Turbulence – and related problems

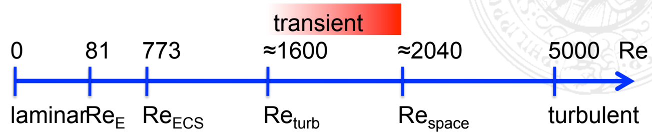
Lecture 1: Transition: Coherent structures
Lecture 2: Transition: Transient turbulence
Lecture 3: Global transport

Bruno Eckhardt

Philipps Universität Marburg TU Delft 3TU.

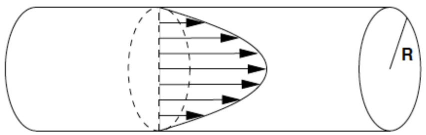
Reynolds numbers for pipe flow

(not to scale)



Re_E	Energy stability
Re_{ECS}	Coherent structures appear
Re_{turb}	Turbulence appears in experiments
Re_{space}	Turbulence becomes spacefilling

Pipe flow

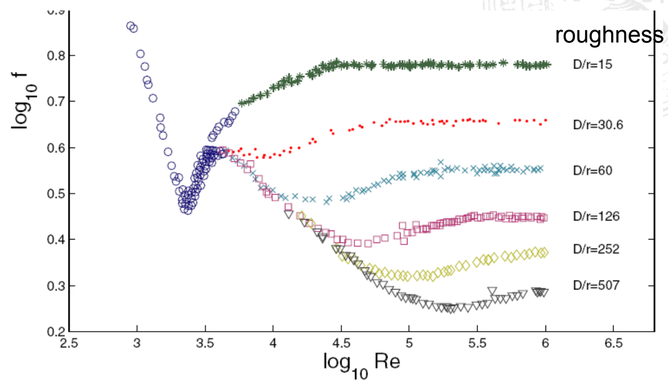


- Laminar: Hagen-Poiseuille law

volume flow $$Q = \frac{\pi}{8\eta}\frac{\Delta p}{L}R^4$$

- Turbulent: Friction law $$\frac{\Delta p}{\rho L} = f\frac{U^2}{4R}$$

Nikuradse 1932/1933

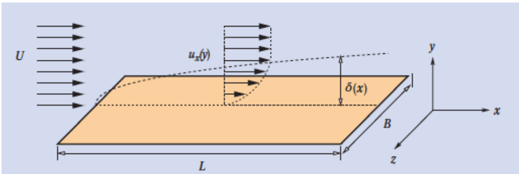


from Goldenfeld, PRL **96**, 044503 (2006)

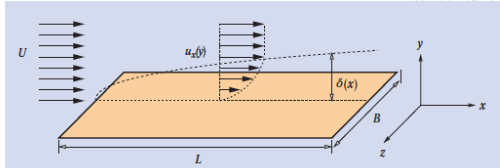
Mean profiles: Blasius

$$\cancel{\partial_t u_x} + \partial_x(u_x u_x) + \partial_y(u_y u_x) + \cancel{\partial_z(u_z u_x)} = -\partial_x p + \nu(\cancel{\partial_{xx} u_x} + \partial_{yy} u_x + \cancel{\partial_{zz} u_x})$$

$$\partial_x(u_x u_x) + \partial_y(u_y u_x) = -\partial_x p + \nu \partial_{yy} u_x$$
$$\partial_x u_x + \partial_y u_y = 0$$



Prandtl 1904:



$$\delta(x) = \sqrt{x\nu / U}$$

Local force proportional to

- free stream velocity
- 1/boundary layer thickness
- width . dx

$$dF \propto \rho\nu \cdot \frac{U}{\delta(x)} \cdot B dx$$

$$F \propto 1{,}1 \rho B L U^2 / \sqrt{\mathrm{Re}}$$

$$\mathrm{Re} = \mathrm{LU}/\nu$$

Mean profiles: Prandtl-von Karman

$$\cancel{\partial_t u_x} + \cancel{\partial_x(u_x u_x)} + \partial_y(u_y u_x) + \cancel{\partial_z(u_z u_x)} = -\cancel{\partial_x p} + \nu(\cancel{\partial_{xx} u_x} + \partial_{yy} u_x + \cancel{\partial_{zz} u_x})$$

$$\partial_y\left(\langle u'_y\, u'_x \rangle - \nu \partial_y u_x\right) = 0$$ Momentum balance

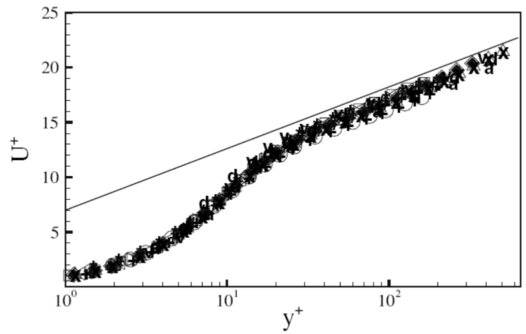
Mixing length model

$$\langle u'_y\, u'_x \rangle = -l^2 \left(\partial_y u_x\right)^2$$
$$l = \kappa y$$

$$u_x = A \ \ln\ y + B$$ Law of the wall

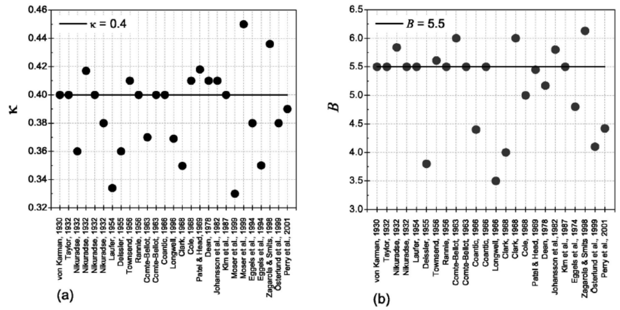
Turbulent boundary layer

$$U^+ = \frac{1}{\kappa} \ln y^+ + B$$



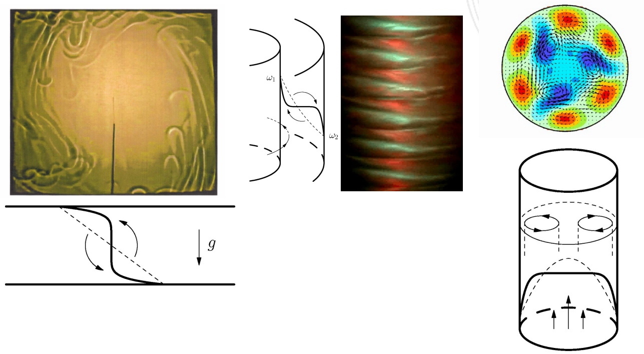
Law of the wall

$$U^+ = \frac{1}{\kappa} \ln y^+ + B$$



Zanoun+Durst+Naghib, Phys Fluids 15 (2003) 3079

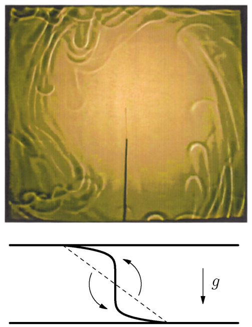
Rayleigh-Benard | Taylor-Couette | Pipe flow



Obs: External forcing induces transverse velocity fluctuations

Q: what can one say about the global scaling?

The Rayleigh-Bénard problem



Input:	Temp. Diff.	Rayleigh number	Ra
	Material	Prandtl number	Pr
Output:	„Wind“	Reynolds number	Re
	Heat flux	Nusselt number	Nu

Wanted:

the response of the fluid

$$\mathrm{Re}(Ra, \mathrm{Pr}) \approx Ra^{\alpha}\,\mathrm{Pr}^{\beta}$$

$$Nu(Ra, \mathrm{Pr}) \approx Ra^{\gamma}\,\mathrm{Pr}^{\delta}$$

Nu(Ra)

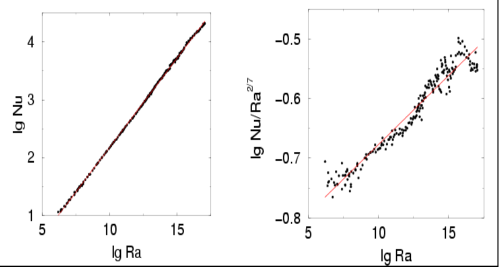
$$Nu \sim Ra^{\beta}$$

data:
cryogenic helium
approximate
power laws only

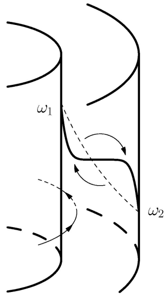
$\beta = 2/7 \approx 0.286$ (Castaing et al., JFM 204, 1 (1989), Siggia, ARFM. 26, 137 (1994))

$\beta \approx 0.309$ (Niemela et al., Nature 404, 837 (2000))

exponent varies
with Ra: β(Ra)



The Taylor-Couette problem



Fluid between indendently rotating cylinders

Input:	Rotation rates	Reynolds numbers Re_{inner}, Re_{outer}
	Geometry	„Prandtl number" Pr
Output:	„Wind"	Reynolds number Re_w
	Angular momentum flux	Nusselt number Nu

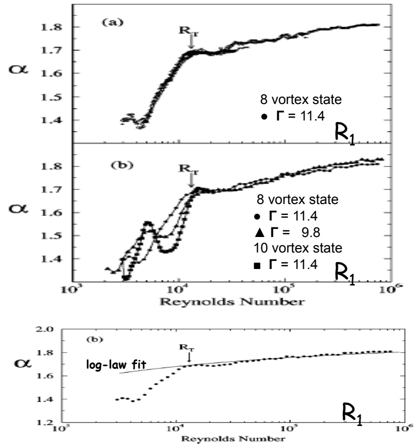
Wanted:

(Outer cylinder at rest)

$$Re_w(Re_{inner}, Pr) \approx Re_{inner}^{\alpha} Pr^{\beta}$$

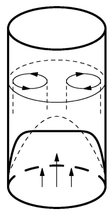
$$Nu_{\omega}(Re_{inner}, Pr) \approx Re_{inner}^{\gamma} Pr^{\delta}$$

Local exponents $\alpha(R_1)$ in $G \sim R_1^{\alpha}$



Lewis Swinney PRE59 5457(1999)

Pipe flow



Pressure driven flow down a circular pipe

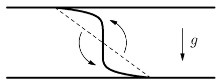
Input:	Mean flux	Re
Output:	„Wind"	Reynolds number Re_w
	Momentum flux	Nusselt number Nu_z

Wanted:

$$Re_w(Re)$$

$$Nu_z(Re)$$

Rayleigh-Bénard



Heat transport enhanced by flow

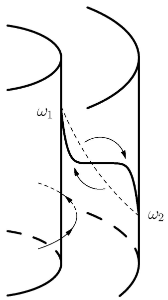
Transport measured by Nusselt number:

$$Nu = \langle u_z \theta \rangle_{x,y,t} - \nu \partial_z \langle \theta \rangle_{x,y,t}$$

Energy dissipation in velocity field

$$\varepsilon = Pr^{-2} Ra(Nu-1) \qquad Pr = \nu / \kappa$$

Taylor-Couette



Transport of angular velocity

Dimensionless Nusselt number

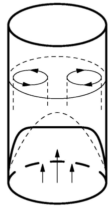
$$Nu_\omega = r^3\left(\langle u_r \omega \rangle - \nu \partial_r \langle \omega \rangle \right)$$

Energy dissipation in excess of laminar dissipation:

$$\varepsilon = \varepsilon_{tot} - \varepsilon_{lam} = \sigma^{-2} Ta (Nu_\omega - 1)$$

$\sigma = \left(\frac{(1+\eta)/\sqrt{2}}{\sqrt{\eta}} \right)^4$ „Prandtl number"

Pipe flow



Transport of momentum

Dimensionless Nusselt number

$$Nu_z = r^{-1}\left(\langle u_r u_z \rangle - \nu \partial_z \langle u_z \rangle \right)$$

Energy dissipation in excess of laminar dissipation:

$$\varepsilon = \varepsilon_{tot} - \varepsilon_{lam} = \mathrm{Re}^2 (Nu_\omega - 1)$$

Analogies

	Transport	Dissipation
RB	$Nu = \langle u_z \theta \rangle_{x,y,t} - \nu \partial_z \langle \theta \rangle_{x,y,t}$	$\varepsilon = \mathrm{Pr}^{-2} Ra (Nu - 1)$
TC	$Nu_\omega = r^3 \left(\langle u_r \omega \rangle - \nu \partial_r \langle \omega \rangle \right)$	$\varepsilon = \sigma^{-2} Ta (Nu_\omega - 1)$
Pipe	$Nu_z = r^{-1} \left(\langle u_r u_z \rangle - \nu \partial_z \langle u_z \rangle \right)$	$\varepsilon = \mathrm{Re}^2 (Nu_\omega - 1)$

Rayleigh-Bénard (I)

Two equations:

$$\varepsilon = \mathrm{Pr}^{-2} Ra (Nu - 1) \qquad \lambda \propto 1/Nu \quad \text{thermal BL}$$

$$Nu = \langle u_z \theta \rangle_{x,y,t} - \kappa \partial_z \langle \theta \rangle_{x,y,t} \qquad \delta \propto 1/\sqrt{\mathrm{Re}} \quad \text{velocity BL}$$

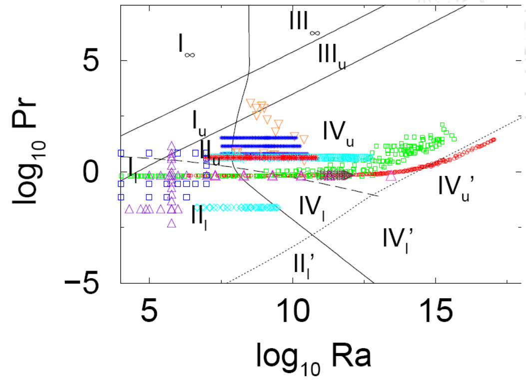
Two models:

$$\varepsilon = \mathrm{Pr}^{-2} Ra\, Nu = c_2 \mathrm{Re}^3 + c_1 \mathrm{Re}^{5/2}$$

$$Nu = c_4 \mathrm{Re}\,\mathrm{Pr}\, f(\lambda/\delta) + c_3 \sqrt{\mathrm{Re}\,\mathrm{Pr}\, f(\lambda/\delta)}$$

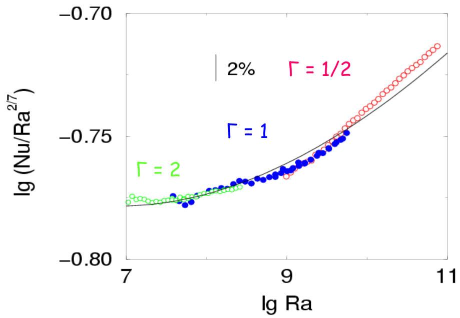
$f(\lambda/\delta)$ switches between 1 and λ/δ for $\lambda > \delta$ and $\lambda < \delta$ respectively

Rayleigh-Bénard (II)



Grossmann and Lohse, JFM 407, 27 (2000) and later papers

Rayleigh-Bénard (III)



Data: acetone,
Pr = 4.0
$10^5 < Ra < 10^{10}$

G. Ahlers, SGn, D. Lohse
Physik Journal 1 (2002) Nr 2, 31-37

Taylor-Couette

Two equations: TC | RB

TC:

$$\varepsilon = \sigma^{-2} Ta (Nu_\omega - 1)$$

$$Nu_\omega = r^3 \left(\langle u_r \omega \rangle - \nu \partial_r \langle \omega \rangle \right)$$

RB:

$$\varepsilon = \mathrm{Pr}^{-2} Ra (Nu - 1)$$

$$Nu = \langle u_z \theta \rangle_{x,y,t} - \nu \partial_z \langle \theta \rangle_{x,y,t}$$

Two models:

$$\varepsilon = c_2 \mathrm{Re}_w^{\,3} + c_1 \mathrm{Re}_w^{\,5/2}$$

$$Nu_\omega / \sigma = c_4 \mathrm{Re}_w \, g(s) + c_3 \sqrt{\mathrm{Re}_w \, g(s)}$$

„Prandtl number:“

$$\sigma = \left(\frac{(1+\eta)/\sqrt{2}}{\sqrt{\eta}} \right)^4$$

$$s = \frac{\lambda}{\delta} = \frac{1}{2a} \frac{\sqrt{\mathrm{Re}_w}}{Nu_\omega}$$

Predictions .....

Modelling transport and excess dissipation

All quantities determined by transverse velocity field: „wind“

Reynolds number Re_w

Energy dissipation

$$\varepsilon = c_2 \mathrm{Re}_w^{\,3} + c_1 \mathrm{Re}_w^{\,5/2}$$

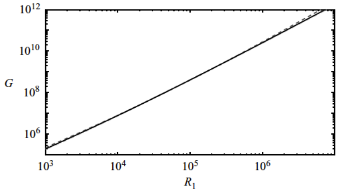
Momentum transport

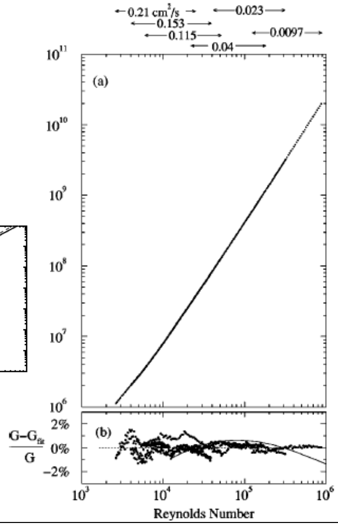
$$Nu_\omega / \sigma = c_4 \mathrm{Re}_w \, g(s) + c_3 \sqrt{\mathrm{Re}_w \, g(s)} \qquad s = \frac{\lambda}{\delta} = \frac{1}{2a} \frac{\sqrt{\mathrm{Re}_w}}{Nu_\omega}$$

Fit coefficients and compare ….

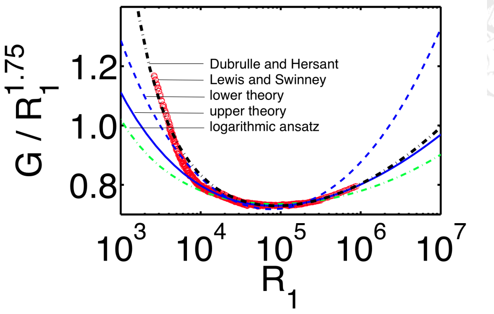
Comparison to data

Lewis and Swinney,
PRE 59, 5457 (1999)





Taylor-Couette



Data: Lewis and Swinney, PRE 59, 5457 (1999)

Pipe flow

Two equations: Pipe

$$\varepsilon = \mathrm{Re}^2 (Nu_\omega - 1)$$

$$Nu_z = r^{-1} \left(\langle u_r u_z \rangle - \nu \partial_z \langle u_z \rangle \right)$$

Two models:

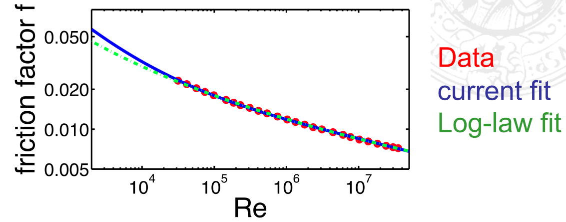
$$\varepsilon = c_2 \,\mathrm{Re}_w^{\;3} + c_1 \,\mathrm{Re}_w^{\;5/2}$$

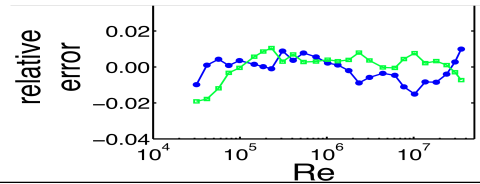
$$Nu_\omega = c_4 \,\mathrm{Re}_w \, g(s) + c_3 \sqrt{\mathrm{Re}_w \, g(s)}$$

$$s = \frac{\lambda}{\delta} = \frac{1}{2a} \frac{\sqrt{\mathrm{Re}_w}}{Nu_\omega}$$

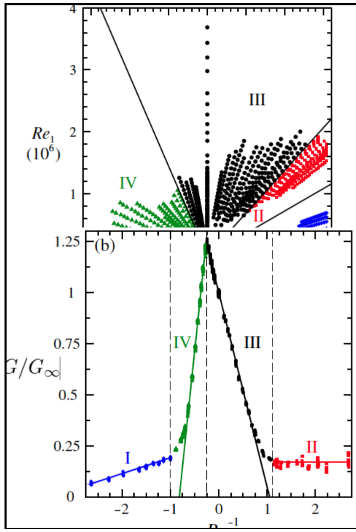
Predictions .....

Comparison to data

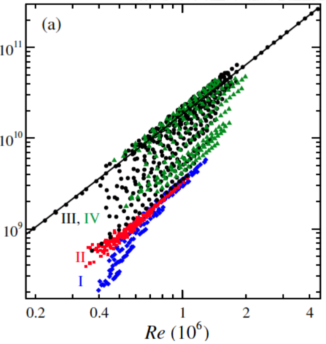


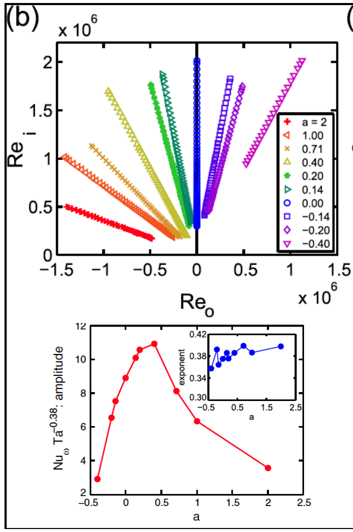


Smits & Zagarola

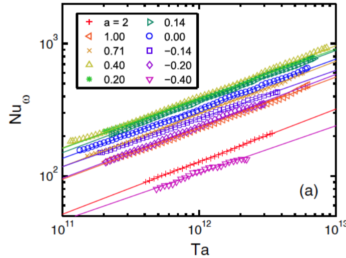


Paoletti + Lathrop,
PRL 2011





Van Gils et al
PRL 2011



Conclusions:

- Analogy between RB, TC and pipe clarified
- Corresponding currents and wind dissipation identified
- Analogous modelling assumptions successfully tested
- To do: Analysis of most recent data