BOULDER JULY 3, 2001 MARK BOWICK

The FORMATION of TOPOLOGICAL DEFECTS

- M.B, L. CHANDAR, E. SCHIFF & A. SRIVASTAVA:
  SCIENCE 263 (1994) 943 hep-ph/9208233
- M.B & A. MOMEN,
  PHYS. REV. D58 (1998) 085014 hep-ph/9803284
- M.B & A. CACCIUTO, A. TRAVESSET
  cond-mat/0107188

KEY INGREDIENTS

- SPONTANEOUS SYMMETRY BREAKING (SSB)
- TOPOLOGICALLY NON-TRIVIAL VACUUM / GROUND STATE MANIFOLD $\mathcal{M}$
- CONTINUOUS PHASE TRANSITION OR FIRST ORDER TRANSITION PROCEEDING VIA **SPINODAL DECOMPOSITION**

- FINITE QUENCH RATE / EXPANSION RATE OF UNIVERSE

▲ SSB

$$G \xrightarrow{\langle \vec{\Phi} \rangle \neq 0} H$$

G	H
DISORDERED	ORDERED
HIGH TEMP.	LOW TEMP.
EARLY TIME	LATER TIME
MAXIMALLY SYMMETRIC	REDUCED SYMMETRY

▲ VACUUM MANIFOLD

$$\mathcal{M} = G/H$$

NON-TRIVIAL TOPOLOGY for $\mathcal{M}$

⇒ TOPOLOGICAL DEFECTS

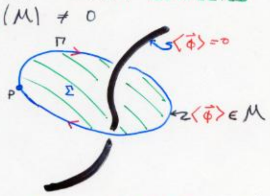
Exs: 1. $\pi_0(\mathcal{M}) \neq 0 \Leftrightarrow$ DOMAIN WALLS

i.e. $\mathcal{M}$ IS NOT CONNECTED e.g. $\underset{\text{(ISING)}}{\mathcal{M}} = \mathbb{Z}_2$

dim(DEFECT) = 2

2. $\mathcal{M}$ IS NOT SIMPLY-CONNECTED

$\pi_1(\mathcal{M}) \neq 0$



$\Leftrightarrow$ LINE/LOOP/STRING DEFECT $(d=1)$

EXAMPLES:

- COSMIC STRINGS
- SUPERFLUID He^4 VORTEX LINES
- SUPERCONDUCTING FLUX TUBES
- NEMATIC LIQUID CRYSTAL DISCLINATIONS
- SUPERFLUID He^3 VORTEX LINES

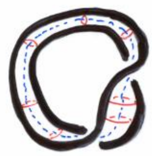
3. $\pi_2(\mathcal{M}) \neq 0$

$\Leftrightarrow$ POINT (MONOPOLE) DEFECT

dim = 0

4. $\pi_3(\mathcal{M}) \neq 0$

$\Leftrightarrow$ TEXTURE



dim = 3

"SKYRMEON"

GLOBAL COSMIC STRINGS

- $G = U(1) \longrightarrow 1$

  $\mathcal{M} \equiv S^1 \qquad \pi_1(S^1) \equiv \mathbb{Z}$

- $G = SO(10) \xrightarrow{126} SU(5) \times \mathbb{Z}_2$

  $\mathcal{M} \equiv SO(10)/SU(5) \times \mathbb{Z}_2$

  $\pi_1(\mathcal{M}) = \mathbb{Z}_2$

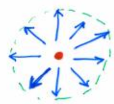
FORM AT $t \sim 10^{-37}\,s$ AFTER
BIG BANG $T \sim 10^{28}\,K$
$\mu \sim 10^{17}\,kg/cm$

Planar Spins

$\mathcal{M} = S^1 \quad \pi_1(S^1) = \mathbb{Z} \Rightarrow$ VORTICES

Ordinary Spins

$\mathcal{M} = S^2 \quad \pi_2(S^2) = \mathbb{Z} \Rightarrow$ MONOPOLES

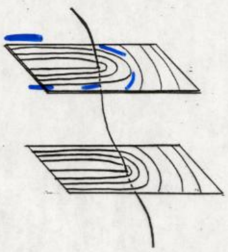


Nematics

$$\mathcal{M} = S^2/\mathbb{Z}_2 \equiv RP^2 \equiv O(3)/D_{\infty h}$$

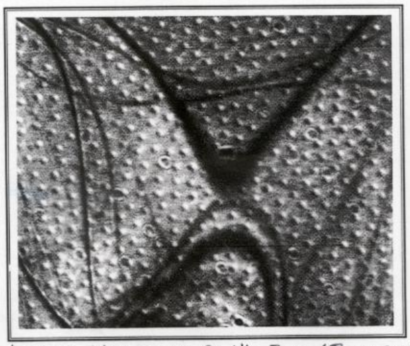
$\pi_1(\mathcal{M}) = \mathbb{Z}_2$ DISCLINATIONS $\quad \pi_2(\mathcal{M}) = \mathbb{Z}/\mathbb{Z}_2$ MONOPOLES $\quad \pi_3(\mathcal{M}) = \mathbb{Z}$ TEXTURE

LINE DEFECTS IN NEMATICS (M. KLÉMAN)



BULLETIN

OF THE AMERICAN PHYSICAL SOCIETY

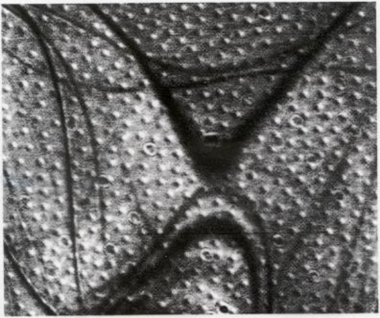


LORENTZ MICROGRAPH OF Nb FILM (B = 100G)
(HITACHI ADVANCED RESEARCH LAB, JAPAN)

Program of the 1994 March Meeting

March 1994
Volume 39, No. 1

BULLETIN
OF THE AMERICAN PHYSICAL SOCIETY



LORENTZ MICROGRAPH OF Nb FILM (B = 100 G)
(HITACHI ADVANCED RESEARCH LAB, JAPAN)

Program of the 1994 March Meeting

March 1994
Volume 39, No. 1

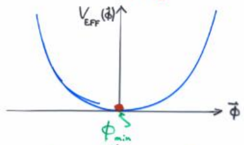
CONTINUOUS
PHASE TRANSITION

COMPUTE $V_{EFF}(\vec{\Phi})$

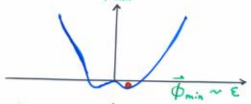
FOR $T > T_c$ $\quad \langle \vec{\Phi}_{min} \rangle = 0$ DISORDERED

$T < T_c$ $\quad \langle \vec{\Phi} \rangle \neq 0$ ORDERED

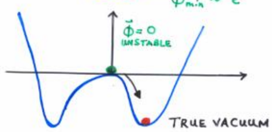
$\langle \vec{\Phi} \rangle_{T = T_c} \equiv 0$ CRITICAL POINT



$T > T_c$



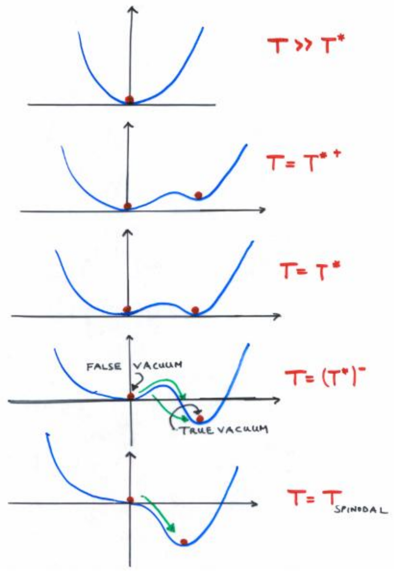
$T = T_c^- = T_c - \varepsilon$



$T << T_c$

FIRST ORDER

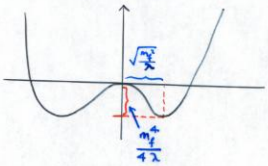
OR



Let $\varepsilon = \frac{T - T_c}{T_c}$ REDUCED TEMPERATURE

$m^2 \simeq m_0^2\,\varepsilon$ IN VICINITY OF TRANSITION

$\lambda = \text{CONST.}$



$t \geq \tau_Q$

$$|\vec{\phi}^2|_{min} = v^2 = m_f^2/\lambda$$

$$\Delta V = V(0) - V(\vec{\phi}_{min}) = m_f^4/4\lambda$$

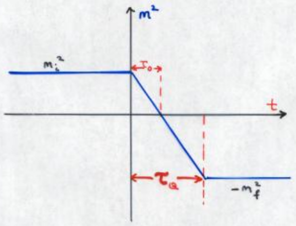
MODEL PHASE TRANSITION VIA

LANDAU-GINZBURG EFFECTIVE POTENTIAL / FREE ENER

$$V_{eff}(\vec{\phi}) = \frac{1}{2}|\nabla\vec{\phi}|^2 - \frac{1}{2}m^2(t)\,|\vec{\phi}|^2 - \frac{\lambda}{4}|\vec{\phi}^2|^2$$

with

$$m^2(t) = \begin{cases} m_i^2 & t \leq 0 \\ m_i^2 - \left(\frac{m_i^2 + m_f^2}{\tau_Q}\right)t & 0 \leq t \leq \tau_Q \\ -m_f^2 & t \geq \tau_Q \end{cases}$$



$$m_i^2 = \mu^2\left(\frac{T_i^2}{T_c^2} - 1\right)$$

$$m_f^2 = \mu^2\left(1 - T_f^2/T_c^2\right)$$

E.O.M :

$$\left[\frac{d^2}{dt^2} + \vec{k}^2 + m_i^2\right] \underbrace{U_k(t)}_{\text{SINGLE-PARTICLE WAVE FUNCTION}} = 0 \qquad t \leq 0$$

$$\underbrace{U_k(t) = e^{-i\omega(k)t}}_{\text{INITIAL CONDITION}} \qquad \omega^2(k) = \vec{k}^2 + m_i^2$$

$\tau_Q \to 0$ SUDDEN QUENCH

$$\left[\frac{d^2}{dt^2} + \vec{k}^2 - m_f^2\right] U_k(t) = 0 \qquad t \geq 0$$

LONG-WAVELENGTH MODES w/ $\vec{k}^2 < m_f^2$

ARE UNSTABLE (SPINODAL)

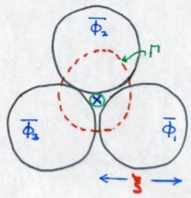
$\Longrightarrow$ FORMATION OF ORDERED DOMAINS

GUTH & PI (1985) E. WEINBERG & A. WU (1988)

$$U_k(t) = A_k e^{W(k)t} + B_k e^{-W(k)t}$$

w/ $$W(k) = \sqrt{m_f^2 - \vec{k}^2}$$

Domain Formation
à la Kibble (1976, 1980)



Assumptions

- $\bar{\Phi}_i$ uniform within domain of size ξ
- $\bar{\Phi}_i$ randomly chosen from $\mathcal{M}$
  i.e. domains are uncorrelated
- Along path Γ order paramer $\langle \phi \rangle$ smoothly interpolates from

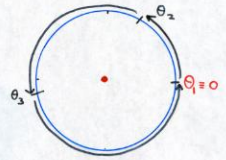
$$\bar{\Phi}_1 \to \bar{\Phi}_2 \to \bar{\Phi}_3 \to \bar{\Phi}_1$$

$\equiv$ **Geodesic Rule**

$\xi_{max} \to$ Horizon size (cosmology)

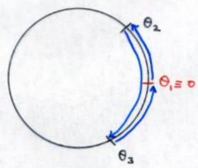
$\xi_{max} \to$ Mean correlation length (stat. mech.)

EX. $\mathcal{M} = S^1$ IN 2D



+1 VORTEX

NON-CONTRACTIBLE LOOP FROM GEODESIC RULE



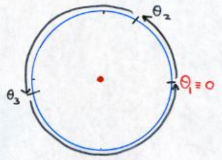
NO VORTEX

CONTRACTIBLE LOOP

VORTEX IFF $\theta_3 > \pi$ & $|\theta_3 - \theta_2| < \pi$

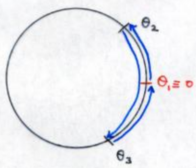
PROBABILITY $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

EX. $\mathcal{M} = S^1$ IN 2D



+1 VORTEX

NON-CONTRACTIBLE
LOOP FROM
GEODESIC RULE



NO VORTEX

CONTRACTIBLE
LOOP

VORTEX IFF $\theta_3 > \pi$ & $|\theta_3 - \theta_2| < \pi$

PROBABILITY $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$n_{defects}$ = # DEFECTS/VOLUME

$\sim 1/(\xi^*)^d$ d = dimension

$\xi^* = \xi$ (FORMATION)

Q: WHAT DETERMINES ξ^*

A: KIBBLE (1980), ZUREK (93)

ORDER FIELD(S) CAN EQUILIBRATE (REACH A UNIFORM VACUUM) PROVIDED THE INTRINSIC DYNAMICS IS FASTER THAN THE QUENCH RATE

$$\Gamma_{in} \gg \Gamma_Q$$

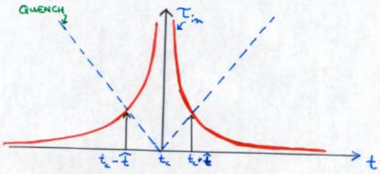
FOR $\Gamma_{in} < \Gamma_Q$ ORDERING IS FROZEN

BUT (!) IN A CONTINUOUS TRANSITION THE INTRINSIC DYNAMICS BECOMES ARBITRARILY SLUGGISH

AS $T \rightarrow T_c$

TAKE $\varepsilon = \dfrac{T - T_c}{T_c} = t/\tau_Q$ QUENCH

$\tau_{in} = \Gamma_{in}^{-1} = \dfrac{\tau_0}{|\varepsilon|^{\mu}}$ CRITICAL EXPONENT $\mu = 1$ IN MFT



At time $\hat{t}$ $\quad \tau_{in}(\hat{t}) = \hat{t}$ FREEZE-OUT

$$\Rightarrow \hat{t} = \frac{\tau_0}{(\hat{t}/\tau_Q)^{\mu}}$$

$$\Rightarrow \hat{t} = (\tau_0 \tau_Q^{\mu})^{\frac{1}{1+\mu}}$$

$$\varepsilon(\hat{t}) = \left(\frac{\tau_0}{\tau_Q}\right)^{\frac{1}{1+\mu}}$$

$$\xi = \xi_0 / |\varepsilon|^{\nu}$$

$$\xi^* = \xi(\hat{t}) = \xi_0 (\tau_0/\tau_Q)^{\frac{\nu}{1+\mu}}$$

MFT $\quad \nu = 1/2 \quad \mu = 1$

$$\xi^* = \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{1/4}$$

FOR LINE DEFECTS

$$\rho_{\text{DEFECTS}} = \left(\text{length}/\text{volume} \right)_{\text{defects}}$$

$$\sim \frac{1}{(\xi^*)^2}$$

$$\sim \frac{1}{\xi_0^2} \sqrt{\frac{\tau_0}{\tau_Q}}$$

BUT

- THIS IS AN INTERACTING FIELD THEORY!
- WHAT EFFECT DO THE INTERACTIONS HAVE ON THE DEFECT DENSITY AT FORMATION?

- INTUITION — EXPECT AN ATTRACTION BETWEEN DOMAINS WITH SMALL PHASE DIFFERENCE RELATIVE TO DOMAINS WITH A LARGE PHASE DIFFERENCE

  — MORE REFINED VERSION OF THE GEODESIC RULE

Simulation

$$\frac{\partial \vec{\Phi}}{\partial t} = \vec{\nabla}^2 \vec{\Phi} - \frac{\partial V(\vec{\Phi})}{\partial \vec{\Phi}} + \vec{\eta}(\vec{r}, t)$$

Gaussian noise

$$\langle \vec{\eta}(\vec{r}, t) \rangle = 0$$

$$\langle \eta_a(\vec{r}_1, t_1) \, \eta_b(\vec{r}_2, t_2) \rangle = 2 T \delta_{ab} \cdot \delta(\vec{r}_1 - \vec{r}_2) \, \delta(t_1 - t_2)$$

Linear Quench : $m_f^2 = -m_i^2$

$T/m_i = \text{const.}$

$0.001 \leq T/m_i \leq 0.1$

Procedure

1. Equilibrate in Disordered Phase $m^2 = m_i^2$ $\langle \vec{\Phi} \rangle = 0$ with Thermal Noise

2. Measure Largest Thermally Generated Domain $V_{\text{THERM}} = 79$

3. QUENCH : $m^2 : m_i^2 \searrow -m_i^2$

$\langle |\vec{\phi}\cdot\vec{\phi}|^2 \rangle \rightarrow v^2 \neq 0$

4. COMPUTE NO. DEFECTS*

5. AVERAGE OVER INDEPEDENT SIMULATIONS

6. COMPARE w/ RANDOM DOMAIN FORMATION

* NUMERICAL IMPLEMENTATION

DOMAIN DEF^N

- $\theta_i, \theta_j \in D$ provided $|\theta_i - \theta_j| < \theta_\varepsilon$
- θ_ε $\begin{cases} \text{TOO SMALL} \Rightarrow \text{ARTIFICIALLY LARGE \# DOMAINS} \\ \text{TOO LARGE} \Rightarrow \text{DOMAINS LOST} \end{cases}$

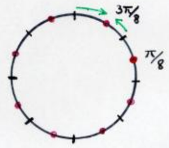
WE CHOOSE θ_ε TO BE THE MAXIMUM VALUE THAT PRESERVES THE TOPOLOGY OF THE ORDER FIELD i.e. REPRODUCES DEFECT STRUCTURE OF THE CONTINUUM

$$S^1 \to \mathbb{Z}_n = \{ e^{2\pi i k/n} ; k = 0, 1, \dots, n-1 \}$$

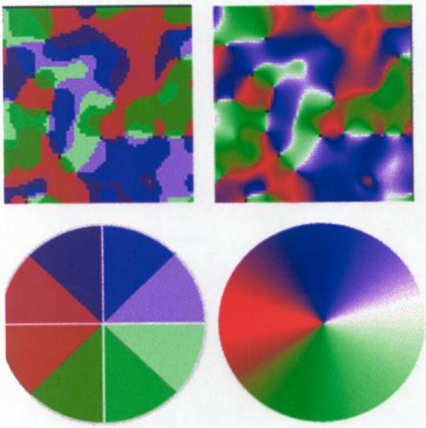
$$\theta_i = \frac{1}{2}\left(k_i \Delta + (k_i + 1)\Delta \right) \qquad \Delta = \frac{2\pi}{n}$$

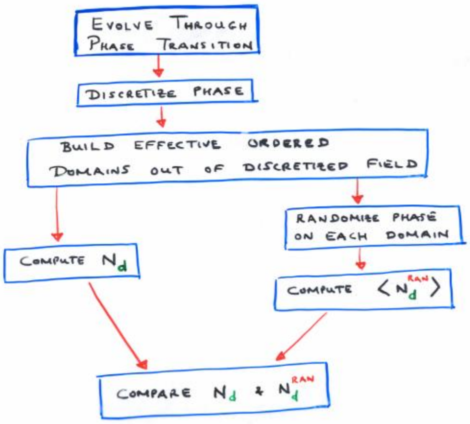
$$\text{if} \qquad \frac{2\pi}{n} k_i < \theta_i < \frac{2\pi}{n}(k_i + 1)$$

$\mathbb{Z}_8$

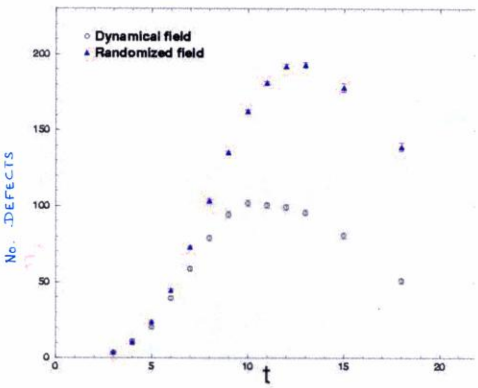


IMPOSE VOL(DOMAIN) > V_{THERMAL} for THERMAL STABILITY




EVOLVE THROUGH PHASE TRANSITION
DISCRETIZE PHASE
BUILD EFFECTIVE ORDERED DOMAINS OUT OF DISCRETIZED FIELD
RANDOMIZE PHASE ON EACH DOMAIN
COMPUTE N_d
COMPUTE $\langle N_d^{RAN} \rangle$
COMPARE N_d & N_d^{RAN}

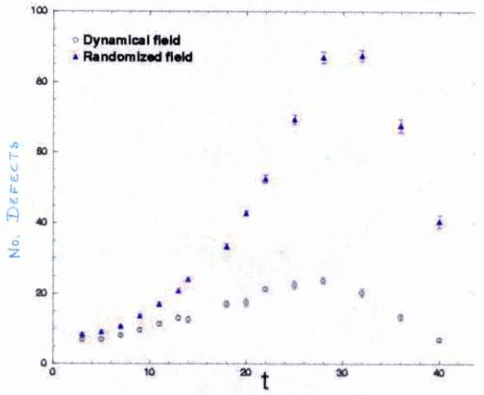
INSTANTANEOUS QUENCH



T= 0.035 $\tau_Q = 0$

Finite Quench Rate

$\tau_Q m_i = 50$



$T = 0.035$

Conclusions

- Naive correlation length ≡ maximal domain size at freeze out does not give the correct topological defect density for finite quench rates

- Phase alignment via domain interaction reduces wandering on the ground state manifold $\mathcal{M}$ i.e. phases / order fields are not random at the time of defect formation

Future

- Determine τ_Q which gives maximal suppression of defect density
- Relativistic dynamics (2nd-order in time)
- Dynamical spacetime