B. Halperin -- Spins 2

Halperin lecture 4 at Boulder Summer School

Origin of spin-orbit coupling parameters.

Relaxation of Spin

Optical production and detection of spin polarization.

Spin Hall Effect and Anomalous Hall Effect

Anomalous Hall Effect (AHE) and Spin Hall Effect in 2D and 3D systems

Based on work done with

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Phys. Rev. Lett., 2004, and unpublished

and

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Anomalous Hall Effect (AHE)

In a system with net spin polarization, either a ferromagnet in zero magnetic field, or a strongly paramagnetic material in an applied magnetic field, there can be a contribution to the Hall conductance, due to spin-orbit coupling, separate from the usual Hall conductance arising from Lorentz force. This AHE can be quite large.

Usually attributed to skew-scattering by impurities, but there can be other "intrinsic" contributions. (A. H. MacDonald and coworkers)

Spin currents

In a system of appropriate symmetry, an electric field can produce a spin current, in the absence of a magnetic field. If one can produce a spin-current with non-zero divergence, and if the spin-relaxation rates are not too large, this can cause build up of spin density in some region of the system, which might be useful.One obtains a spin current, with no charge current, if carriers of opposite spin polarization are moving with opposite average velocities.

A non-zero spin density is forbidden in an equilibrium system with no magnetic field, due to time-reversal symmetry, but it is allowed when there is a transport current.

A non-zero spin current may be allowed in equilibrium, as it is even under time reversal,and is indeed predicted in some cases. So spin currents do not necessarily lead to spin accumulation. Broken inversion symmetry is necessary for a non-zero spin current.

Spin Hall conductivity

For a 2D electron or hole system, an electric field E_x in the x-direction, can cause a spin-current j^z_y , transporting the z-component of spin in the y-direction. We write

$$j^z_y = - \sigma_{sh} \; E_x \;\; ,$$

where σ_{sh} is the "spin-Hall conductivity.". Similar effects can occur in 3D systems. As for AHE, several mechanisms have been proposed.

An "extrinsic" mechanism based on asymmetric spin-dependent scattering by impurties, can occur even if the host crystal is inversion symmetric.(Dyakonov and Perel, 1971)

An additional"intrinsic"mechanism has been proposed when the host crystal or heterostructure is not inversion symmetric.

“Universal” spin Hall conductivity?

Sinova et al showed that for a two-dimensional electron system, in the absence of impurities, an electric field could produce a spin Hall conductivity, for the z-component of spin, given by a “universal value”: $\sigma_{sh} = e / 8\pi$, independent of the strength of the spin-orbit coupling. They speculated that this result would still hold in the presence of weak impurity scattering. If so, this could lead to significant spin accumulations at the boundaries of the sample. We shall see for a (001) 2DEG with pure Rashba or linear Dresselhaus coupling, even a small concentration of impurities can cancel the intrinsic contribution, but the situation can be different in other systems. For 2D hole systems, it is possible that there could be a spin hall conductance close to the predicted intrinsic value..

Experiments which “observe” a spin Hall conductance

Actually, experiments which observe a spin polarization that is attributed to spin-Hall conductance.

Kato et al:, Science (2004): 3D n-type GaAs

Wunderlich et al. PRL (2005), 2D hole gas

Sih et al, (2005) 2D electron gas on (110) surface

More remarks

In dirty limit, impurity scattering always reduces the intrinsic spin Hall effect, typically by a factor $\approx (\Delta\tau)^2$.

Three-dimensional samples are always in the dirty limit.

"Extrinsic" contributions to spin-Hall conductance may be small compared to the intrinsic spin-Hall conductance in the clean limit, but are generally dominant in the dirty case.

We have calculated the effects of extrinsic scattering for the 3D electron system studied by Kato et al., and find reasonable agreement with the observations The intrinsic effect, due to the k^3 Dresselhaus coupling, gives a contribution which is an order of magnitude smaller, because of the large scattering rate, (as found by Bernevig and Zhang) .

Effective Hamiltonian with impurity scattering

(Non-interacting electrons, 2D or 3D)

$$H = (p^2/2m) - \mathbf{b}(\mathbf{p}) \cdot \mathbf{s} + V(\mathbf{r}) + 2\lambda\, \mathbf{s}\cdot(\mathbf{p}\times\nabla V) \;,$$

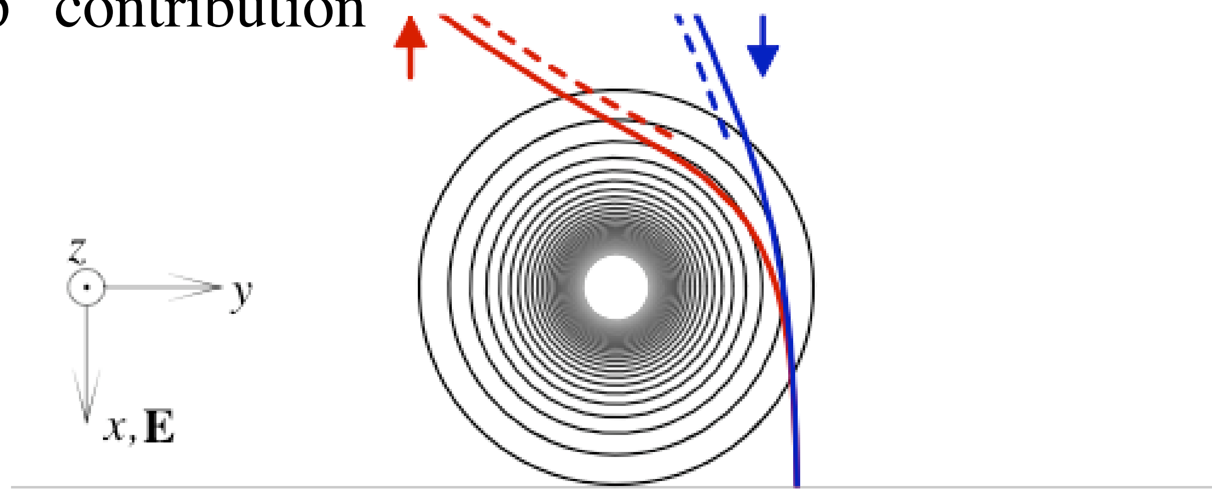
where λ is another spin-orbit coupling constant, which depends on the material. For n-type GaAs, $\lambda = 0.053\text{nm}^2$. This coupling does not require broken inversion symmetry in the host material.

Spin-orbit effects due to λ coupling will be called extrinsic.

Spin-orbit effects due to b term will be called intrinsic.

Skew-scattering and side-jump contributions

Extrinsic scattering gives two contributions to the spin-Hall conductance: "skew scattering" (Mott scattering) and the "side-jump" contribution



To compare with experiments of Kato et al., in 3D n-type GaAs, we represent ionized donors by attractive screened Coulomb potentials, with density of impurities = density of electrons. Calculations are first order in spin-orbit coupling λ; assume Boltzmann equation. Use previous results from analyses of the Anomalous Hall Effect.

Results for extrinsic scattering in 3D n-type GaAs

(H. Engel, E. I. Rashba, and B. I. Halperin)

Skew scattering contribution to spin Hall conductance

$= 1.7\ \Omega^{-1}\,\mathrm{m}^{-1} \times (-2/e)$.

Side-jump contribution to spin Hall conductance

$= -0.8\ \Omega^{-1}\,\mathrm{m}^{-1} \times (-2/e)$

Total contribution to spin Hall conductance

$= 0.9\ \Omega^{-1}\,\mathrm{m}^{-1} \times (-2/e)$

Experimental results of Kato et al. for spin Hall conductance

$= -0.5\ \Omega^{-1}\,\mathrm{m}^{-1} \times (-2/e)$ [question about sign]

Definition of spin current

$$j^m_n(\mathbf{r}) \;\; = (1/4) \sum_i \{ \{v^i_n , s^i_m\} , \delta(\mathbf{r}-\mathbf{r}_i) \}$$

In the absence of spin orbit coupling, the velocity operator is given by $\mathbf{v} = \mathbf{p} / m$ .When spin-orbit is important, the velocity operator is more complicated, and involves the spin.

We use units where h-bar = 1, and $s_m = \pm (1/2)$, and e is the charge of the carrier.

The spin components s_x, s_y, and s_z obey angular momentum commutation rules, but they are not really spins. They are actually a mixture of spin and translational degrees of freedom.

Effective Hamiltonian for a pure electron system with spin-orbit coupling

(Non-interacting electrons, no impurity scattering)

$$H = (p^2/2m) - \mathbf{b}(\mathbf{p}) \cdot \mathbf{s} \quad ,$$

where the effective magnetic field **b** depends on **p**, and satisfies **b**(-**p**) = - **b**(**p**) . There can be several contributions to **b**.

In 2D, if the confining well is asymmetric, so there is a non-zero average of E_z in the well, we expect a "Rashba" term:

$$\mathbf{b}(\mathbf{p}) = \alpha\, \mathbf{z} \times \mathbf{p} \quad .$$

For a material such as GaAs, without a center of inversion, there are additional "Dresselhaus" terms, linear and cubic in **p**.
In 3D GaAs, we have only cubic Dresselhaus term.

Velocity operator: $\mathbf{v} = (\mathbf{p}/m) - \mathbf{s} \cdot \partial\mathbf{b}/\partial\mathbf{p} \Rightarrow (\mathbf{p}/m) - \alpha\, \mathbf{z} \times \mathbf{s}$

Effects of impurities on intrinsic spin Hall conductance in 2D case

$$H = (p^2/2m) - \mathbf{b}(\mathbf{p}) \cdot \mathbf{s} + V(\mathbf{r}) + 2\lambda\, \mathbf{s}\cdot(p\times\nabla V)$$

Effects of impurities on intrinsic spin Hall conductance in 2D case

$$H = (p^2/2m) - \mathbf{b}(\mathbf{p}) \cdot \mathbf{s} + V(\mathbf{r})$$

One-electron eigenstates

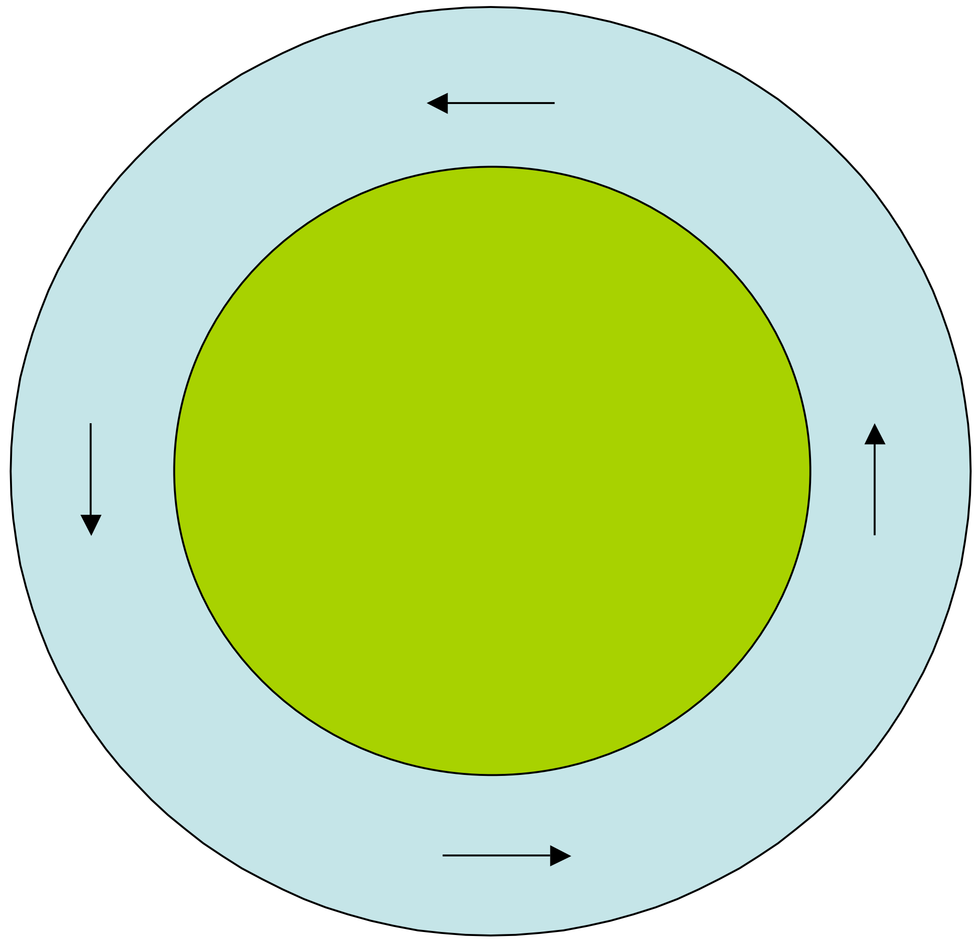
In the clean limit, for any given p, there are two energy eigenstates with energies

$$\varepsilon_{\mathbf{p}} = (p^2/2m) \pm \Delta_{\mathbf{p}} \,, \qquad \Delta_{\mathbf{p}} = |\mathbf{b}(\mathbf{p})| /2 \,.$$

For a given direction of p, there are two states at the Fermi energy, with different values of p, given by $p^{\pm} = p_F{}^0 \pm \Delta / v_F$.

For pure Rashba model: $\Delta=2b=\alpha\, p_F{}^0$, independent of direction, and $p^{\pm} = p_F{}^0 \pm \alpha\, m$. The velocities of the two states are equal, and are the same as the unperturbed Fermi velocity v_F.

Two Fermi Surfaces



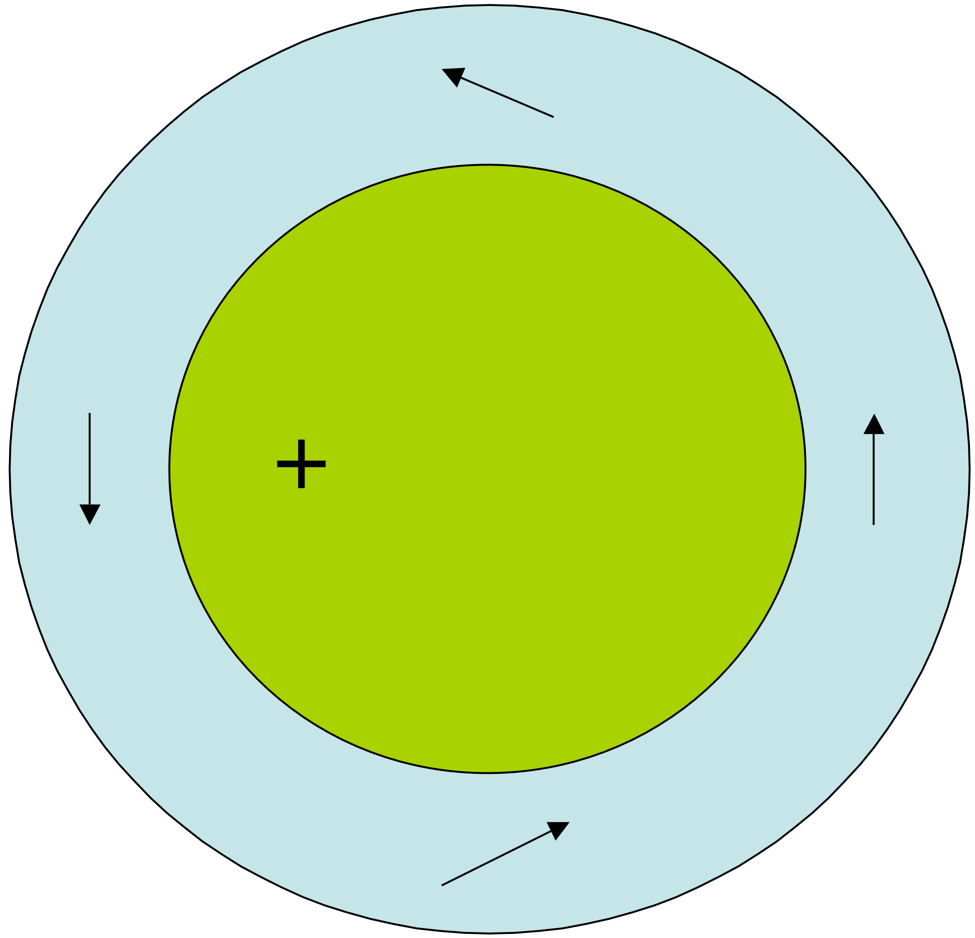
Green region is doubly occupied.
Arrows show spin directions for singly-occupied states.

Effect of a constant electric field E_x .

Momentum is a good quantum number. Momenta of all filled states change at the same rate, $d\mathbf{p}/dt = e\,\mathbf{E}$. Two Fermi circles remain concentric, with centers displaced by amount linear in time.

Spin direction is not a good quantum number. In limit of small E, spins of singly-occupied states reorient adiabatically to follow effective field $\mathbf{b}(\mathbf{p})$ as $\mathbf{p}$ changes in time. Spins remain in the x-y plane.

Effect of a weak electric field E_x. ($\mathbf{E} \rightarrow 0$)



Adiabatic displacement of the Fermi Sea

+ denotes origin of momentum space

But there is a correction to the adiabatic results, linear in E; spins do not follow perfectly a time-varying magnetic field. Equation of motion is $d\mathbf{s}/dt = \mathbf{s} \times \mathbf{b}$. For $\mathbf{b}$ moving slowly in the x-y plane, and $\mathbf{s}$ initially parallel to $\mathbf{b}$, this gives result

$$s_z = (\mathbf{b} \times d\mathbf{b}/dt) / \; 2b^3$$

For E in x-direction, $db_y/dt = 2\alpha\, dp_x d/t = \; 2\alpha\, eE$. If $\mathbf{p}$ makes an angle θ with the x-axis, then $b_x = -2\alpha p_y \; = -2\Delta \sin\theta$. So

$$s_z = -eE \sin\theta \,/\, 4\Delta$$

Contribution of this spin to j^z_y : $-eE\, v_F \sin^2\theta \,/\, 4\Delta$
Number of unpaired spins in angle $\delta\theta$: $\delta\theta\, \Delta\, p^2 / 2\pi^2 \; v_F$

Gives total: j^z_y : $-eE_x / 8\pi$, if no impurity scattering.

Remark

The electric field contribution to the spin Hall conductance has a topological origin,

$\sigma_{sh} = e\, N / 8\pi$,

where N is the winding number of the direction of **b**(**p**) as one moves around the Fermi surface.

For 2D electron systems, with Rashba and /or linear Dresselhaus coupling

$N = 1$, if Rashba > Dresselhaus

$N = -1$, if Dresselhaus > Rashba

For 2D hole system, with Rashba coupling: $N = 3$.

Effect of a small density of impurities

One cannot apply a truly dc electric field in the absence of impurities as one would obtain an infinite drift velocity. For finite impurity density, drift velocity saturates at $\mathbf{v}_D = e\mathbf{E}\tau$. Then the net rate of momentum transfer by impurities is equal and opposite to momentum transferred by electric field.

For the 2DEG with pure Rashba coupling, we find that momentum transfer by impurities gives rise to a spin-Hall current equivalent to that of an electric field producing the same momentum transfer. This cancels the spin current produced by the actual **E** field, so there is no net spin-Hall effect for a dc field.

General circularly symmetric 2D model

For $b_x + i b_y = b(|p|)\, e^{Ni\theta}$, we find

Spin Hall conductance = 0, for N = ± 1

(i e., 2D electron gas with pure Rashba or linear Dresselhaus coupling)

Spin Hall conductance is non-zero for $|N| \neq 1$. Value depends on the angular-dependence of the scattering, and on the energy-dependence of b, can have either sign, and can be larger than the intrinsic spin Hall conductance in clean limit. No renormalization for isotropic impurity scattering. (N=3 is simple model of 2D hole gas in GaAs.)

Spin Hall conductance is strongly suppressed in the dirty limit, $b\tau \ll 1$.

More Results

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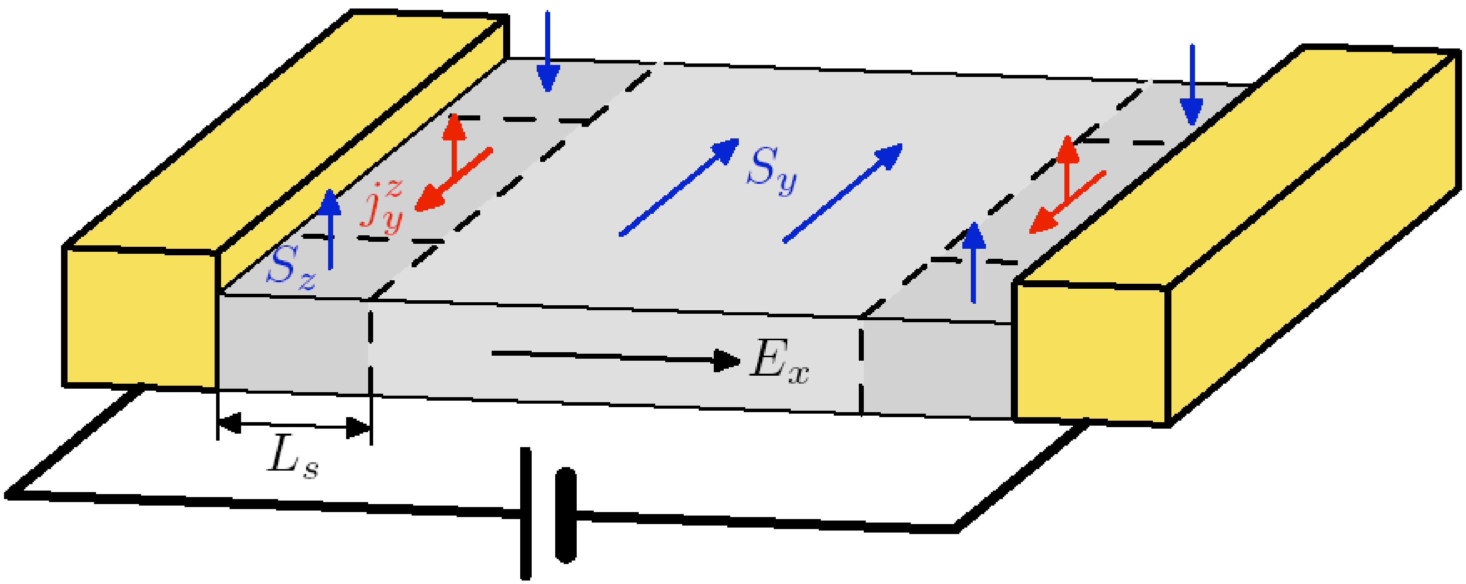
"Universal" spin Hall conductance does occur for ac field, if

$$\tau^{-1} << \omega << \Delta \ .$$

Although there is no bulk dc spin Hall current in the 2D electron system with pure Rashba or linear Dresselhaus coupling, dc spin Hall currents do occur near contacts in a finite sample, could lead to z-polarized spin accumulation in corners of the sample.

Bulk spin-polarization does occur, with polarization in x-y plane

Spin-currents and polarization in a finite sample



Notes. Spin-orbit length $L_S = v_F/2\Delta$ is the distance for spin to rotate by one radian. E_x is the voltage gradient, including both chemical potential and electrostatic contributions.

Remarks

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Our results are based on perturbation theory, and may be derived using a variety of approaches: Kubo formula, Keldysh formalism, semiclassical Boltzmann-equation approach, etc. Results should be valid when the spin orbit coupling energy Δ and the impurity scattering τ^{-1} are both small compared to E_F, but for arbitrary value of $\Delta\tau$.

Summary of Results

For 2D electrons in GaAs: with pure Rashba or linear Dresselhaus coupling, intrinsic spin Hall effect is absent even in clean limit. (The cubic Dresselhaus term could lead to a non-zero result)

For 2D holes in GaAs, intrinsic effect should be present. May be responsible for observations of Wunderlich et al.

For 3D electrons in GaAs, estimates of extrinsic effect seem consistent with experiments of Kato et al. Intrinsic effect not important.