Turbulence – and related problems

Lecture 1: Transition: Coherent structures
Lecture 2: Transition: Transient turbulence
Lecture 3: Global transport

Bruno Eckhardt

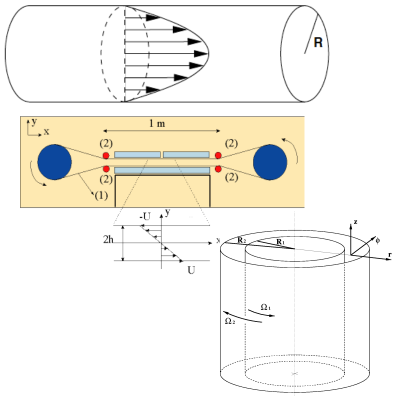
Sir Horace Lamb, Hydrodynamics 1932

It remains to call attention to the chief outstanding difficulty of our subject...

Unless the velocities ... be very small the actual motion in such cases ... is found to be very different from that represented by our formulae.

Although much has been written on the subject, the explanation of the practical instability of linear flow ... and of the manner in which the irregular eddies are maintained against viscosity, has yet to be found.

Shear flows:



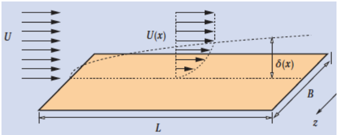
Sir James Lighthills tribute to Ludwig Prandtl

External aerodynamics was a disturbingly mysterious subject before Prandtl solved the mystery with his work on boundary layer theory from 1904 onwards.

The trouble had been by no means a lack of theory, but rather the existence of an overwhelmingly large body of theory, constructed by many of the best mathematical physicists of the nineteenth century, according to the most respectable physical principles.

This theory gave ... the fullest of information, *none of which accorded with the most elementary observation of the facts.*

Prandtl 1904:



$$\delta(x) = \sqrt{x\nu / U}$$

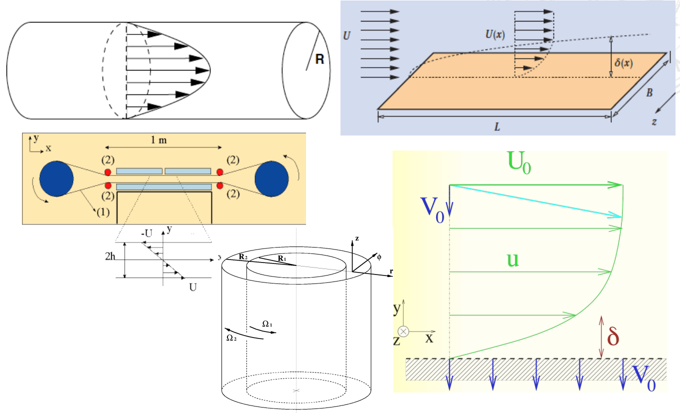
Local force proportional to
- free stream velocity
- 1/boundary layer thickness
- width . dx

$$dF \propto \rho\nu \cdot \frac{U}{\delta(x)} \cdot Bdx$$

$$F \propto 1{,}1 \rho B L U^2 / \sqrt{\mathrm{Re}}$$

$$\mathrm{Re} = LU/\nu$$

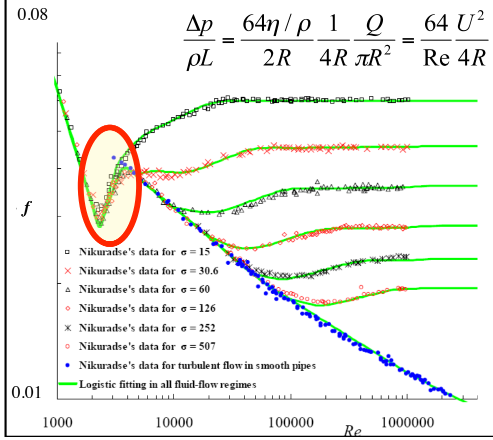
Shear flows:



Friction factors for pipe flow

$$\frac{\Delta p}{\rho L} = \frac{64\eta/\rho}{2R}\frac{1}{4R}\frac{Q}{\pi R^2} = \frac{64}{\mathrm{Re}}\frac{U^2}{4R}$$

$$\frac{\Delta p}{L\rho} = f\,\frac{U^2}{4R}$$



fit
for
different
roughness
values

BH Yang and
DD Joseph (2009)

Fully developed turbulence

- Kolmogorov scale $\eta_K = \left(\nu^3/\varepsilon\right)^{1/4}$
- Energy dissipation per mass:

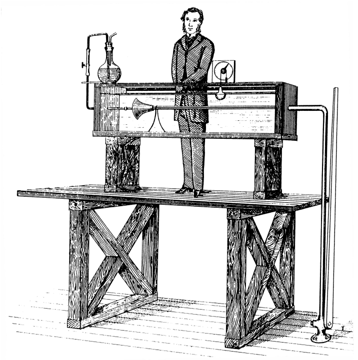
$$\varepsilon = \frac{\Delta p}{\rho L}U = f\frac{U^3}{2D} = \frac{\nu^3}{2D^4} f \,\mathrm{Re}^3$$

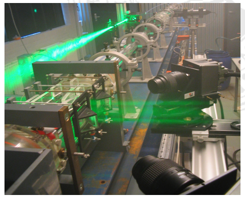
$$\frac{\eta_K}{D} = \left(f\,\mathrm{Re}^3/2\right)^{-1/4}$$

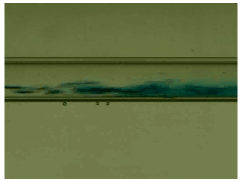
- At Re=5000:

$$\frac{\eta_K}{D} = 0.044 \approx 1/227$$

Reynolds' experiment (1883)







When does pipe flow become turbulent?

• Reynolds	1800
• Gerthsen (physics textbook)	2000
• Wikipedia	2300
• Fluid mechanics books	2000-3000
• Recent experiments (Mullin)	>1650
• Stöcker (Data reference)	1000-2500
• From a lab report: Laminar regime 2100 < Re < 4000	
	? ? ?

Stability against small perturbations … in theory:

Linearisation around laminar profile:

All sufficiently small perturbations decay at all Reynolds numbers !

…and as a patent:

http://www.freepatentsonline.com/3945402.html

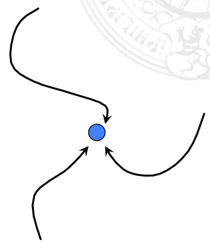
- Title: **Laminar flow pipe system**
- United States Patent 3945402
- Abstract: A pipe system for conveying fluids having a straight pipe section with a Reynolds number exceeding 2200 by reason of the critical values of interior surface roughness of the pipe and of turbulence at the pipe section inlet, such values being defined, by …
- Inventors: Murphy, Peter J. (Ithaca, NY, US)
  Application Number: 518035
- Filing Date: 10/25/1974
- Publication Date: 03/23/1976
- Claims:
  1. A fluid conveying system having **laminar** fluid flow and **a Reynolds number in excess of 2200**, comprising a straight pipe of circular cross-section having an inlet and outlet, and fluid turbulence control means …

  After making a study and a series of experimental investigations, I have found that by using the usual features of a pipe system and by, in addition, properly relating the internal roughness of the pipe and the fluid turbulence at the inlet of the pipe to the Reynolds number, **laminar flows can be obtained consistently at higher values of the Reynolds number than the usual 2200, e.g., 20,000** and higher. …

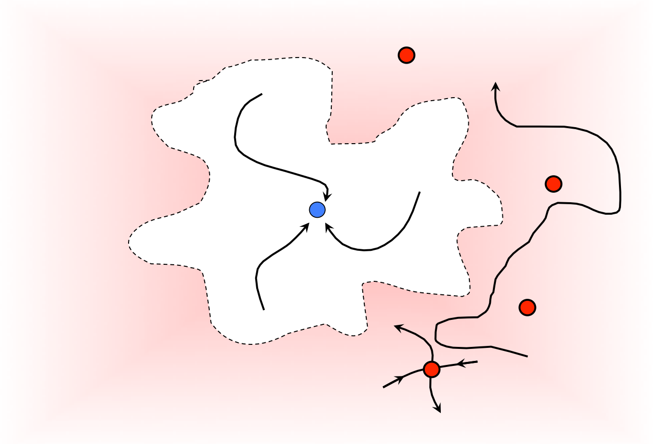
Hagen-Poiseuille in state space

State Space:
Space of all initial conditions for pipe flow

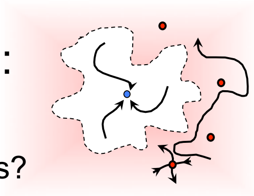
Attractor:
Object to which initial conditions are attracted



Turbulent pipe flow



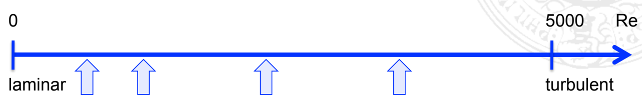
Questions:



- What are the new red states?
- Do they exist for all Reynolds numbers?
- Can they be seen in experiment?
- What is the nature of the boundary between laminar and turbulent?

Reynolds numbers for pipe flow

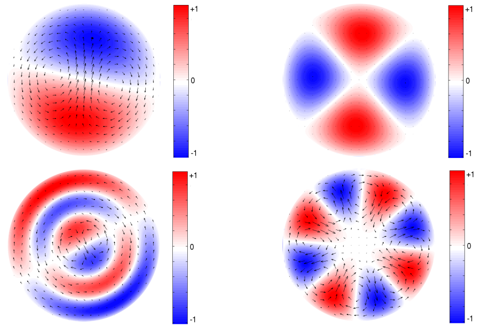
(not to scale)



Linear Analysis

Linear stability analysis

- HP-profile linearly stable for all Re
- eigenmodes show vortex-streak structure



Non-normal amplification
Lift-up effect

- Linearized Navier-Stokes equation

$$\partial_t \vec{u} + (\vec{u}_0 \cdot \vec{\nabla})\vec{u} + (\vec{u} \cdot \vec{\nabla})\vec{u}_0 = -\vec{\nabla} p + \nu \Delta \vec{u}$$

Write as

$$\partial_t \vec{u} = L\vec{u}$$

- Then L is not self adjoint and not normal, i.e.

$$L \neq L^+ \qquad LL^+ \neq L^+ L$$

Non-normal operators

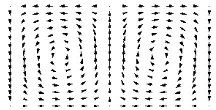
- Eigenvalues need not be real
- Eigenvectors are not orthogonal
- Left and right eigenvectors have to be distinguished
- The representation of small perturbations may need huge components along eigenvectors

Vortex-Streak interaction

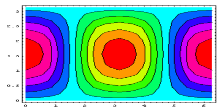
- Downstream vortices create spanwise streaks:

$$\begin{pmatrix} 0 \\ \beta\ \cos(\alpha y)\sin(\beta z) \\ -\alpha\ \sin(\alpha y)\cos(\beta z) \end{pmatrix} \qquad \begin{pmatrix} -\beta\cos(\alpha y)\sin(\beta z) \\ 0 \\ 0 \end{pmatrix}$$

Vortex



Streak



Vortex-Streak interaction

Downstream vortex (amplitude ω)
amplifies spanwise streak (amplitude s)

$$\partial_t \begin{pmatrix} s \\ \omega \end{pmatrix} = \begin{pmatrix} -1 & S \\ 0 & -1 \end{pmatrix} \begin{pmatrix} s \\ \omega \end{pmatrix}$$

$$s(t) = (s_0 + S\omega_0 t)e^{-t}$$

$$\omega(t) = \omega_0 e^{-t}$$

transient
linear growth !

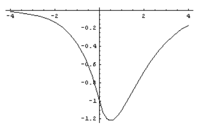
Lift-up in correlation functions

- Consider the noisy system and the correlation functions between downstream (u) and vertical (v) velocity component:

$$C(\tau) = \langle u(x, t+\tau) \quad v(x,\tau) \rangle_{x,t}$$

- Negative
- Asymmetric (streak u forms after vertical v)
- Extremum at positive τ

$$C(\tau) = \begin{cases} -Se^{\tau t} & \tau < 0 \\ -S(1+2\tau)e^{-\tau} & \tau > 0 \end{cases}$$



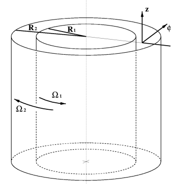
Waleffes turbulent cycle

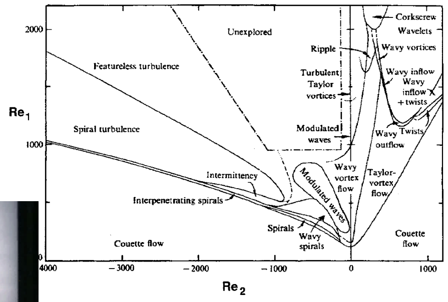
- Downstream vortices induce streaks by non-normal amplification
- Streaks undergo shear flow instability forming vortices in normal direction
- Normal vortices are rotated in downstream direction by background flow

Vortices dominate dynamics

Towards non-linear states

Taylor-Couette flow
Shearflow with centrifugal instability

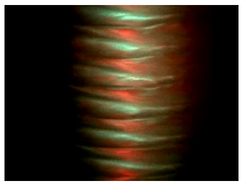


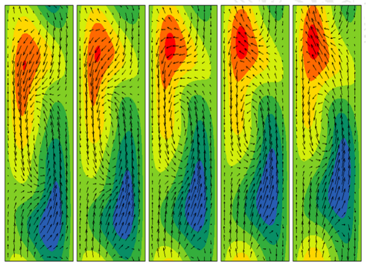


•Sequence of linear instabilities

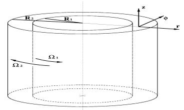
Wavy Vortex Flow

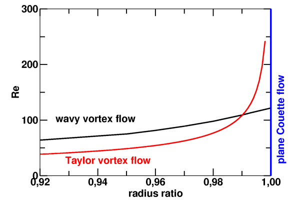
at higher rotation speed

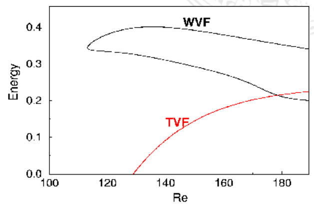




From Taylor-Couette flow to plane Couette flow





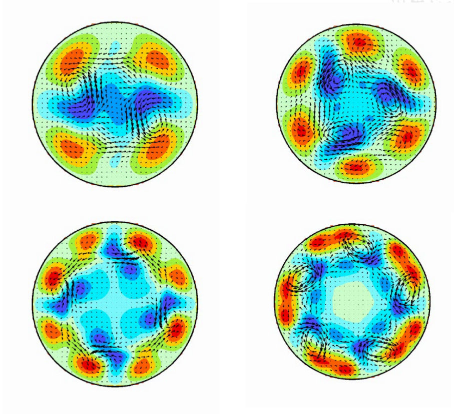


H. Faisst and B.E., PRE 61, 7227 (2000)

•Bifurcations exchange order and become subcritical

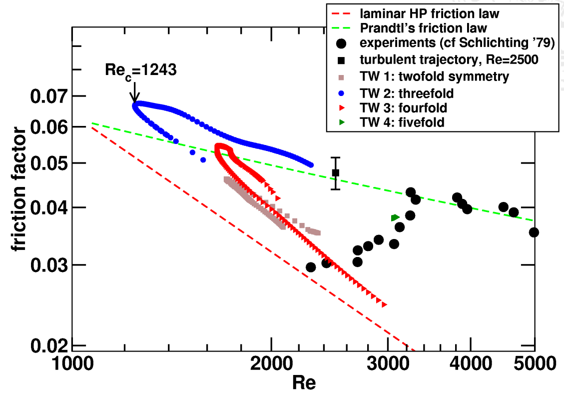
•Transition to turbulence changes its characteristics

Travelling waves in pipe flow

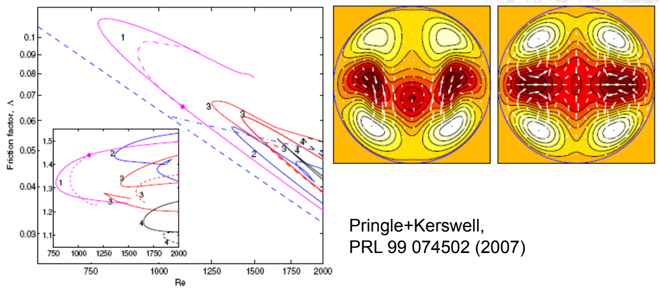


PRL 91 (2003)
224502

Travelling waves and turbulent friction



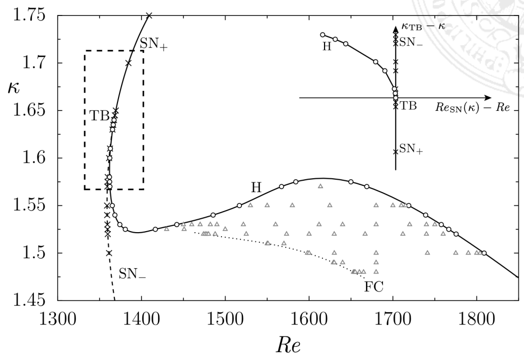
Further states…



Pringle+Kerswell,
PRL 99 074502 (2007)

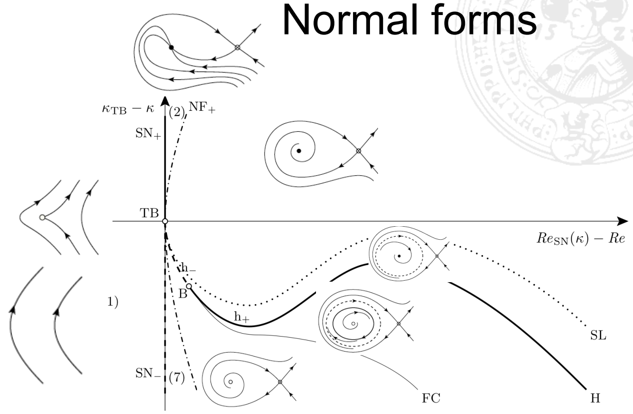
Mirror symmetric mode extends down to Re=773

Takens-Bogdanov bifurcation



F Mellibovsky and BE 2011, JFM

Normal forms



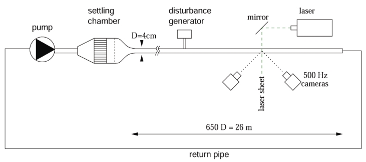
F Mellibovsky and BE 2010, JFM

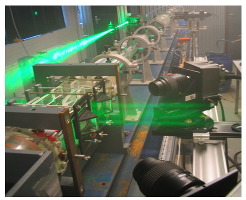
Summary

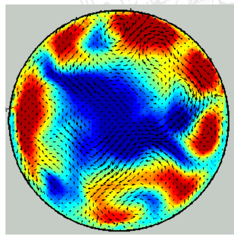
- Turbulence possible, once coherent structures are present in state space
- Open question: 3-d arise at Re as low as 773 why do experiments not see turbulence until Re about 1700?

Experimental evidence

Dynamics in cross sections





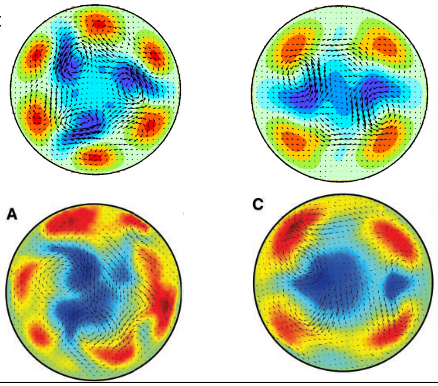


Cas van Doorne
Björn Hof
(Delft)

Coherent structures

Theory:
H. Faisst & B. Eckhardt
(PRL, 2003)

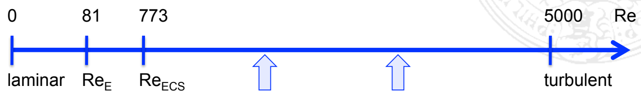
Experiment:
B. Hof et al.
(Science, 2004)



Key Reynolds numbers

	pCf	pPf	pipe
Re(E)	20.7	49.6	81.5
Re(lin)	∞	5772	∞
Re(turb)	320	1500	1800
Re(3D)	125	---	---
Re(TW)	150	~1000	1243
Re(UPO)	<400?	???	???

Reynolds numbers for pipe flow



Re_E Energy stability

Re_{ECS} Coherent structures appear

Literature:

Pipe flow:
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- JFM **504**, 343 (1994)
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Edge of Chaos:
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Models:
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- SIAM Dynamics **4,** 352 (2005)

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- Phys. Rev. E **61**, 7227 (2000)
- Europhys. Lett. **51**, 395 (2000)
- Phys. Rev. E **78,** 037301 (2008)
- Phys. Rev. E **81**, 015301 (2010)
- JFM **646**, 441 (2010)