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Let's add a spin to the electron in a magnetic field problem:

$$H_D = v_F \vec{p} \cdot \vec{\sigma} \longrightarrow H_D^B = v_F \underbrace{\left(\vec{p} + \frac{|e|}{c} \vec{A} \right)}_{\vec{\pi}} \cdot \vec{\sigma}$$

where $\vec{\sigma} = \langle \sigma_x, \sigma_y \rangle$. In the Landau gauge, $\vec{A} = \langle 0, Bx, 0 \rangle$,

$$[\pi_x, \pi_y] = -i \frac{|e| \hbar}{c} B = -i \frac{\hbar^2}{\ell^2} ,$$

where ℓ is the magnetic length. Define ladder operators for $\vec{\pi}$,

$$a = \frac{\ell}{\hbar \sqrt{2}} (\pi_x - i \pi_y) ,$$

which satisfy $\left[a, a^\dagger \right] = 1$. Then $\pi_x = \frac{\hbar}{\ell \sqrt{2}} \left(a + a^\dagger \right)$ and $\pi_y = -i \frac{\hbar}{\ell \sqrt{2}} \left(a - a^\dagger \right)$. Hamiltonian then becomes

$$H_D^B = \hbar \omega \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix} ,$$

where $\omega = \sqrt{2} \frac{v_F}{\ell}$. This is the interaction term in the Jaynes-Cummings model and can be solved to obtain

$$\epsilon_{n,\pm} = \pm \hbar v_F \sqrt{\frac{2 |e|}{\hbar c}} \sqrt{Bn} = O\left(\sqrt{n} \right)$$

We recall the LL problem for two "Schrodinger fermions" $\epsilon_n \propto \pm \left(n + \frac{1}{2} \right)$. Key differences are $\sqrt{n}$ dependence and the presence of a zero energy Landau level for the Dirac fermion case.