The Scientist in the Sand Box

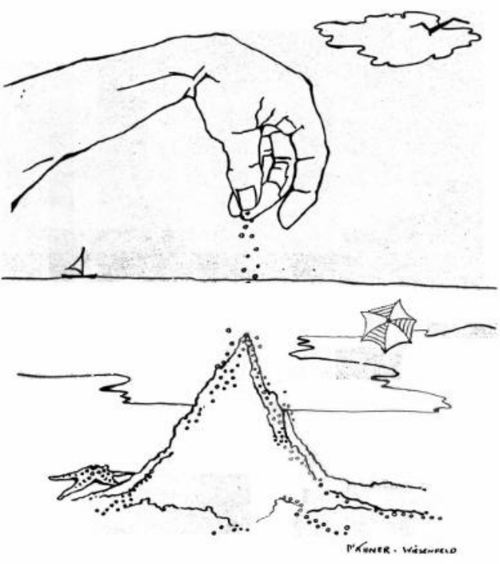

FIG 1

OUTLINE

I. **Introduction**
   - A. Simple Ideas
   - B. Granular Phases

II. **Key Ideas**
   - A. Dilatancy
   - B. Friction
   - C. Janssen Effect
   - D. Indeterminacy
   - E. Nonlinearity of Contacts
   - F. Granular Temperature
   - G. Granular Entropy (?)

III. **Dense Phases**
   - A. Stress Chains
   - B. Fabric
   - C. Force Propagation
   - D. History Dependence
   - E. Stick Slip
   - F. Fluctuations
   - G. Jamming
   - H. Compaction
   - I. Shearing

Overview
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IV. Gas Phase
- **A. Collapse**
- **B. Clustering**
- **C. Granular Temperature**
- **D. Hydrodynamics**
- **E. Melting in 2D**
- **F. Velocity Distributions**

V. "Phase Transitions"
- **A. Horizontal Shaking**
- **B. Shear Banding**
- **C. Fluidization**

VI. Pattern Formation
- **A. Shaking**
- **B. Segregation**
- **C. Avalanching**
- **D. Hopper Flow**

VII. Conclusions/Summary

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E. Aharonov and D. Sparks, Phys. Rev. E 60, 6890 (1999)

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Rotating Drums

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Jamming

M. Cates et al. Phys. Rev. Lett. 81, 1841 (1998)

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Force Fluctuations

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A. Ngadi and J. Rachenbach, Phys. Rev. Lett. 80, 273 (1998)

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Rotating Drums

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A. Ngadi and J. Rachenbach, Phys. Rev. Lett. 80, 273 (1998)

D. Howell et al. Phys. Rev. Lett. 82, 5241 (1999)

1. Consists of a large number of individual solids

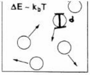
2. Grain-grain interactions are purely classical

$\lambda = \frac{h}{mv} \sim 10^{-27}$ m $\lambda \sim$ system size, quantum important

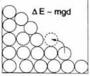
3. Grains exert forces only when in contact

4. Collisions between grains are generally inelastic

5. Grains are surrounded by a fluid or a vacuum



Fluid



Granular Material

For a fluid: $mgd << k_bT$ ($mgd \sim 10^{-13}\ k_b\,T$)

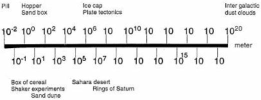
Thermal energy allows exploration of many different states.

Fluid finds equilibrium lowest free energy state,
Thermodynamics applicable.

For a granular material : $mgd >> k_bT$ ($mgd \sim 10^{12} k_bT$)

Consequently, the material does not sample other configurations unless energy is put into the system.

This can lead to metastable states that persist indefinitely.



colloids
suspensions
cohesive materials
ceramics

Fluctuations, hydrodynamic limit, e.g. of jamming system, self-organized criticality…

2. Continuum dynamics

   What are right models? When do they apply?…

3. Member of class of similar systems:

   Foams, Glasses, Colloids, Quantum flux lattices

4. Important applications

   - Coal and grain transport…
   - Pharmaceuticals
   - Xerography
   - Soils
   - Avalanches
   - Mixing
   - Earthquakes and mudslides…

5. $$$!! Lots of them Claim: ~1 $Trillion/year in US for bulk solids handling

   Industrial facilities operate well below design efficiency

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■ FROM THE GEOGRAPHIC ARCHIVES

Changing the Channel

Construction crews in 1913 had to clear frequent landslides (top)
Culebra Cut (later called Gaillard Cut) across the Continental Di
ama Canal navigable. In William Joseph Showalter's February 191

The Herald-Sun/BILL [illegible]

DAMAGED FREEWAY: A truck passes the damaged section of the Durham Freeway near Hillandale Road. A section of Durham's newest

determined a landslide posed a threat to motorists. Both southbound lanes from Hillandale Road to Elba Street probably will stay closed

Slow landslide forces freeway section closed

Southbound repairs to take 2 months

By ROCKY ROSEN

The Herald-Sun

One of Durham's newest sections of freeway was abruptly closed Monday afternoon when engineers determined a slow-moving landslide posed a threat to motorists.

An embankment supporting the southbound lanes of the Durham Freeway near Hillandale Road began sliding about four weeks ago, causing the freeway's right lane to cave in slightly. Highway crews had closed the lane while engineers planned a way to halt the slide.

But steady rain this weekend and a downpour Monday morning apparently made the embankment more unsteady. When the earth kept moving Monday, engineers decided it wasn't safe to leave even one lane open.

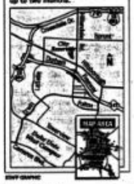
By Wednesday afternoon, the depression in the right lane was about 20 feet long and 3 feet deep — and growing.

"It represents an immediate hazard to the travelling public," Ken McEntire, district engineer for the state Division of Highways, said just before closing both southbound lanes. "It's still sliding. It slid two inches just today."

Both lanes will probably stay closed during the two-month repair project. McEntire said the state might be able to open one

FREEWAY DETOUR

Heavy rains accelerated a landslide on an embankment supporting the southbound lanes of the Durham Freeway, forcing the lanes (shown in red) to be closed. Both lanes may be closed up to two months.



STAFF GRAPHIC

but he doubts it.

Drivers headed from U.S. 15-501 Bypass toward Durham's Research Triangle Park or Raleigh will have to exit the freeway at the Hillandale Road/Fulton Street off ramp, follow detour signs to the nearby Elba Street on ramp and get back on the freeway.

The freeway problem — technically called a "slope failure" — stems from a thin layer of clay about 10 to 15 feet below the surface of the embankment, McEntire said. The layer of clay is sloped about 11 degrees, and


C'EST FINI !
DÉMONSTRATION :
TU DIS "HAUT LES MAINS TOUT L'MONDE !"
DIS-LE !
TRRRGUIDIGUIDIC
?
TRRRGUIDIGUIDIC
DES BILLES D'ACIER !
HUNGTUD'
'DJIÛÛ !

I. Different regimes or states:

A. 'Solid' regime
- **Persistent contacts, slow flows**
  - **Compact structure**
    - **Strong force fluctuations**
      - **Spatially variable**
        - **Temporally intermittent**

    Industrial applications-hoppers…

B. 'Fluid' or 'Gas' regime
- **Energetic flows**
  - **Binary dissipative collisions**
    - **Granular 'temperature'** $T_g \sim \langle V^2 \rangle$

      E.g. avalanches, shaken systems…

C. Transition regimes
- **'Gas'** $T_g \to 0$**: clustering/collapse**
  - **'Solid' → Plastic under shear (Jamming)**
    - **'Solid' + Energy → fluid**
      - **'Solid' & 'Fluid' often close spatially**

II. Statistical properties

A. Solid

1. Disorder and fluctuations

Strong force fluctuations—only weak packing fluctuations

Small grain displacements ➔ large force changes

Length and time scales of fluctuations an open question

Continuum Limit?

2. Friction and contacts

Coulomb friction is indeterminate➔History Dependence

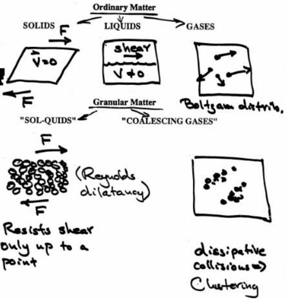
Persistent contacts ➔ limited sampling of phase space

B. 'Fluid'

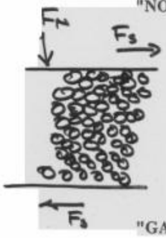
Instabilities: clustering, hydrodynamics
Existence of T_g?

Segregation and mixing phenomena

1. New states of matter



"NOT-SO-SOLID SOLIDS"

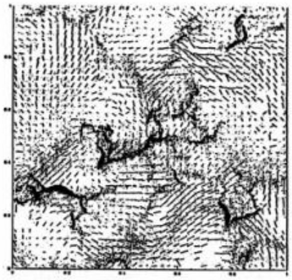


Solid-like up to yield

Then dilates + flows

O. Reynolds (Plastic?)

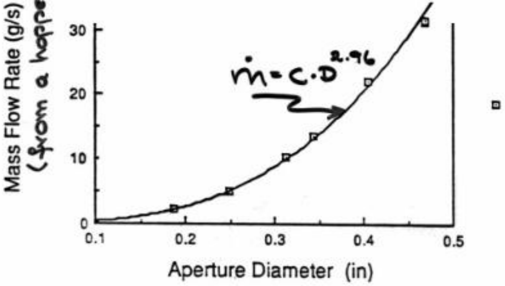
"GASES THAT CLUSTER"



Clustering in 2-D system,
from M-D distribution?

Goldhirsch, Tan, Zanetti
J. Sci. Computing
8, 1 (1993).

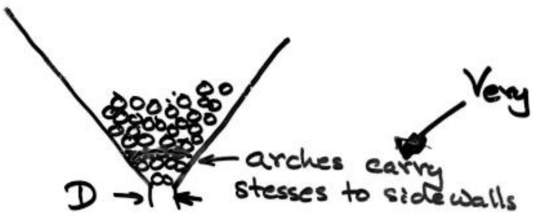
Physical Systems: Gravity compacts



$\dot{m}$ independent of fill height!

No!

Why is $\dot{m}$ height independent?

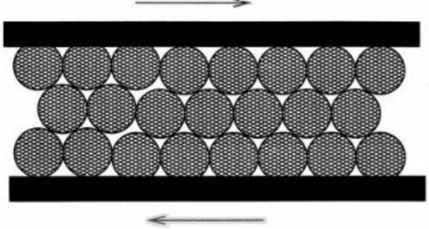


grains fall out near exit

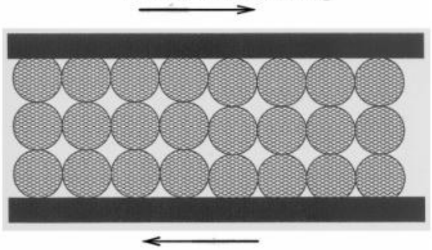
$$v \sim (gD)^{1/2}$$

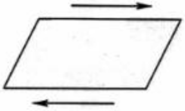
$$m \sim D^2 \; v \sim D^{5/2}$$

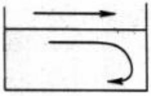
(+ Boundary layer effect)

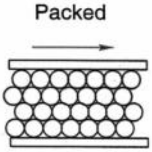


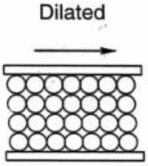
Dilation after shearing











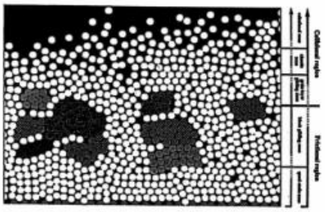
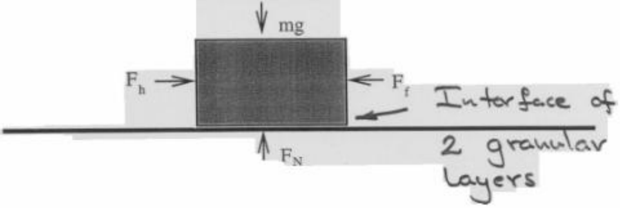


Figure 3

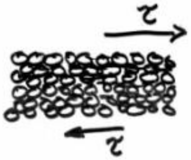
T.G. Drake, Journal of Geophysical Research, 1989

Coulomb Friction

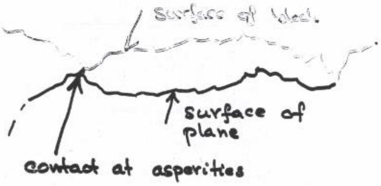


$$F_N = mg$$

$$F_f = \begin{cases} F_h & F_h < \mu_s F_N \\ \mu_k F_N & \text{Otherwise} \end{cases}$$



"The Real" Coulomb Friction



(See Cevoli + Nozières.)

ESSAI

Sur une application des règles de Maximis & Minimis *à quelques Problèmes de Statique, relatifs à l'Architecture.*

Par M. COULOMB, Ingénieur du Roi.

INTRODUCTION.

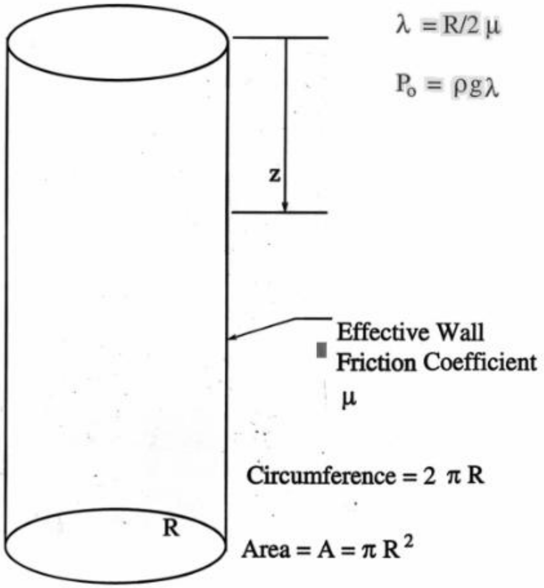
CE Mémoire eſt deſtiné à déterminer, autant que le mélange du Calcul & de la Phyſique peuvent le permettre, l'influence du frottement & de la cohéſion, dans quelques problèmes de Statique. Voici une légère analyſe des différens objets qu'il contient.

Après quelques obſervations préliminaires ſur la cohéſion, & quelques expériences ſur le même objet, l'on détermine la force d'un pilier de maçonnerie; le poids qu'il peut porter, preſſé ſuivant ſa longueur; l'angle ſous lequel il doit ſe rompre. Comme ce problème n'exige que des conſidérations aſſez ſimples, qui ſervent à faire entendre toutes les autres parties de cet Eſſai, tâchons de développer les principes de ſa ſolution.

Si l'on ſuppoſe un pilier de maçonnerie coupé par un plan incliné à l'horizon, en ſorte que les deux parties de ce pilier ſoient unies dans cette ſection, par une cohéſion donnée, tandis que tout le reſte de la maſſe eſt parfaitement ſolide, ou lié par une adhérence infinie; qu'enſuite on charge ce pilier d'un poids: ce poids tendra à faire couler la partie ſupérieure du pilier ſur le plan incliné, par lequel il touche la partie inférieure. Ainſi, dans le cas d'équilibre, la portion de la peſanteur, qui agit parallèlement à la ſection, ſera exactement égale à la cohérence. Si l'on remarque actuellement, dans le cas de l'homogénéité, que l'adhérence du pilier eſt réellement égale

Memoirs de Mathématique de
l'Academie Royale des Sciences
(Paris) 7, 343 (1773)

Janssen Effect $P = P_o \, 1 \quad \exp(-z/\lambda))$

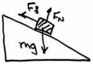


$$\lambda = R/2\,\mu$$

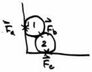
$$P_o = \rho g \lambda$$

it rm nac

y dependence



$F_f \leq \mu F_N$



For each disk:

$\sum_i F_i = 0$

$\sum \tau = 0$

3 Unknown $\vec{F}$'s

6 constraints

6 unknowns } ✓

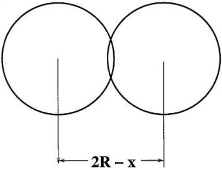
9 constraints

12 unknowns

Materials

$$F = 0 \qquad x < 0$$

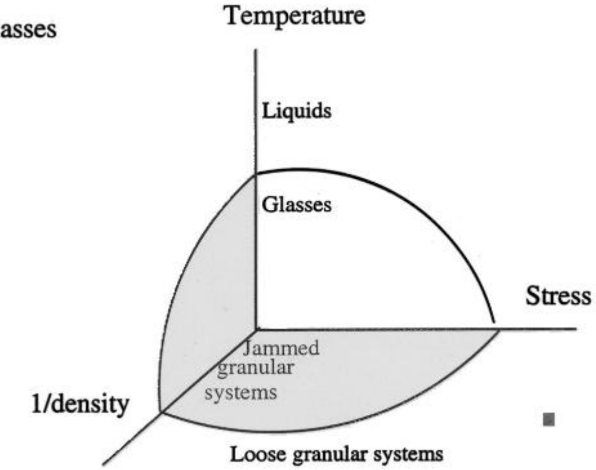
$$F \simeq x^{3/2} \qquad x > 0$$



A. Liu and S. Nagel
(e.g. Nature 396, 21 (1998)

Class of systems that are constrained or jammed

- Granular Materials
- Foams
- Colloids
- Glasses



1. Liu and Nagel, Phys. Rev. Lett. *68*, 2301 (1992).

2. Baxter, Leone and Behringer, Europhys. Lett. *21*, 569 (1993).

Note also

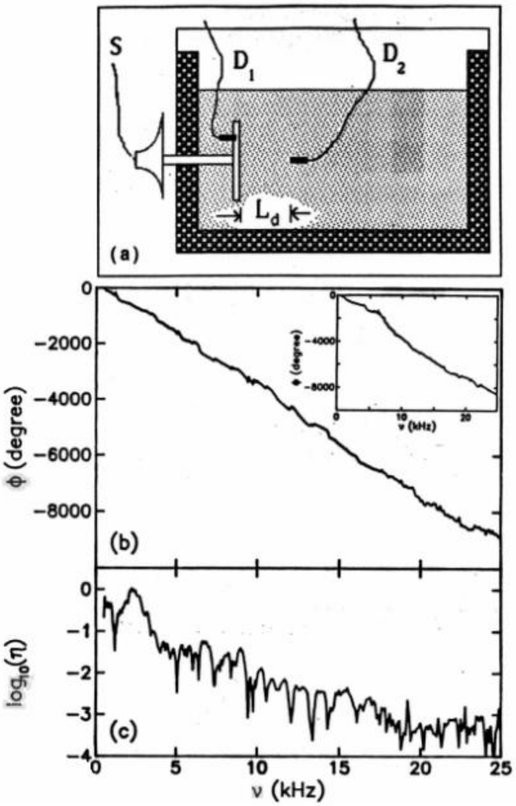
Ngadi + Rajchenbach
PRL 81, 1841 (1998)

Chicago group
PRE 53, 4673 (1996)

Durian et al.
Science, 275, 1920
(1997)

F. Radjai, M. Jean,
J.-J. Moreau, S. Roux
PRL 77, 264 (1996)

Liu + Nagel
PRL 68, 2301 (1992)



Intermittencies in the Compression Process of a Model Granular Medium

A. Ngadi and J. Rajchenbach
toire des Milieux Désordonnés et Hétérogènes (URA 800 CNRS), Case 86, Université Pierre et Marie Curie,
4 place Jussieu, 75252 Paris Cedex 05, France

(Received 28 July 1997)

Measurements of the force required by a bidimensional piling to be compressed are reported. This force is shown to exhibit sudden fluctuations of large amplitude which strongly deviate from the mean value. We determine the scaling laws followed by the mean force and by the fluctuations as a function of the grain diameter, the grain mass, the aspect ratio, and the container filling. The mechanism of fluctuations is identified, and the passage to the Mohr-Coulomb plasticity behavior is recognized in the limit of very large systems. [S0031-9007(97)04983-1]

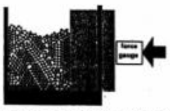


FIG. 1. Experimental setup for a compression test of two-dimensional packing of beads. The bead collection is composed of either 2.0, 2.4, or 3 mm diameter balls, and the left lateral wall is moved inwards with a constant velocity $v = 10\ \mu m/s$. The aspect ratio height/width is varied from 0.2 to 4.

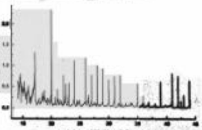


FIG. 2. An example of force variation as a function of the container width, obtained by pushing the lateral wall inwards: A series of pulses of very large amplitude superimposed to a base line can be observed.

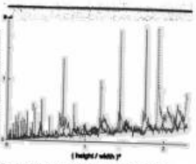


FIG. 3. Pushing force plotted as a function of the square of the pile height, for different fillings (- - - -: 250 grains, 3 mm diameter; ——: 500 grains, 2 mm diameter). For a low height/width ratio, the base lines follow a (height)² dependence, in contrast to the large fluctuation peaks. The straight line is a guide for the eyes.

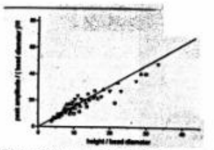
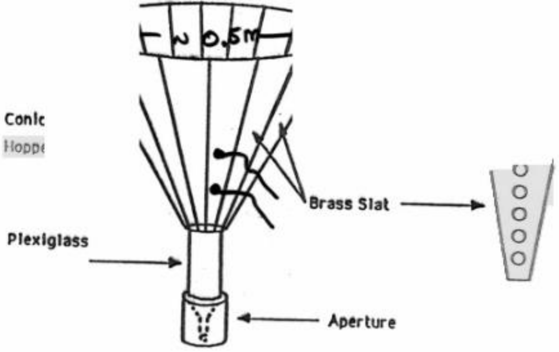
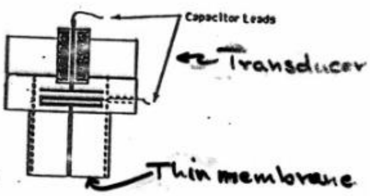


FIG. 6. Rescaled peak magnitude $F/d^{1/2}$. The points are taken from Fig. 5, corresponding to experiments conducted with different bead diameters. All points merge on a single straight line.



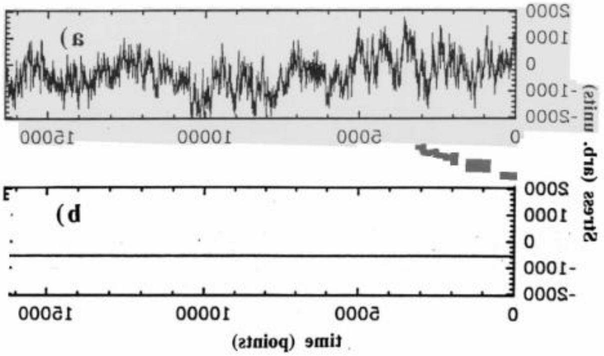
Sand Types

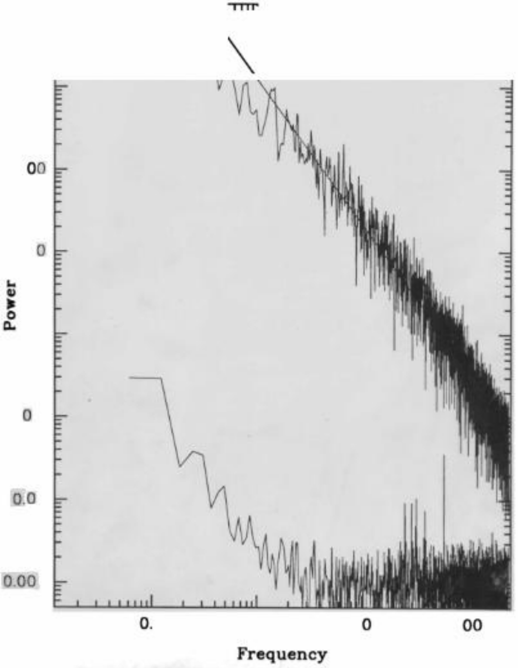
1. Smooth, spherical
   d = 0.07cm
   (Ottowa)
2. Rough d= 0.06cm



3. Booming Dune - rough, d= 0.02 cm

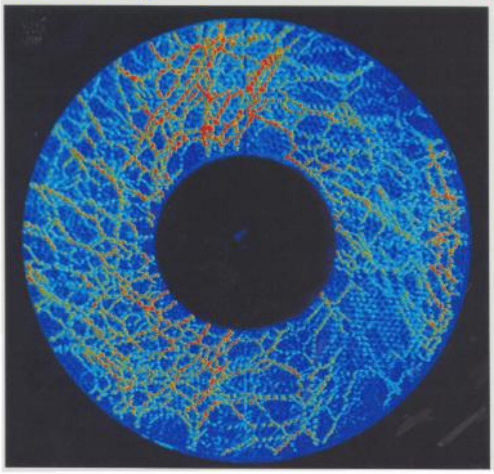
Baxter et al
Europhys. Lett. 21, 569 (1993)


a)
b)
Stress (arb. units)
2000
1000
0
-1000
-2000
0
5000
10000
15000
time (points)


00
0
0
0.0
0.00
Power
0.
0
00
Frequency

D

2D Shearing 🟦 small F 🟥 Large F



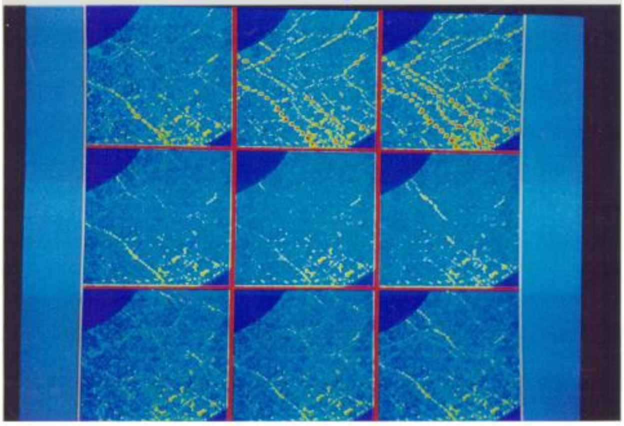
Stress chains, vaults,
arches

(From D. Howell
et al.)

1. **Chains deform, break, reform**
2. **Result: large spatio-temporal force fluctuations**

Intermittency:

Time series
$\sim \frac{1}{5}$ shear cell



Some Models for Stress Propagation
In Granular Materials
(Static)

1. Elasto-Plastic (Classical theories of soils—Coulomb)

   Plus variations: Rigid Plastic
   Incipient Failure Everywhere (IFE)
   Mixed Elastic and plastic
   …

2. Q-model (Coppersmith et al.)

   Microscopic model---introduces randomness
   Improvements by Socolar, Bouchaud et al.

   In continuum limit, Q-model is diffusional

3. Oriented Stress Linearity---OSL
   Bouchaud, Cates Wittmer, Claudin

   Builds on intuitive idea of stress chains

   Continum model—Hyperbolic Partial Differential Eqs.

EQUATIONS OF MOTION FOR A GRANULAR MATERIAL

- **conservation of mass:**

  $$\partial\rho/\partial t + \partial_i(\rho v_i) = 0.$$

- **conservation of energy:**

  material properties not affected by energy dissipation.

Dissipation by sliding friction

Typical Elasto-plastic Model

EQUATIONS OF MOTION FOR A GRANULAR MATERIAL - slow flow

(Compare to Newtonian Fluid)

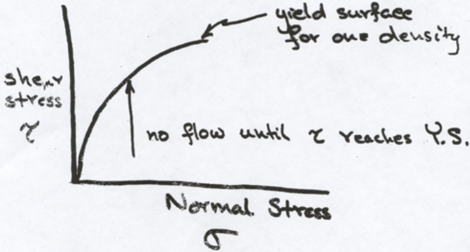
- **conservation of mass:**

  $\partial\rho/\partial t + \partial_i(\rho v_i) = 0.$

- **conservation of energy:**

  material properties not affected by energy dissipation.

Dissipation by sliding friction



- conservation of momentum:

$$\underline{\rho\, dv_i/dt = -\partial_j T_{ij}}$$

(T_{ij} = stress tensor)

- For a "simple" model: <u>Flow by plastic deformation</u>

$$T_{ij} = P\delta_{ij} + kP\underbrace{V_{ij}/|V|}_{\text{0'th order homogeneous}} . \quad \Rightarrow \text{rate invariant}$$

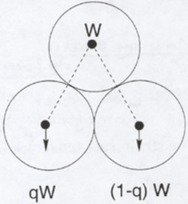
$$V_{ij} = -(\partial_j v_i + \partial_i v_j)/2 \quad \text{(The strain rate tensor)}$$

$$|V|^2 = \Sigma V_{ij}^2. \quad (|V| = \text{norm}(V))$$

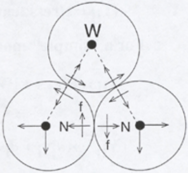
- compare to Newtonian fluid:

$$T_{ij} = P\delta_{ij} + \underbrace{2\eta[V_{ij} - (1/3)\mathrm{Tr}(V)] + (2\zeta/3)\mathrm{Tr}(V)}_{\text{1st order homogeneous}}.$$

Fluctuations and lattice-based models



q-model



Suppose the following distribution of forces P(f):

$P(f) = A f^2 \exp(-f/f_{avg})$ (Coppersmith et al.)

where f_{avg} = mean force , σ = rms about f_{avg}

$P(f > f_{avg} + \sigma) = 0.149$

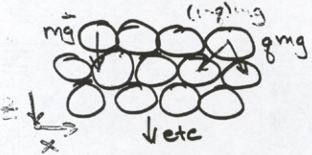
$P(f > 2 f_{avg}) = 0.062$

$P(f > 3 f_{avg}) = 0.0062$

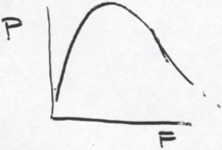
Continuum equations used in engineering design only model the mean behavior.

Q-Model

Model for static stress propagation and fluctuations



q = random variable



$$P(F) = C\,F^2 \exp(-F/F_o)$$

Liu, Nagel, Schecter, Coppersmith, Majumdar, Narayan and Witten, Science 269, 513 (1995).

*** **Experiments support exponential distribution at large F**

Improvements by Socolar (torque and vector force bal.)
P.R.E. 57, 3204 (1998)

Similar results by 'Contact Dynamics' Radjai et al. PRL 77, 264 (1996).

PHYSICAL REVIEW E VOLUME 57, NUMBER 3 MARCH 1998

Force distribution in a granular medium

Daniel M. Mueth, Heinrich M. Jaeger, and Sidney R. Nagel
The James Franck Institute and Department of Physics, The University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637

(Received 18 August 1997)

We report on systematic measurements of the distribution of normal forces exerted by granular material under uniaxial compression onto the interior surfaces of a confining vessel. Our experiments on three-dimensional, random packings of monodisperse glass beads show that this distribution is nearly uniform for forces below the mean force and decays exponentially for forces greater than the mean. The shape of the distribution and the value of the exponential decay constant are unaffected by changes in the system preparation history or in the boundary conditions. An empirical functional form for the distribution is proposed that provides an excellent fit over the whole force range measured and is also consistent with recent computer simulation data. [S1063-651X(98)02603-8]

PACS number(s): 81.05.Rm, 46.10.+z, 05.40.+j, 83.70.F

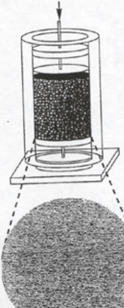


FIG. 1. Sketch of the apparatus used for experiments with "floating walls." The lower piston is fixed and the cylinder is supported by friction with the bead pack. A load is applied to the upper piston and the beads press the carbon paper into white paper, leaving marks which are used to determine the contact forces. A detail of the obtained raw data is shown in the photograph (field of view: 76 mm across).

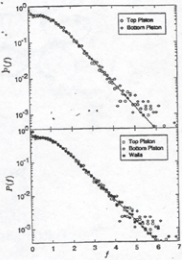
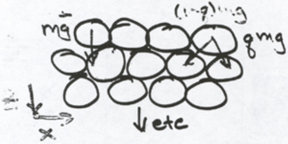


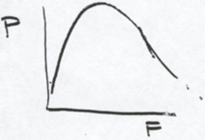
FIG. 3. The distribution $P(f)$ of normalized forces f against the top piston (open circles), the bottom piston (diamonds), and the walls (solid circles). The upper panel shows $P(f)$ for the pistons, averaged over fourteen identical experiments. The curve drawn is a fitting function as explained in the text [Eq. (1)]. The lower panel shows the same data, but with data from the walls included as well.

Q-Model

Model for static stress propagation and fluctuations



q = random variable



$$P(F) = C\,F^2 \exp(-F/F_o)$$

Liu, Nagel, Schecter, Coppersmith, Majumdar, Narayan and Witten, Science 269, 513 (1995).

***** Experiments support exponential distribution at large F**

Improvements by Socolar (torque and vector force bal.)
PRE 57, 3204 (1998)

Similar results by 'Contact Dynamics' Radjai et al. PRL 77, 264 (1996).

New Continuum Models

1. Q-model → in continuum limit:

Diffusion of Forces/stresses $\frac{\partial F}{\partial z} = D \frac{\partial^2 F}{\partial x^2}$

2. **Oriented Stress Linearity (OSL): based on idea that chains can slip wrt each other.**

Claudin, Cates, Wittmer: J. Phys. I 5, 639 (1995)

Combines static stress balance plus

$$T_{xx} = \eta\, T_{zz} + \mu\, T_{xz}$$

η and μ history-dependent

$$(\partial_z + c_+ \partial_x)(\partial_z + c_- \partial_x)\, T_{ij} = 0$$

$c_\pm(\mu, \eta)$

(hyperbolic—propagating stresses)

3. **Kenkre, Scott, Pease and Hurd—suggest mixed diffusion/propagation**

Note: Soil mechanics models can have elliptic behavior

PHYSICAL REVIEW E VOLUME 57, NUMBER 4 APRIL 1998

Models of stress fluctuations in granular media

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Service de Physique de l'Etat Condensé, CEA, Orme des Merisiers, 91191 Gif-sur-Yvette, Cedex, France

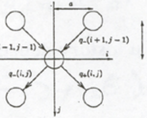
M. E. Cates and J. P. Wittmer
Department of Physics and Astronomy, JCMB King's Buildings, University of Edinburgh, Mayfield Road, Edinburgh EH9 3JZ, United Kingdom

(Received 19 November 1997)

We investigate in detail two models describing how stresses propagate and fluctuate in granular media. The first one is a scalar model where only the vertical component of the stress tensor is considered. In the continuum limit, this model is equivalent to a *diffusion equation* (where the role of time is played by the vertical coordinate) plus a randomly varying convection term. We calculate the response and correlation function of this model and discuss several properties, in particular related to the stress distribution function. We then turn to the tensorial model, where the basic starting point is a wave equation that, in the absence of disorder, leads to a raylike propagation of stress. In the presence of disorder, the rays acquire a diffusive width and the angle of propagation is shifted. A striking feature is that the response function becomes negative, which suggests that the contact network is mechanically unstable to very weak perturbations. The stress correlation function reveals characteristic features related to the raylike propagation, which are absent in the scalar description. Our analytical calculations are confirmed and extended by a numerical analysis of the stochastic wave equation. [S1063-651X(98)07004-4]

PACS number(s): 81.05.Rm, 46.10.+z, 05.40.+j, 83.70.Fn

$$w(i,j)=w_g+q_+(i-1,j-1)w(i-1,j-1)$$
$$+q_-(i+1,j-1)w(i+1,j-1). \qquad (1)$$



Proposed Model:

(after Claudin et al.)

- Two-branch Convection-Diffusion Equation. Assuming the stress satisfies the following PDE:

  $$O^{+}O^{-}\sigma = 0\ ,$$

  where $O^{\pm} = \partial h - D\partial xx \pm c\partial x$, with c and $D \geq 0$.

  For a $\delta(x,0)$ initial condition, the solution to this equation is:

  $$\sigma_{zz} = \frac{F}{2}\left(\frac{1}{2\sqrt{\pi Dz}}e^{-\frac{(x-cz)^2}{4Dz}} + \frac{1}{2\sqrt{\pi Dz}}e^{-\frac{(x+cz)^2}{4Dz}}\right)$$

- Biased Random-Walk Model and DoubleY Model (Bouchaud et al. Ludwig)

Next generation.

$D \to 0 \Rightarrow$ OSL model.

Since then.

$$O^{+}O^{-} = (\partial h + c\partial x)(\partial h - c\partial x)$$

Contrast types of P.D.E.'s

1. Φ-model (continuum limit):
   Parabolic
2. OSL hyperbolic:
3. Ellasto-plastic:
   elliptic up to yield.

Other Features

1. Hindrance Effects

2. Transition regimes

$T = 0$ Intermediate $T>>0$

|------------------------|------------------------|

Many flows contain all ranges of granular "temperature" simultaneously

Other Features

1. Hindrance Effects

Dense ordered

Low Density
Disordered

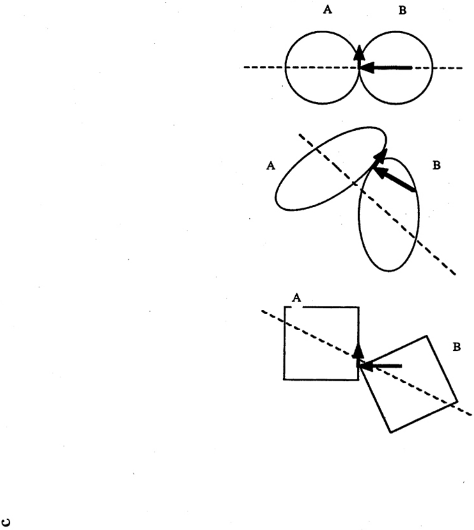
2. Transition regimes

T = 0 Intermediate T>>0

|------------------------|------------------------|

Plastic ? Gas

Many flows contain all ranges of granular "temperature" simultaneously


A
B
A
B
A
B
c

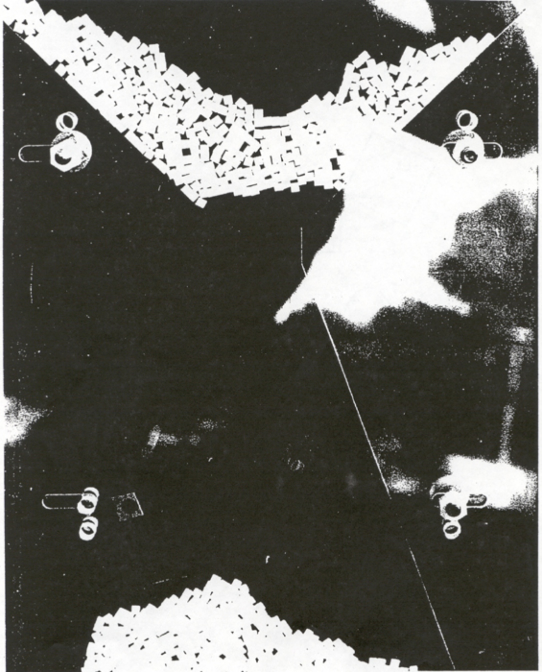


Fig. 1. A 16-mm camera operating at about 1440 frames per second or 60 times the normal projection rate documents a typical disperse flow in the experimental chute. Plastic spheres 6 mm in diameter contained in a hopper above and behind the upper end of the apparatus feed onto a gently inclined tray, organizing into a single layer that spills into the head of the 6.7-mm-wide chute, where they form an essentially two-dimensional grain flow. The chute is 3.7 m long and 0.5 m deep and can be tilted to any inclination. The bed for the flows is the light-colored strip inside the chute, which can be independently adjusted through a small range of angles to fine tune the inclination. The flows approach a nominally steady, uniform condition rapidly, typically in 10-15 flow depths. The center of the camera field is about 70 cm upstream of the chute outlet.

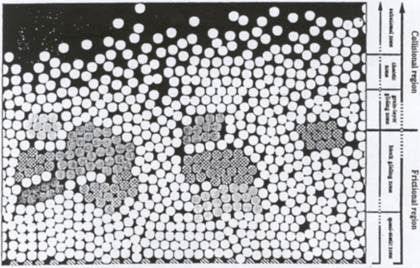


Fig. 3. Flows can be divided into a frictional region in which contacts between neighboring particles are enduring and frictional and an overlying collisional region in which binary collisional contacts predominate. The regions are further subdivided into zones based on characteristics of the collective and individual motions of particles: the frictional region consists of quasi-static and block-gliding zones, and the collisional region consists of grain-layer-gliding, chaotic, and saltational zones. The boundaries between regions and zones are gradational, and some flows do not exhibit all the zones. Dashed lines on vertical axis indicate both temporal variation and uncertainty in boundary positions. Groups of shaded particles are blocks (see text). Drawing traced from a single film frame of 6-mm-diameter particles flowing from right to left over the 6-mm bed inclined 38° to the horizontal. The flux is about 2070 particles s^{-1}, determined by counting the number of particles crossing a line perpendicular to the bed in 0.42 s.

Conclusions

Granular systems:

Are complex, rich arena for physics and other disciplines

Have phases analogous to solids and fluids, but…

Require new statistical approaches (no 'canonical ensemble' for example)

Have unresolved questions of very simple issues such as how forces 'propagate'

Show large force fluctuations

Offer great intellectual challenges, but also the potential for real economic payoff

Note also Numerical Studies

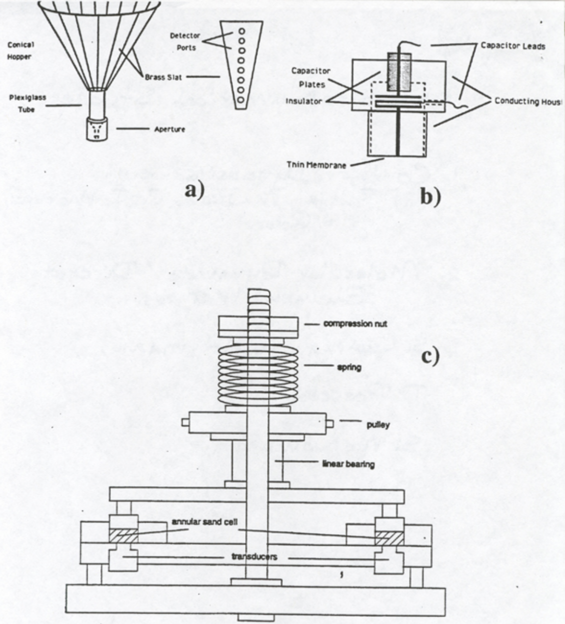
1. Contact Dynamics - e.g.
   F. Radjai, M. Jean, J.-J. Moreau,
   S. Roux

2. Molecular Dynamics / Discrete
   Element Methods

   S. Luding, H. Herrmann,

   T. Poeschel

   S. McNamara


Conical Hopper
Brass Slat
Plexiglass Tube
Aperture
Detector Ports
a)
Capacitor Leads
Capacitor Plates
Insulator
Conducting Housi
Thin Membrane
b)
compression nut
spring
c)
pulley
linear bearing
annular sand cell
transducers

"Definition"

- Consists of a large number of interacting macroscopic grains.
- Grain-grain interactions are purely classical.
- Grains interact only when in contact.
- Collisions are generally inelastic / dissipative
- Grains are surrounded by a fluid or vacuum.

Fig: E. van Doorn

Why Granular Flows?

1. Interesting statistical questions

   Fluctuations, hydrodynamic limit, e.g. of jamming system, self-organized criticality…

2. Continuum dynamics

   What are right models? When do they apply?…

3. Member of class of similar systems:

   Foams, Glasses, Colloids, Quantum flux lattices

4. Important applications

   Coal and grain transport…
   Pharmaceuticals
   Xerography
   Soils
   Avalanches
   Mixing
   Earthquakes and mudslides…

5. $$$!! Lots of them Claim: ~1 $Trillion/year in US for bulk solids handling

   Industrial facilities operate well below design efficiency