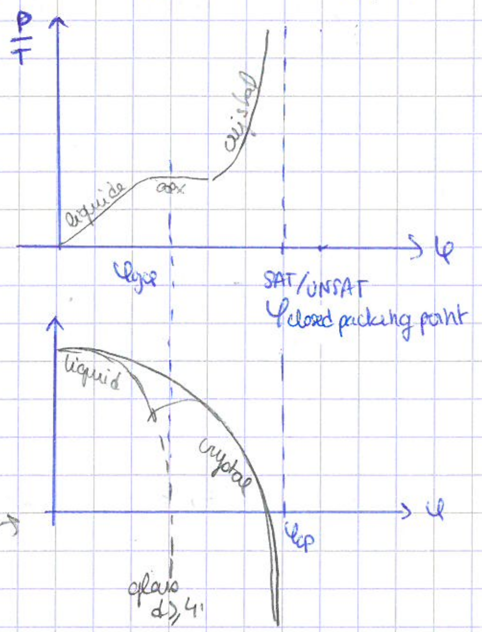
2 spheres in $d\to\infty$ 3 Criticality of jamming.

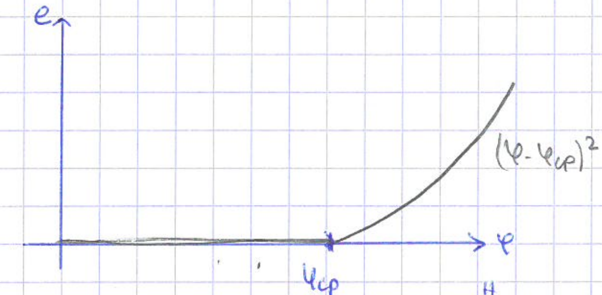
1b Problems: crystal, disorder.

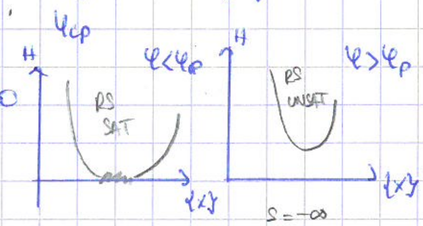
Two important differences with perceptron → no quenched disorder

→ optimal will have packing fraction of the crystal.



→ CRYSTAL LIKE PERCEPTRO





How to study instead disordered phase?

① Introduce back some disorder in the system: → polydispersity $h_\mu(\vec{x}) = |x_i - x_j| - \sigma_{ij}$

→ disorder in the constraints $h_\mu(\vec{x}) = |\vec{x}_i - \vec{x}_j + A_{ij}| - \sigma$ (MKK) Flo Jorge

② Increase dimension $d \geq 4$ → very hard to crystallize, will never nucleate.

20/07/2017 entropy of quantum systems > 0
entropy classicaly 1 harmonic oscillator → $-\infty$, volume = 1.

entropy crisis = Kauzmann → configurational entropy → 0.

1c Franz Parisi potential "replica without disorder".

Take equilibrium configuration R:

$$V(Q) = -T\int d\vec{R}\, \frac{e^{-\beta H(\vec{R})}}{Z} \log \int dx\, e^{-\beta H(x)}\, \delta(Q - Q(X,R))$$

→ can be defined without any disorder
→ need replicas for ⟨log⟩!

⇒ we do a replica computation, R playing the role of the quenched disorder – which we average over finally.

II 2 Hard spheres (SAT phase) in dimension $d\to\infty$.

[Frisch, Rivier, Klein, Wyler '80s
Frisch, 1 . '99 PRE]

2a

Virial expansions (low density expansions)

$$-\beta F(\rho(x)) = \int dx\, \rho(x)\left[\log \rho(x) - 1\right] + \int dx dy\, \rho(x)\rho(y)\, f(x-y) + \triangle + \square + \boxtimes + \boxtimes + \dots$$

ideal gas

mayer function $e^{-\beta v(x-y)} - 1$

↳ plugging a uniform density in space $\rho(x) = \rho$, impose translational invariance → liquid ≠ crystal.

2b

(lower bound to the radius of convergence)

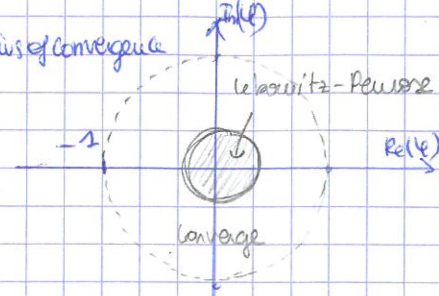
1) Lebowitz – Penrose '64: Virial series converges $\bar{\varphi}_{conv} \geqslant 0.144$ → density below → liquid

2) $-\beta F = -\beta F^1 + \text{—} + \triangle + \square + \boxtimes + \boxtimes + \ldots$

~ Georges-Yedidia $\quad \rho^2 \quad \rho^3 \quad \rho^4$

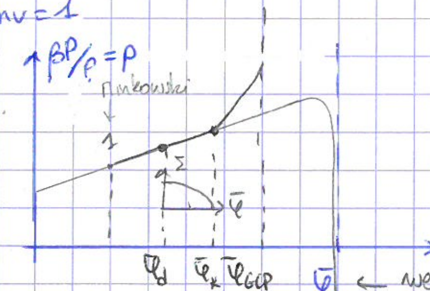
RING DIAGRAMS DOMINATE AT EACH ORDER

radius of convergence



conjecture 1: $\varphi_{conv} = 1$

conjecture 2:



$p = 1 + \frac{\bar{\varphi}}{2} +$ something exponentially small for $\bar{\varphi} < \bar{\varphi}_0 = 2^{0.22135d}$

← weird that $p \to -\infty$ and not $+\infty$.

→ is maximum the point of existence of the liquid? (Sal Torquato) beating exponentially the Minkowski bound.

conjecture 3:

↳ actually much before this point → dynamical, Kautzman transition. (result of replica computation).

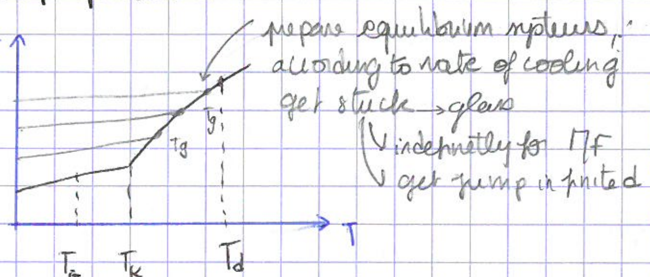
→ the part of the curve in gray above $\bar{\varphi}_K$ is thus unphysical.

→ scaling: $\bar{\varphi}_d = 4.8d$ → much better $\quad \varphi_d = 2^{-d}4.8d$

$\bar{\varphi}_K = d\log d + \ldots \quad \varphi_K = 2^{-d} d\log d + \ldots$

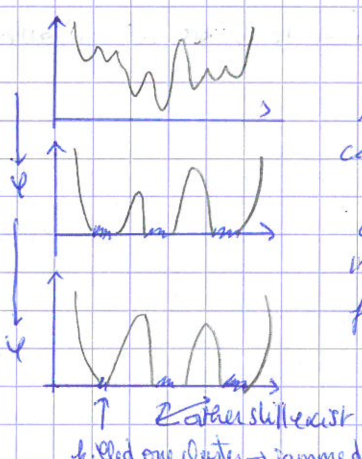
$\bar{\varphi}_{GCP} = d\log + \ldots \quad \varphi_{GCP} = d\log d \ldots$

2c Out of equilibrium

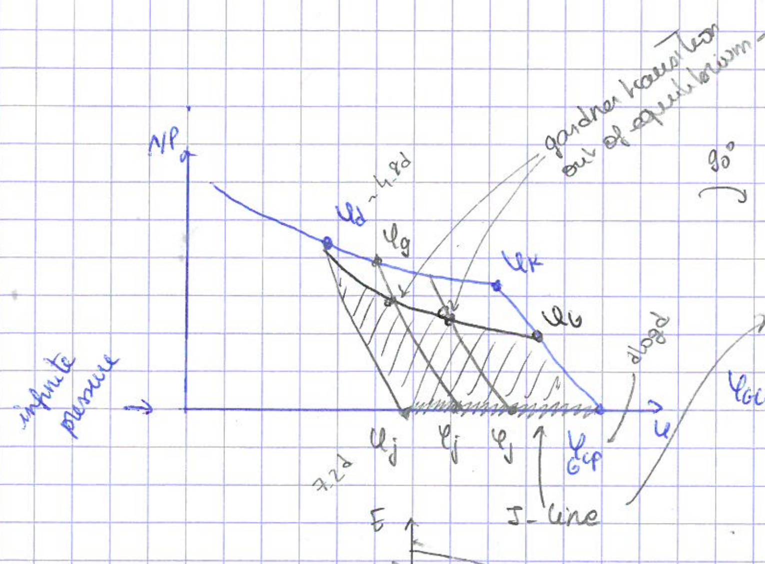


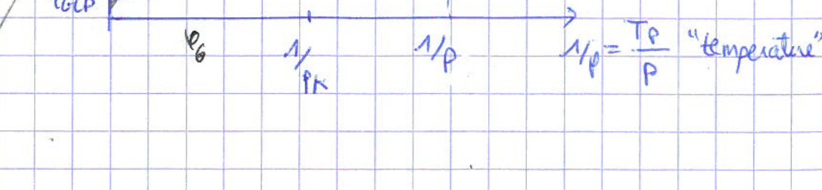
RECALL

"p spin landscape"



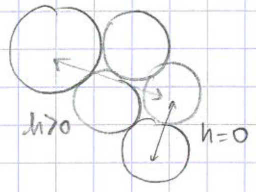
so that if in equilibrium, can always visit the remaining states, but out of equilibrium, stuck in one state, can never jump barriers → jammed state.



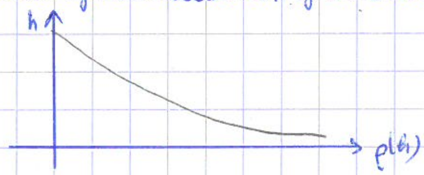


II 3 Criticality of the J-line

1) Isos aticity of all points of the J-line -

2) 

distribution of the realisation of the constraints: $p(h) = \overset{\text{pick}}{z}\,\delta(h) + h^{-\gamma}$



3) Compute in some procedure forces corresponding to contacts $\rightarrow P(f) \sim f^{\theta}$

4) Matthieu Wyart with scaling arguments $\frac{1}{2+\gamma} = \theta$, ok with results of replica computation