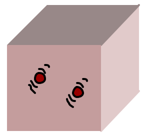
## Microrheology using microscopy

Eric R. Weeks
Emory University (Physics)

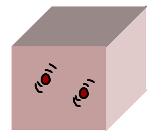
Daniel Chen (UPenn)
∗John Crocker (UPenn)
Alex Levine (UCLA)
Vikram Prasad (Emory)
David Weitz (Harvard)
Arjun Yodh (UPenn)
… others



Funding by NSF-CAREER, NASA/PECASE,
http://www.physics.emory.edu/~weeks/lab/

## Key Idea:

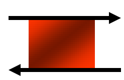
Use embedded tracer particles to measure material properties (viscoelastic moduli)



## Rheology: measuring response of material to stress

Elastic (Hookean) solid:
$\sigma \sim \gamma$ (stress proportional to strain)

Viscous (Newtonian) fluid:
$\sigma \sim \dot{\gamma}$ (stress proportional to strain rate)



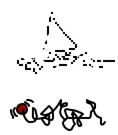
Viscoelastic material:
apply oscillating strain $\gamma_0 \sin(\omega t)$, measure stress

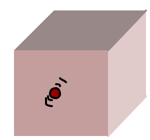
$$s(t) = g_0 [G'(w)\sin(wt) + G''(w)\cos(wt)]$$

elastic modulus — viscous modulus

## Microrheology: observe tracer particle motion

TG Mason & DA Weitz, PRL '95





Newtonian fluid:

$\langle \Delta r^2 \rangle = 6 D \Delta t$

$D = k_B T / 6\pi\eta a$

Elastic medium:

$\langle \Delta r^2 \rangle \sim k_B T / Ga$

as $\Delta t \to \infty$

## Microrheology: observe tracer particle motion

use generalized Stokes-Einstein equation

 — $\langle \Delta r^2 \rangle (\Delta t)$ — $G'(w), G''(w)$

- Measure $\langle \Delta r^2 \rangle (\Delta t)$ using microscopy or light scattering
- Frequencies $w$ are set by longest & shortest $Dt$ measured
- If $G'$, $G''$ differ by more than factor of 10, hard to measure smaller component

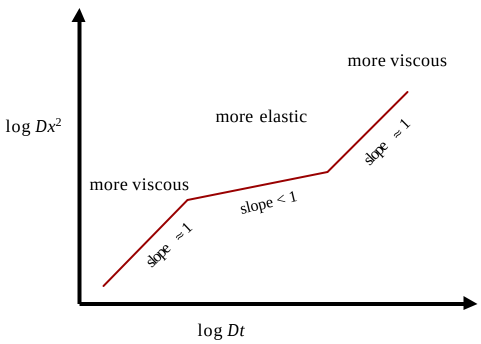
## Calculating mean square displacement

Measure $x_i(t)$ for different particles $i$

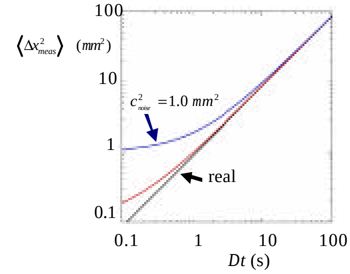
$$\Delta x_i(t, \Delta t) \equiv x(t + \Delta t) - x(t)$$

$$MSD(\Delta t) = \left\langle \Delta x_i^2 (t, \Delta t) \right\rangle_{t,i}$$

average over all particles $i$, all initial times $t$



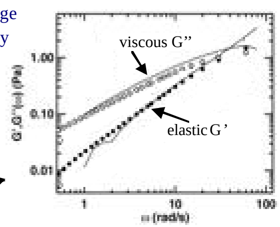

# Don't forget noise: $\langle \Delta x^2_{measured} \rangle = \langle \Delta x^2_{real} \rangle + \varepsilon^2_{noise}$




simulated diffusion of 1.0 μm diameter particles

# Advantages of microrheology

- Can use small samples
- Local probe of viscoelasticity
- Can have higher frequency range
- Confocal or regular microscopy



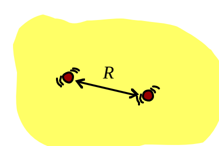

data from 0.25% weight guar solution

lines = rheometer
symbols = microrheology

graph from JC Crocker et al, Phys Rev Lett **85**, 888 (2000)

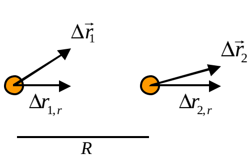
# Two-particle microrheology

*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*




Look at correlated motion of pairs of tracer particles – how does it depend on $R$?

# Two-particle microrheology: longitudinal correlations

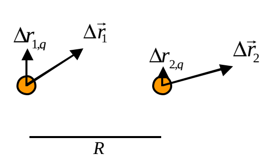



$$D_{rr}(R) = \langle \Delta r_{1,r} \cdot \Delta r_{2,r} \rangle_{1,2}$$

As $R \rightarrow \infty$, $D_{rr} \rightarrow 0$

*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*

# Two-particle microrheology: perpendicular correlations

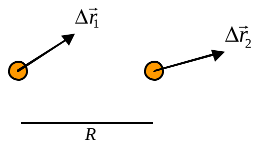



$$D_{\theta\theta}(R) = \langle \Delta r_{1,\theta} \cdot \Delta r_{2,\theta} \rangle_{1,2}$$

As $R \rightarrow \infty$, $D_{\theta\theta} \rightarrow 0$

*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*

# Two-particle microrheology: homogeneous medium is simple



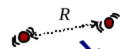

$$D_{rr}(R) \sim 2D_{\theta\theta}(R) \sim 1/R$$

True for fluid, elastic material, or viscoelastic!

*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*

# Finding moduli via 2-particle MSD

(Mean Square Displacement)




$D_{rr}(R)$

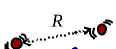
$$\langle \Delta r^2(\Delta t) \rangle_D = \frac{2R}{a} D_{rr}(R, \Delta t)$$

*G'(w), G"(w)*

via the generalized Stokes-Einstein formula, again

# Finding moduli via 2-particle MSD

(Mean Square Displacement)




$D_{rr}(R)$

*this is R independent, can average over R to improve data*

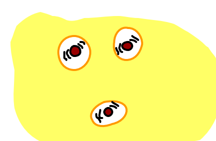
$$\langle \Delta r^2(\Delta t) \rangle_D = \frac{2R}{a} D_{rr}(R, \Delta t)$$

*G'(w), G"(w)*

via the generalized Stokes-Einstein formula, again

# What about an inhomogeneous medium?

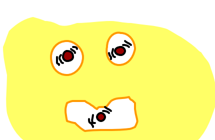
*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*



What if particles sit in pores?

# What about an inhomogeneous medium?

*JC Crocker et al, PRL 2000*
*AJ Levine & TC Lubensky, PRL 2000, PRE 2001*



What if particles sit in pores?

This is OK: Particle motion =
(uncorrelated local motion) + (correlated long-range motion)

For large *R*, correlation will decay as 1/*R*: this is a good sign

# Inhomogeneous medium

Example: polymer network

True no matter what the inhomogeneity:

For large *R*, correlation will decay as 1/*R*: this is a good sign

Also true if tracers aren't spheres, or are different sizes

## Applications

## Application #1: finding rheological properties

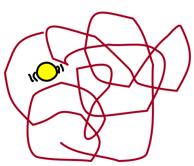
You already know about this.

Hopefully Tom Mason and Dave Weitz convinced you that this can be done.

## Application #2: rheological microscopy

see: DT Chen et al., PRL **90**, 108301 (2003)

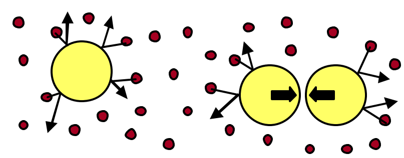


Study particles in polymer gel, to learn <u>microscopic</u> properties of polymer gel.

Key idea: compare 1-particle and 2-particle microrheology

## <u>Depletion Force & Concentrated Polymer Solutions</u>

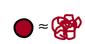
*S. Asakura & F. Oosawa, J. Polym. Sci. (1958)*



Osmotic pressure due to small particles is isotropic

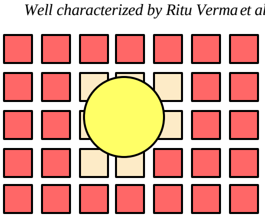
Osmotic pressure pushes large particles together

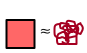
Range of force ≈ diameter of small particles

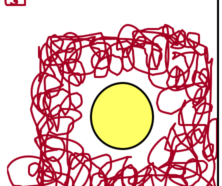
Small particles can be polymers 

## "Depletion Cavity" of particles in concentrated polymer solutions

*Well characterized by Ritu Verma et al, PRL 1998*





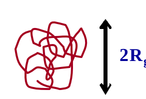


Particle blocks polymer which would otherwise be near its surface

## <u>Concentrated λ-DNA solutions</u>

*Ritu Verma et al, PRL 1998*

λ-DNA: $L_{contour} = 16\,\mu m$
$L_{persistence} = 50$ nm,
$R_{gyration} = 0.55\,\mu m$
$c^* \approx 40\,\mu g/ml$

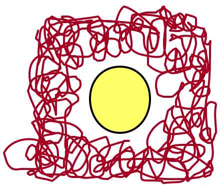



$c < c^*$ : size = $R_g$
$c > c^*$ : effective size = correlation length $\xi$

<u>Our experiments:</u>

- *cDNA* = 0 – 5c*
- polymer size (radius) = 0.55 – 0.20 µm
- particle radius = 0.23 – 1.0 µm

## How does particle motion reveal information about depletion cavity?



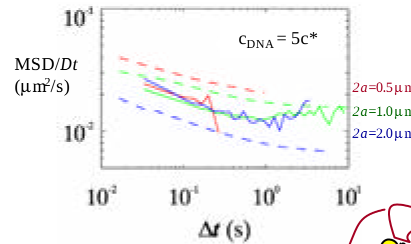
We think cavity "size" ~ polymer size

Our experiments:

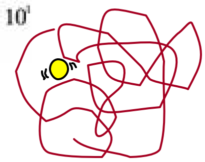
- polymer size (radius) = 0.55 – 0.20 μm
- particle radius = 0.23 – 1.0 μm

## Small particles more mobile

dashed lines = MSD1, solid lines = MSD2

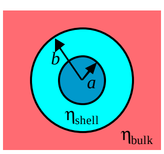



ξ = 0.20 μm
$a$ = 0.23 μm




## Very Simple Model

Alex Levine & Tom Lubensky




Parameters:

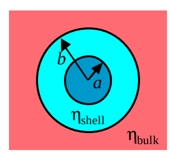
- $a$: particle radius (known)
- $b = a + \Delta$ ($\Delta$ fixed for each $c_{DNA}$)
- $\eta_{shell}$ (fixed to be water)
- $\eta_{bulk}$ (fixed for each $c_{DNA}$, found from 2 particle measurements – MSD2)

12 measurements of MSD1:

- 3 different $a$, 4 different $c_{DNA}$
- Focus on Δ$t$ ® ¥ behavior (viscous)
- Goal: find four free parameters $D$

## Very Simple Model finds shell thickness

Alex Levine & Tom Lubensky



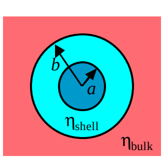

Parameters:

- $a$: particle radius (known)
- $b = a + \Delta$ ($\Delta$ fixed for each $c_{DNA}$)
- $\eta_{shell}$ (fixed to be water)
- $\eta_{bulk}$ (fixed for each $c_{DNA}$, found from 2 particle measurements)

Result: $\Delta \approx 2\xi$ (effective radius of polymer)

Excellent agreement with full $G'(w)$, $G''(w)$ behavior

## Conclusion from application #2: "rheological microscopy"



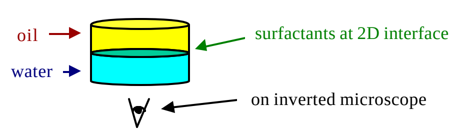

Use difference between MSD1, MSD2 to learn about particle motion.

In this case, shell model is a good simple way to think about particle motion. Thickness of shell related to known polymer size.

*DT Chen et al., PRL **90**, 108301 (2003)*

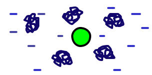
## Application #3: Microrheology on surfaces

Vikram Prasad, Stephan Koehler, & Eric Weeks (Emory)
cond-mat/0604262

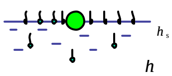



Are tracer particles at interface more influenced by "2D surfactant fluid" or by 3D oil & water reservoirs?

## Microrheology of viscous solutions



$< \Delta r^2 > = \frac{k_B T}{pha} t$  3-D bulk viscous solution

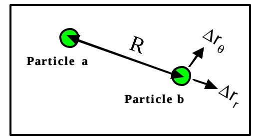



$< \Delta r^2 > = \frac{k_B T}{p h_s} (\ln [2h_s/(ha)] - 0.577)\ t$

2-D interfacial viscosity

L= $\eta_s / \eta$   length scale dependence for 2-D systems

## Two-particle microrheology




$D_{rr}(R,t) = < \Delta r_r^a(t,t)\, \Delta r_r^b(t,t) >_t$  Longitudinal

$D_{qq}(R,t) = < \Delta r_q^a(t,t)\, \Delta r_q^b(t,t) >_t$  Transverse

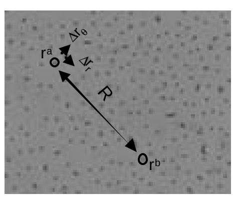
$D_{rr}, D_{qq} \sim \frac{1}{R}, \frac{1}{h}$  for 3-D viscous systems, **not measured for 2-D**

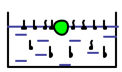
## Experimental Details

PS beads, a=0.85 µm, spread at interface

20× objective, N.A=0.5, frame rate=30 frames/s

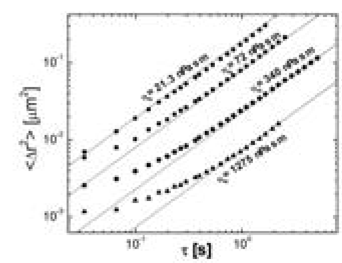
Human Serum Albumin at air-water interface (bulk c≅0.03-0.45 mg/ml)



- Measure vector displacements of particles $\Delta r$ for 200 frames
- Determine $< \Delta r^2(\tau)>$ (1-particle MSD)
- Determine $D_{rr}(R,\tau)$ and $D_{\theta\theta}(R,\tau)$ from displacements for different $R$, $\tau$

## One-particle MSD: see viscous behavior

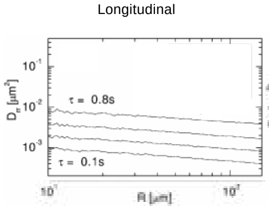



2D viscosities from Saffman formula.
Are these local measurements accurate?

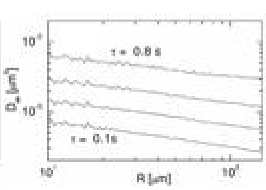
## 2-particle correlation functions

($\eta_s$ = 340 nPa s m)

Longitudinal




Transverse

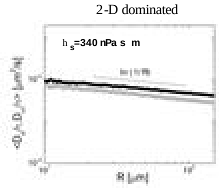



$D_{rr}, D_{\theta\theta} \sim \tau$

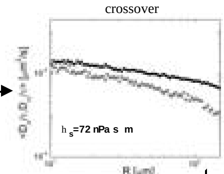
- τ-scaling used to determine averaged quantities $<D_{rr}/\tau>$, $<D_{\theta\theta}/\tau>$
- Behavior different from what is observed in 3-D systems

## Effect of surface viscosity $\eta_s$

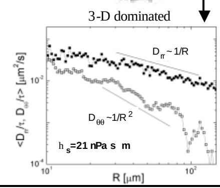
2-D dominated




crossover

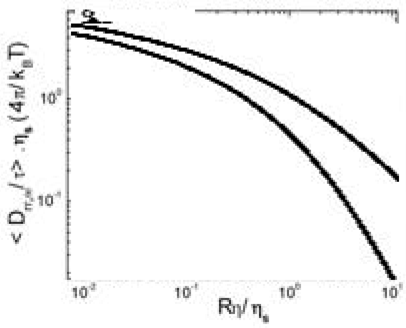



3-D dominated



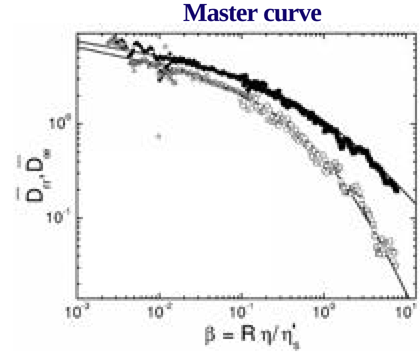

- Changing $\eta_s$ has a large effect on $D_{rr}$, $D_{\theta\theta}$
- Transition from 2-D(large $\eta_s$) to 3-D(small $\eta_s$)
- Different from 3-D at all $\eta_s$

## Master curve



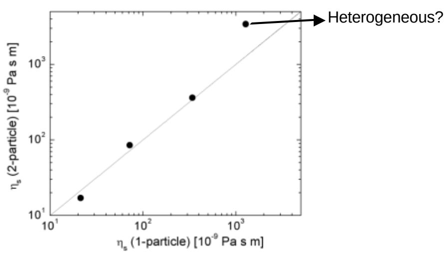

•Fits are from theory - A.J. Levine and F.C. MacKintosh, *Phys Rev E* 66, 061606(2002)
•Characterizes flow/strain fields over different length scales

## Master curve




•Fits are from theory - A.J. Levine and F.C. MacKintosh, *Phys Rev E* 66, 061606(2002)
•Characterizes flow/strain fields over different length scales

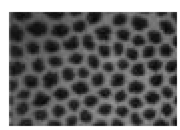
## Albumin is homogeneous




2-particle and 1-particle MSD's agree reasonably well

## Summary – Application #3

- Must consider full 2D + 3D system to understand 2-particle measurements
- Implications for measurements on cell membranes
- Heterogeneous systems will be interesting…

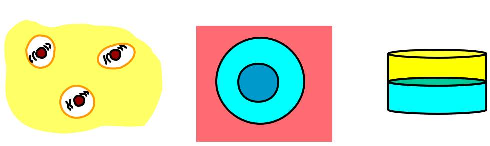


DPPC/POPG monolayer under compression; heterogeneous !
(A. Gopal and Ka Yee C. Lee, *J Phys Chem B 105* (2001))

*V. Prasad, S.A. Koehler and Eric R. Weeks, cond-mat/0604262*

## Overall Summary:

- Microrheology useful for small systems – scattering & microscopy
- Microscopy helpful when system is heterogeneous; can study local properties
- 2-particle microrheology can infer bulk properties
- Idea of "rheological microscopy"



## References:


Microrheology:
• TG Mason & DA Weitz, PRL **74**, 1250 (1995)
• "Microrheology", ML Gardel et al., in: *Microscale Diagnostic Techniques*, K. Breuer (Ed.) Springer Verlag (2005) (see Weitz lab website to download)

Two-particle microrheology:
• JC Crocker et al., PRL **85**, 888 (2000)
• AJ Levine & TC Lubensky, PRL **85,** 1774 (2000)

Applications:
• Rheologicalmicroscopy: DT Chen et al., PRL **90**, 108301 (2003)
• 2D/3D microrheology : V Prasad et al., cond-mat/0604262