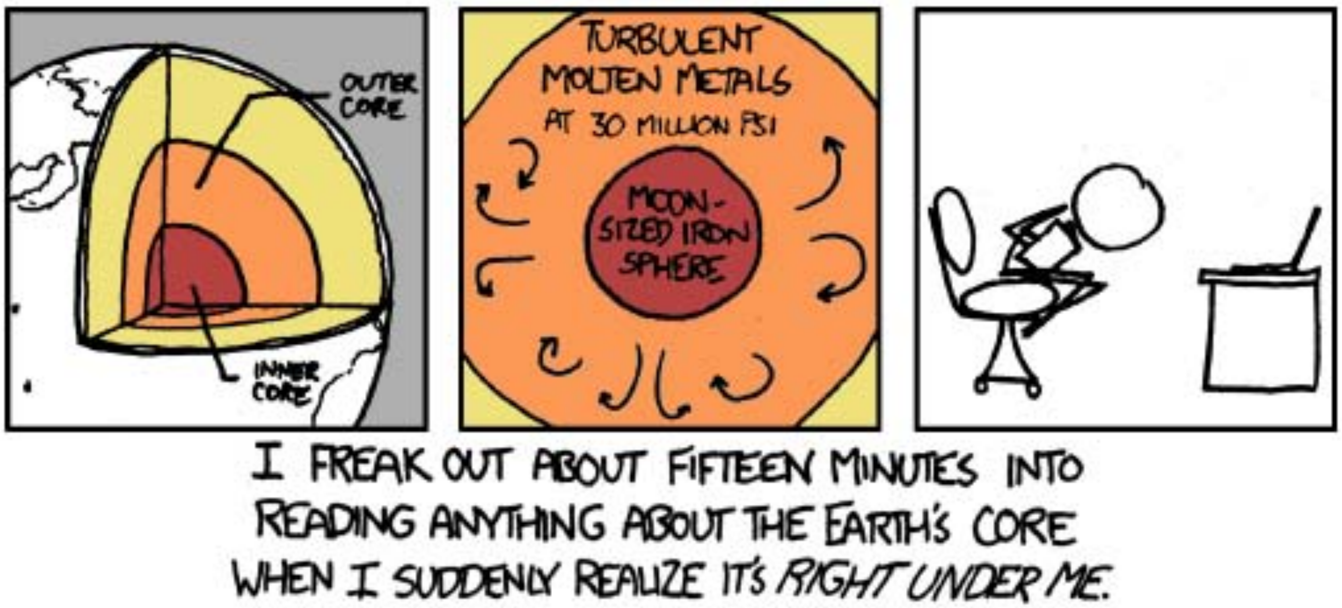
# TURBULENT
# Core



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Permanent link to this comic: http://xkcd.com/913/
Image URL (for hotlinking/embedding): http://imgs.xkcd.com/comics/core.png

# What is turbulence?

Daniel P. Lathrop
University of Maryland

Goal: grant an intuitive sense of turbulence and its importance in nature, science, and engineering

Outline:

1. Definitions
2. Examples
    1. Geophysics
    2. Astrophysics
    3. Engineering
    4. Quantum turbulence
    5. Optical turbulence

## Turbulence: an emergent phenomena

Rivers!
Niagra Falls

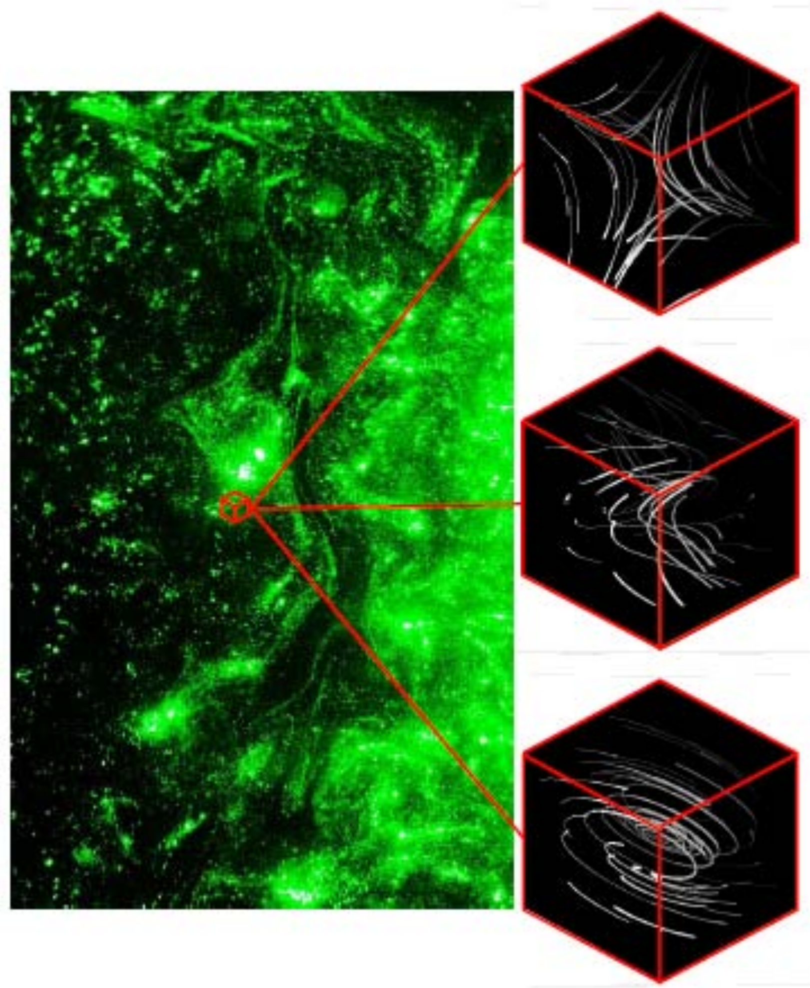
# Reynolds number

$R$ or $Re = U L / \nu$

$Re \sim$ Inertia / viscosity

Large R → Turbulence

# Turbulence definitions



**Turbulence:** A Spatially extended field with a large, continuous range of aperiodic spatial and temporal dynamics

**Newtonian fluid turbulence:** Turbulence in a classical, Newtonian viscous fluid

**Kolmogorov turbulence:** Turbulence exhibiting spatial power spectra with

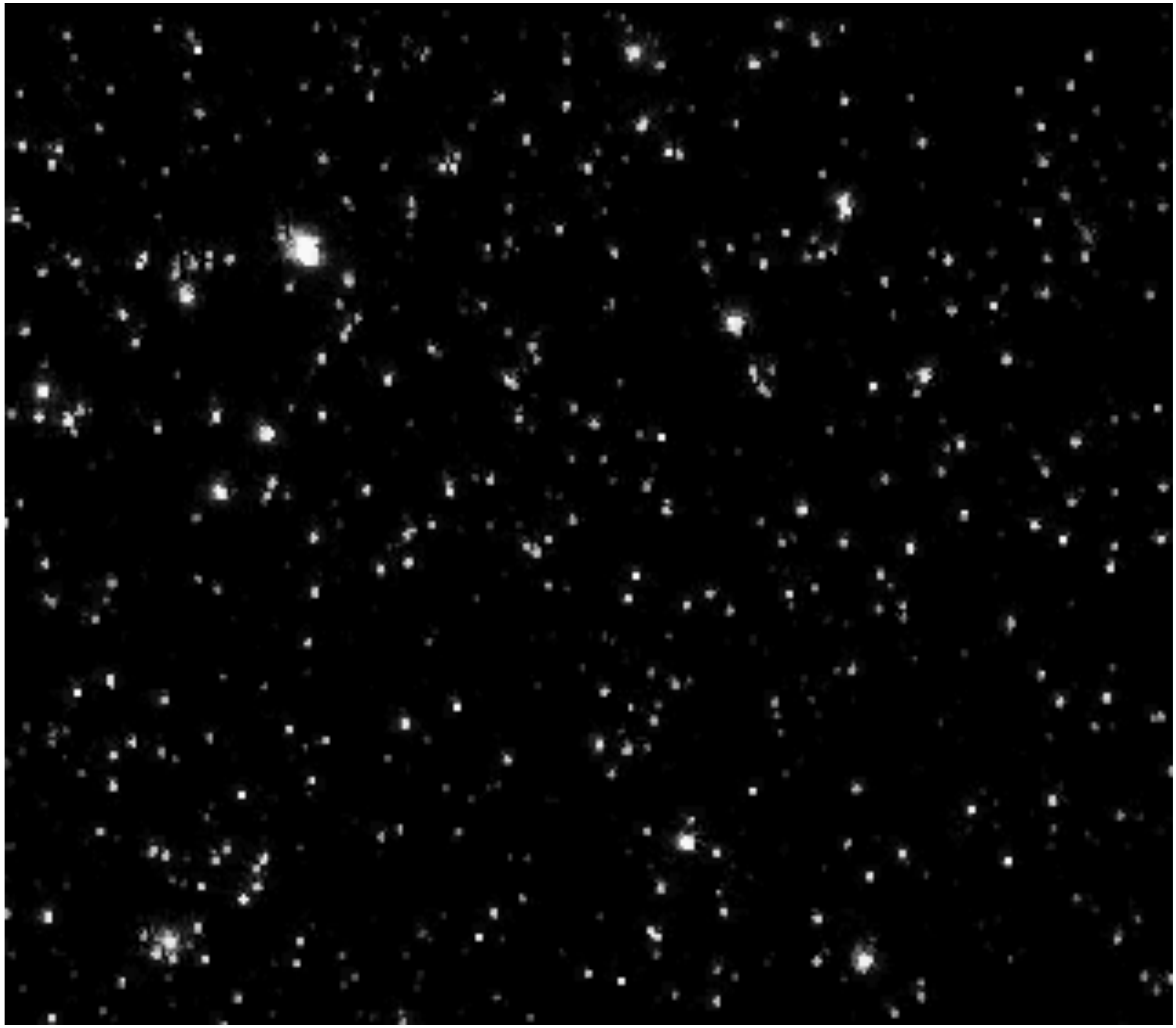
$$E(k) \sim k^{-5/3}$$

**Quantum turbulence:** A turbulent complex field or interacting set of complex fields

**Optical turbulence:** A turbulent field of electromagnetic amplitudes

Turbulence at small scale:

strain
vorticity



Watson:  You have to admit Holmes that a supernatural explanation to this case is theoretically possible.
Holmes: Agreed, but it is useless to theorize before one has data. Inevitably one begins to twist facts to suit theories instead of theories to suit facts.

## Computational speed of experiments
## (for equivalent Direct Numerical Simulation)

**Finest scale** $l_K = L\, R^{-3/4}$ $R \sim 5 \times 10^6$

$L / l_K \sim 100{,}000$ **linear resolution needed**

$N_g = (L/ l_K)^3 = R^{9/4}$ **number of grid points**

$N_f = 6$ **number of fields**

$1/T = U_{max} / l_K = R^{3/4}\, U_{max} /L$ **update rate** $U_{max} / L \sim 250\ s^{-1}$

**Rate of updating field values** $N_f\, N_g/T = 6\, R^3\, U_{max} / L = 3 \times 10^{23}$

**.3 yottaflops**

**300,000 exaflops**

***300 million petaflops***

So many natural flows are necessarily turbulent, but we cannot simulate them.

Turbulence:

- Fluctuating in time and space
- Aperiodic
- Large range of time and length scales
- Deterministic

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Turbulence:

- Fluctuating in time and space
- Aperiodic
- Large range of time and length scales
- Deterministic

Anyone who attempts to generate random numbers by deterministic means
is, of course, living in a state of sin.
John von Neumann

# Eyjafjallajokull 2010

# Cleveland volcano

Atmospheric flows

Cumulous

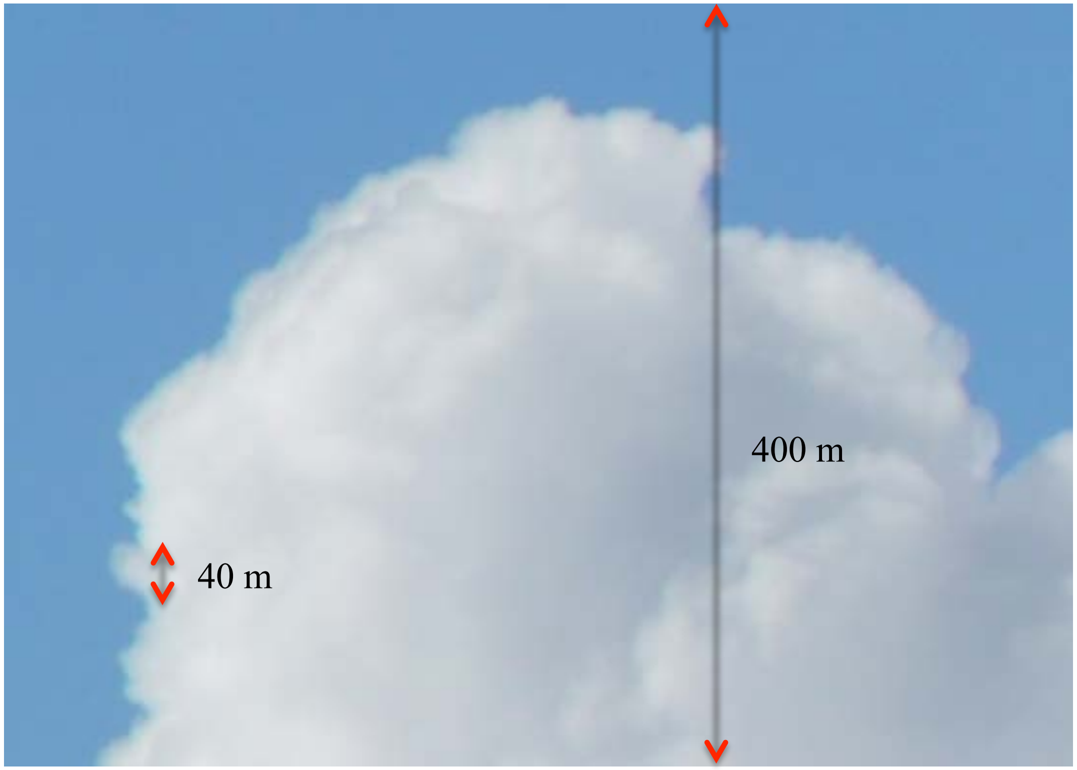
# Cumulonimbus

# Cumulonimbus

Atmospheric flows

# Cumulonimbus

# Cumulonimbus

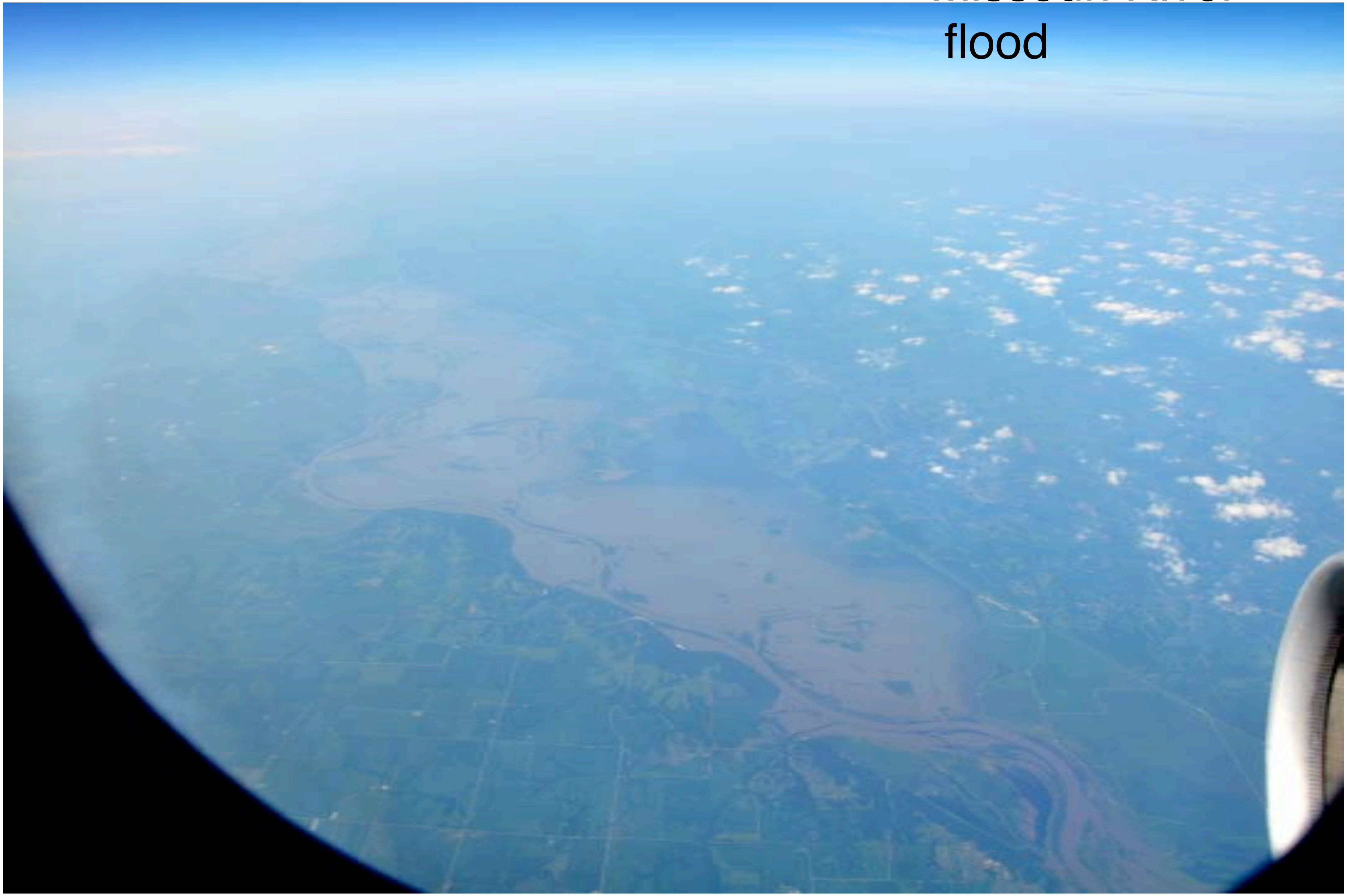


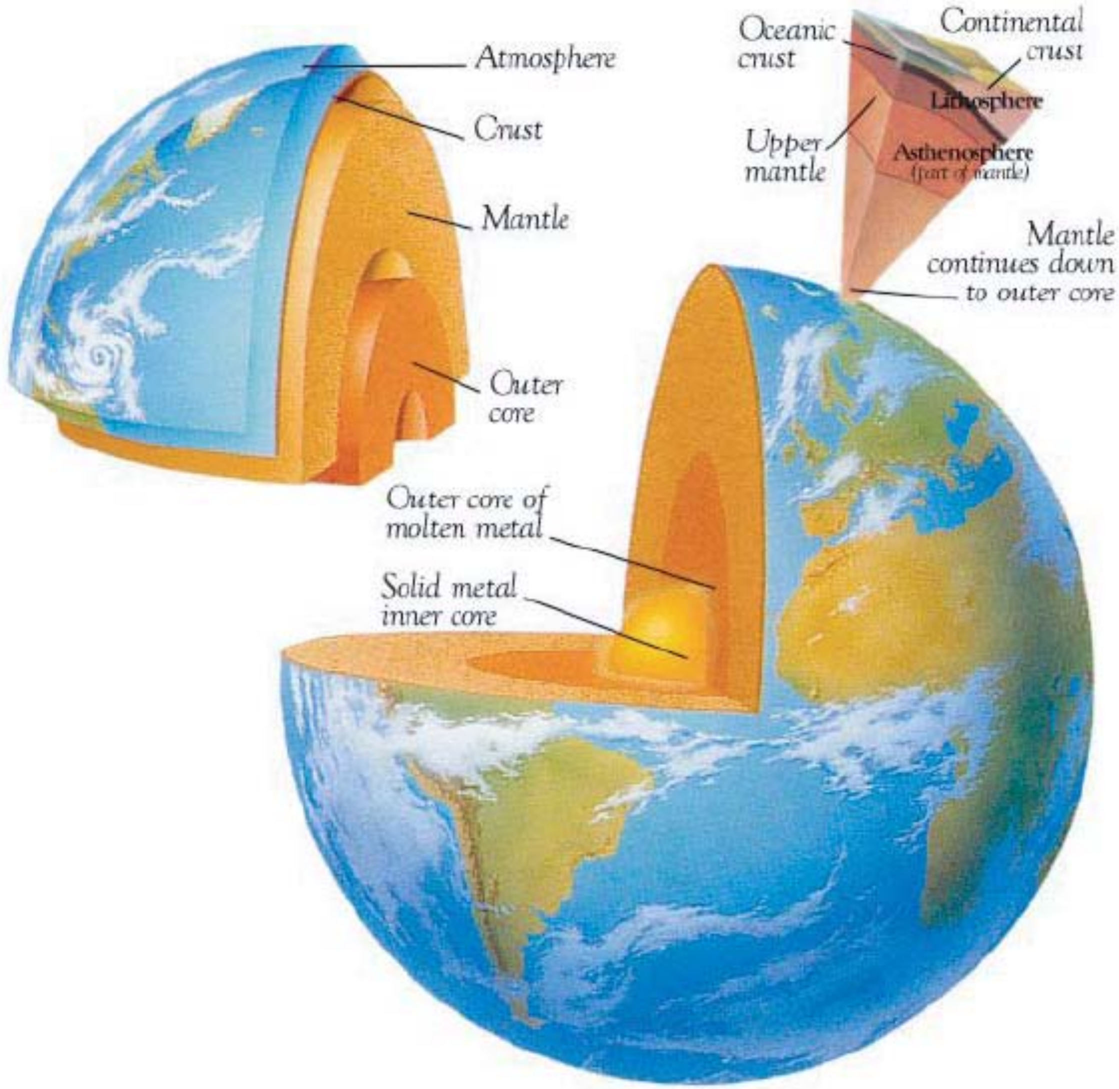
Geophysical turbulence

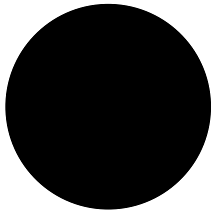
# Rivers!

# Geophysical turbulence

Rivers!
Missouri River
flood




Atmosphere
Crust
Mantle
Outer core
Oceanic crust
Continental crust
Lithosphere
Upper mantle
Asthenosphere (part of mantle)
Mantle continues down to outer core
Outer core of molten metal
Solid metal inner core



# Simple Earth

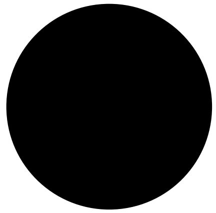
Nonrotating; Spherical

Isothermal $T = 2.725\ ^{o}K$

Zero axial tilt

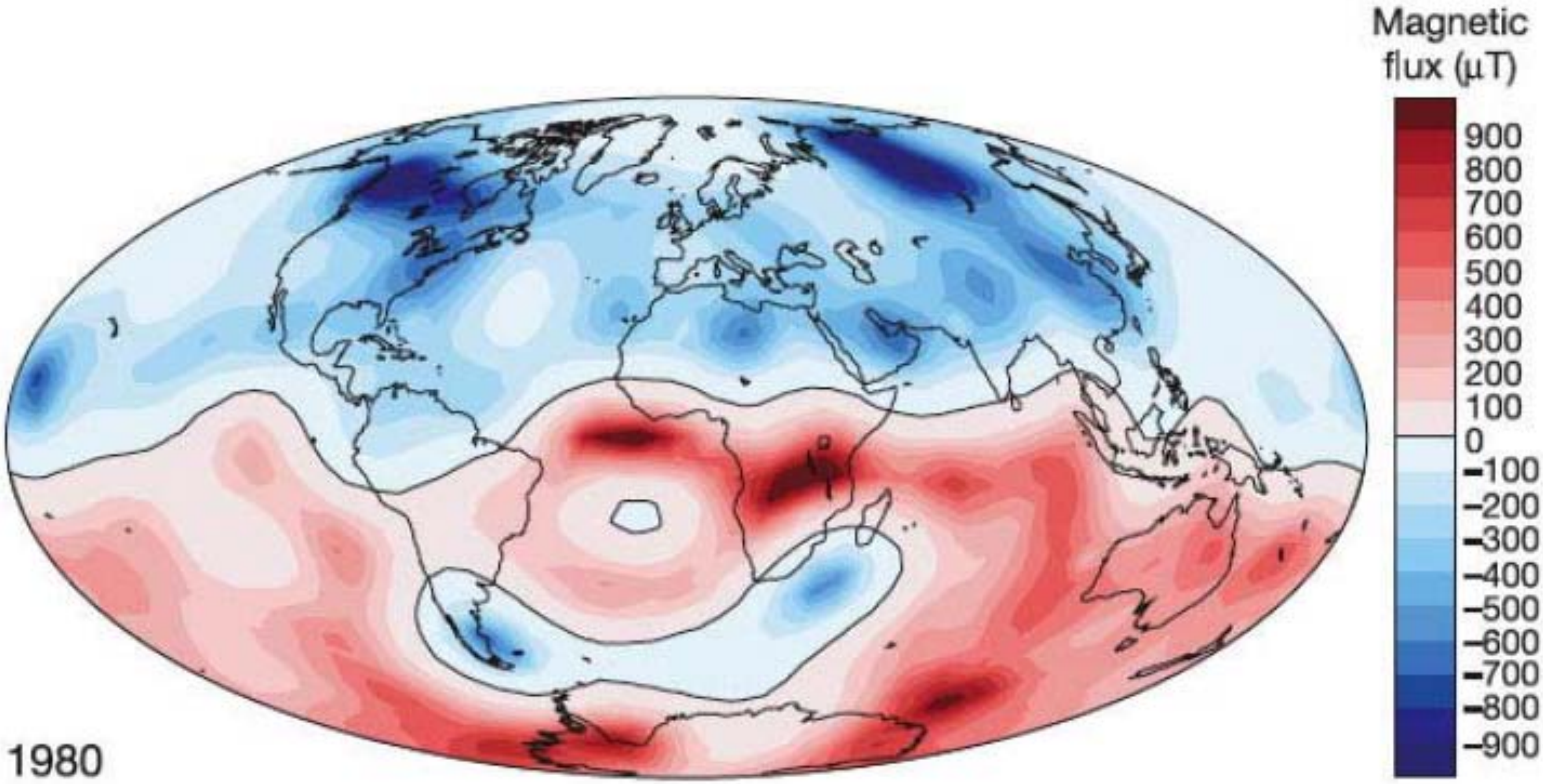
Quiet atmosphere
Quiet crust
Quiet mantle
Quiet core

Core magnetics → $\underline{B} = 0$

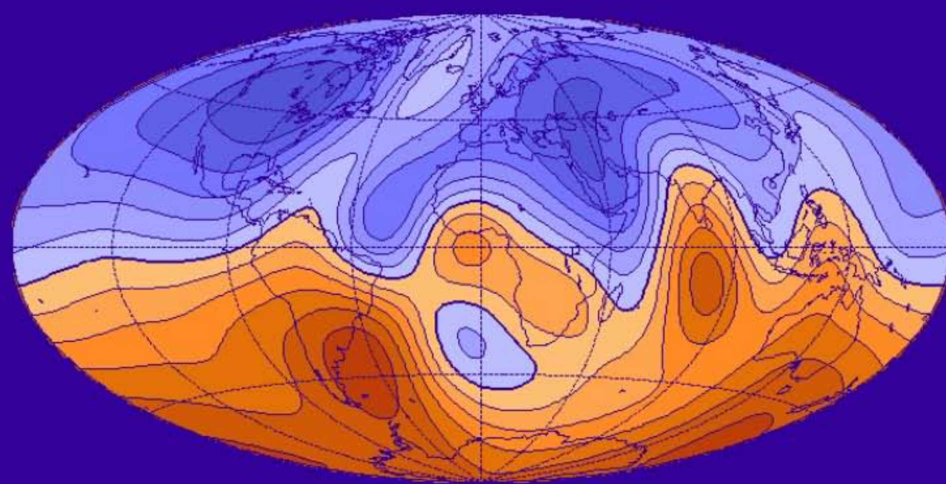


| Simple Earth | Habitable Home |
|---|---|
| Nonrotating; Spherical | One Day Period; Oblate spheroid |
| Isothermal $T = 2.725\,^{o}K$ | $T_{core} / T_{space} \sim 10^3$ |
| Zero axial tilt | $23.5^{o}$, precessing |
| Quiet atmosphere | Turbulent, zonal atmosphere |
| Quiet crust | Brittle quaking crust |
| Quiet mantle | Complex convecting mantle |
| Quiet core | Turbulent convecting core |
| Core magnetics → $\underline{B} = 0$? | Active self-inducing geodynamo |

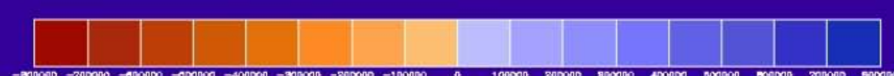
A. Jackson, et al., Phil. Trans. Roy. Soc. A 358, 957 (2000)



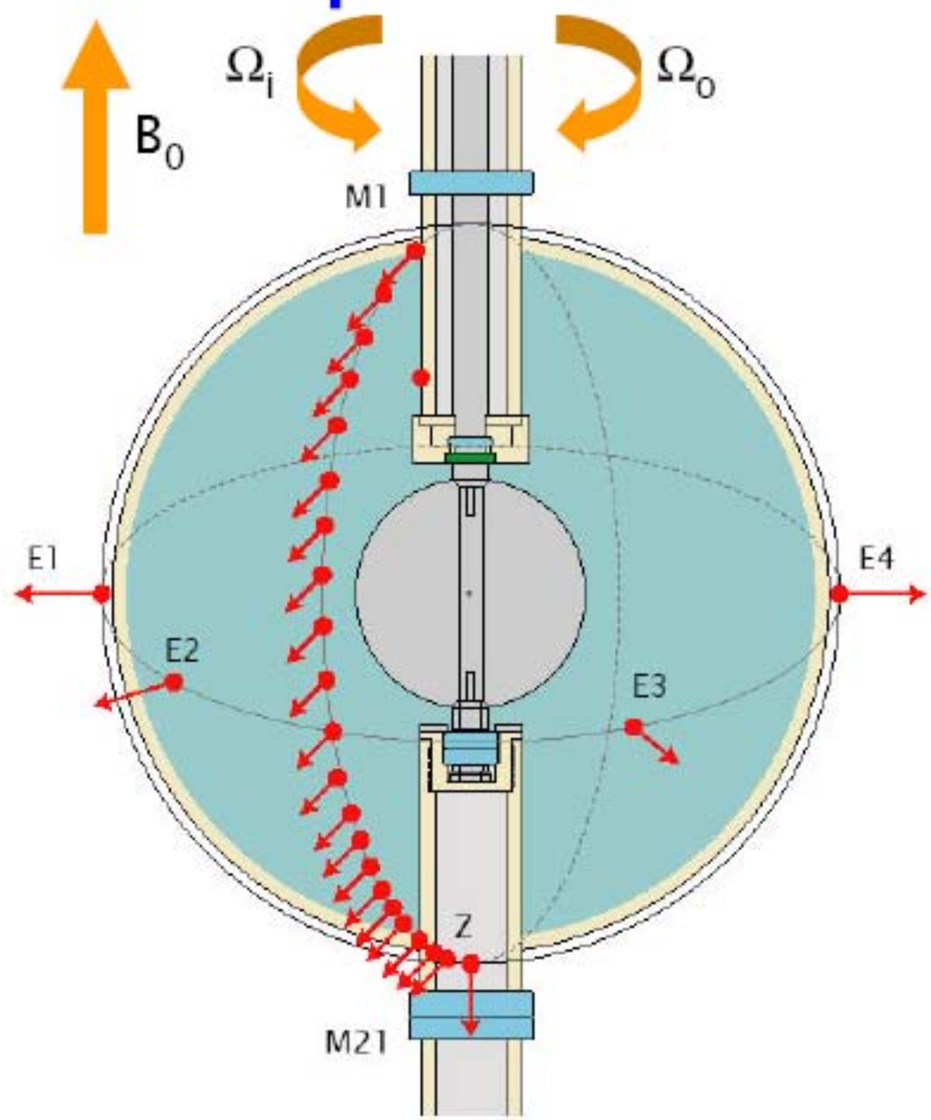
1590



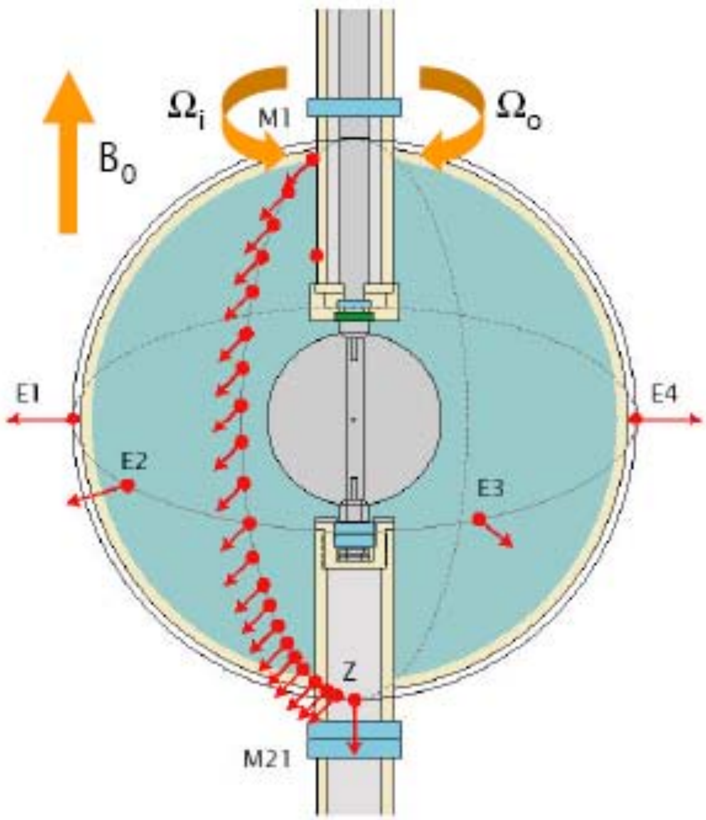
Contour interval = $10^5$



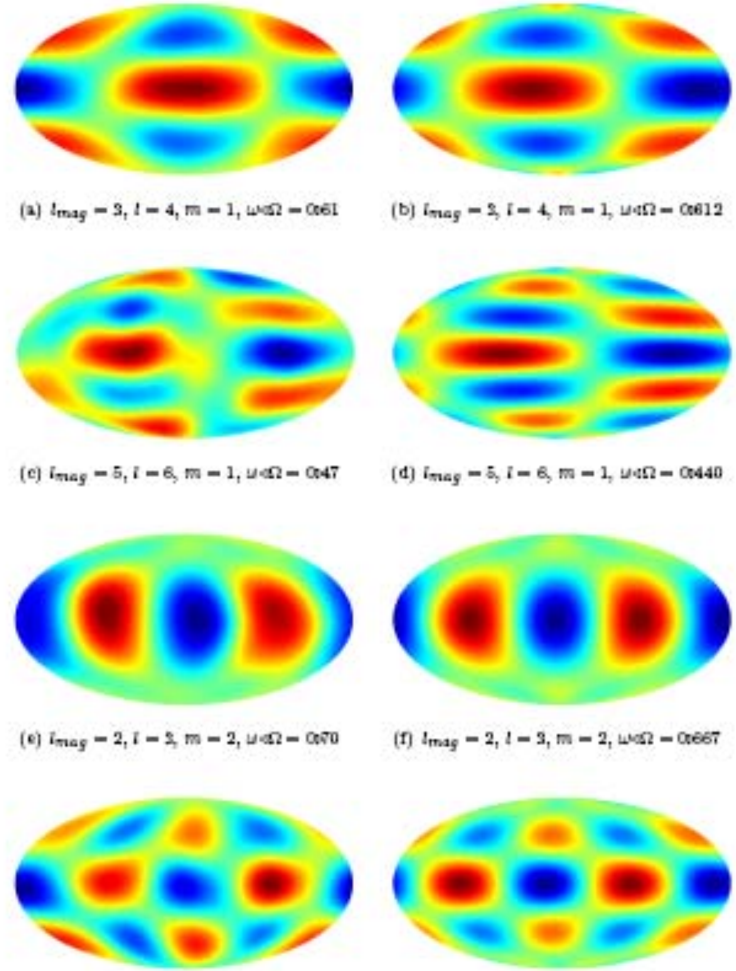
# 60 cm experiment




# Inertial Modes - Low *Ro*, Low *E*




$$\frac{\partial \vec{U}}{\partial t} + 2\hat{z} \times \vec{U} = -\nabla p$$



(a) $l_{mag} = 3$, $l = 4$, $m = 1$, $\omega/\Omega = 0.61$

(b) $l_{mag} = 3$, $l = 4$, $m = 1$, $\omega/\Omega = 0.612$

(c) $l_{mag} = 5$, $l = 6$, $m = 1$, $\omega/\Omega = 0.47$

(d) $l_{mag} = 5$, $l = 6$, $m = 1$, $\omega/\Omega = 0.440$

(e) $l_{mag} = 2$, $l = 3$, $m = 2$, $\omega/\Omega = 0.70$

(f) $l_{mag} = 2$, $l = 3$, $m = 2$, $\omega/\Omega = 0.667$

(g) $l_{mag} = 4$, $l = 5$, $m = 2$, $\omega/\Omega = 0.49$

(h) $l_{mag} = 4$, $l = 5$, $m = 2$, $\omega/\Omega = 0.467$


Douglas H. Kelley, Santiago Andrés Triana, Daniel S. Zimmerman, Andreas Tilgner, and Daniel P. Lathrop.
Geophysical and Astrophysical Fluid Dynamics 101 5/6: 469-487 (2007)

Rapidly Rotating -- Coriolis Large

$$\partial_t \vec{v} + \left(\vec{v} \bullet \vec{\nabla}\right)\vec{v} + 2\vec{\Omega}\times \vec{v} = -\tfrac{1}{\rho}\vec{\nabla} P + \nu \nabla^2 \vec{v}$$

$$\vec{\nabla} \bullet \vec{v} = 0$$

$$\partial_t \vec{v} + 2\vec{\Omega}\times \vec{v} = -\tfrac{1}{\rho}\vec{\nabla} P$$

$$2\vec{\Omega}\times \vec{v} = -\tfrac{1}{\rho}\vec{\nabla} P$$

$$(\vec{\Omega}\bullet\vec{\nabla})\vec{v} = 0$$

Taylor-Proudman theorem

$$\partial_t \vec{v} + 2\vec{\Omega}\times \vec{v} = -\tfrac{1}{\rho}\vec{\nabla} P$$

$$\partial_t \vec{\omega} = 2(\vec{\Omega}\cdot\vec{\nabla})\vec{v} = 2\Omega_o \partial_z \vec{v}$$

Plane wave solutions

$$\vec{v} = \vec{v}_o e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\omega = \pm 2\Omega_o \tfrac{k_z}{k}$$

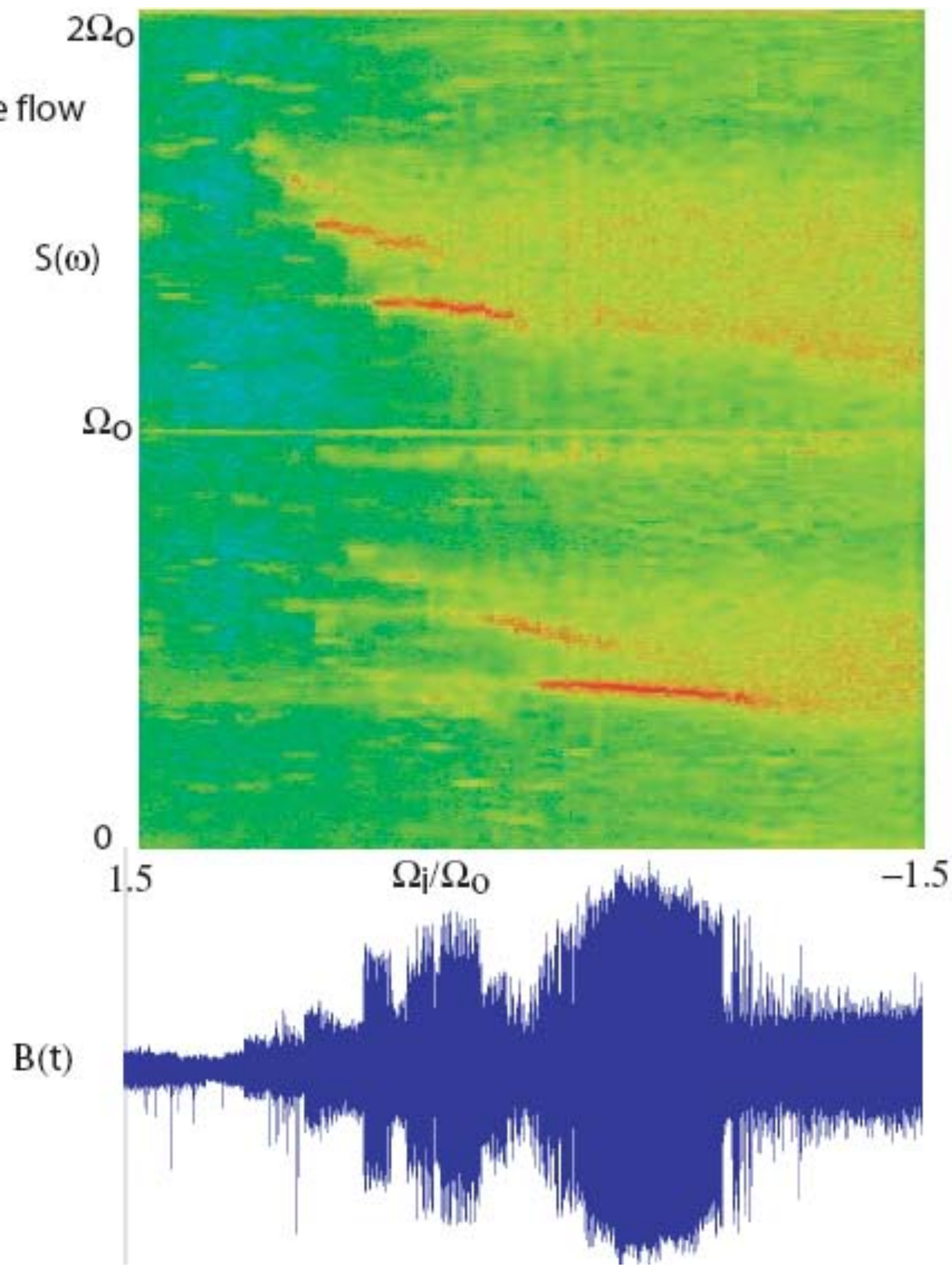
$$0 < |\omega| < 2\Omega_o$$

Modes of Containers

$$Q \sim E^{-1/2} = (\nu/2\Omega l^2)^{-1/2}$$

Modes in spherical Couette flow

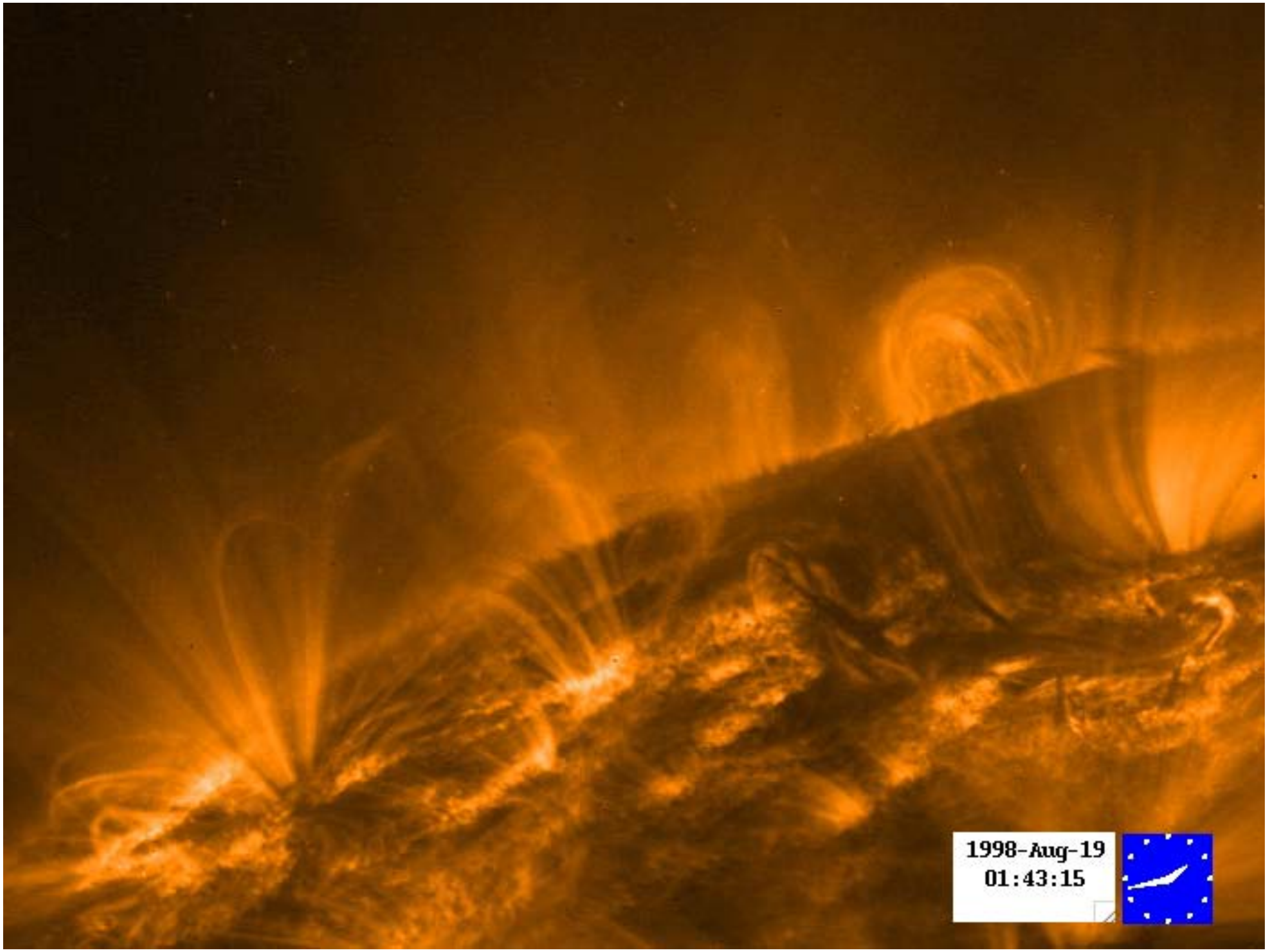
$\Omega_O = 30$ Hz

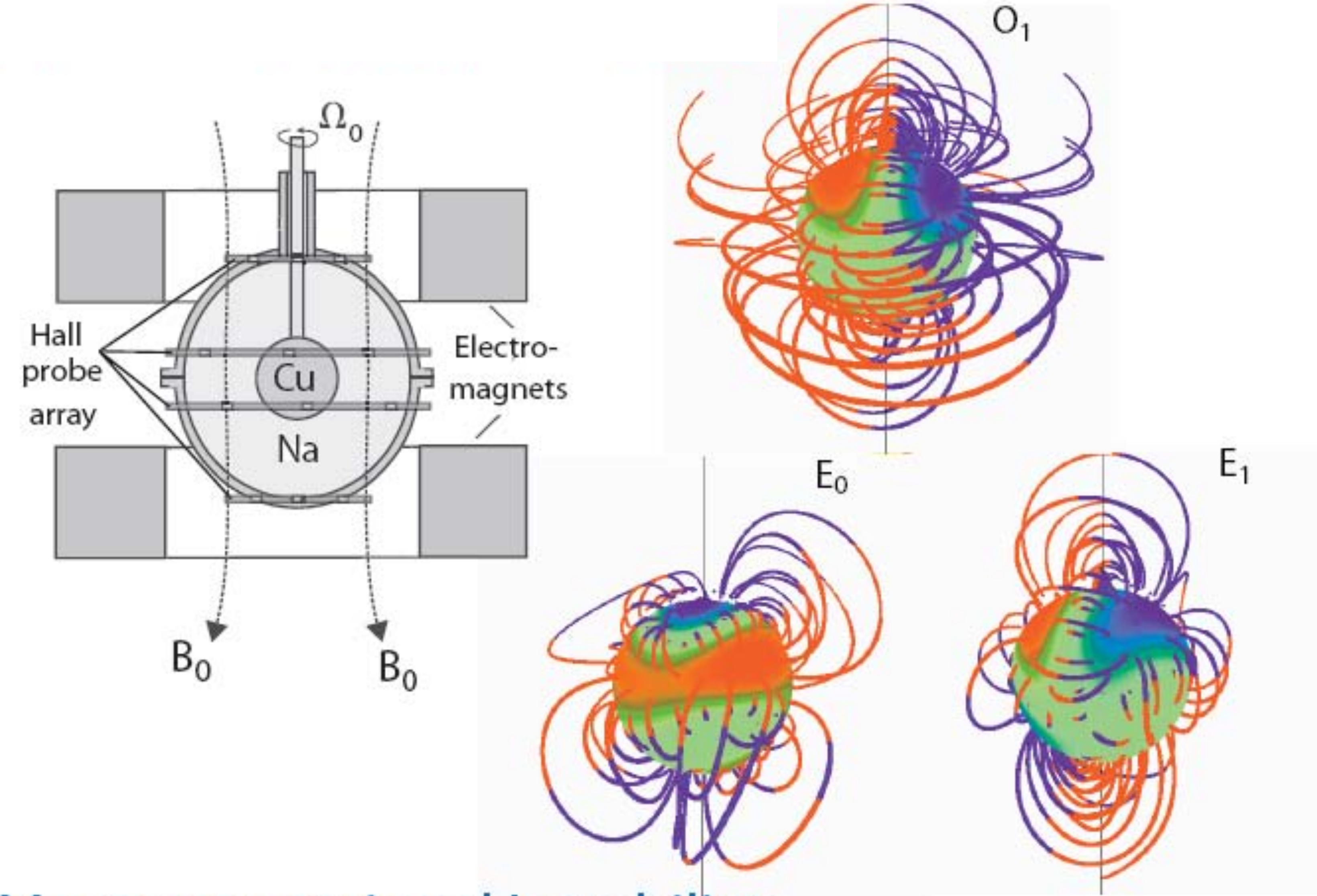


Astrophysical turbulence

Galaxy: M51

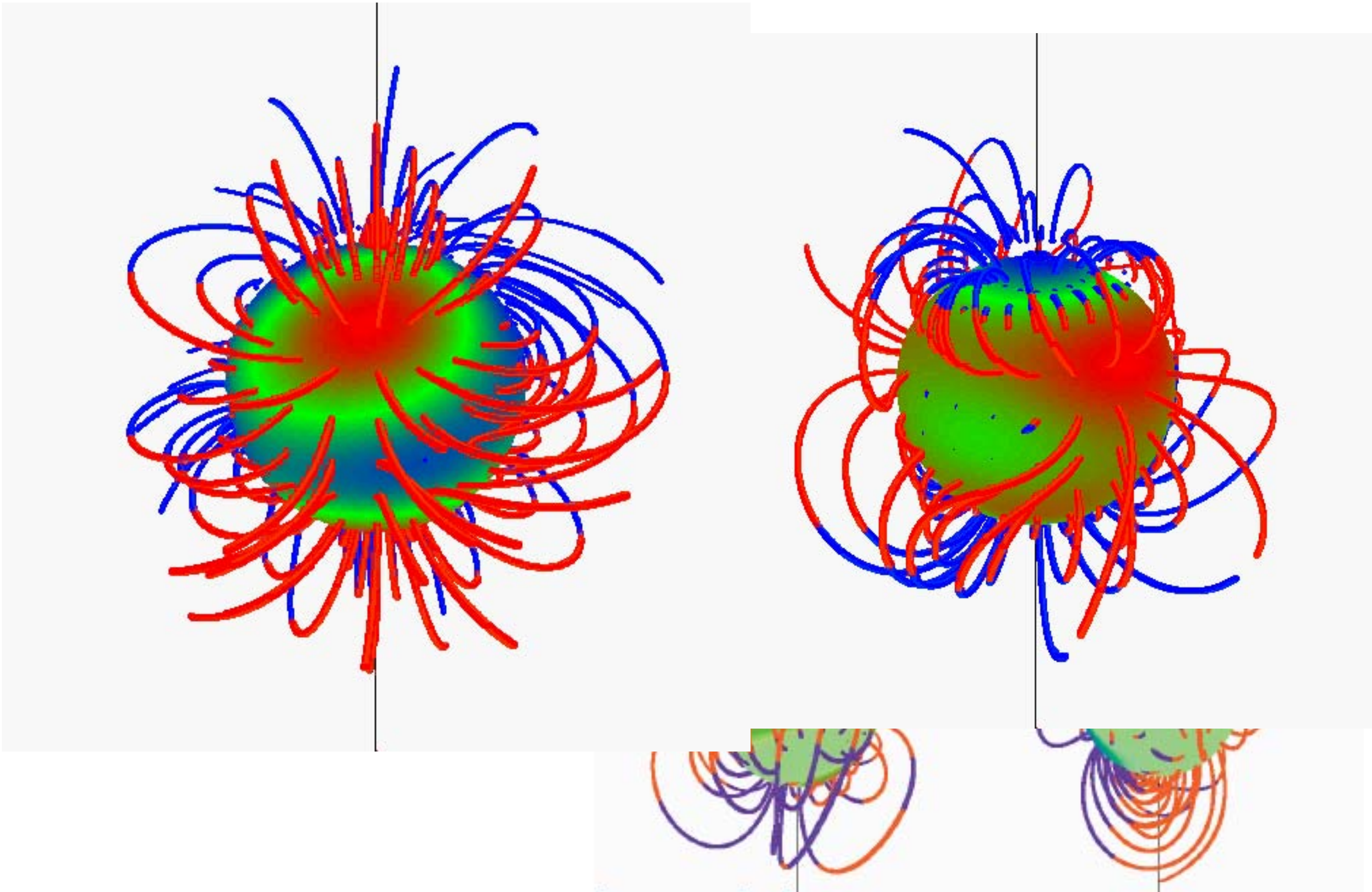
Solar turbulence:






# Magnetorotational Instability

Sisan et al., Phys. Rev. Lett. 93, 114502 (2004).



# Magnetorotational Instability

Sisan et al., Phys. Rev. Lett. 93, 114502 (2004).

## Magnetorotational instability

Velikhov (1959)
Chandrasekhar (1960)
Donnelly and Ozima (1960)

Balbus and Hawley
Astrophys. J. 374, 214 (1991)
Rev. Mod. Phys. 70,1 (1998)

Accretion disks

Active Galactic Nuclei

NGC 7052
van der Marel & van den Bosch
HST

Rotation rate $\Omega(s)$ with $d\Omega/ds < 0$
Differential motion --> growing magnetic energy
Angular momentum moves outward (toward positive s)


Aerospace Engin. turbulence
Rocket Lauch
DARKLORD

## Aerospace Engin. turbulence

Turbofan engine design

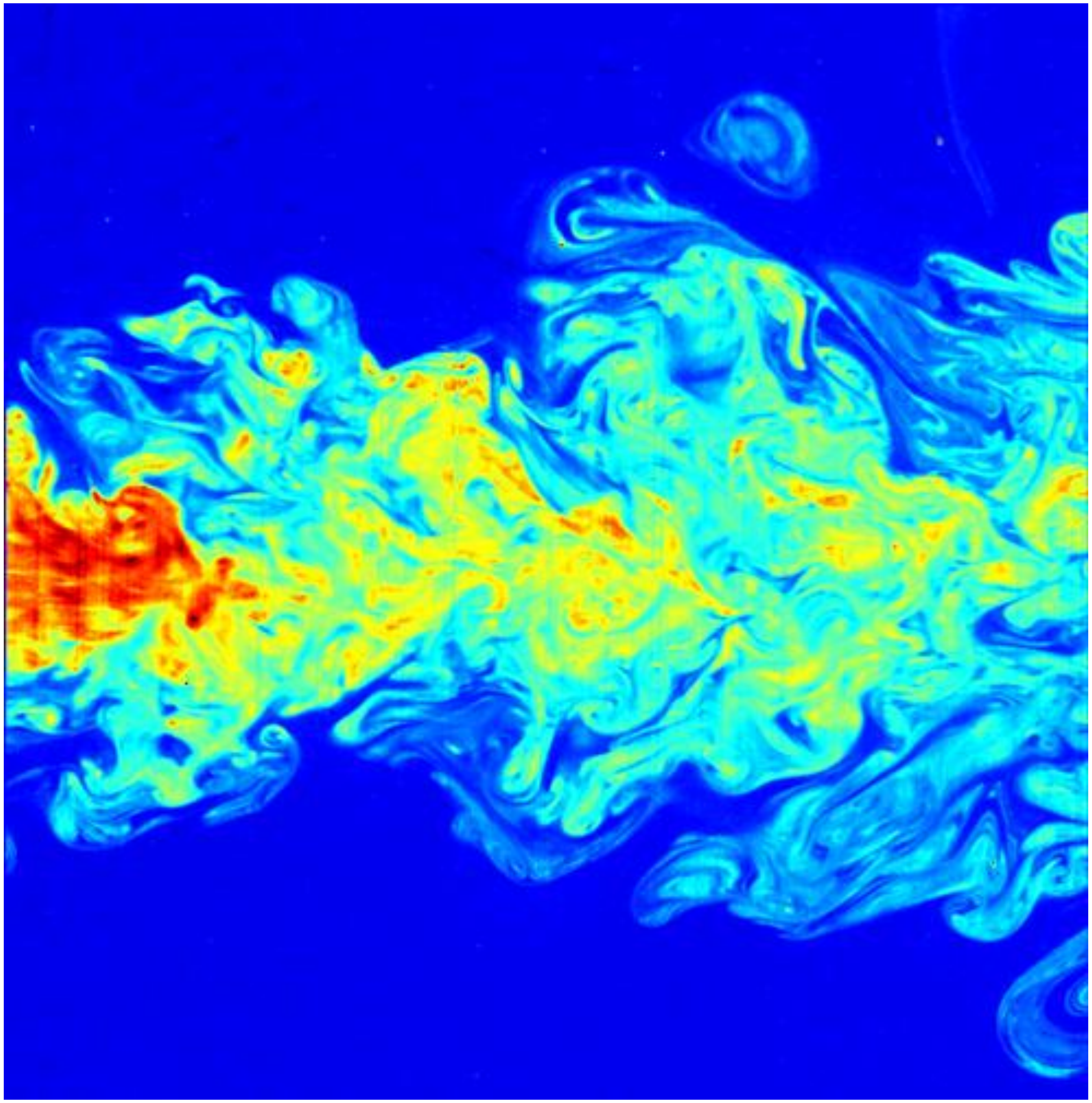
Boundary layer turbulence

Subcritical instabilities
noise driven

Turbulence control:
suction!

# What is meant by JET in fluid dynamics?

Turbulent jet
C. Fukushima and J. Westerweel



# Quantum Fluids

A state of matter with long range quantum order

Type of synchronization

partial phase sync of the individual atomic wave functions

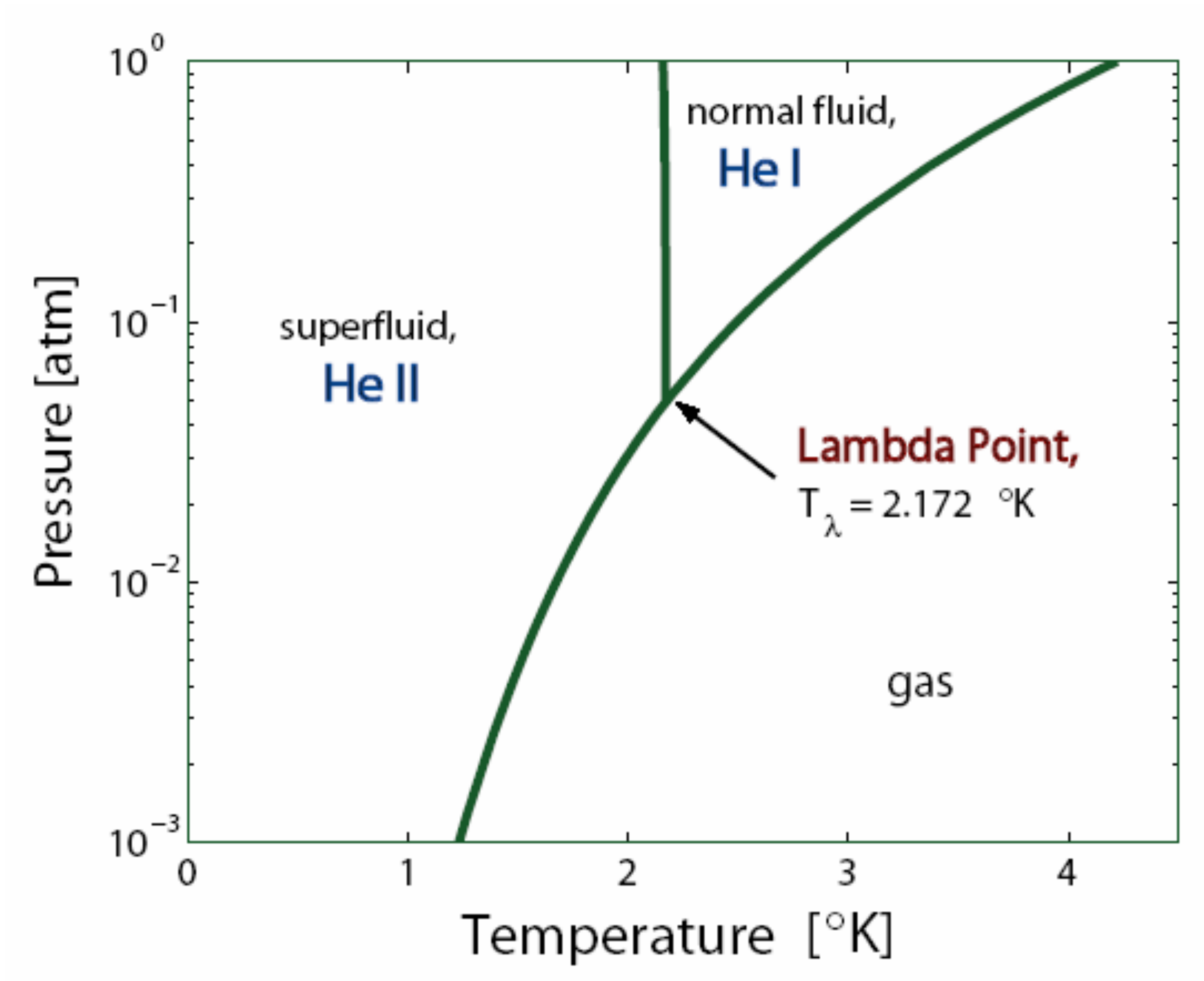
E.g.

- BEC atomic systems
- $^4He$
- $^3He$
- Cooper pair electrons in superconductors
- Vacuum

Quantum Turbulence –> turbulence in a quantum fluid

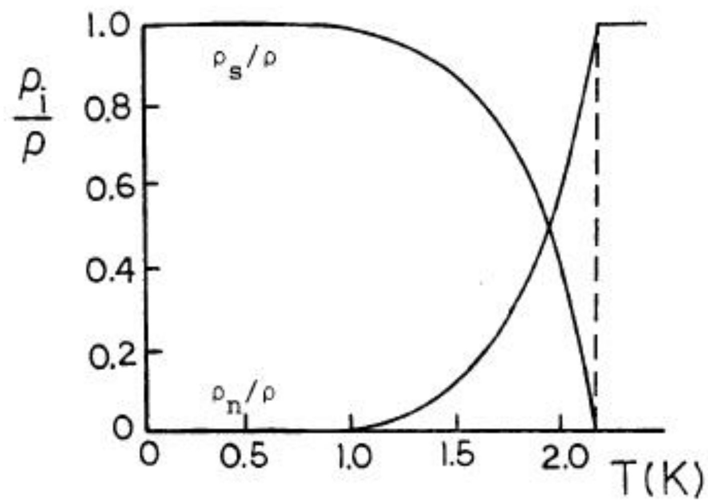
Why does it matter?

# Background: Superfluid Helium



# Two-Fluid Model

- Superfluid helium acts as if it's a mixture of a normal and superfluid component
- Normal and superfluid have separate velocity fields $\mathbf{v}_n$ and $\mathbf{v}_s$, respectively



# Superfluid Order Parameter

- Order parameter for superfluid helium is a complex field,

$$\Psi(\mathbf{x}) = Ae^{i\phi}$$

A is amplitude, and $\phi$ is the phase

- Superfluid velocity given by

$$v_s = \kappa \nabla \phi \qquad \kappa = \frac{h}{m}$$

h = Planck's constant

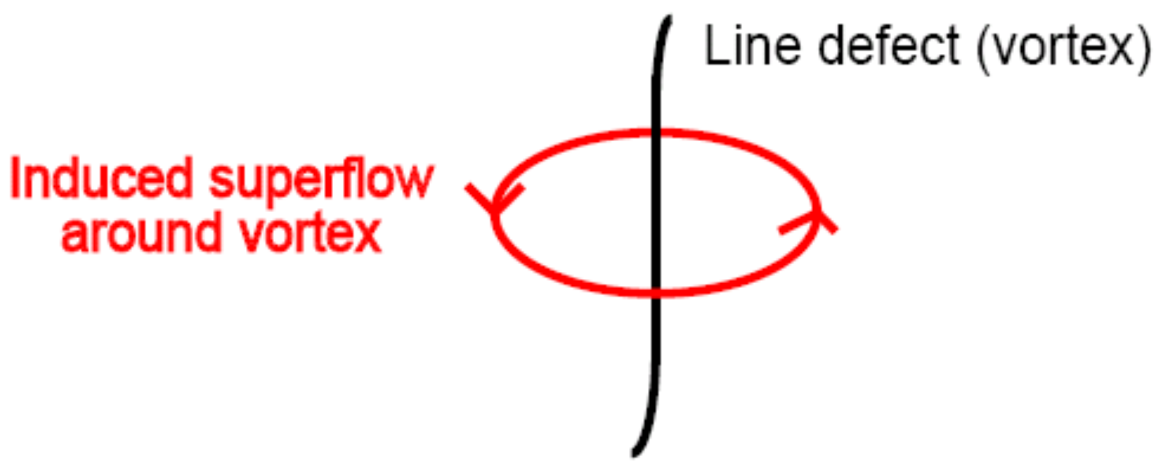
m = mass of helium atom

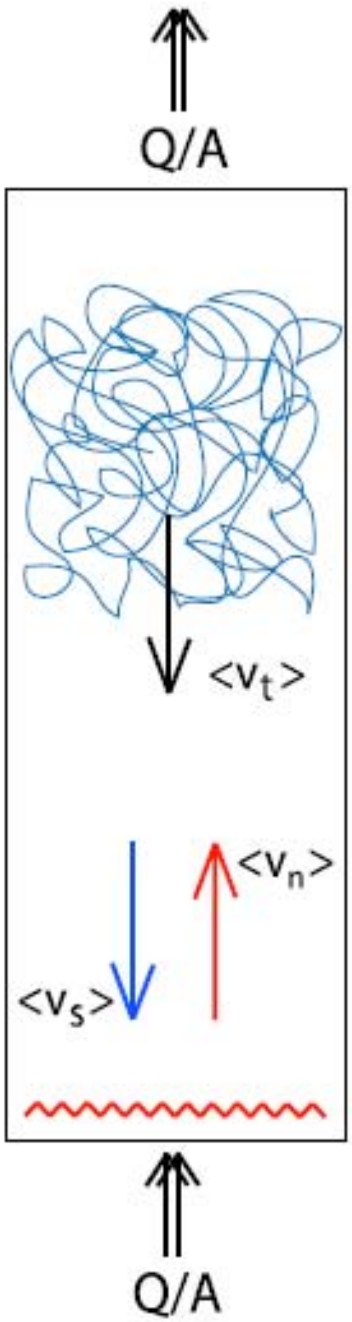
# Quantized Vortices

- Lowest energy state: n=1, so $\phi$ wraps $2\pi$ around a defect
- Induces a superflow around the line:

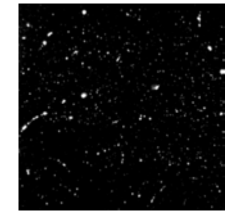
$$\mathbf{v}_s = \frac{\kappa}{s}$$

$s$ is distance from defect






# Thermal Counterflow



- Drive the system from equilibrium by applying a heat flux Q/A to the bottom of the channel

$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$

$$L = \frac{\text{vortex line length}}{\text{volume}}, \mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$$

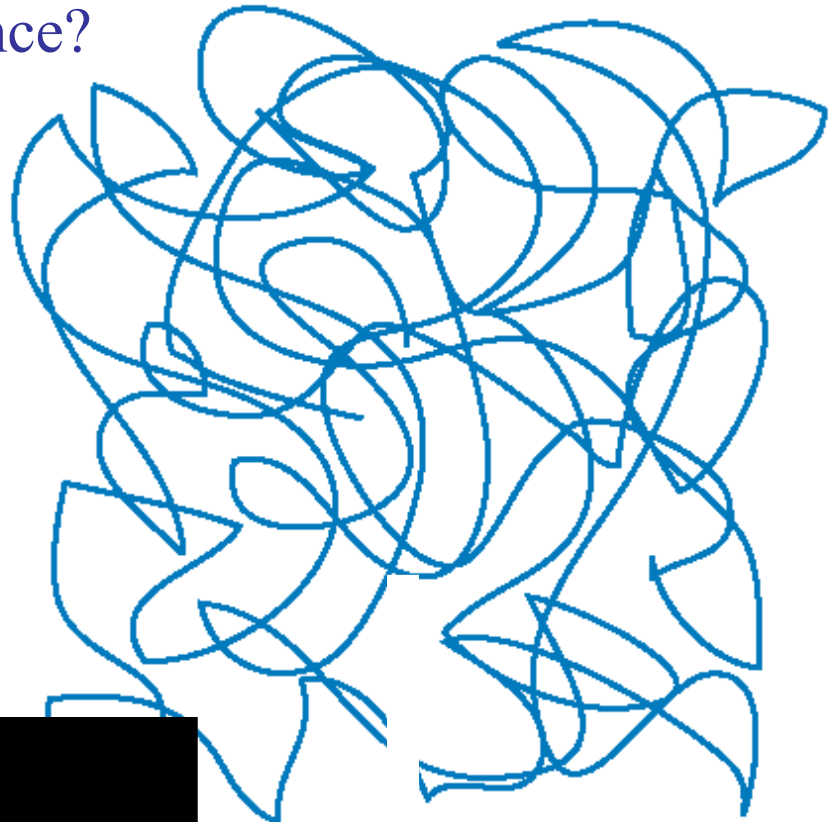
The spatially averaged velocites are of:
$\langle v_n \rangle$ - the viscous component,
$\langle v_s \rangle$ - the superfluid,
$\langle v_t \rangle$ - the quantized vortex tangle

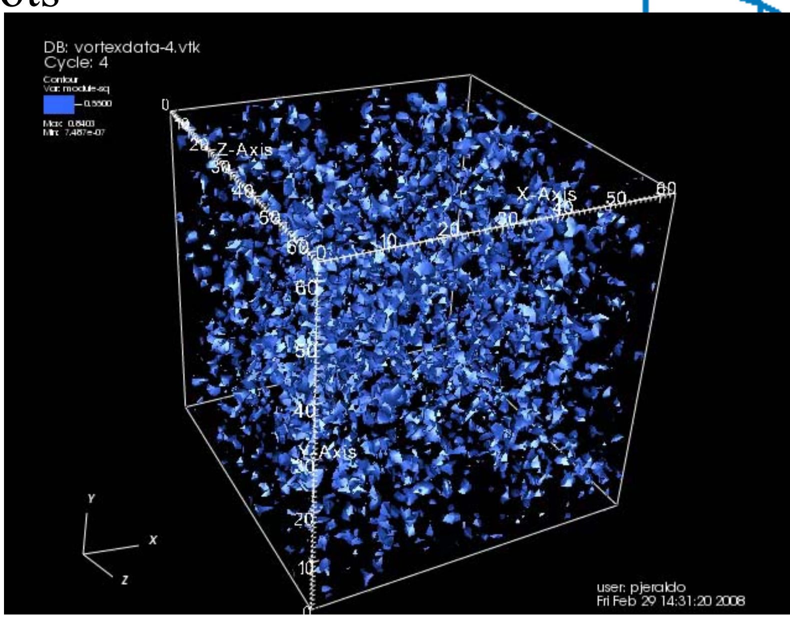
WF Vinen: Proc. R. Soc. London Ser. A 242, 493 (1957)

# What is quantum turbulence?

An evolving set of
quantized vortices:

aperiodic
large range of length scales
and curvatures
rings
vortices ending at walls
knots






Simulations by Patricio Jeraldo
and Nigel Goldenfeld

# Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades

Jeffrey Yepez,[1] George Vahala,[2] Linda Vahala,[3] and Min Soe[4]

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