recipes for preparing quantum states

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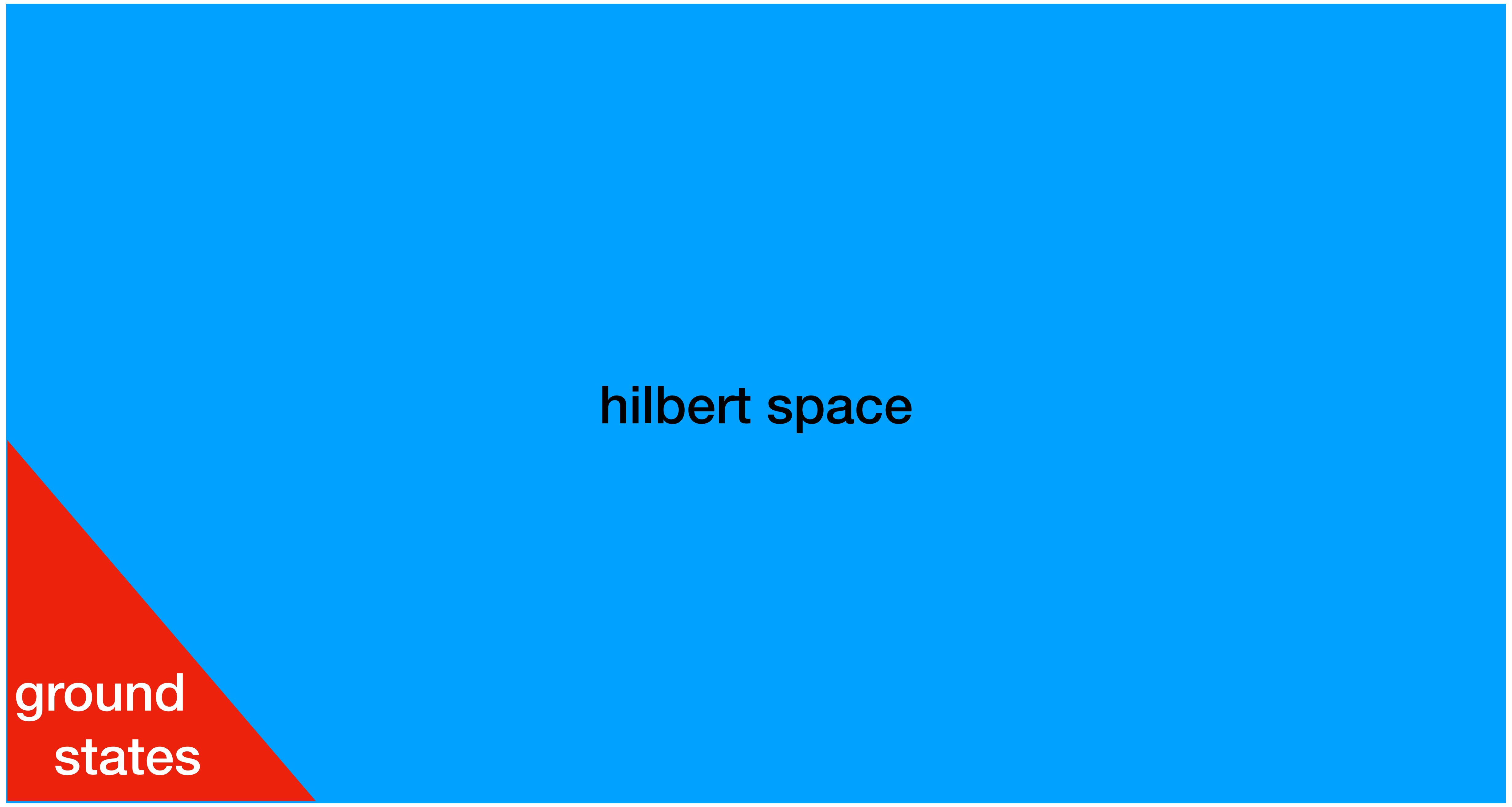
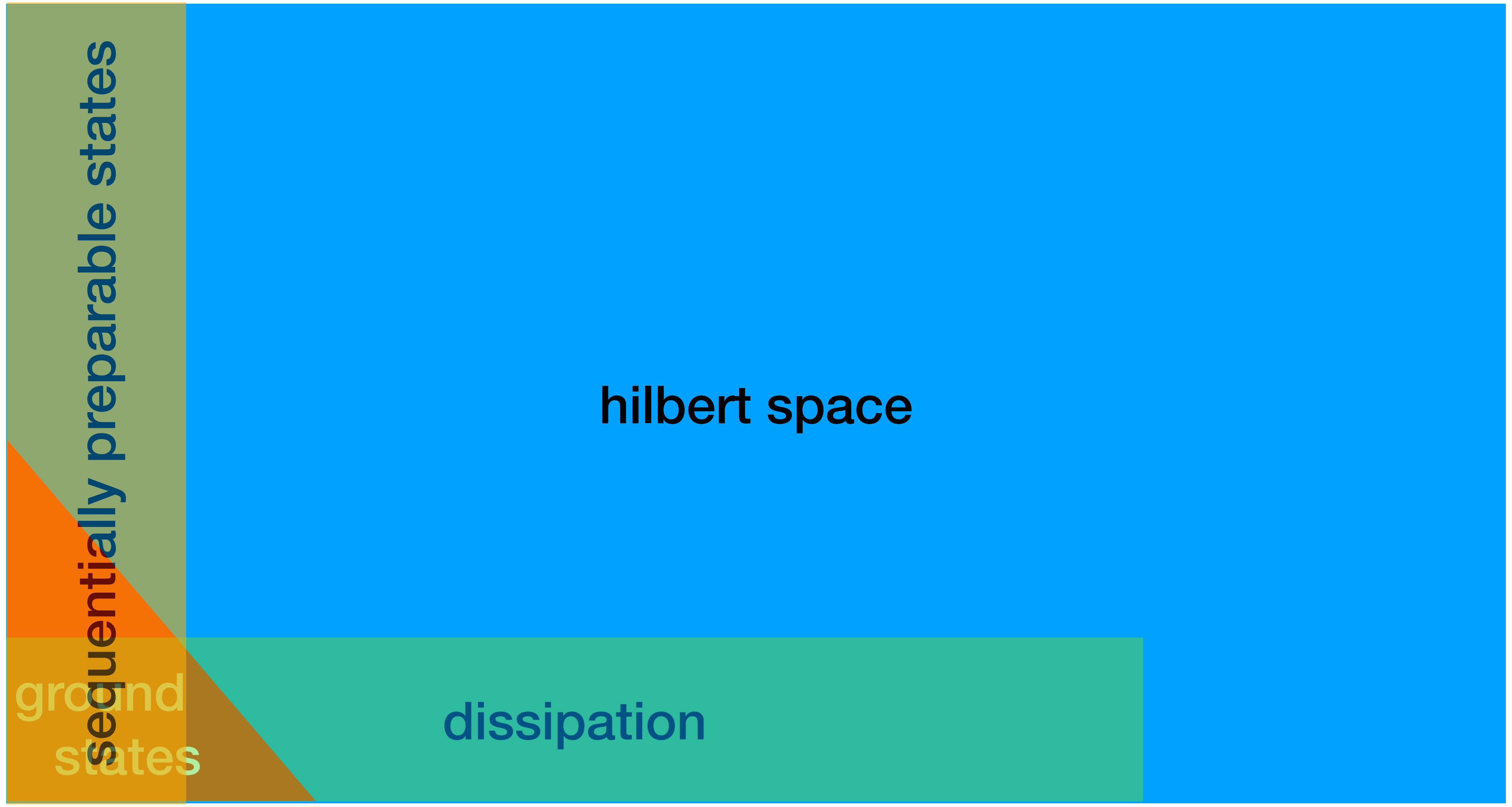
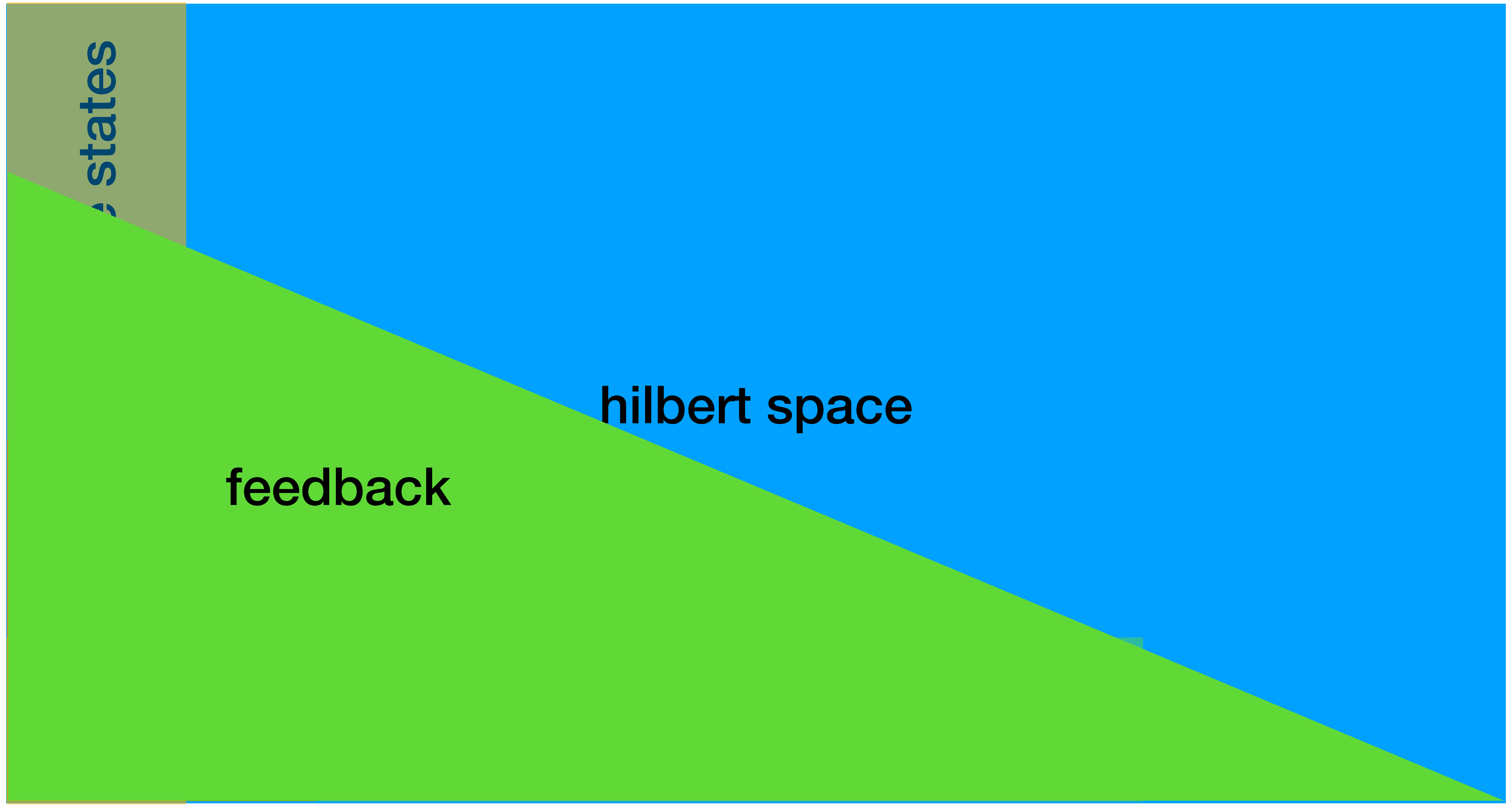
affordances of measurement

in design, affordance has a narrower meaning, it refers to possible actions that an actor can readily perceive.
(wikipedia)

So far, we've been concentrating mostly on types of matter found in nature, like neutron stars. But we can turn this around, and ask questions about design: what kinds of matter configuration can we stabilize in principle? In other words, instead of looking for naturally occurring substances, let's ask the question: what types of matter do the laws of physics enable us to design? ***In this designer's viewpoint you ask how you can engineer reality to fit what you want. And the emphasis isn't so much on the "engineer" as on "what you want".*** *Because it turns out that figuring out what is possible requires immense imagination.*
(michael nielsen, "maps of matter")


hilbert space
ground
states


hilbert space
sequentially preparable states
ground states
dissipation


states
hilbert space
feedback

measurements and (classical) feedback

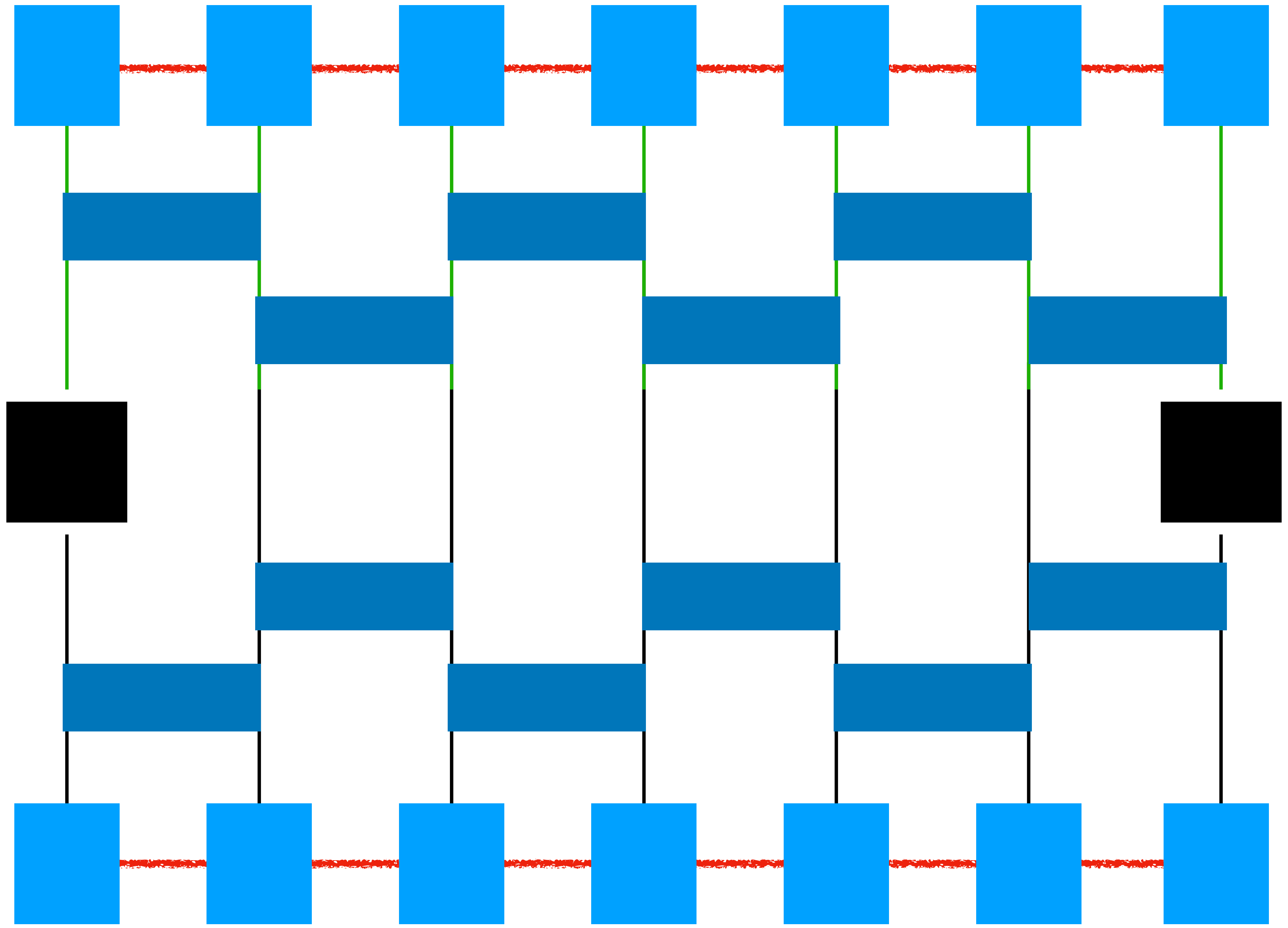
- As components in finite-depth circuits: what power do they have that unitaries don’t?
- As ways of defining processes that lead to steady states: what steady states are possible?
- As ways of manipulating the entanglement pattern of a preexisting quantum state

light cones

the “finite-depth” question

- Given an initial state, what can you prepare using a finite-depth circuit (i.e., a circuit that remains finite depth in the thermodynamic limit)?

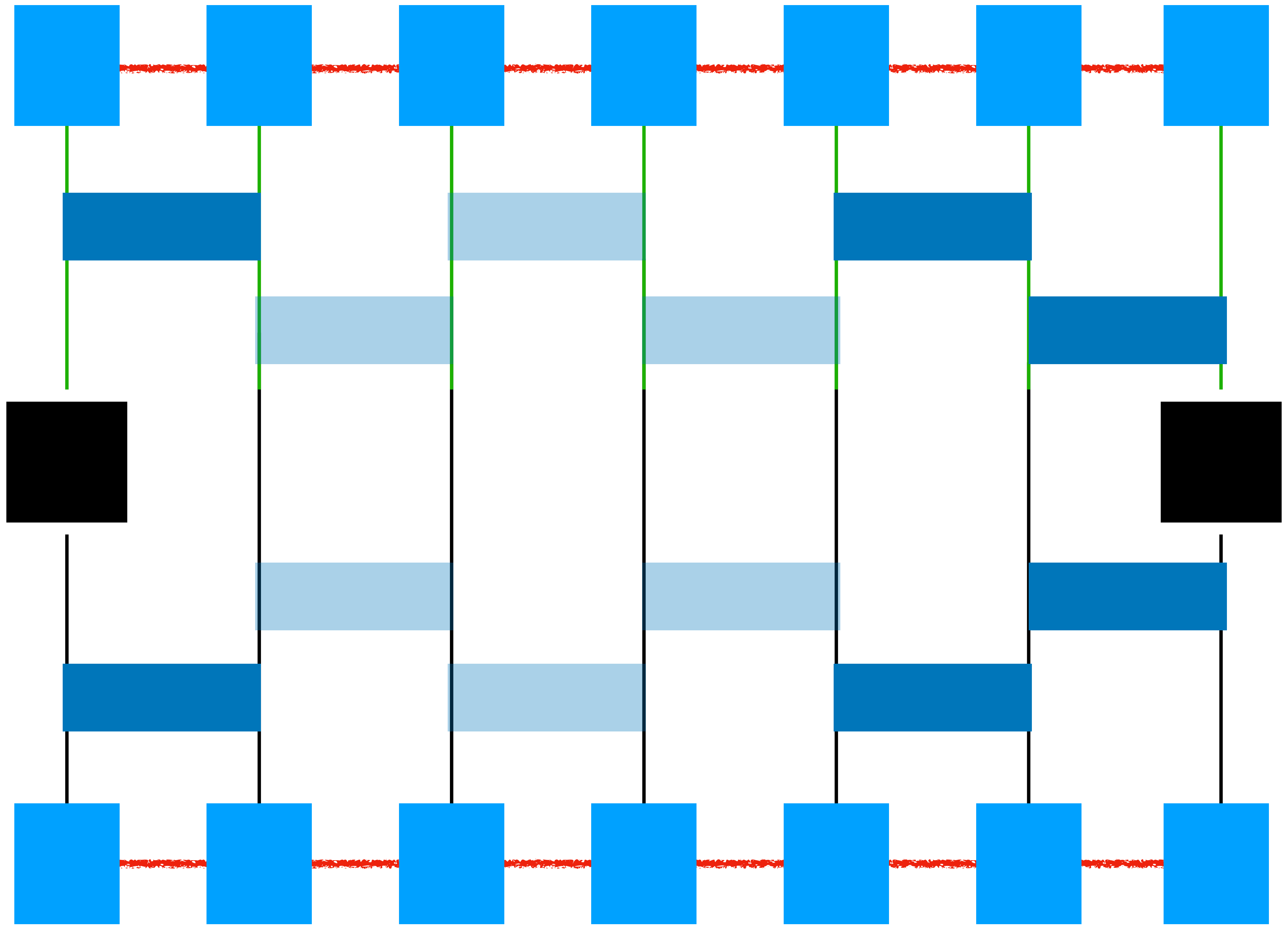
- Simple constraints for unitary circuits starting from MPS:



the “finite-depth” question

- Given an initial state, what can you prepare using a finite-depth circuit (i.e., a circuit that remains finite depth in the thermodynamic limit)?

- Simple constraints for unitary circuits starting from MPS: light-cones



finite-depth question for channels

- CPTP property for channels leads to Kraus form

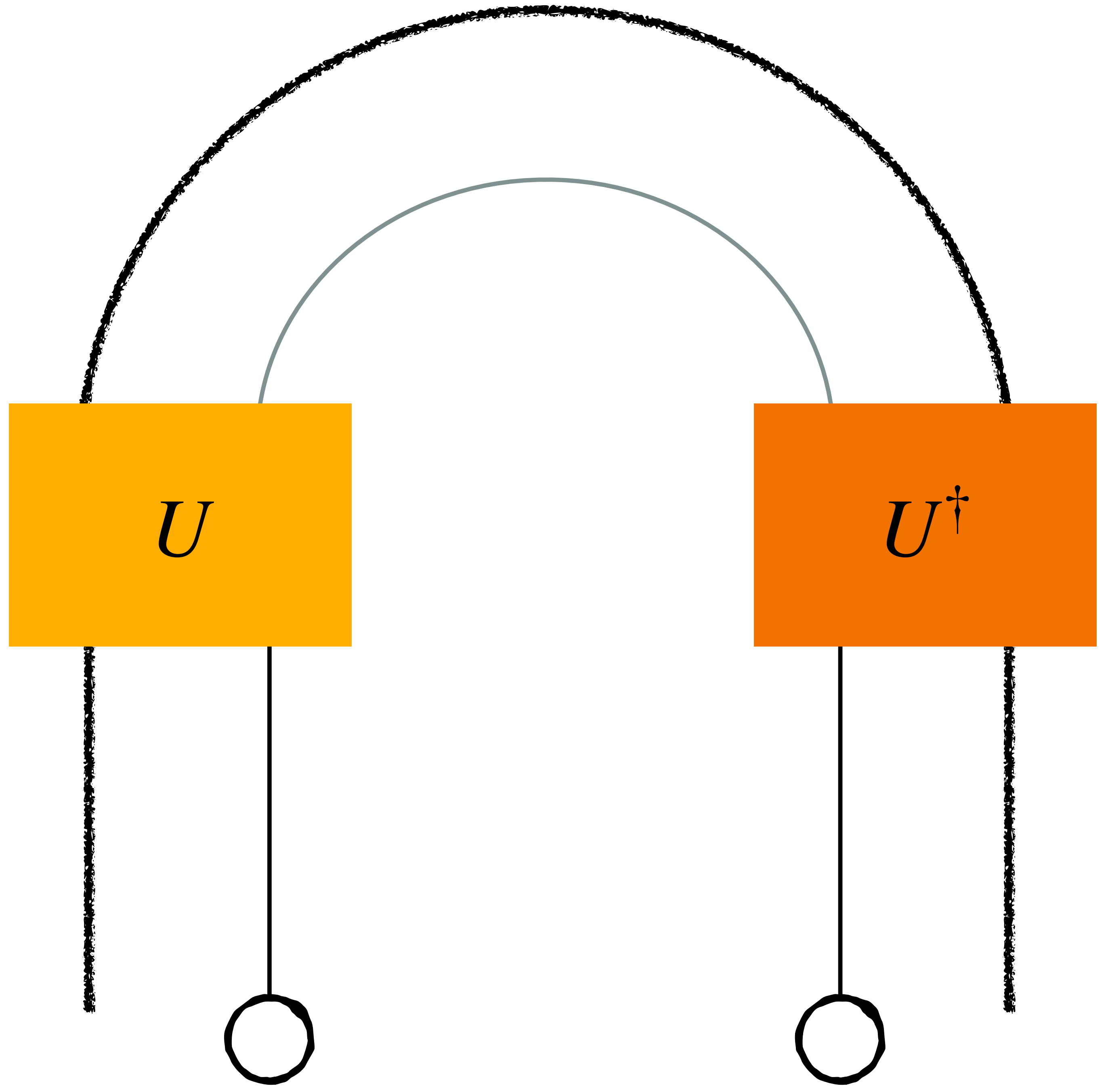
$$\mathcal{E}(\rho) = \sum_i M_i \rho M_i^\dagger, \quad M_i^\dagger M_i = \mathbb{1}$$

- Operator expectation values evolve as

$$\mathrm{Tr}(O\mathcal{E}(\rho)) = \sum_i \mathrm{Tr}(OM_i\rho M_i^\dagger) = \mathrm{Tr}\left[\left(\sum_i M_i^\dagger O M_i\right)\rho\right] \equiv \mathrm{Tr}(\mathcal{E}^*(O)\rho)$$

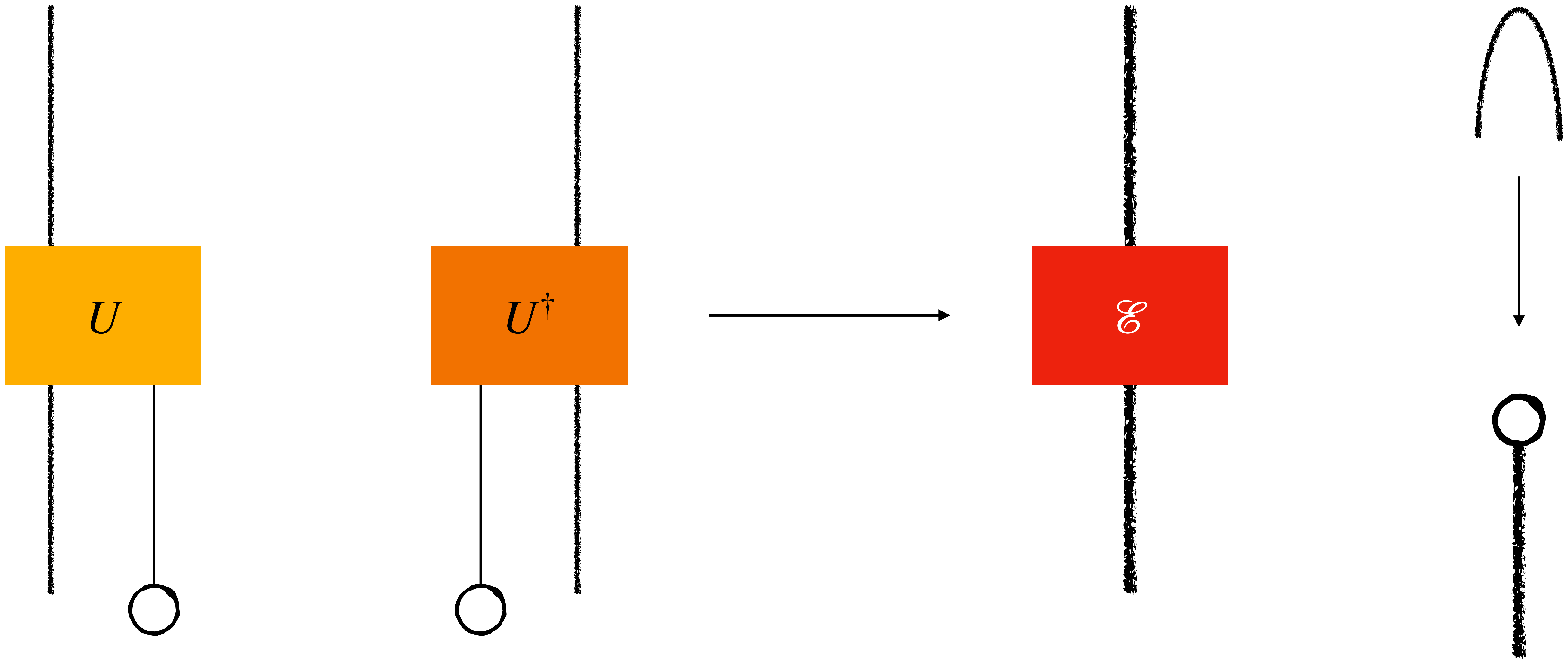
- Operator evolution under channels is “unital” i.e. preserves the identity

- Circuits composed of local channels also have light cones



finite-depth question for channels

- Folded circuit notation for channels



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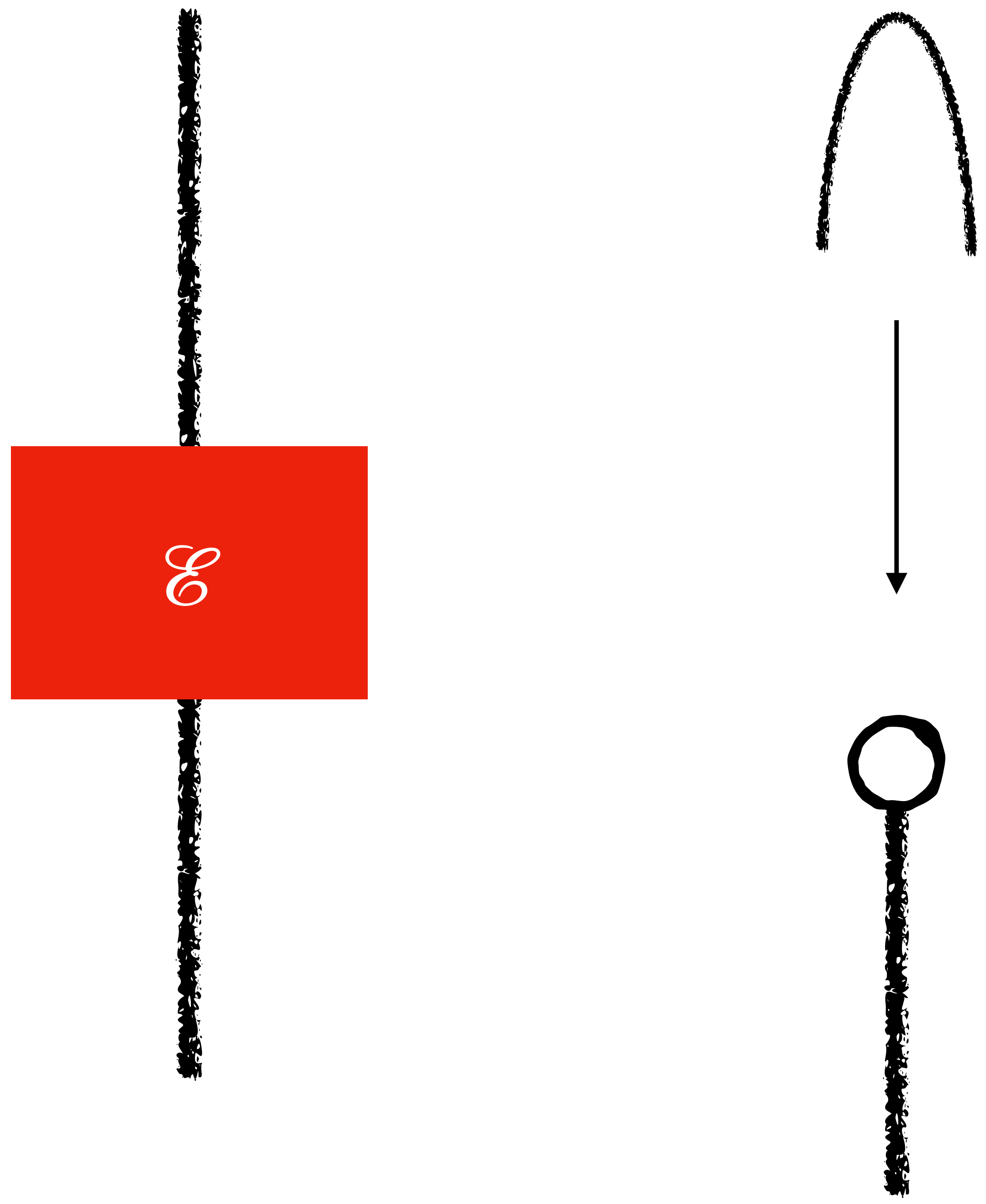

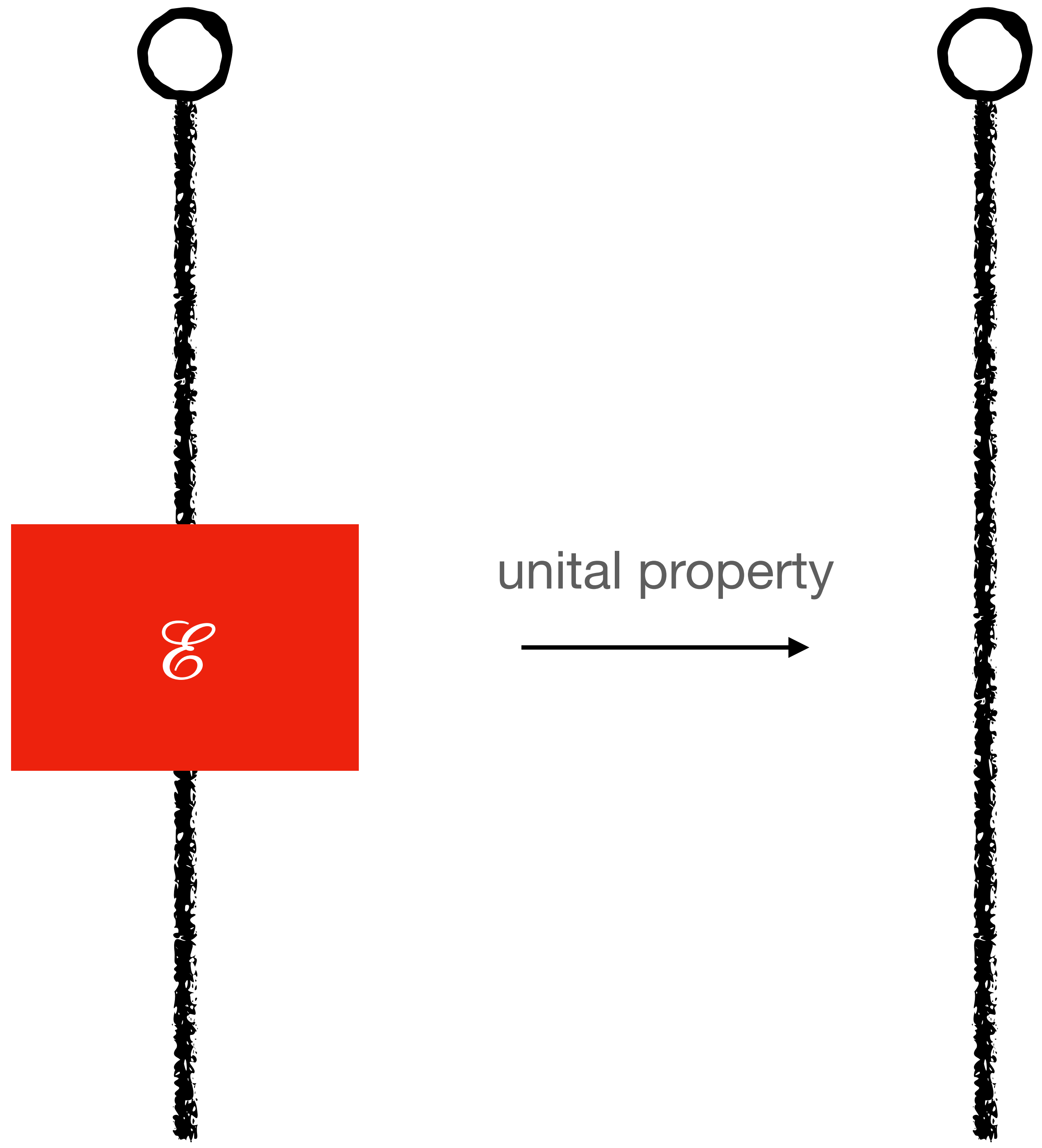

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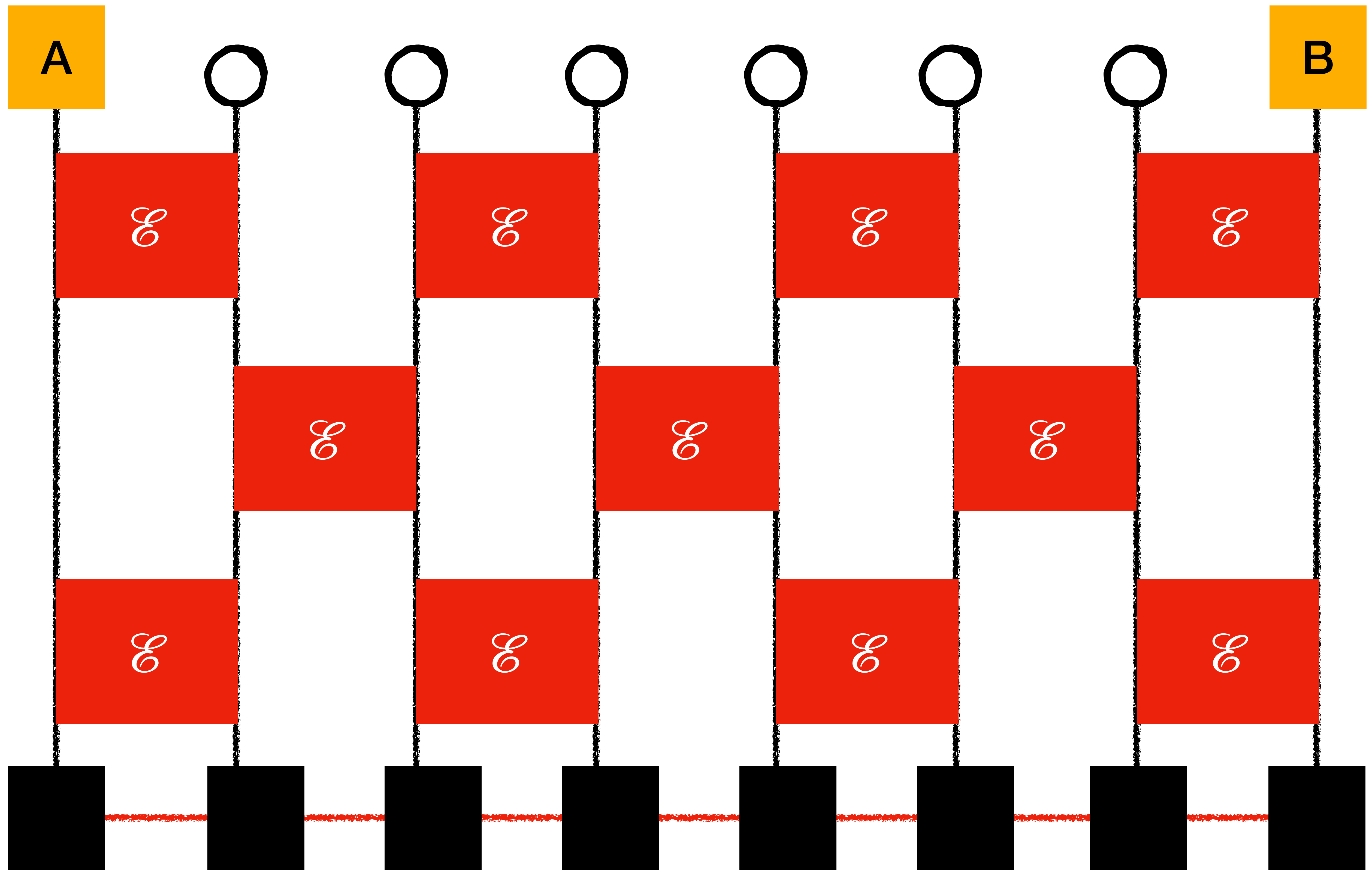

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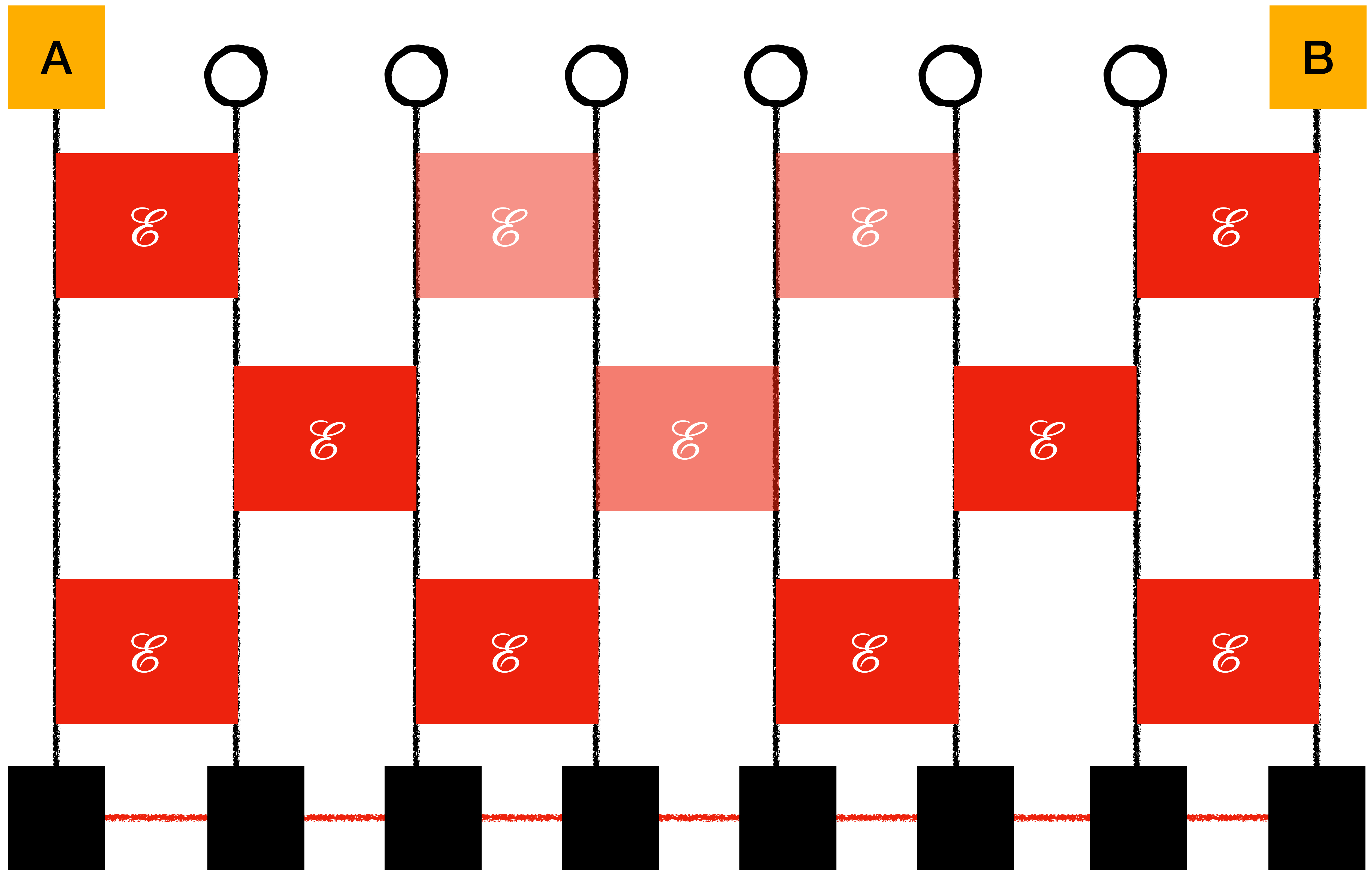
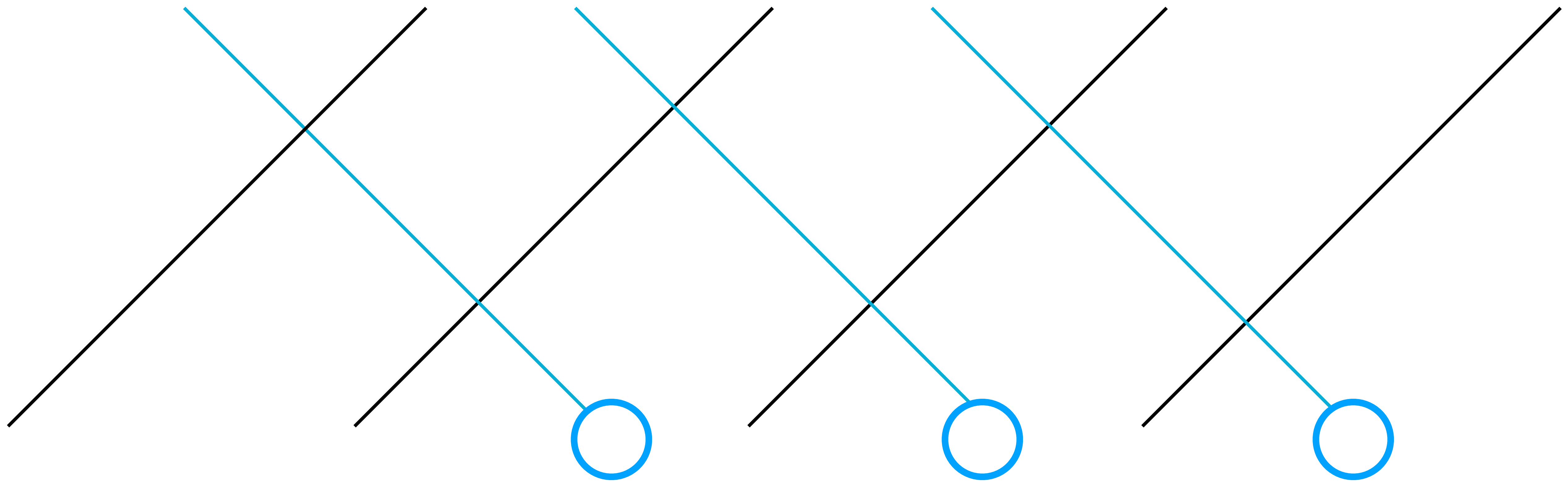


channels have light cones and cannot change asymptotics of correlations at finite depth

nontrivial QCAs as channels

- In the Heisenberg picture, operator evolution under local channels is constrained by light cones: so they cannot create nontrivial states at finite depth
- But channels can implement operator evolution that corresponds to nonlocal unitaries
- General family: “QCAs”, nontrivial unitaries such that many copies are trivial (e.g. translation) [I learned this from Ruben Verresen]



beating light cones with nonlocal classical communication

quantum teleportation

Consider a state consisting of Bell pairs:

Now perform (simultaneous) Bell measurements between neighboring pairs.

For each outcome of all the Bell measurements, you create a Bell state on the end qubits.

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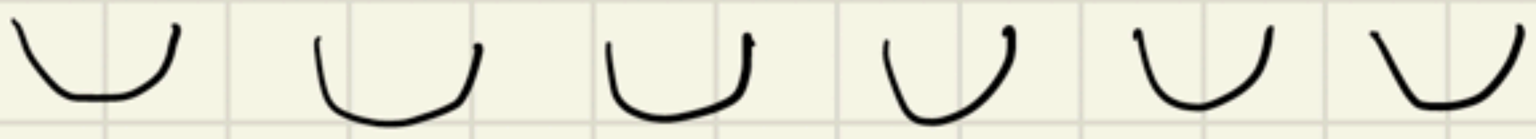

Now perform (simultaneous) Bell measurements between neighboring pairs.

For each outcome of all the Bell measurements, you create a Bell state on the end qubits.

But you need to use the full measurement record to know <u>which</u> Bell state

Once you know <u>which</u> state you can rotate it back to a reference Bell state.

→ using measurements & <u>nonlocal</u> classical processing you can beat light cones.

teleportation (and feedback) as a channel

- Each trajectory consists of Bell measurements followed by a (classically) conditioned unitary

$$U(\vec{i})_n \Pi^{i_{n-1}}_{n-1,n-2} \Pi^{i_{n-3}}_{n-3,n-4} \ldots \Pi^{i_3}_{32}$$

- The full channel can be written as

$$\mathcal{E}(\rho) = \sum_{\vec{i}} U(\vec{i})_n \tilde{\Pi}^{\vec{i}}_{2\ldots n-1}$$

- Although each trajectory is a product, the channel is not a product because of the classical dependency structure
- Channels like this are called separable
- An important class of separable channels can be implemented via local operations and classical communication (LOCC)
- Note: the measurements seem to do the hard work of moving entanglement around, but the “CC” is essential to get a determinate state at the end of the process (and to avoid the light cone bound)

nonlocal info and quantum error correction

- Recovery channels for QEC tend to be of this form: measure all the syndromes, process classically, find a unitary (e.g., matching anyons)
- Can one correct errors without nonlocality (with “cellular automaton decoders”)? Currently unclear, examples exist mostly in cases where passive error correction is possible
- Classically, there are fault-tolerant Markov chains even in 1D [Gacs 1983], so the space of nontrivial steady states does not coincide with finite-temperature order
- Many open questions about the space of steady states of quantum channels…

measurement-induced entanglement

bell pairs revisited

- In the pre-measurement state, $I(A : B) = 0$ since there are no correlations across M

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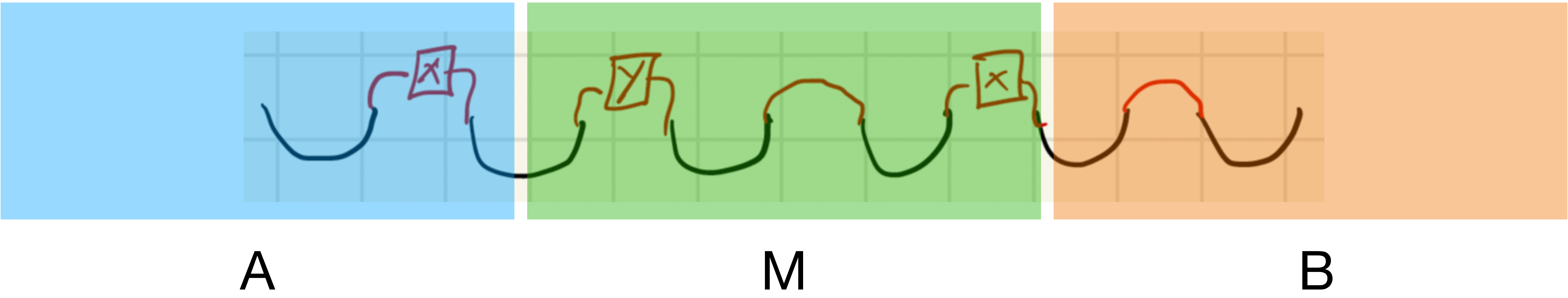
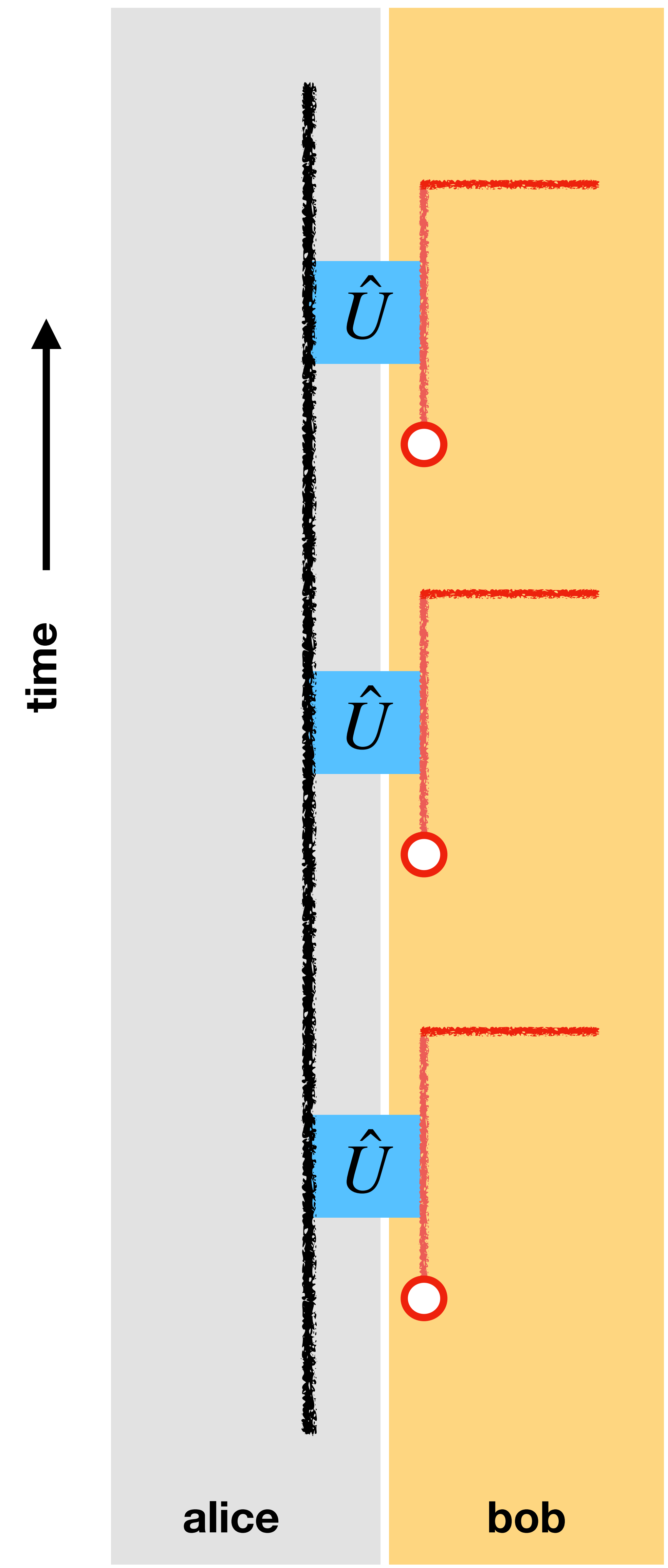


- Post-measurement, $I(A:B) = 2S(A) = 2$ (in bits): measurements moved correlations across M

channels revisited



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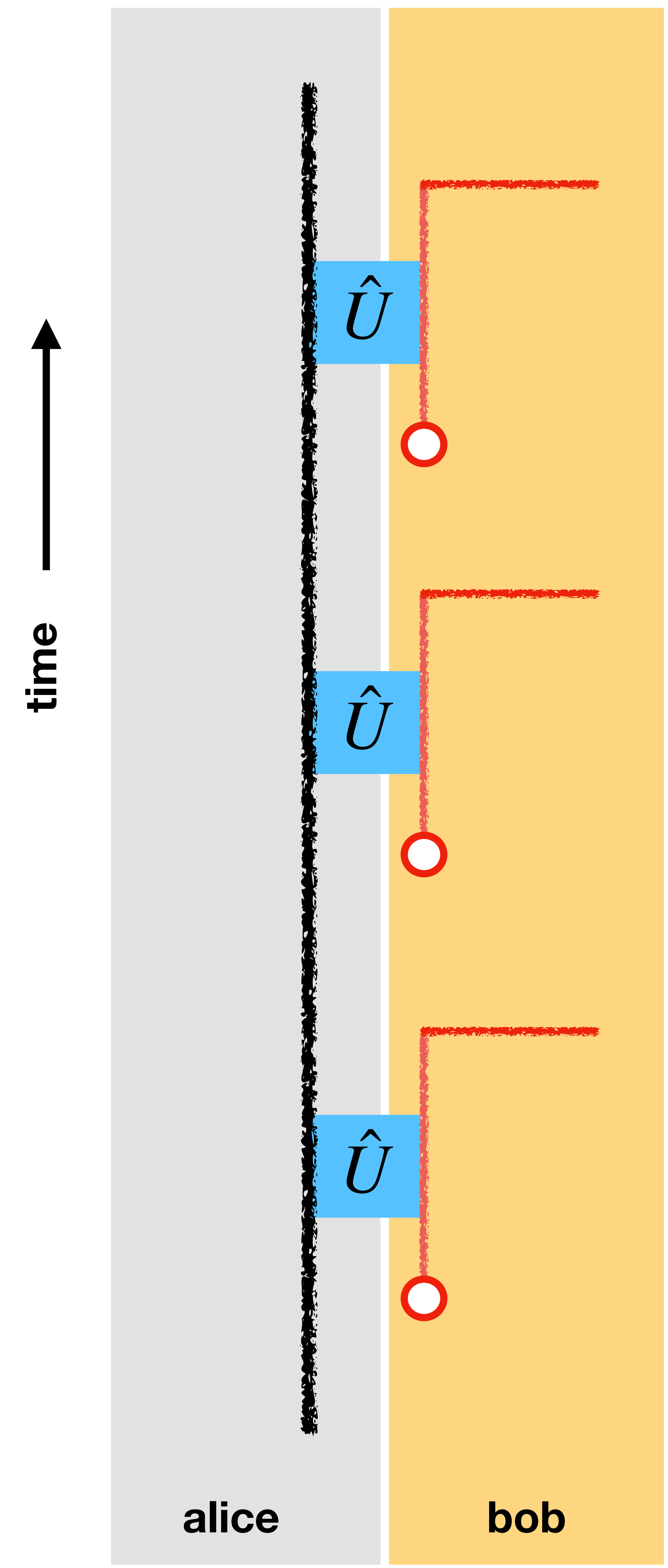
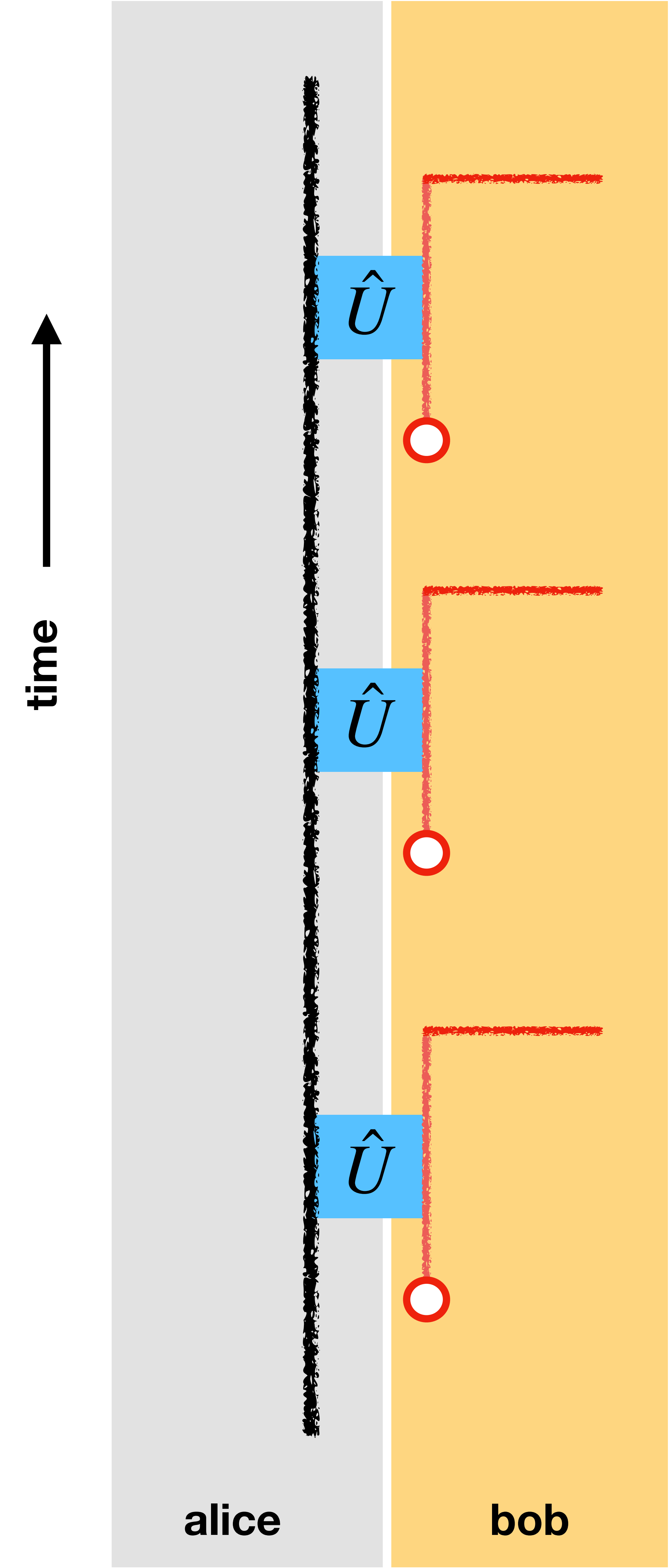


channel perspective: Bob's qubits are lost, have to be traced over

channels revisited



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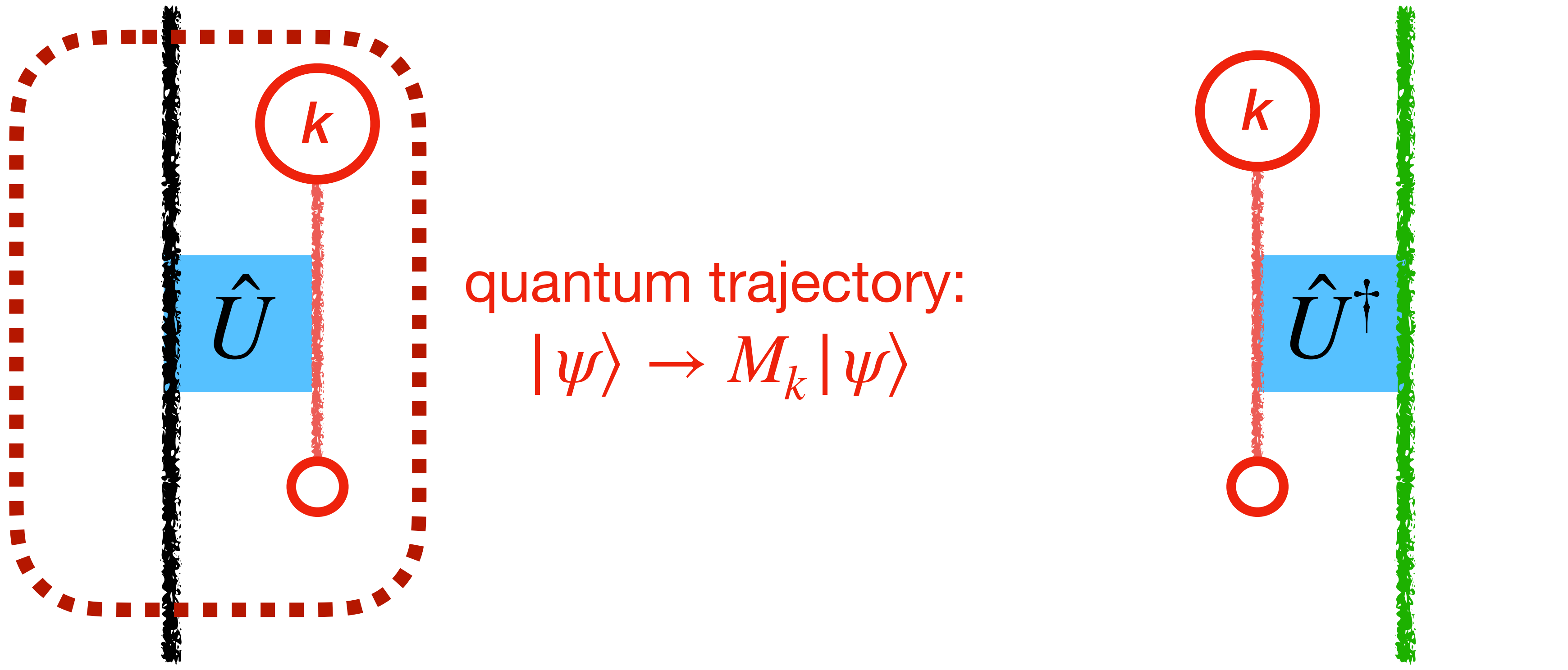
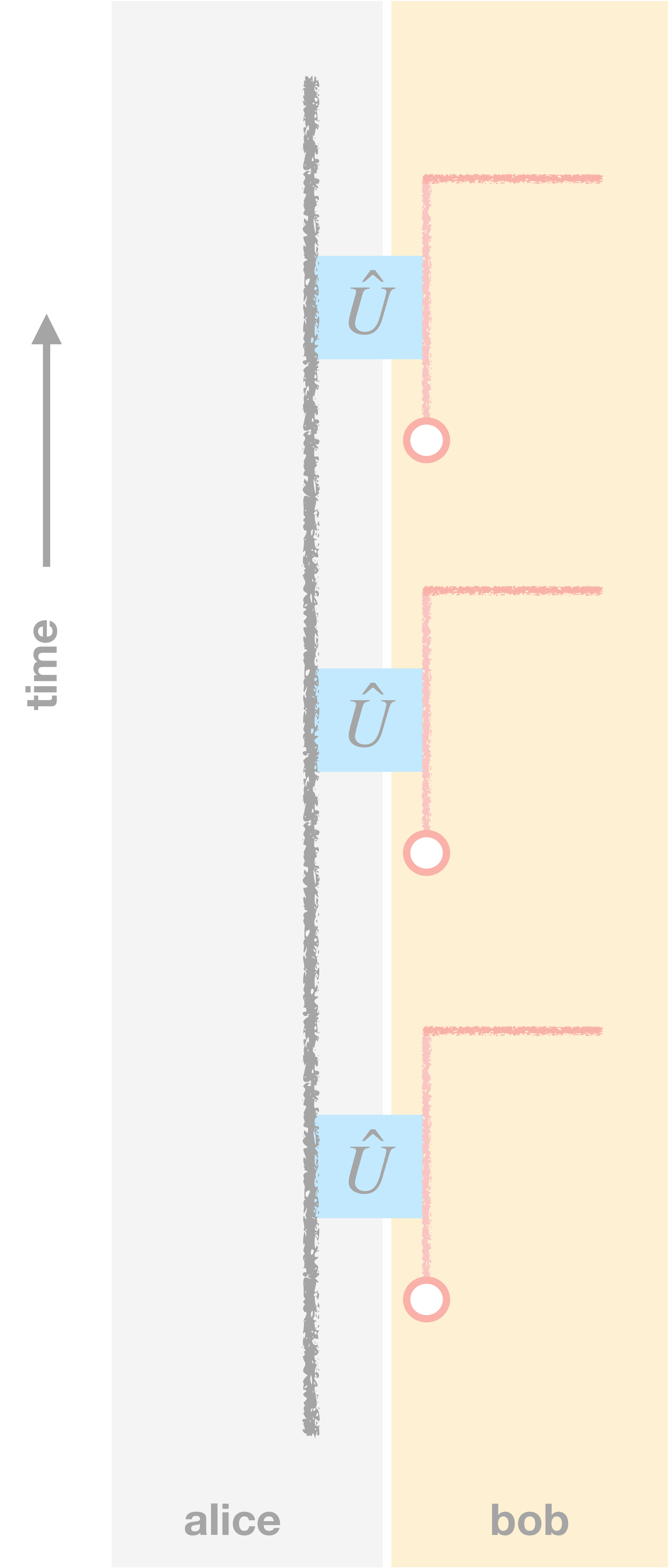
POVM perspective: Bob measures



channels revisited (and seen as MPSs)



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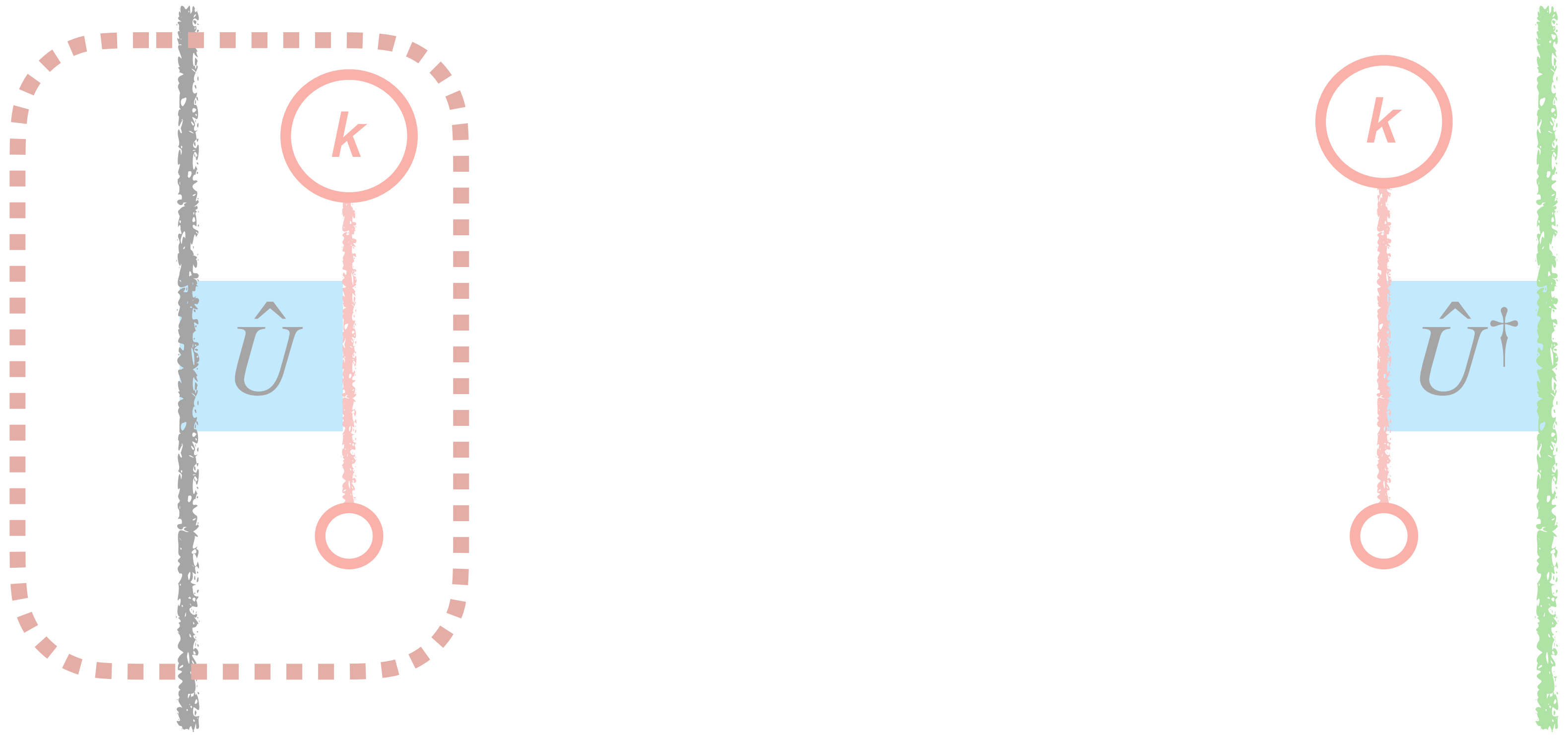
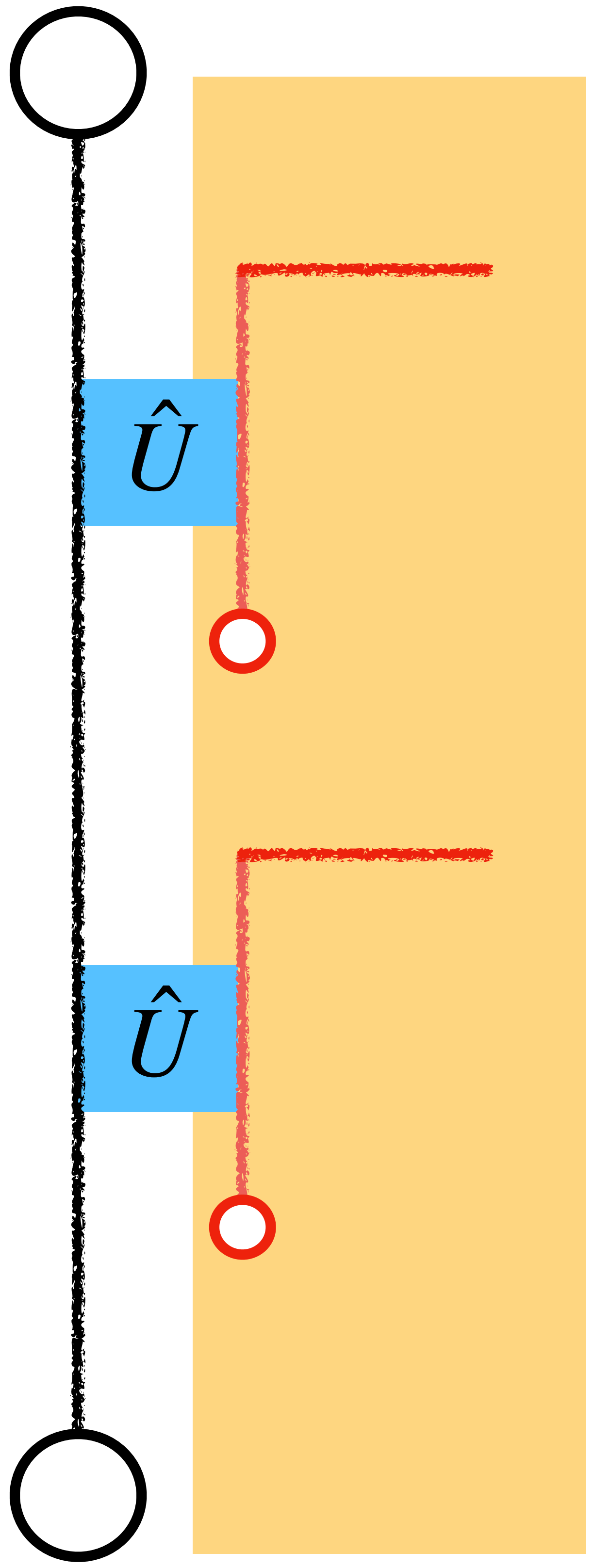
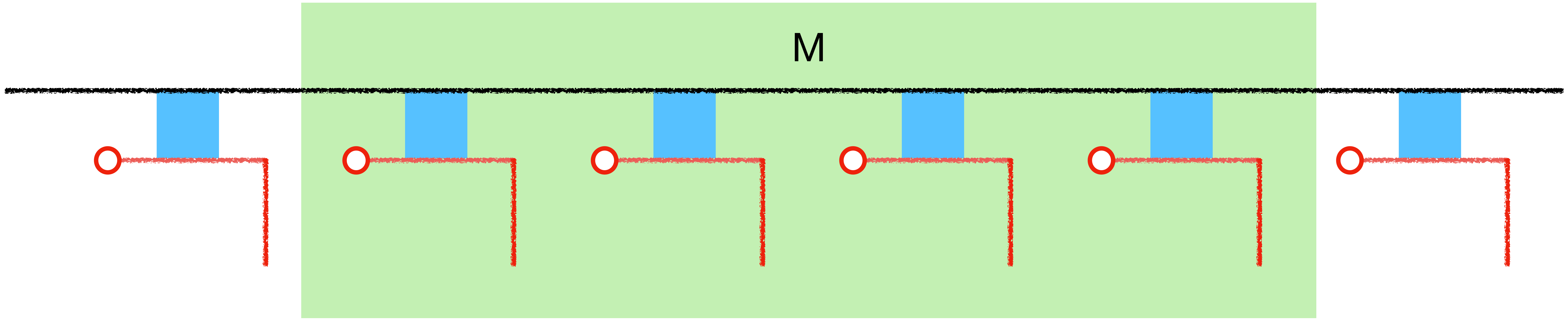


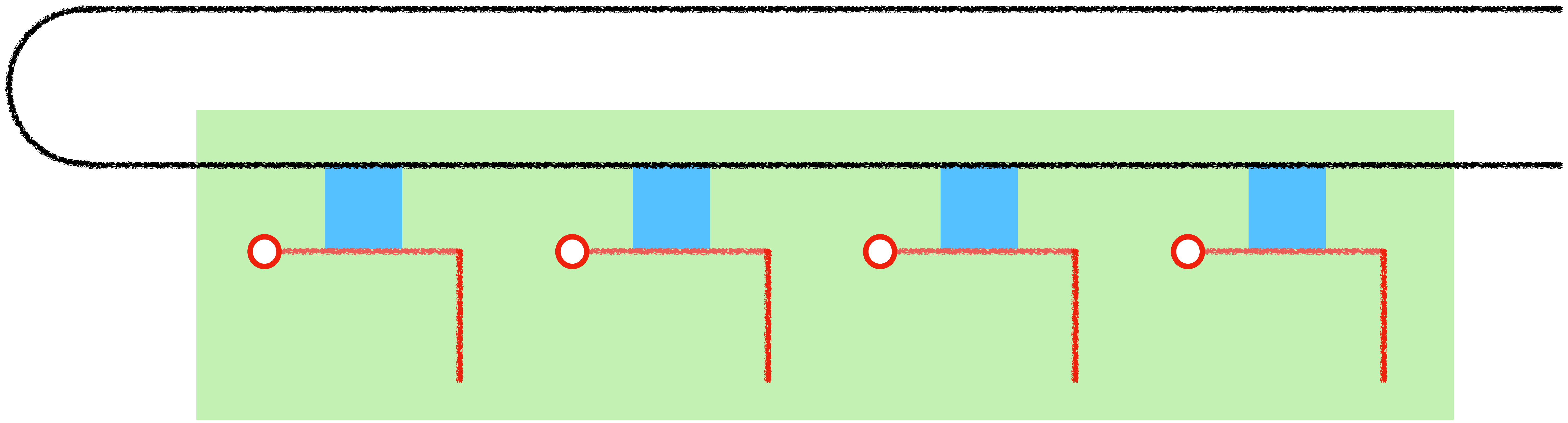
dual perspective: Bob harvests qubits and collects an entangled state (MPS)



MIE for an MPS



Correlations across M flow through the black wire, so you can think about MIE in terms of this diagram instead



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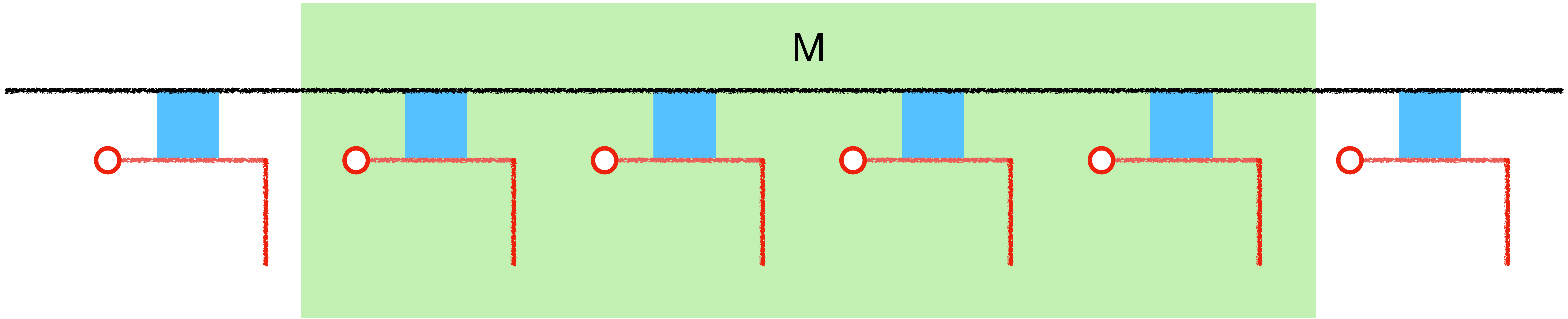

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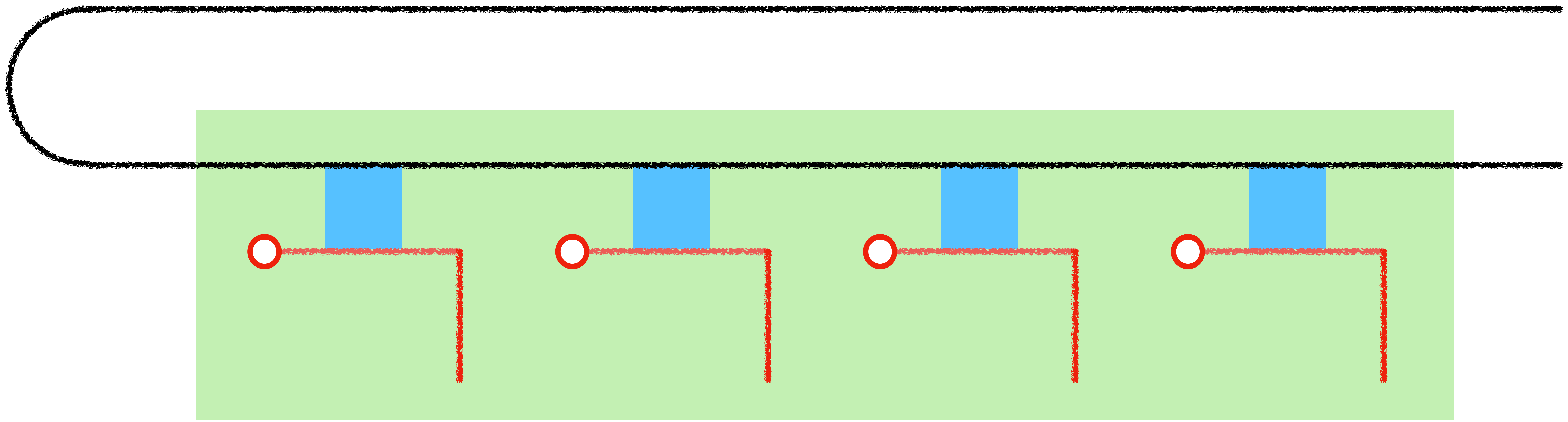
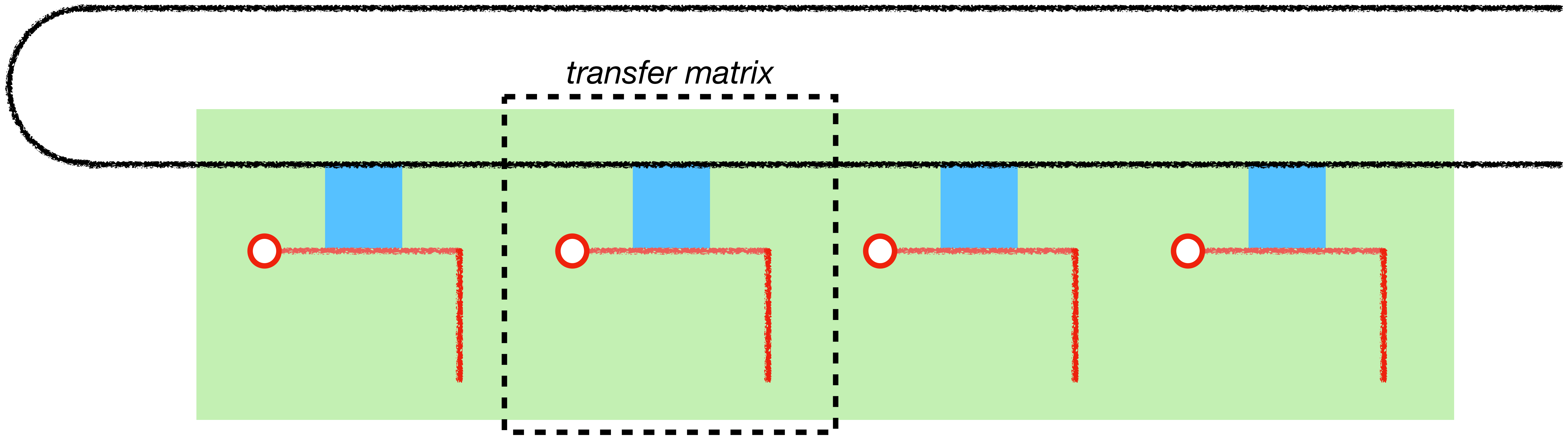


But this is just purification (in terms of the bond space)

some implications



- For a 1D MPS the transfer matrix is 0D, so iterating the transfer matrix gives a pure state (except for “resource states” like SPTs)

- For a 2D state sliced up into an MPS, the transfer matrix is 1D and this maps (more or less) onto the 1+1D circuit, which has a purification transition

- “Sideways” understanding of the mixed phase: long-range MIE (“teleportation”; Bao et al.), highly entangled transfer matrix (hard to sample measurement outcomes; Napp et al.), “volume-law” entanglement if you measure everything but the last slice

- Connection to “deep thermalization” (Choi, Ho, Ippoliti…)

some loose ends…

- General relationship between quantum states and lower-dimensional processes
- Spacetime dualities, temporal entanglement, and related questions
- Non-Markovianity
- Connections to classification of phases of matter