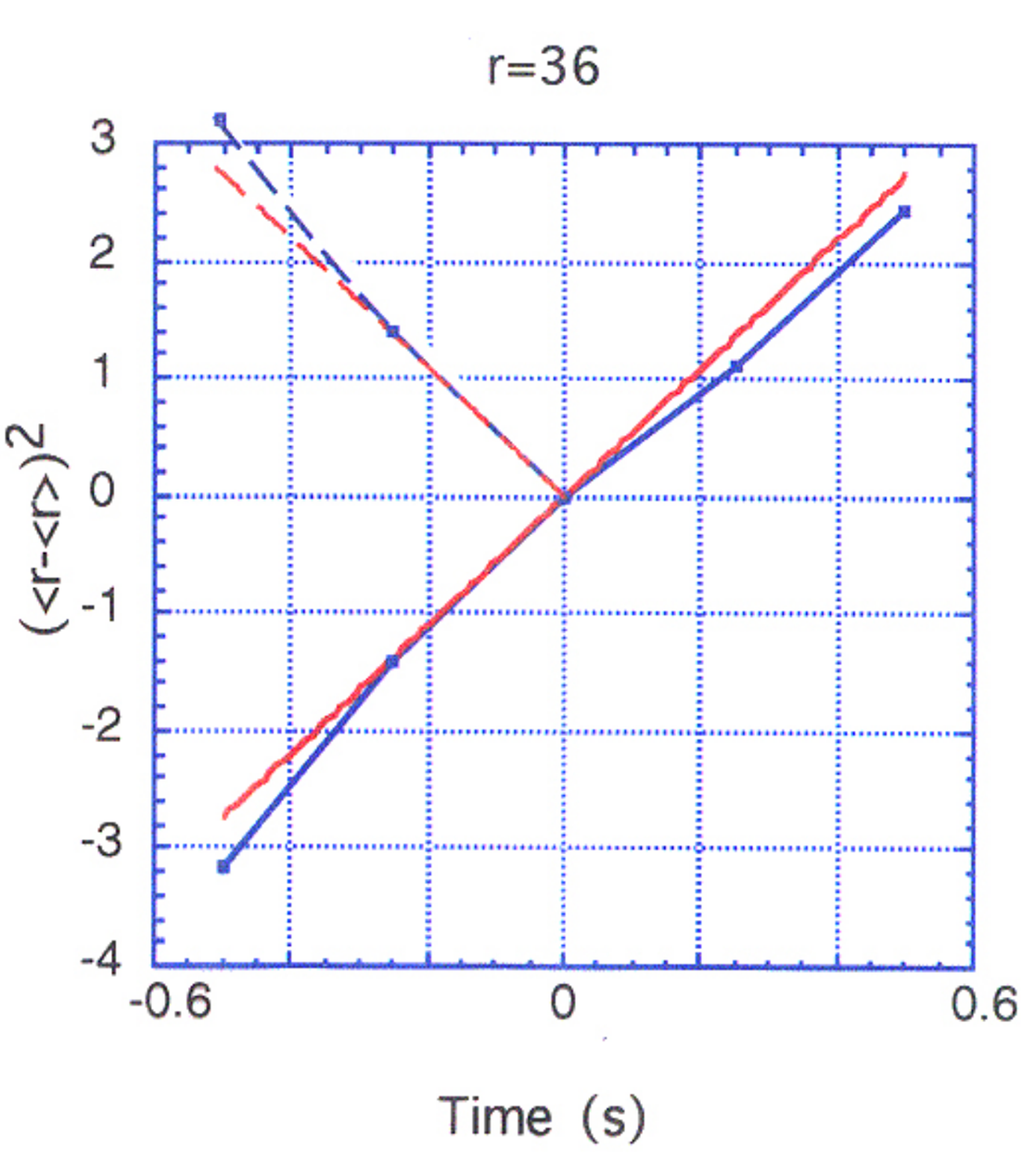
$$\dot{r} = \frac{\mu_0 k}{r} \left(\ln \frac{r}{r_{core}} \right)^{-1}$$

What Information can be obtained about the deterministic nature of the attractive force from data like this?

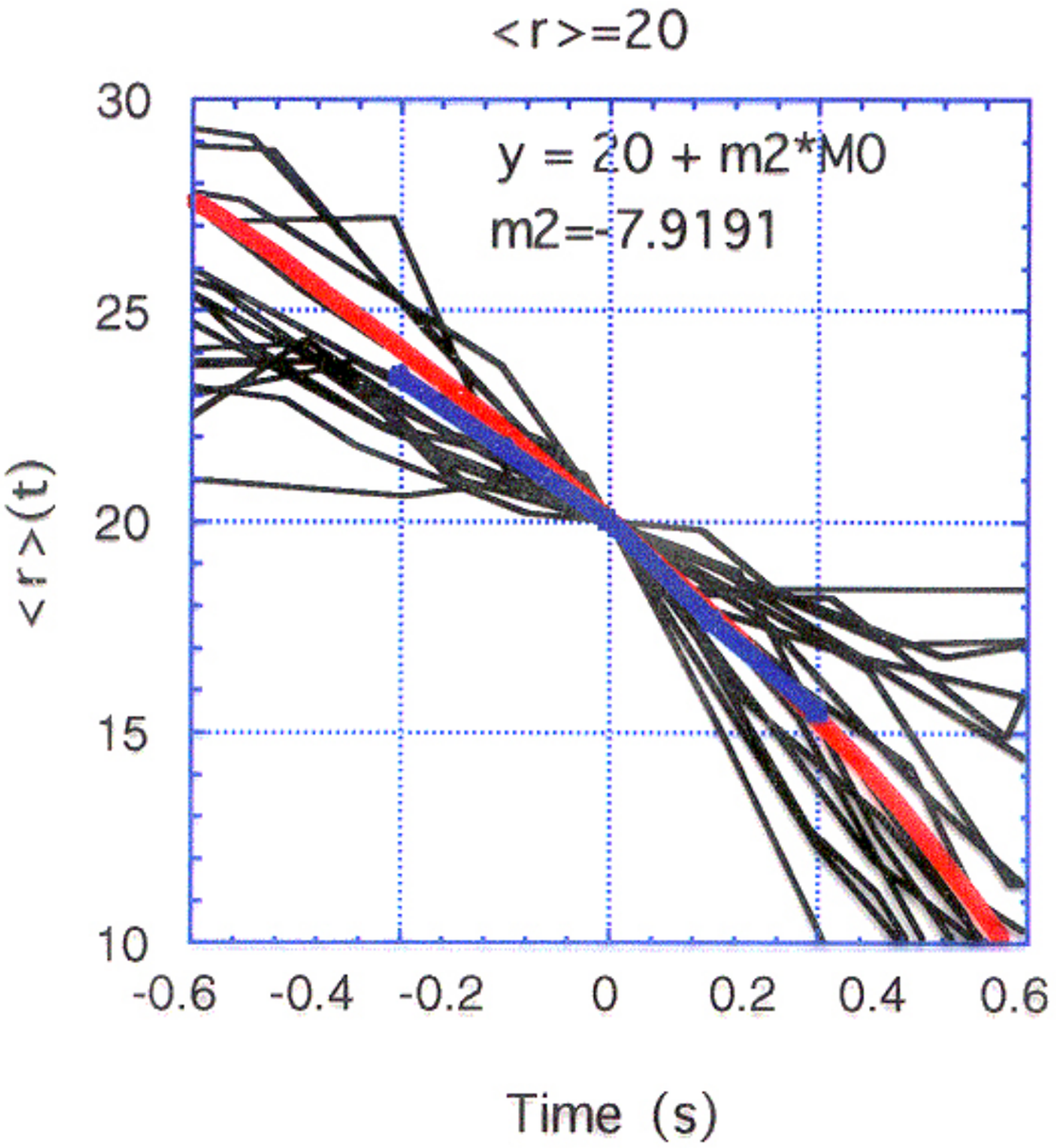


Measuring the velocity and the Diffusion Constant





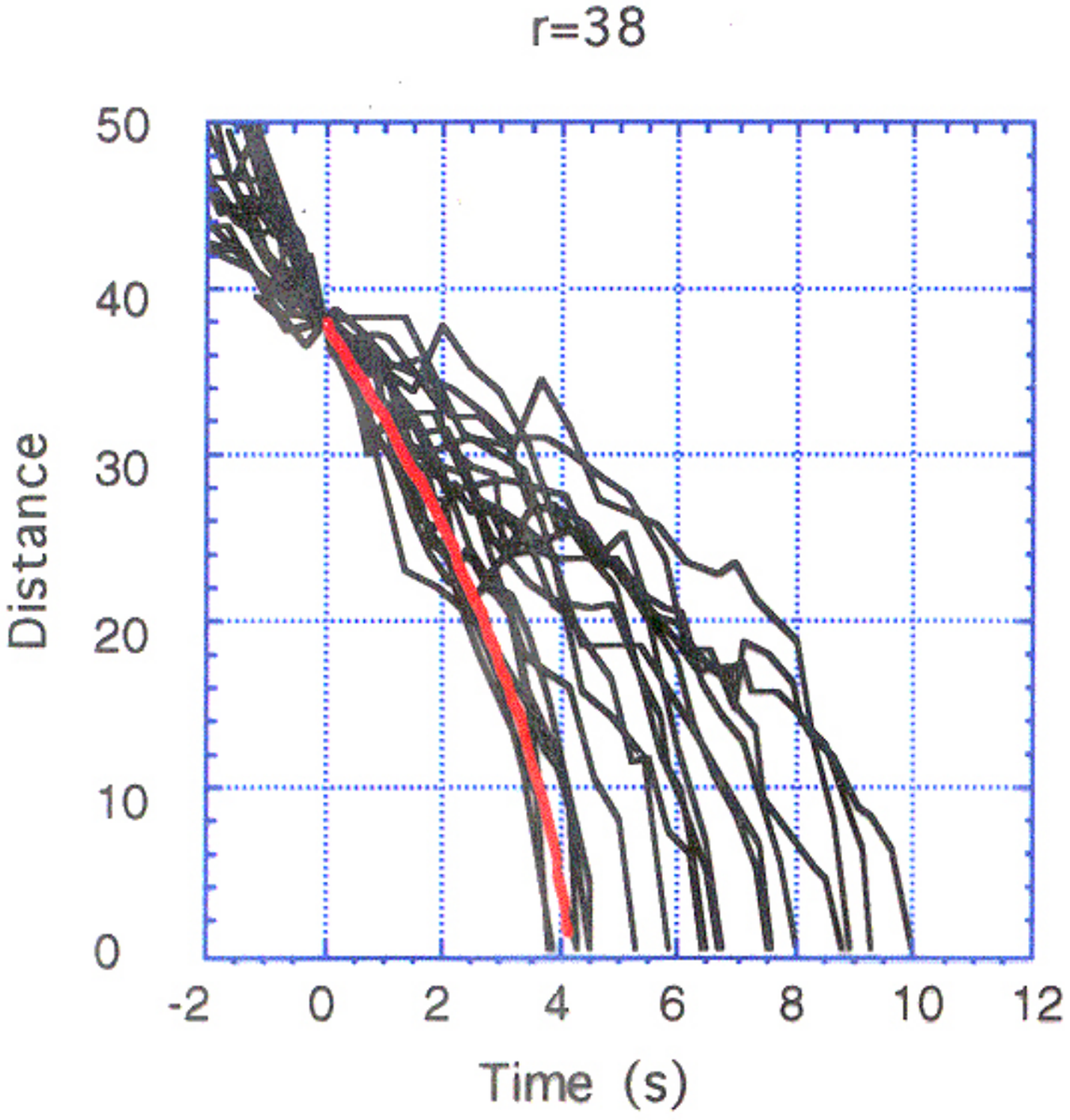
$(\langle r-\langle r\rangle)^2 = Dt$

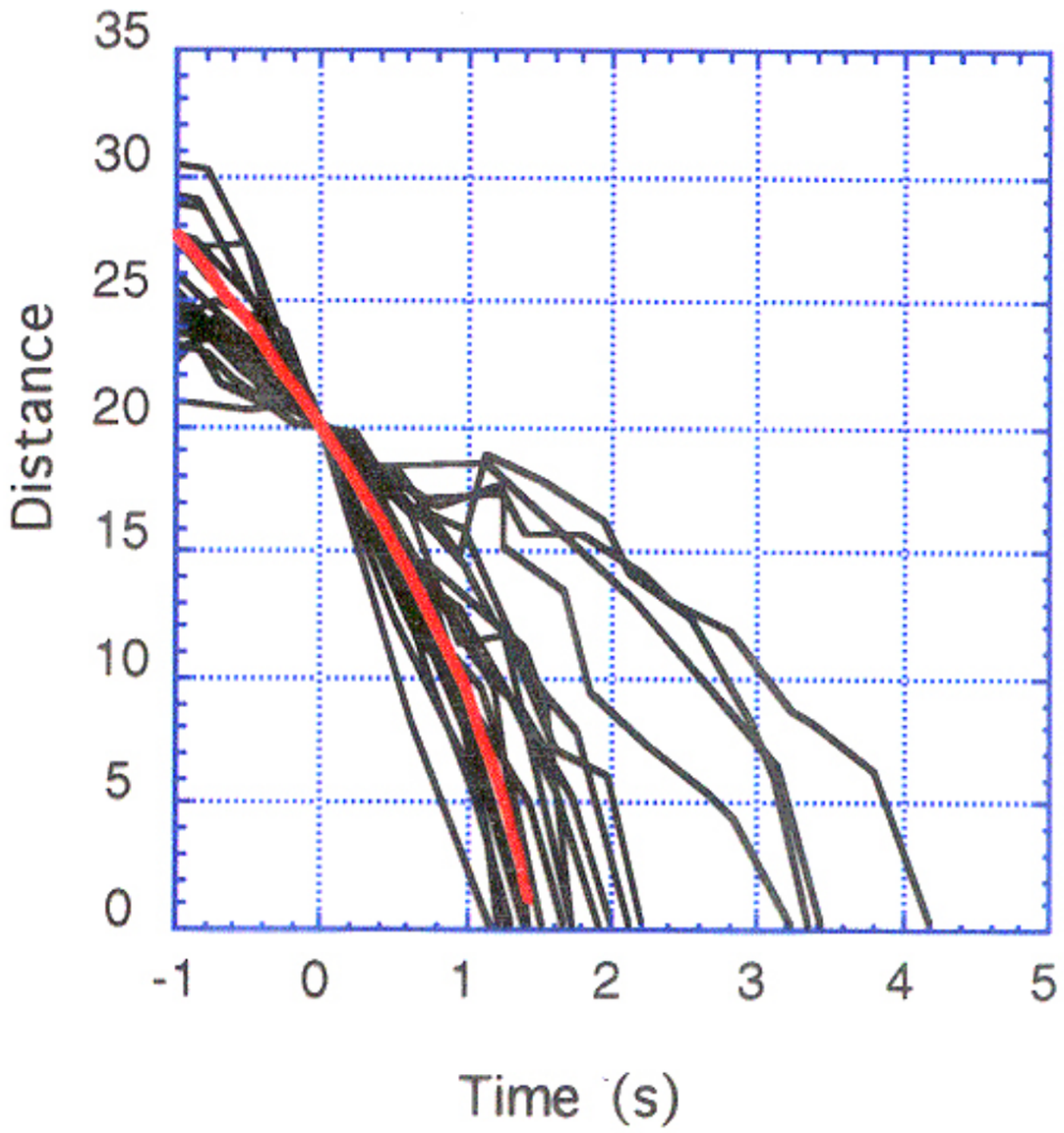


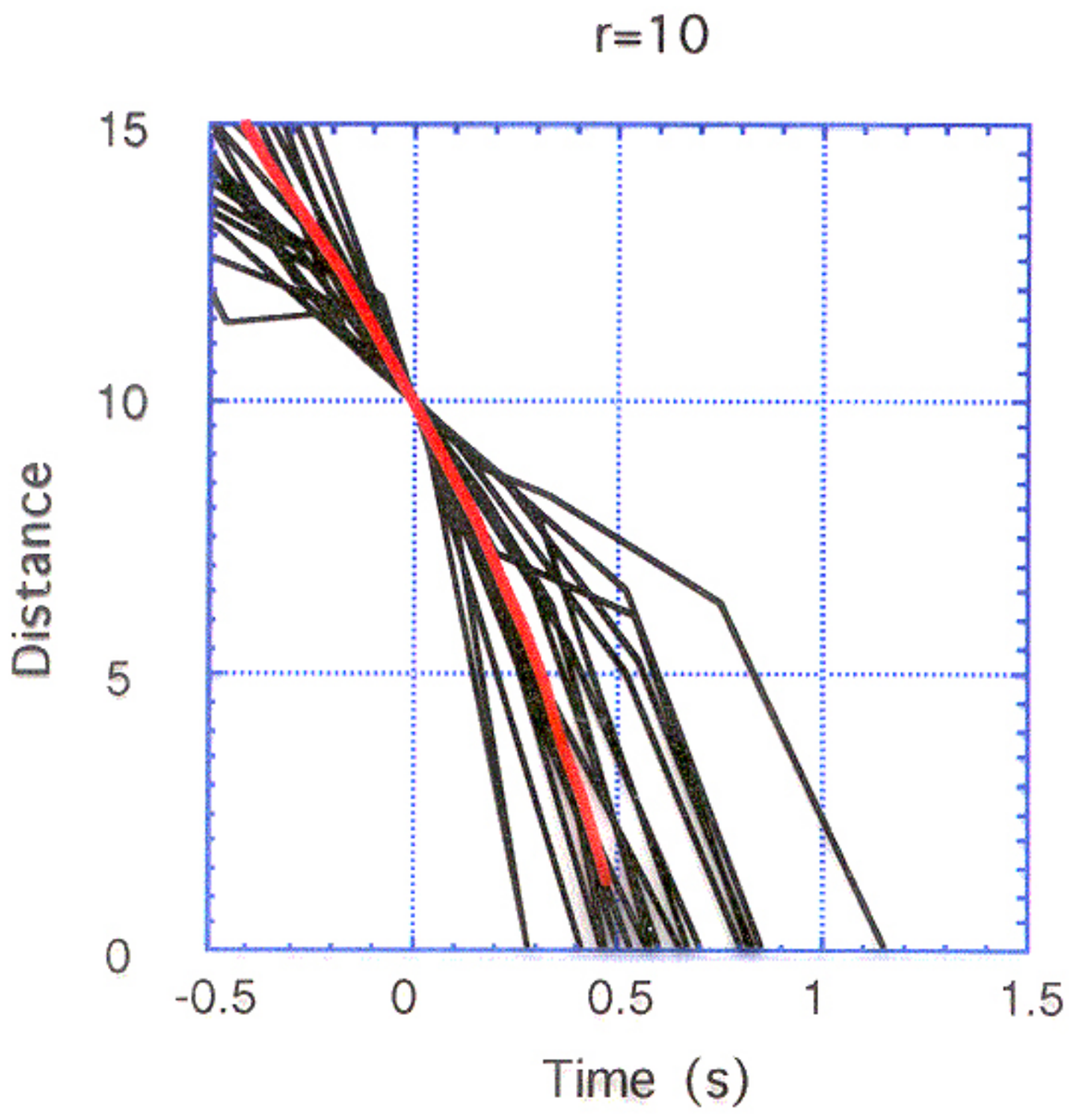
<r>(t)

Fit to all data

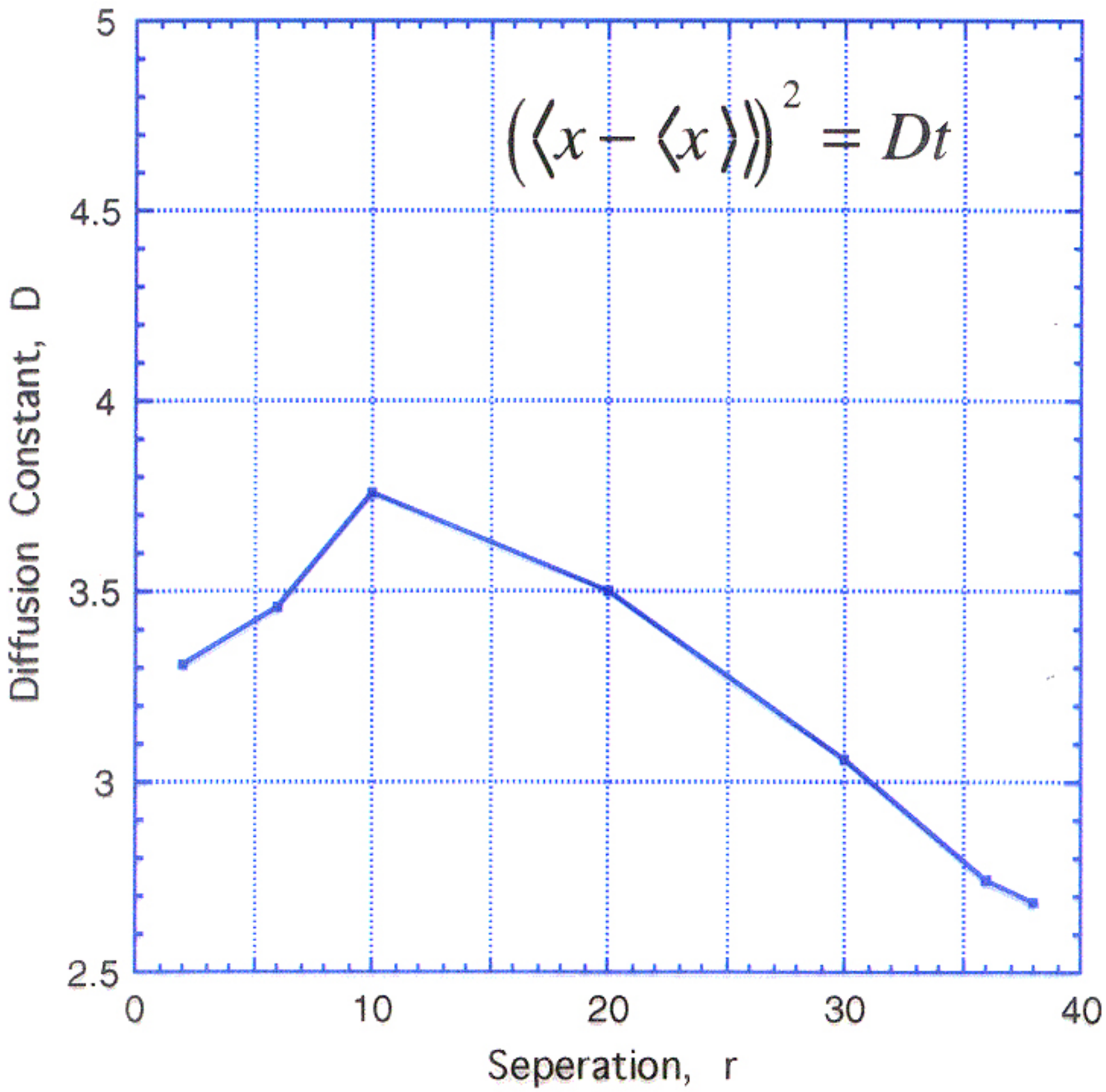
Observed Paths and Calculated Trajectory



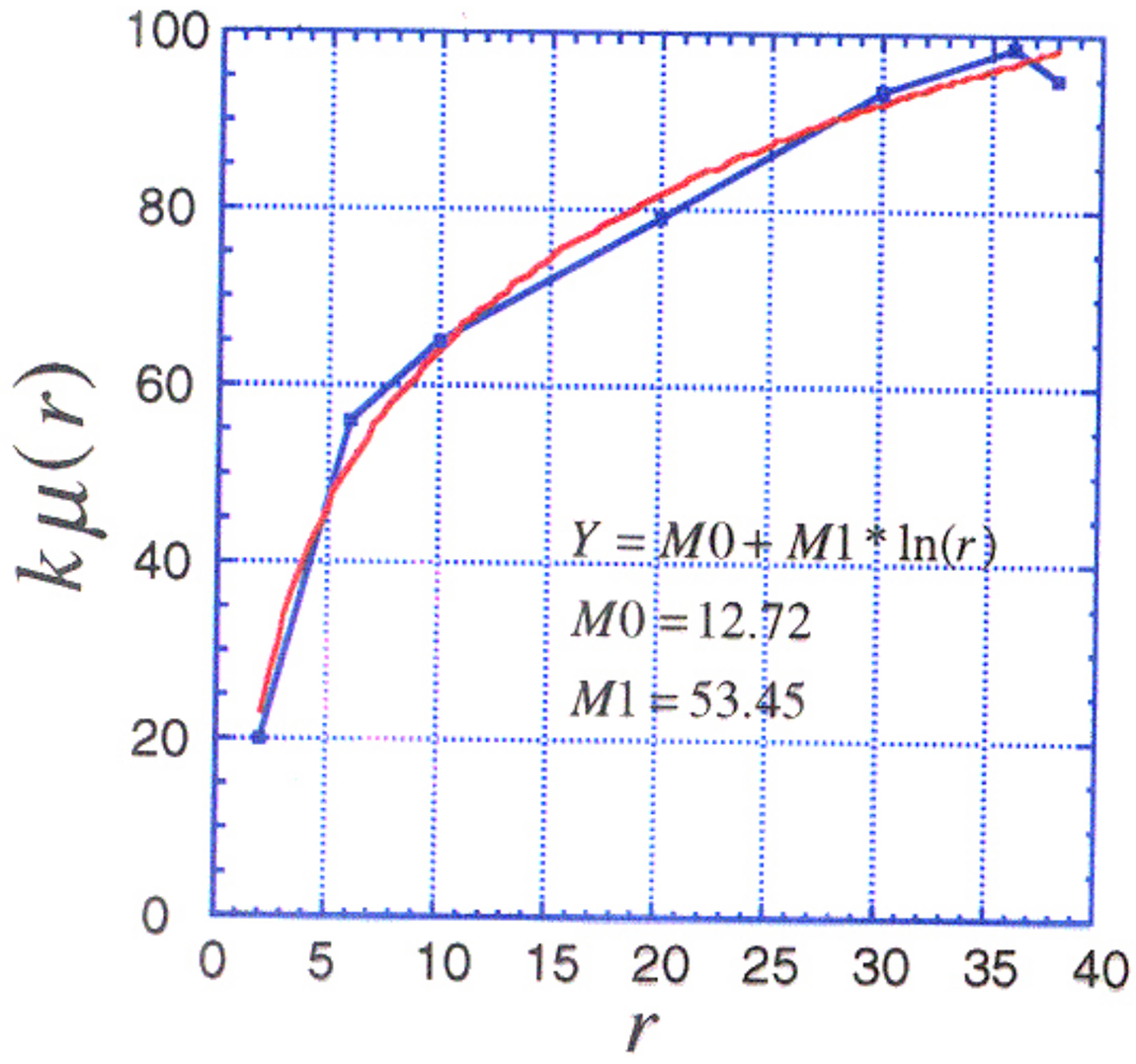




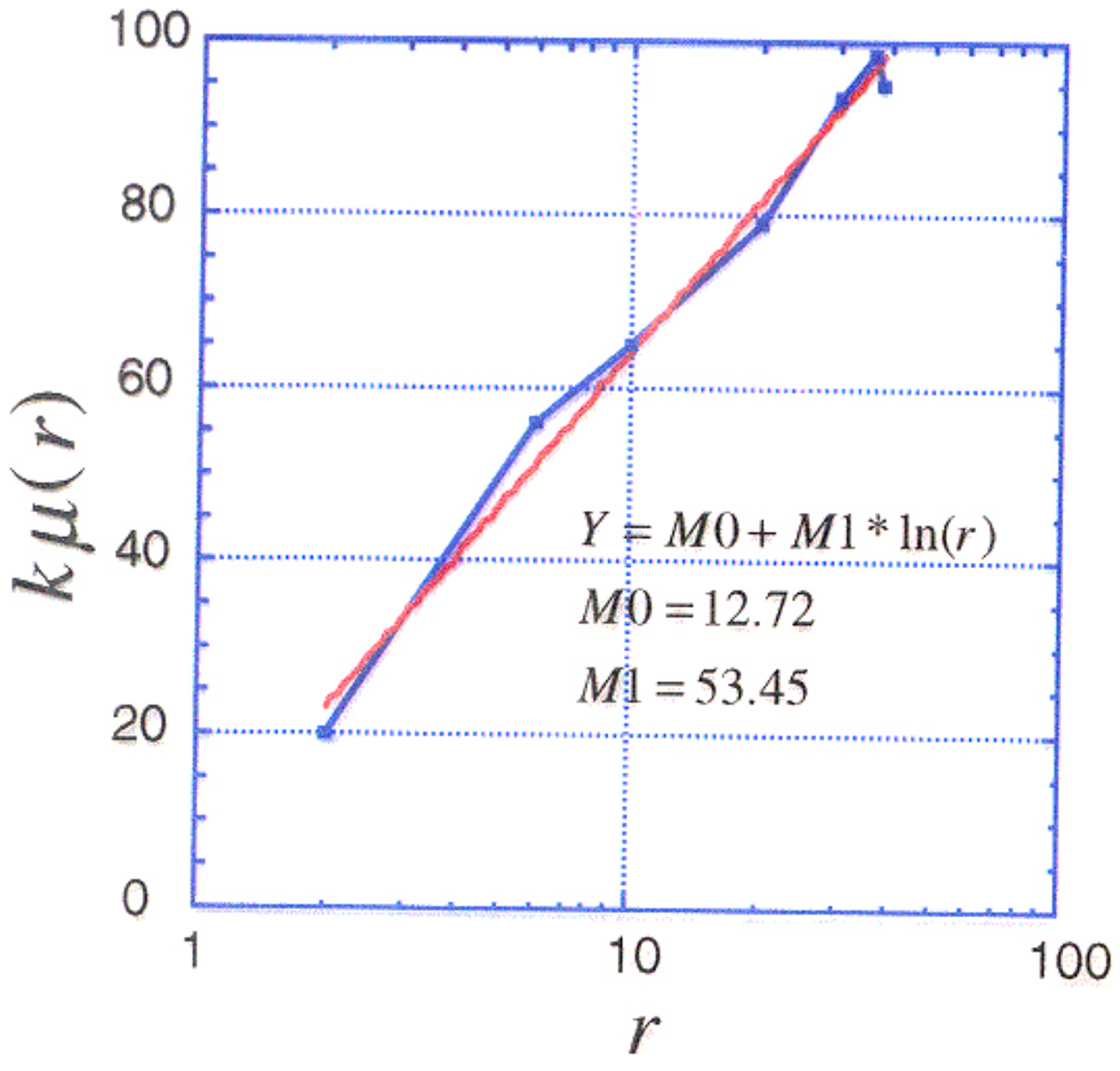
Separation Dependence of the Diffusion Constant



Separation dependence of $k\mu(r)$



$$r\frac{dr}{dt} = k\mu(r)$$



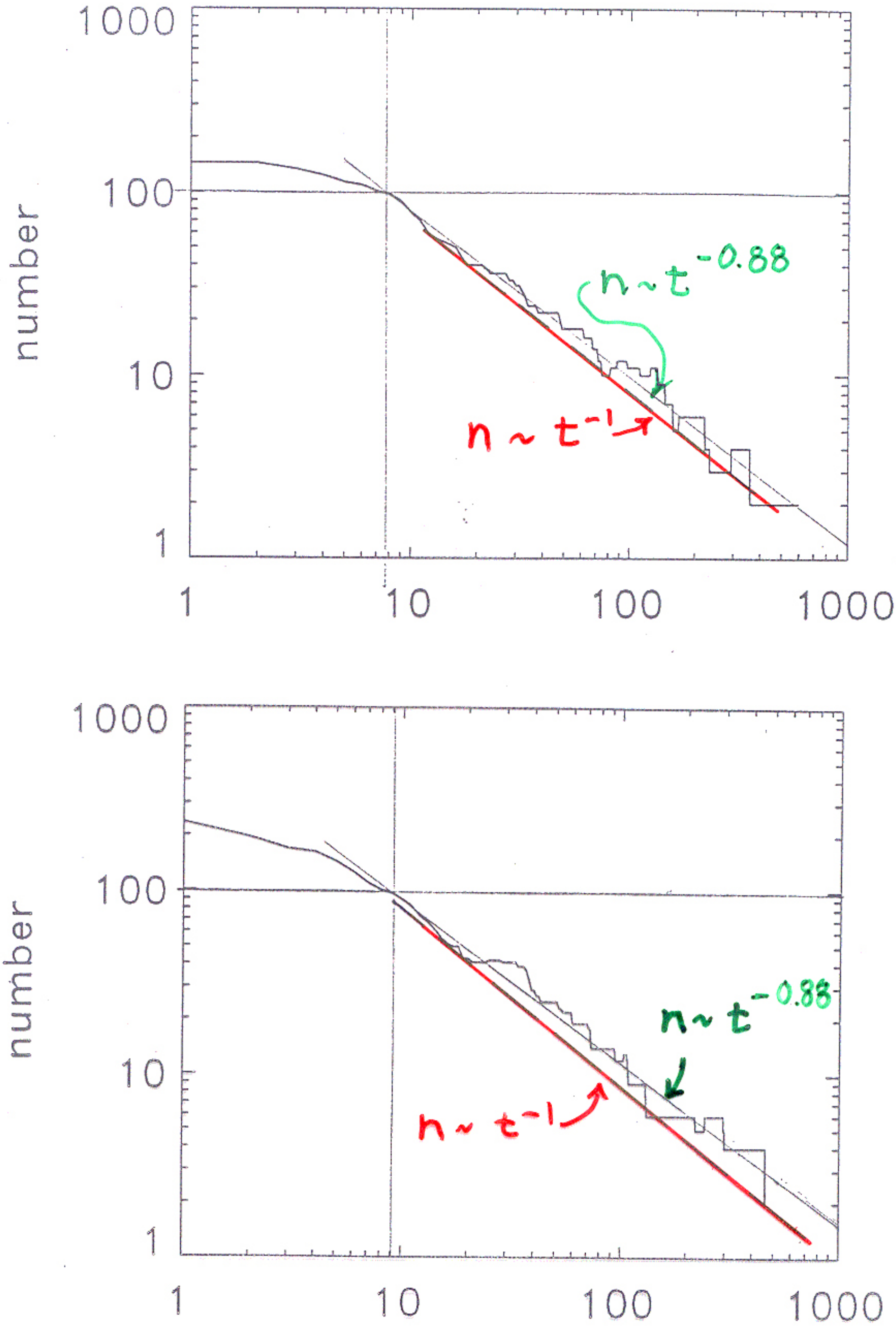
From Yurke et. al. and Pleiner, we were expecting to find:

$$k\mu(r) = \frac{k\mu}{\ln\left(\frac{r}{r_{core}}\right)}$$

But what we found was:

$$k\mu(r) = k\mu \ln\left(\frac{r}{r_{core}}\right)$$

OF DEFECTS REMAINING VS. TIME { # OF FRAMES



$$\frac{dn}{dt} = -4\pi\gamma n^2 g(r_c,t)$$

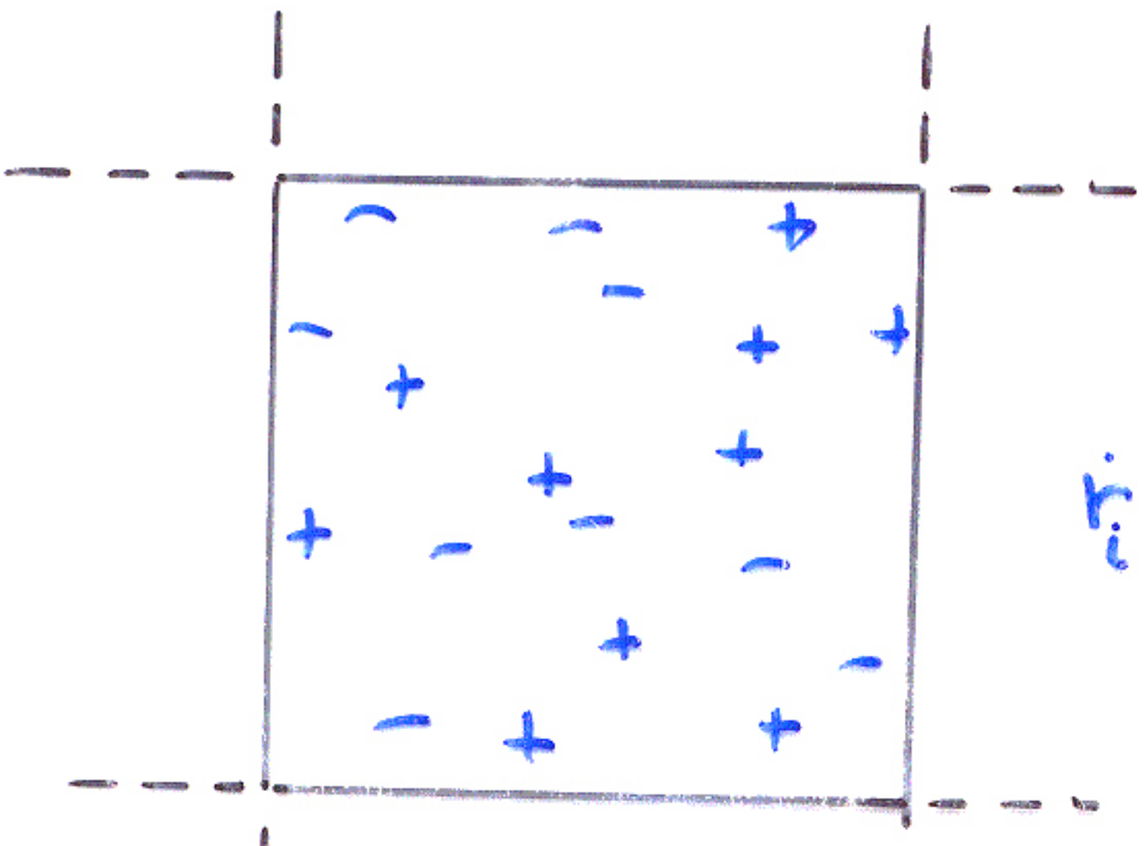
$\gamma = D$ DIFFUSIVE

$\gamma = \mu K$ DETERMINISTIC

if $g(r_c,t) = 1$ (NO CORRELATIONS)

then $n(t) \sim t^{-1}$

COMPUTER SIMULATION – BROWNIAN DYNAMICS OF DEFECTS



$$\dot{r}_i = \mu\left[\sum_j \frac{\pm K}{|\bar{r}_i - \bar{r}_j|} + \lambda\,\delta(t)\right]$$

at $t = 0$: $20{,}000 \pm$

$\delta\rho(L) \sim L^{1/2}$

~ 150

DETERMINISTIC SCALING

$n(t) \sim t^{-6/7}$

$\sim t^{-0.86}$

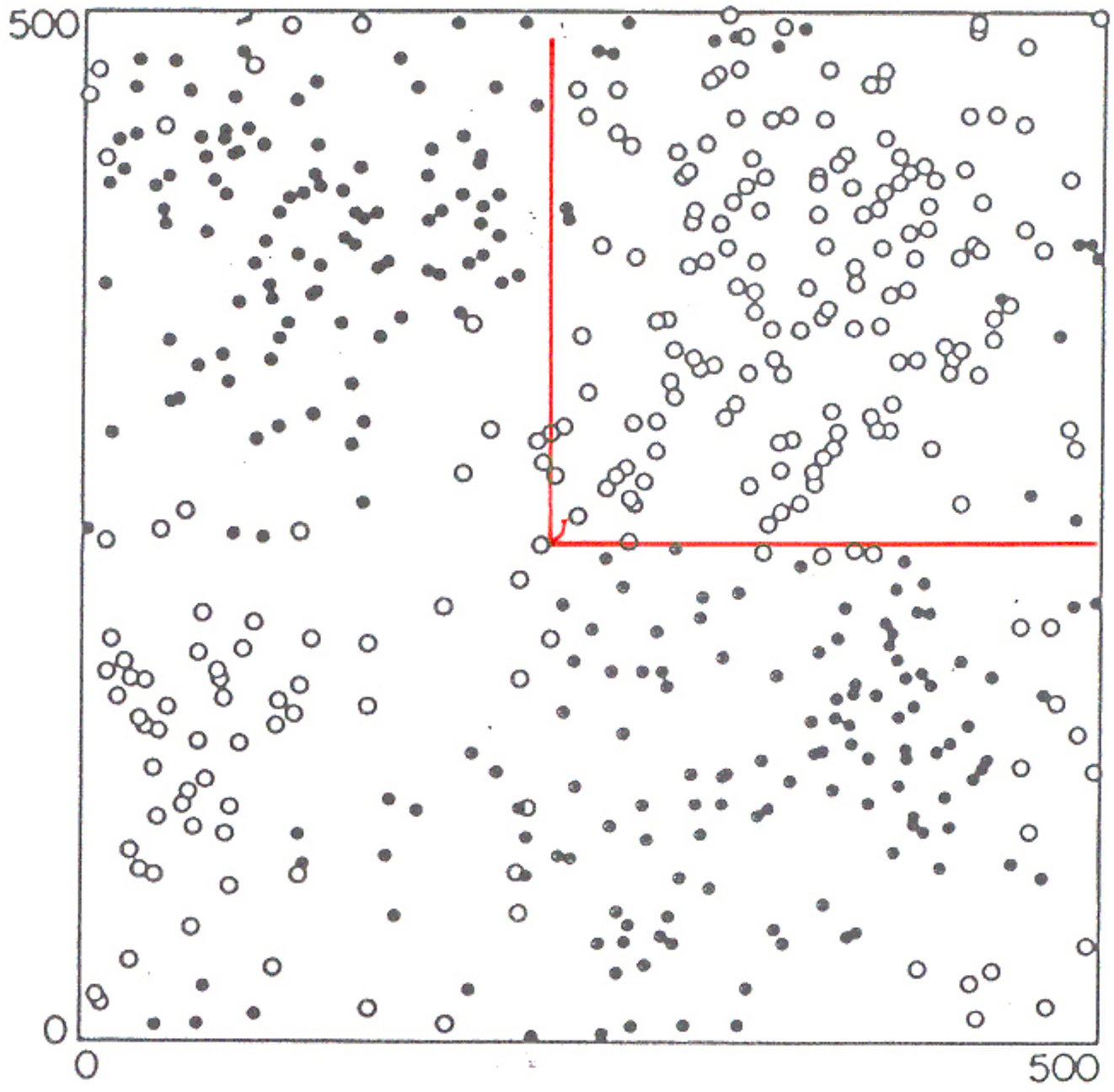
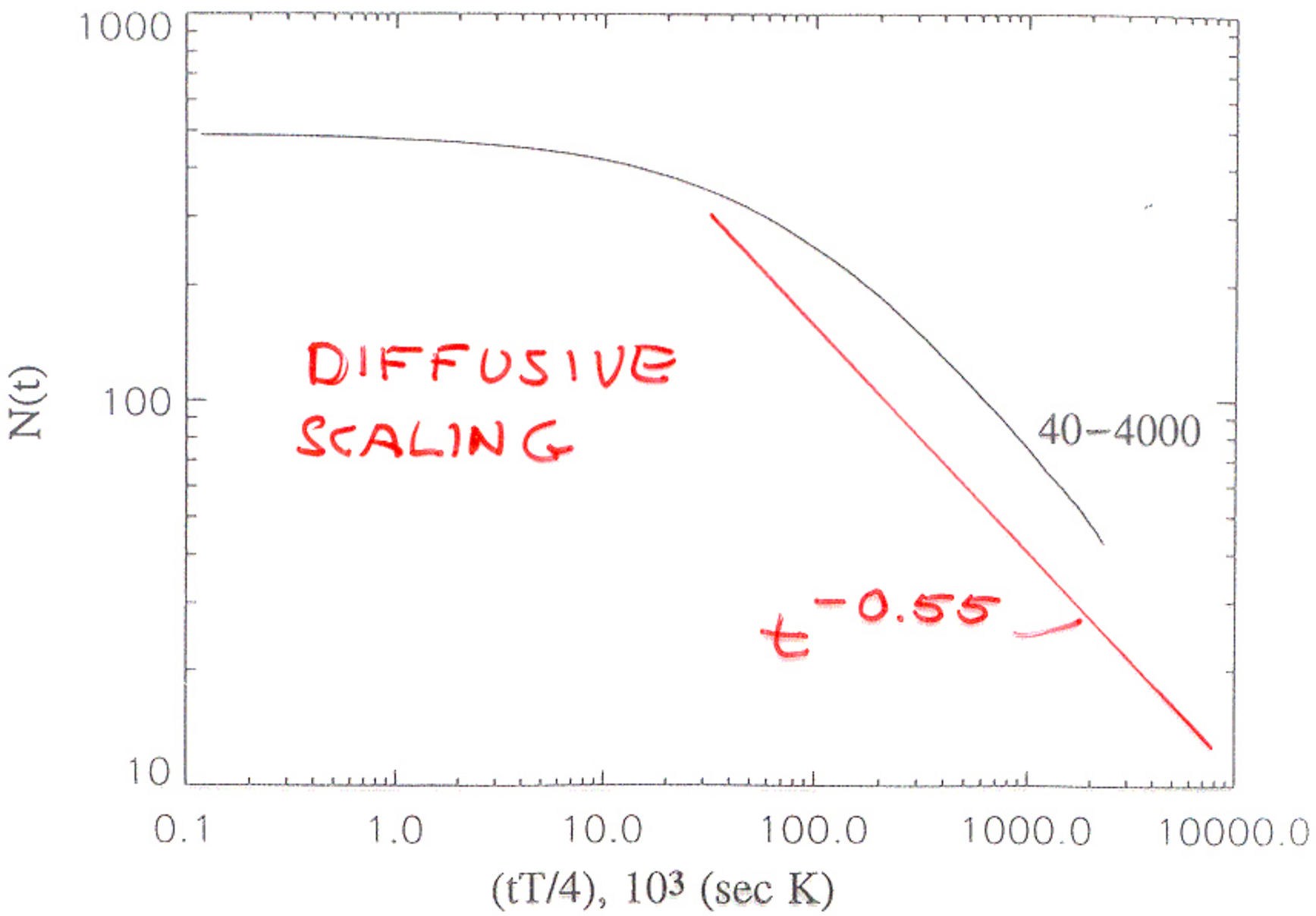
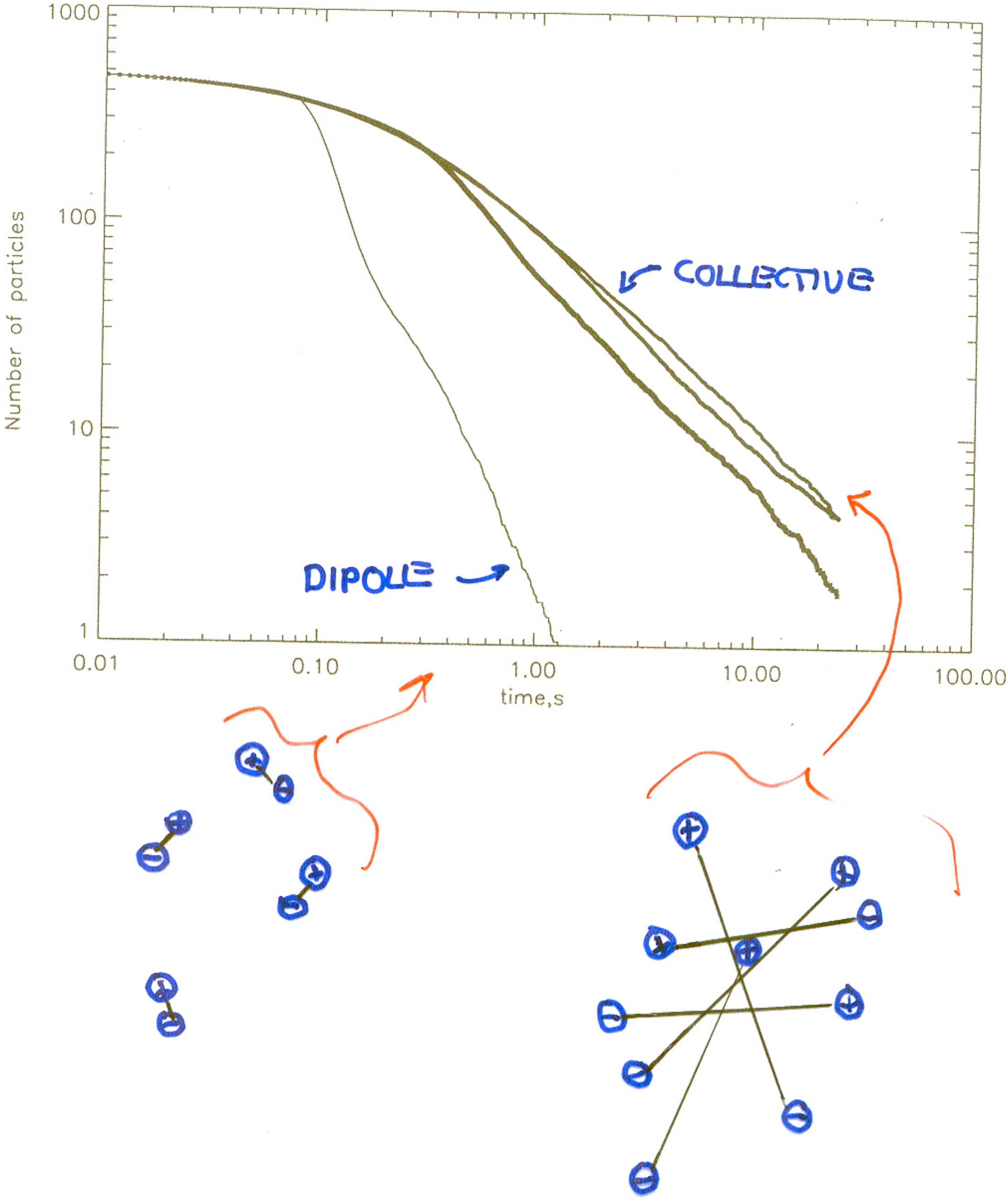


FIG. 5. Particle and antiparticle positions at $Dt = 1000$ on a 500×500 system in two dimensions. The initial condition was a uniform random distribution with $\rho_0 = 0.05$ as in Fig. 4.




1000
100
10
1
Number of particles
0.01
0.10
1.00
10.00
100.00
time,s
COLLECTIVE
DIPOLE

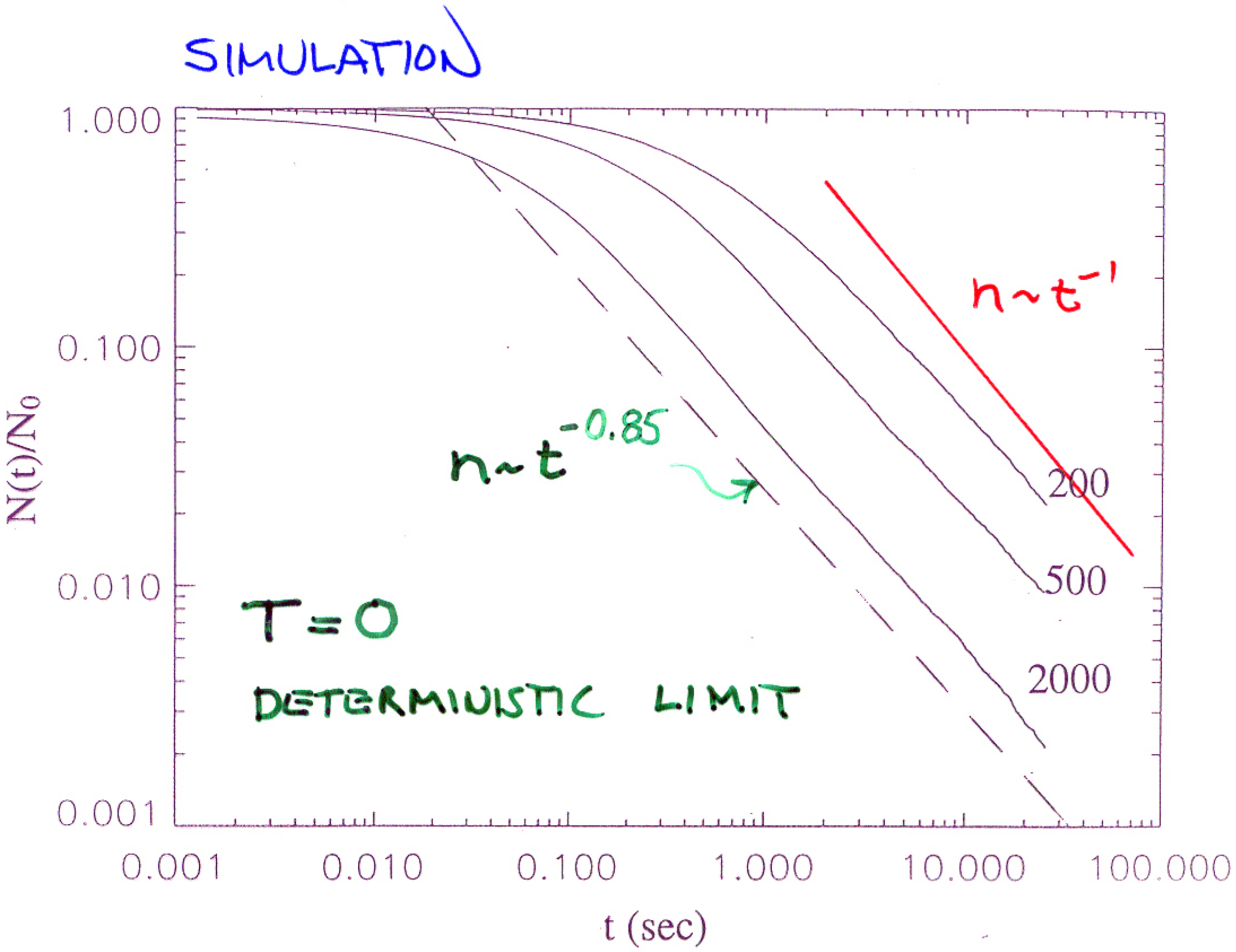

SIMULATION
1.000
0.100
0.010
0.001
N(t)/N₀
n~t⁻¹
n~t^-0.85
200
500
2000
T=0
DETERMINISTIC LIMIT
0.001
0.010
0.100
1.000
10.000
100.000
t (sec)

Figure 1a

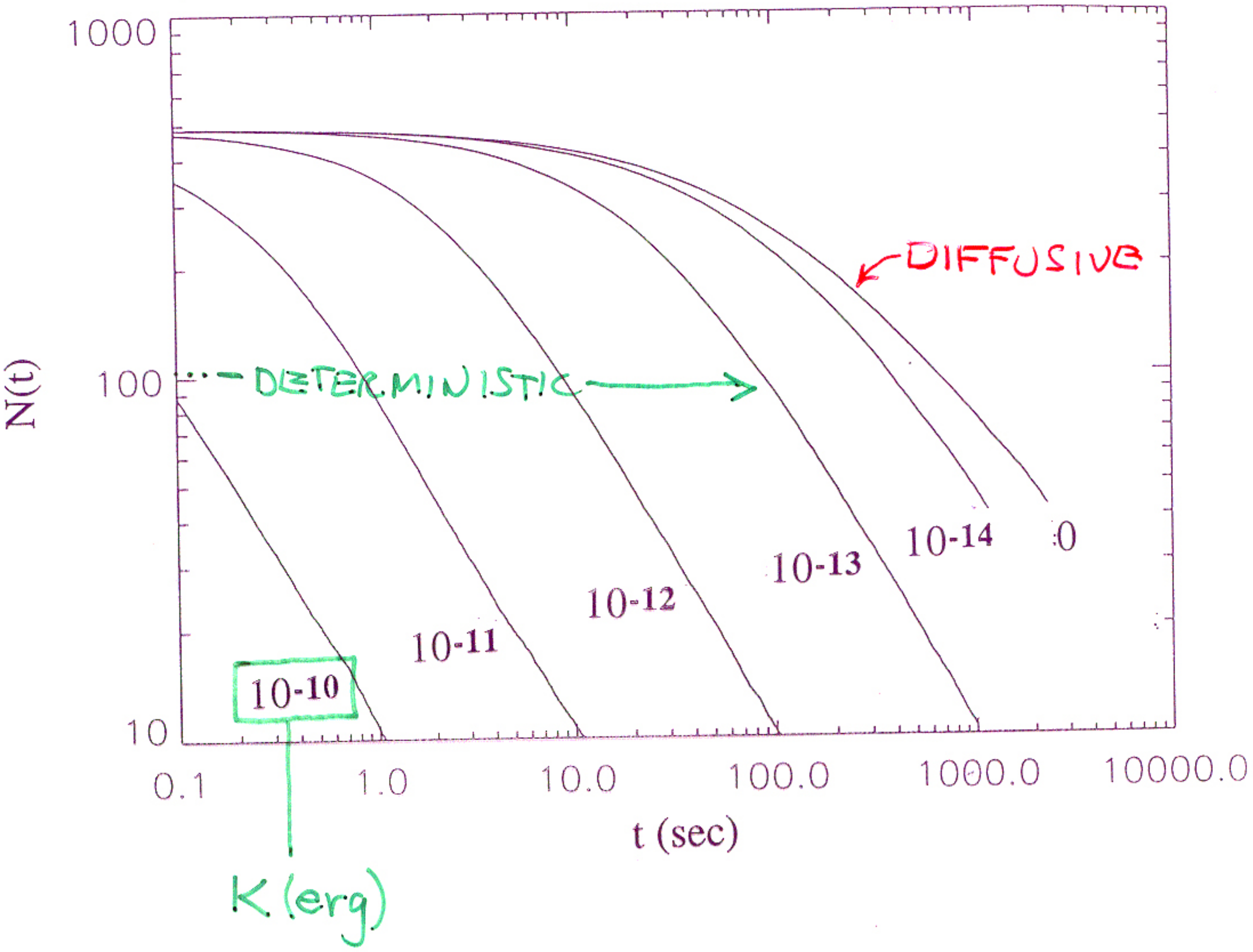

1000
100
10
N(t)
DIFFUSIVE
DETERMINISTIC
10-10
10-11
10-12
10-13
10-14
0
0.1
1.0
10.0
100.0
1000.0
10000.0
t (sec)
K (erg)

Figure 2a

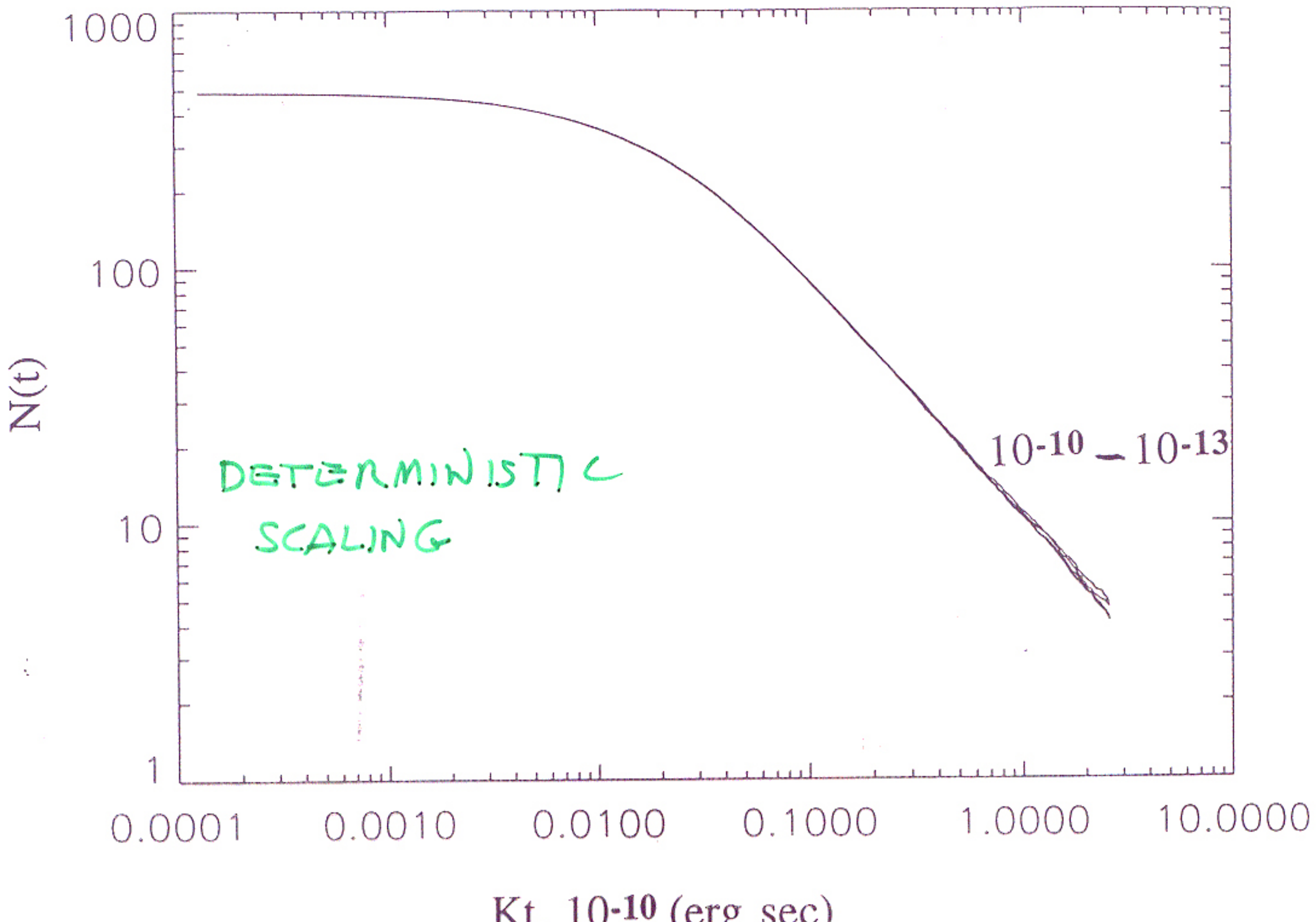

1000
100
10
1
N(t)
DETERMINISTIC
SCALING
10-10 – 10-13
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000
Kt, 10-10 (erg sec)

Figure 3b