Evolutionary Game Theory

non-equilibrium and non-linear dynamics of interacting particle systems

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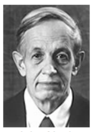
Game Theory

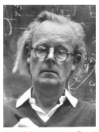
John Nash:
"An equilibrium is reached as soon as no party can increase its profit by unilaterally deciding differently."

John Maynard-Smith and George R. Price:
"A strategy is called evolutionary stable if a population of individuals homogenously playing this strategy is able to outperform and eliminate a small amount of any mutant strategy introduced into the population."


John von Neumann


Oskar Morgenstern


John Nash


John Maynard-Smith

Strategic Games

Mathematical description of strategic situations, in which an individual's success in making choices depends on the choices of others.

Prisoner's Dilemma:

P	Cooperator (C)	Defector (D)
C	1 year	10 years
D	0 years	5 years

(D,D) is a Nash equilibrium where unilateral deviation does not pay off.

Classical Formulation of Prisoner's Dilemma

``Two suspects of a crime are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal. If one testifies (defects from the other) for the prosecution against the other and the other remains silent (cooperates with the other), the betrayer goes free and the silent accomplice receives the full 10-year sentence. If both remain silent, both prisoners are sentenced to only 1 year in jail for a minor charge. If each betrays the other, each receives a five-year sentence. Each prisoner must choose to betray the other or to remain silent. Each one is assured that the other would not know about the betrayal before the end of the investigation. How should the prisoners act?''

Social Dilemmas

The fundamental problem of cooperation:

P	Cooperator (C)	Defector (D)
C	$b-c$	$-c$
D	b	0

General two-player games

P	Cooperator (C)	Defector (D)
C	$\mathcal{R}$eward	$\mathcal{S}$uckers payoff
D	$\mathcal{T}$emptation	$\mathcal{P}$unishment

Social Dilemmas

The fundamental problem of cooperation:

P	Cooperator (C)	Defector (D)
C	$b-c$	$-c$
D	b	$\boxed{0}$

The snowdrift game:

P	Cooperator (C)	Defector (D)
C	$b-c/2$	$\boxed{b-c}$
D	$\boxed{b}$	0

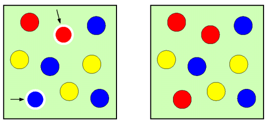
Evolutionary Game Theory

Consider a population of size N

N_i individuals play strategy A_i: $a_i = N_i/N$ (frequency)

Composition of the population is updated by some (evolutionary) rules: $N_i\,(t) \longrightarrow N_i\,(t+dt)$

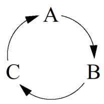
Moran process:



- pick two at random
- the fitter wins

Rate Equations

"Chemical" reactions:



$$A + B \xrightarrow{k_A} A + A$$
$$B + C \xrightarrow{k_B} B + B$$
$$C + A \xrightarrow{k_C} C + C$$

Rate equations:

$$\begin{aligned} \partial_t a &= a(k_A b - k_C c) \\ \partial_t b &= b(k_B c - k_A a) \\ \partial_t c &= c(k_C a - k_B b) \end{aligned}$$

Fitness and replicator equations

Payoff matrix:

$\mathbf{P}$	A	B
A	$p_{11} := \mathcal{R}$	$p_{12} := \mathcal{S}$
B	$p_{21} := \mathcal{T}$	$p_{22} := \mathcal{P}$

Frequencies: $a = N_A/N\,,\quad b = N_B/N = (1-a)$

Fitness = expected payoff:

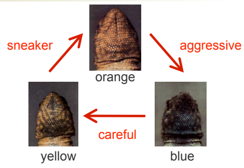
$f_A(a) = \mathcal{R}a + \mathcal{S}(1-a)\,,\quad f_B(a) = \mathcal{T}a + \mathcal{P}(1-a)$

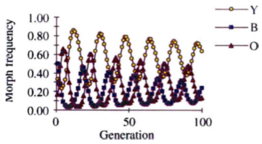
$\bar{f}(a) = a f_A(a) + (1-a) f_B(a)$

Replicator dynamics:

$$\partial_t a = \left[f_A(a) - \bar{f}(a)\right] a \qquad \partial_t a = \frac{f_A(a) - \bar{f}(a)}{\bar{f}(a)}\, a$$

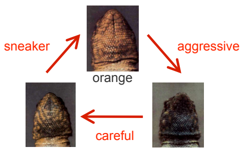
Mating Strategies of Lizards

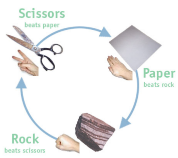




B. Sinervo and C.M. Lively, Nature 380, 240 (1996)

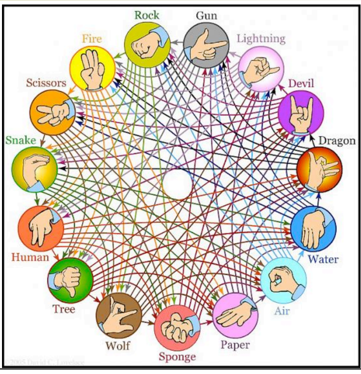
Rock-Scissors-Paper Game





Note that in populations the strategy of an individual is fixed.
What varies is the number of individuals with a particular strategy.

The „latest“ Rock-Scissors-Paper Game



Microbial Laboratory Communities:

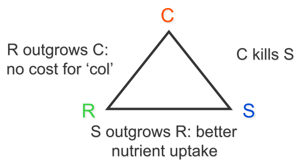
model systems for competition, cooperation, …

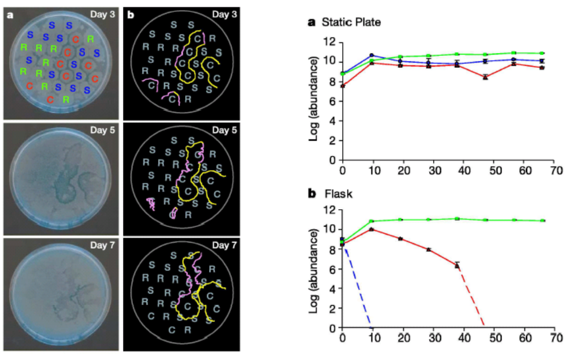
Colicinogenic Bacteria

Toxin producing (colicinogenic) E.coli (C) carry a <u>'col' plasmid:</u> genes for colicin, colicin specific immunity proteins, lysis protein

Colicin-sensitive bacteria (S)

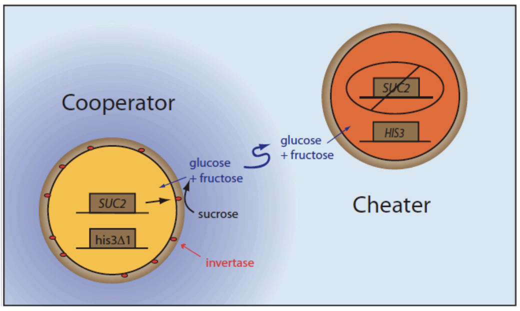
Colicin-resistant bacteria (R) are mutations of S with altered cell membrane proteins that bind and translocate cocilin





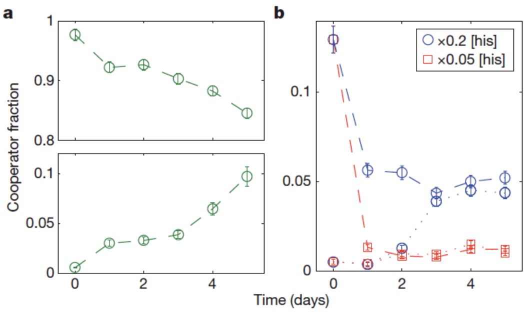
B. Kerr et al., Nature 418, 171 (2002)

Cheating in Yeast

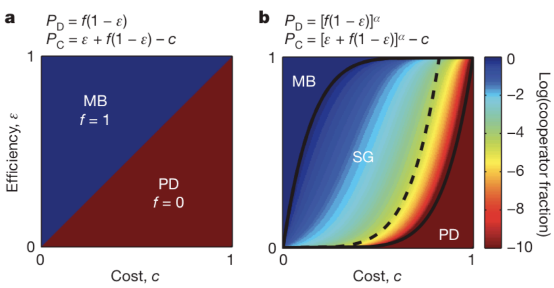


J. Gore et al., Nature 07921 (2009)

Experiments imply „Snowdrift Game“



J. Gore et al., Nature 07921 (2009)



Nonlinear Dynamcis of 2-Player Games

P	Cooperator (C)	Defector (D)
C	$\mathcal{R}$eward	$\mathcal{S}$ uckers payoff
D	$\mathcal{T}$emptation	$\mathcal{P}$ unishment

Replicator dynamics:

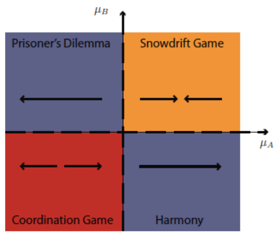
$$\partial_t a = \left[f_A(a) - \bar{f}(a)\right] a \ = a(1-a)(f_A - f_B)$$
$$= a(1-a)\left[\mu_A(1-a) - \mu_B a\right] =: F(a)$$

$\mu_A := \mathcal{S} - \mathcal{P}\,, \quad \mu_B := \mathcal{T} - \mathcal{R}\,.$

P	A	B
A	1	$1+\mu_A$
B	$1+\mu_B$	1

	A	B
A	1	$1+\mu_A$
B	$1+\mu_B$	1

- $\mu_A < 0$, $\mu_B > 0$ strategy B is always better off
- $\mu_A < 0$, $\mu_B < 0$ unstable interior fixed point

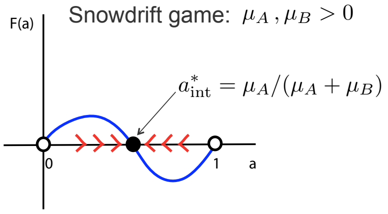


- $\mu_A > 0$, $\mu_B > 0$ stable interior fixed point
- $\mu_A > 0$, $\mu_B < 0$ strategy A is always better off

Nonlinear Dynamics

$$\partial_t a = a(1-a)\left[\mu_A(1-a) - \mu_B a\right] =: F(a)$$

Snowdrift game: $\mu_A\,,\mu_B > 0$



Zeros of F(a) are fixed points a*
Slope of F(a*) determines stability

Recommended Reading:

Examples for game theory problems in biology:

B. Sinervo and C.M. Lively, Nature 380, 240 (1996)
B. Kerr et al., Nature 418, 171 (2002)
J. Gore et al., Nature 07921 (2009)

Background in nonlinear dynamics:

S.H. Strogatz, Nonlinear Dynamics and Chaos, Westview; chapters 2&3