Quantum Hall Effect – part II

N. Read

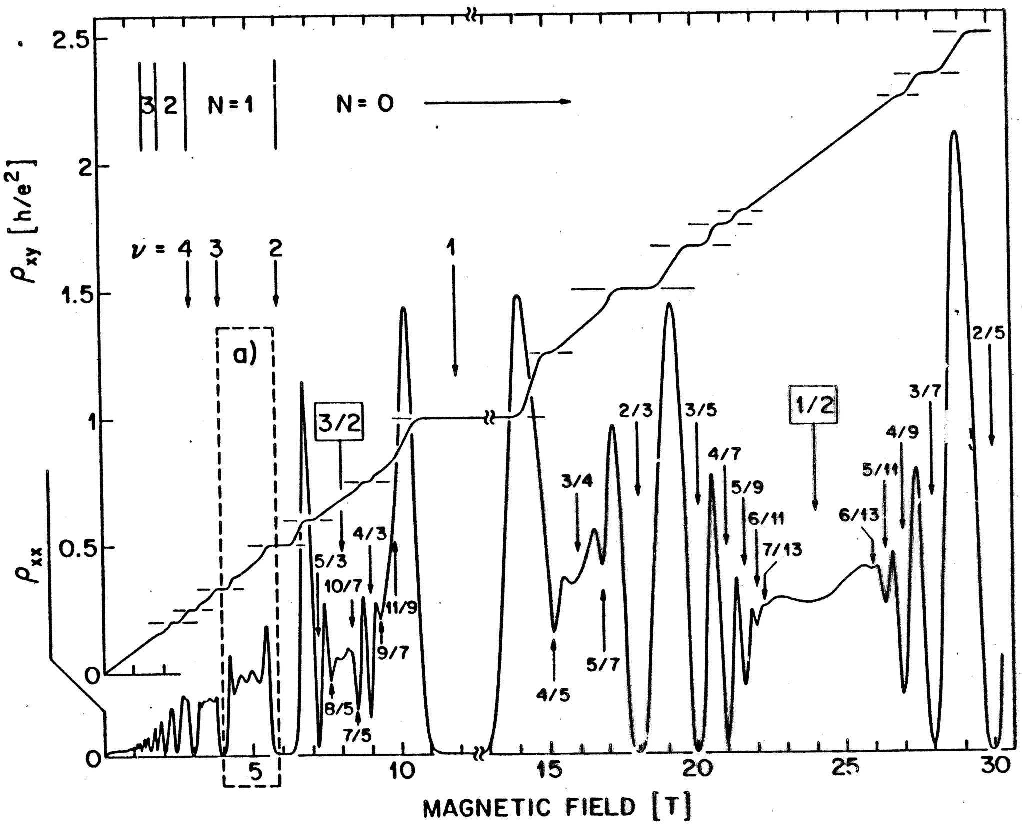
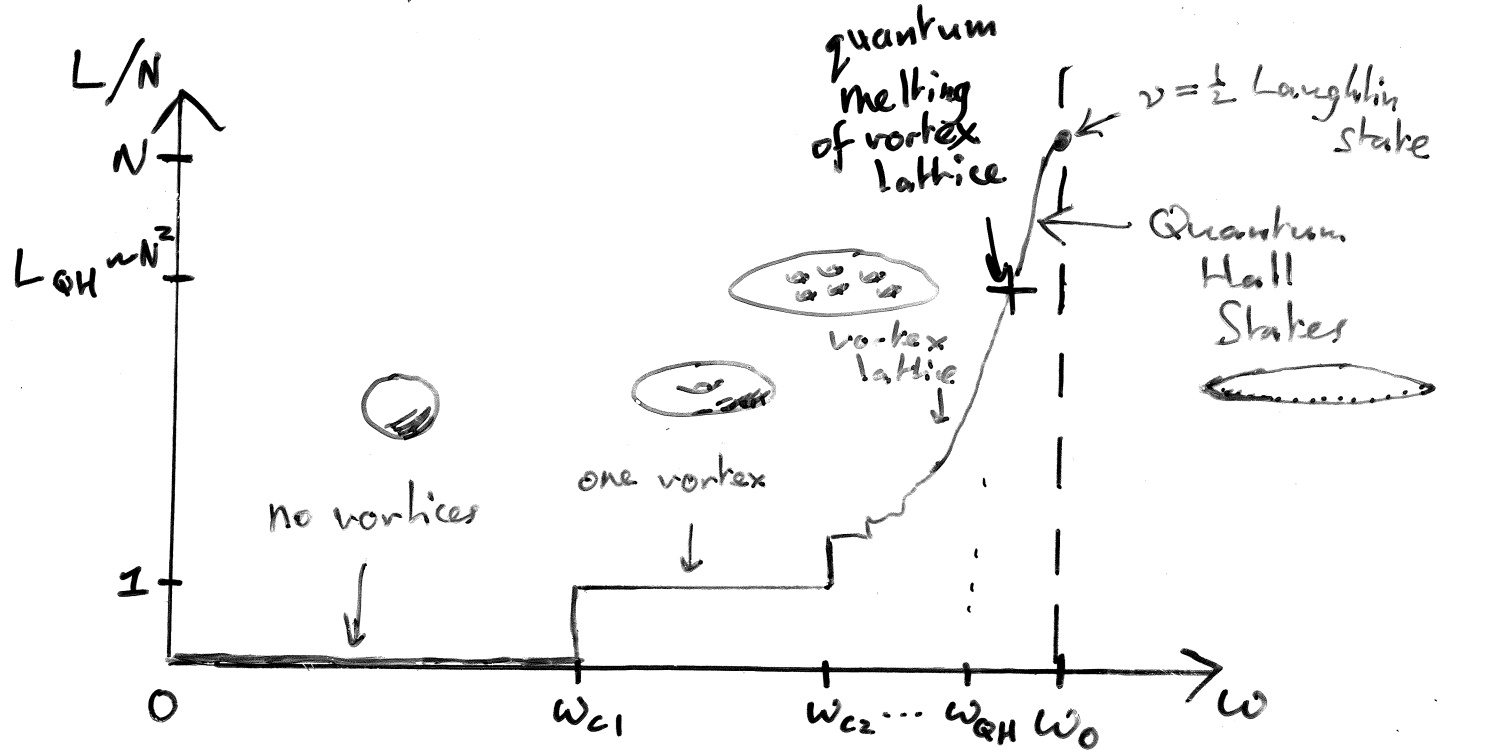


FIG. 1. Overview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} of sample described in text. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at $\simeq 12$ T). Temperatures were ≈ 150 mK except for the high-field Hall trace at $T = 85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. Filling factor ν and Landau levels N are indicated.

Willett et al

Bosons in traps — a few cartoons

$T=0$, isotropic trap, repulsive interaction, free rotation



N spinless/spin-polarized bosons

$L = L_z$ is total angular momentum

$\underline{\omega} = |\omega| \hat{\underline{z}}$

E.g. Butts + Rokhsar 1999 for mean-field theory of condensate with vortex lattice

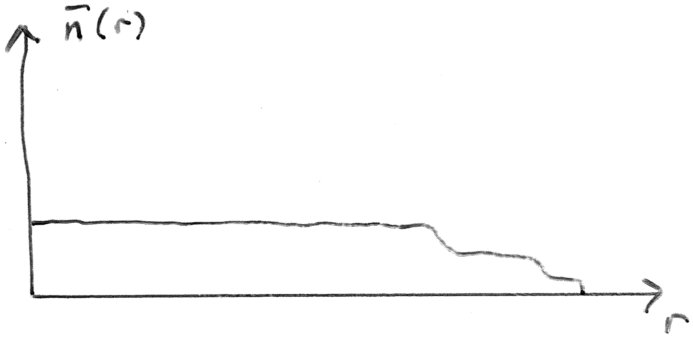
As repulsive interaction → zero, all critical ω's $(\omega_{c1}, \omega_{c2}, \ldots, \omega_{QH}, \ldots) \to \omega_0$ (trap frequency)

ω_0 is only possible free rotation frequency for noninteracting bosons

$\omega > \omega_0$ — particles fly out of trap

Numerical Studies in LLL

To avoid edge effects:



use an edgeless geometry:

complex plane $\mathbb{C}$ → sphere
torus (periodic boundary conditions)

Cooper, Wilkin, Gunn (2001) used torus

$$\nu = \frac{N}{N_v} \qquad N_v = \# \text{ of LLL states},$$

found evidence of quantum Hall (incompressible) fluids at $\nu = \frac{1}{2}, 1, \frac{3}{2}, \ldots < \nu_c \simeq 6-10$
(Ground states of H_{int})

$$\Delta(N) = N\left[\frac{E_0(N+1)}{N+1} + \frac{E_0(N-1)}{N-1} - 2\frac{E_0(N)}{N}\right]$$

Cf $\frac{d\mu}{dN} = \frac{d^2E_0}{dN^2} \to \infty$ if incompressible

Ground states at $\nu = \frac{k}{2} < \nu_c$ have excellent overlaps with Read-Rezayi QH states.

Vortex lattice at $\nu > \nu_c$

dication of the transition to the vortex lattice, perhaps re-entrant around $\nu = 6$). It is not immediately apparent how to construct incompressible states for this sequence of ν. One possibility is that the vortices themselves are forming Laughlin states. This would provide a set of states with vortex filling fraction $\nu_V = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots$ [19], and hence $\nu = 1/\nu_V = 2, 4, 6, \ldots$. However, this construction does not account for states at $\nu = 1, \frac{3}{2}, \frac{5}{2}, 3, \frac{7}{2}, \frac{9}{2}, 5$ [20]. Moreover, the trial wave functions of this form that we have tested (on a disk) have high interaction energies: while they keep the vortices apart, they do not introduce favorable correlations between the bosons. States of this type are likely to describe systems in which the underlying interactions can be described as repulsive two-body

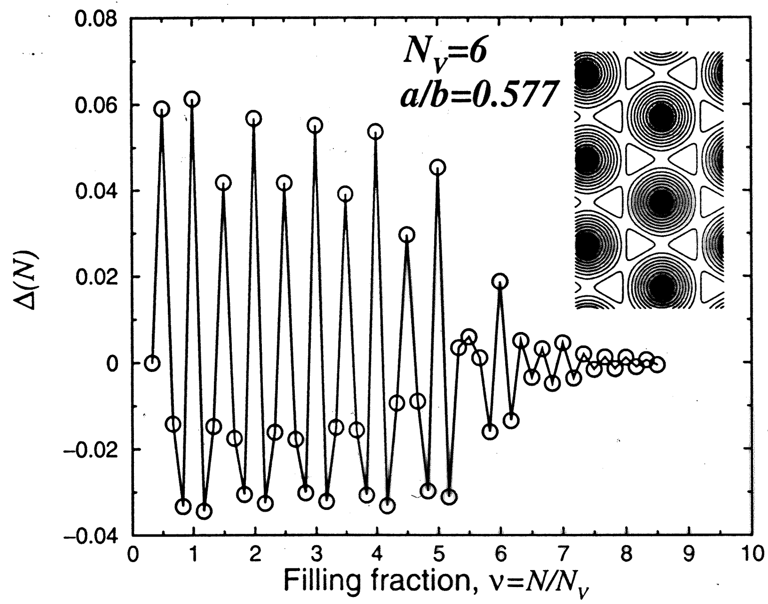


FIG. 2. Energy gap (3) as a function of ν for $N_V = 6$ vortices, at $a/b = 1/\sqrt{3}$. Upward spikes signal values of ν for which the ground state is incompressible. The collapse of the gaps at $\nu \sim 6$ indicates the transition to the vortex lattice phase. (Inset shows the density of the GP ground state.)

where $z = x +$
lowest Landau l
cates symmetriz
sets $A, B, \ldots,$ o
visible by k). T
the Laughlin an
eral k, the wave
ing fraction $\nu^{(k}$
eigenstate of a (

The Read-Re
tion of the inco
the sequence of
($\nu = \frac{k}{2}$ with int
exact wave func
for which we ha
the Read-Rezayi
($k + 1$)-body fo
eigenstates. In g
eigenstate, and r
a torus of $k +$ 1
of the exact grou
the Read-Rezayi
comparison, the
shown.

In conclusion,
weakly interacti
vortex lattices a
distinction betwe
ber of bosons, N
the filling fractic
pear for $\nu \lesssim \nu_c$
A Lindemann cr
agonalizations in
with $N \sim 10^5$ a

Cooper, Wilkin, Gunn, 2001

Read-Rezayi "parafermion" states

NR + E. Rezayi
1999

New phases of QH matter

- Exact ground states of k+1-body int

$$H_{int} = \sum_{i_1<i_2<\dots i_k} \delta(\underline{r}_{i_1} - \underline{r}_{i_2}) \dots \delta(\underline{r}_{i_k} - \underline{r}_{i_{k+1}})$$

- Simpler form of wavefunctions for ground states: (disk)

$$\Psi_{(k)} = \mathcal{S}\Big[\prod_{\substack{i<j \\ i,j\in A_1}} (z_i - z_j)^2 \prod_{\substack{l<m \\ l,m\in A_2}} (z_l - z_m)^2 \dots \prod_{A_k} (\ )^2 \Big]$$

$A_1, \dots, A_k$ each have N/k elements

$\Rightarrow$ vanishes whenever k+1 coincide

Cappelli, Georgiev, Todorov (2001)

- $\nu = \frac{k}{2}$

- Elementary excitations have "charge" $\pm\frac{1}{2}$, each k

- Degeneracy of ground states on torus $= k+1$ (for $k|N$)

- Special cases:
  - $k=1$: Laughlin state (1983) (Haldane 1983)
  - $k=2$: Moore-Read "Pfaffian" paired state (1991) (Greiter, Wen, Wilczek 1992)
  - $k>2$: RR, 1999

- Nonabelian statistics for $k>1$

Recent numerical work

MR state at $\nu = 1$ becoming well-established
Hierarchy/CF states at other ν
esp $\nu = 2/3$

Regnault + Jolicoeur (2003)

RR at $k = 3$ ($\nu = \frac{3}{2}$)?

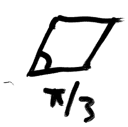
Torus, $H_{int} = \frac{1}{2} g \sum_{ij} \delta(r_i - r_j)$

w./ E. Rezayi

Energy gaps: "charged"
(N ≤ 18)

"neutral"

Overlaps: hexagonal unit cell

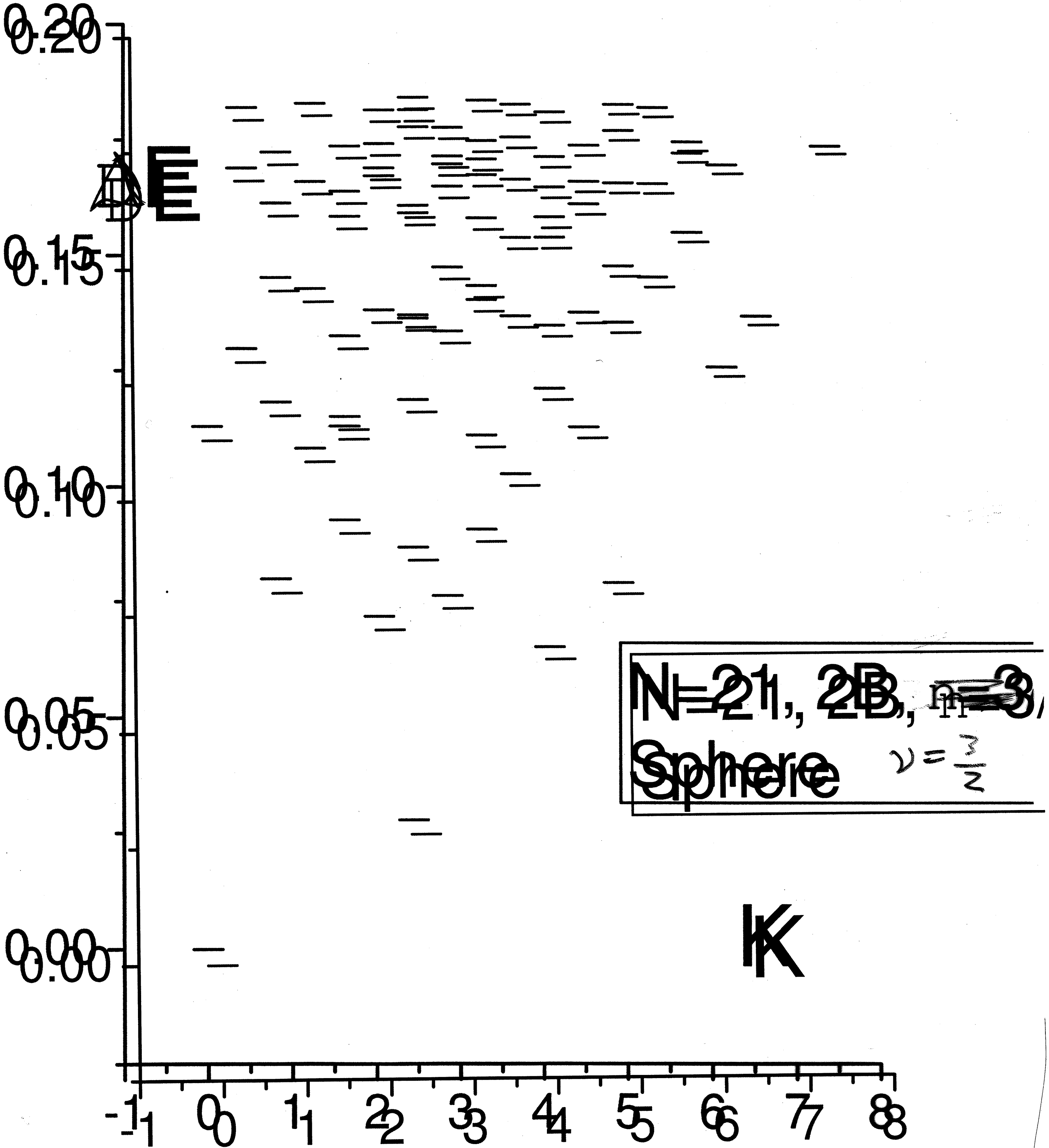


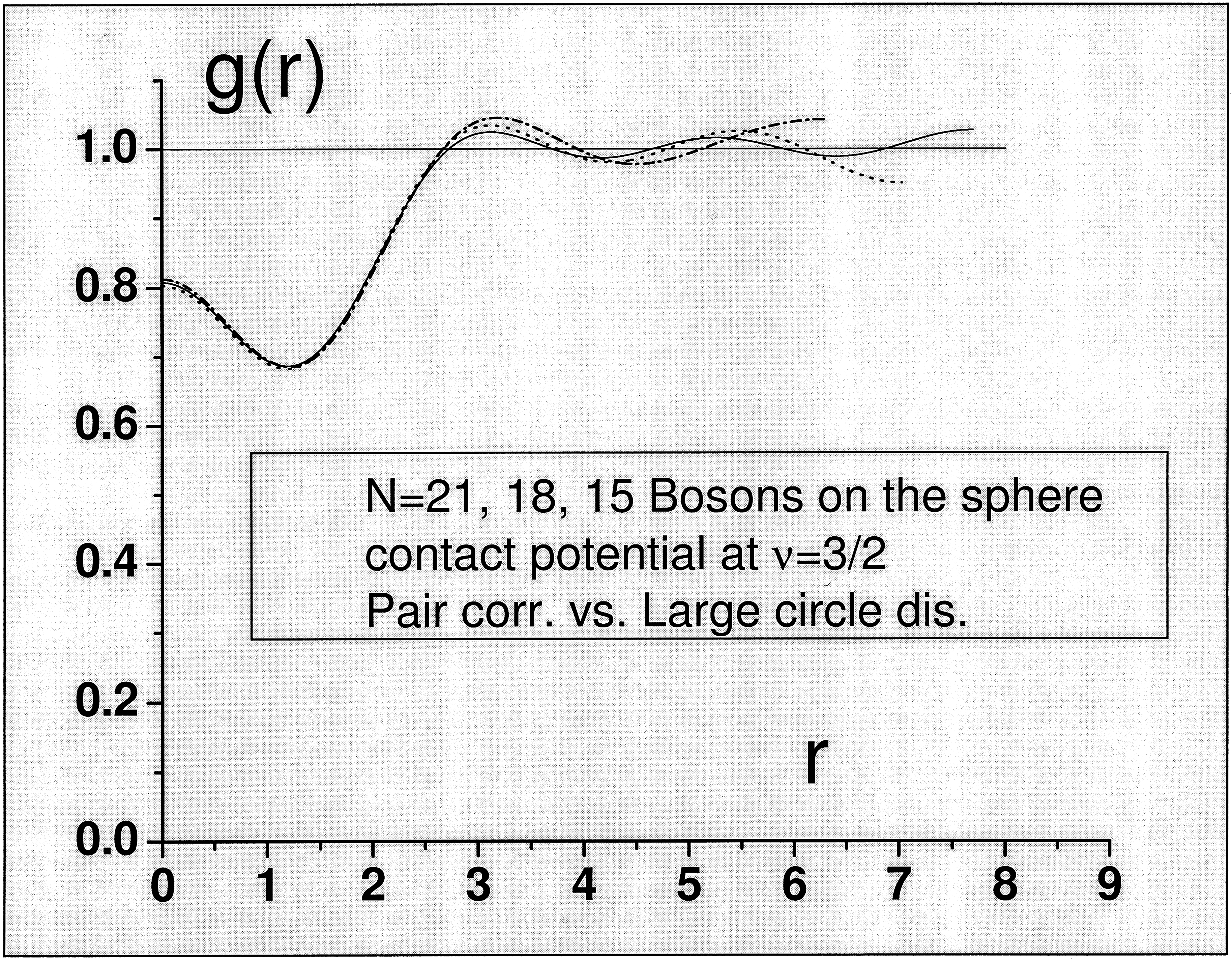
Take out trivial 2-fold, should see two states degenerate as $N \to \infty$. Take overlaps with two exact trial RR states, Ψ_i^{RR}:

$$0 \leq \sum_{ij} |\langle \Psi_i^{2\text{-body}} | \Psi_j^{RR} \rangle|^2 \leq 2$$

N	overlap (hex.)	Hilbert space dim (symmetry reduced)	overlap (sq.)
12	1.22	~250	1.14 (GS not k=0)
15	1.4	~5,000	1.32
18		~110,000	1.10

Impressive but larger sizes still needed


0.20
0.15
0.10
0.05
0.00
ΔE
K
-1
0
1
2
3
4
5
6
7
8
N=21, 2B, n=3
Sphere


g(r)
1.0
0.8
0.6
0.4
0.2
0.0
N=21, 18, 15 Bosons on the sphere
contact potential at ν=3/2
Pair corr. vs. Large circle dis.
r
0
1
2
3
4
5
6
7
8
9