

Glasses and Gels

Dave Weitz
Harvard

Boulder summer school 7/5/06

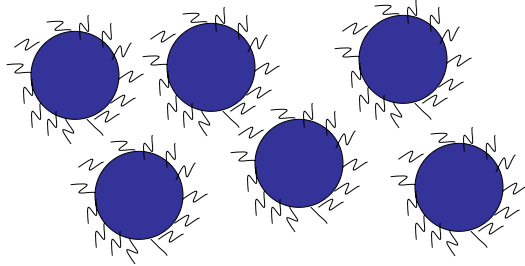
- General introduction
- Experimental techniques
- Colloidal glasses
- Colloid Aggregation
- Colloidal gels

<http://www.deas.harvard.edu/projects/weitzlab>

Colloidal particles

- Good model system
 - Control size, shape
 - Control strength, range of interactions
- Statistical properties of phase behavior
- Model for complex fluid systems
- Equilibrium properties
- Non-equilibrium properties (often dominate)

Colloidal Particles



Stability:

Short range repulsion

Sometimes a slight charge

Colloid Particles are:

•Big

• $a \sim 1$ micron

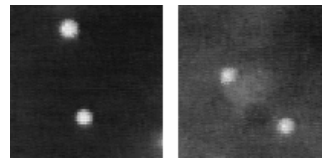
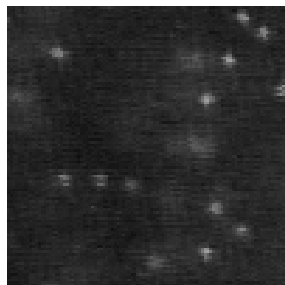
• Can “see” them

Slow

• $\tau \sim a^2/D \sim$ ms to sec

• Follow individual particle dynamics

Colloidal particles undergoing Brownian motion



Thermal motion ensures particles are
always equilibrated with the fluid

They can explore phase space

Soft Materials

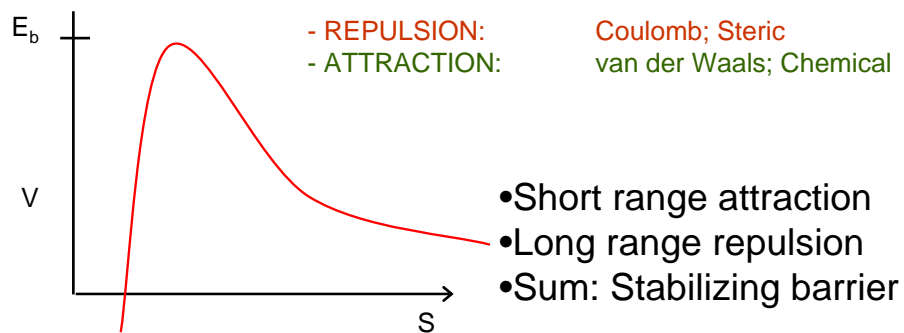
Easily deformable → Low Elastic Constant:

Elastic Constant: → Pressure $\frac{\text{Force}}{\text{Area}}$ $\frac{\text{Energy}}{\text{Volume}}$

Atoms: $\frac{eV}{\text{\AA}^3}$ ~GPa

Colloids: $\frac{k_B T}{\mu\text{m}^3}$ ~Pa

Colloidal interactions – stabilizing



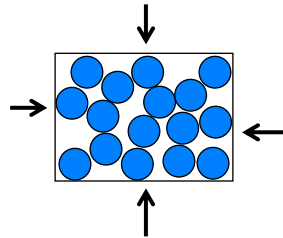
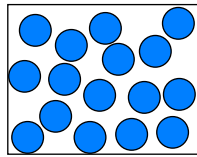
$E_b > k_B T \rightarrow$ Colloid stable against aggregation

OTHER POSSIBLE REPULSIVE INTERACTIONS

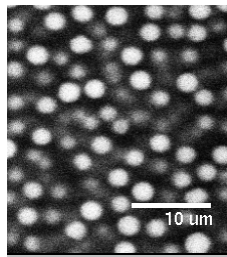
Hard Spheres: Only Volume Exclusion

Volume Fraction Controls Phase Behavior

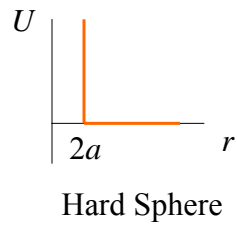
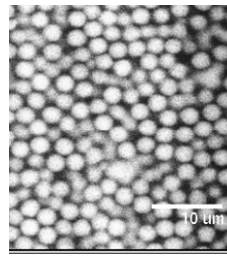
Increase ϕ => Decrease Temperature



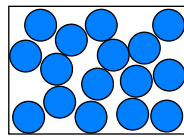
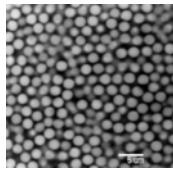
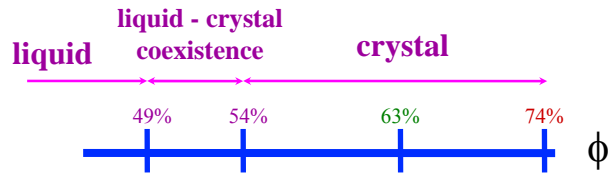
$$F = U^0 - TS$$



?

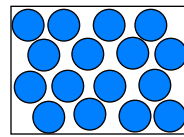


Hard Sphere Phase Diagram



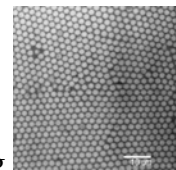
maximum packing

$$\phi_{RCP} \approx 0.63$$



maximum packing

$$\phi_{HCP} = 0.74$$

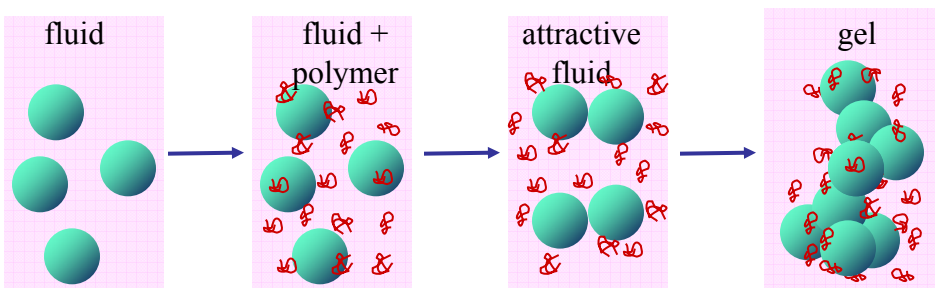


Colloidal interactions – destabilizing

- Add electrolyte → screen repulsion
- Reduce steric repulsion
- Always have van der Waals attraction
 - Colloids are inherently unstable
- Can also induce controlled attraction

Controlled Attraction of Colloidal Particles

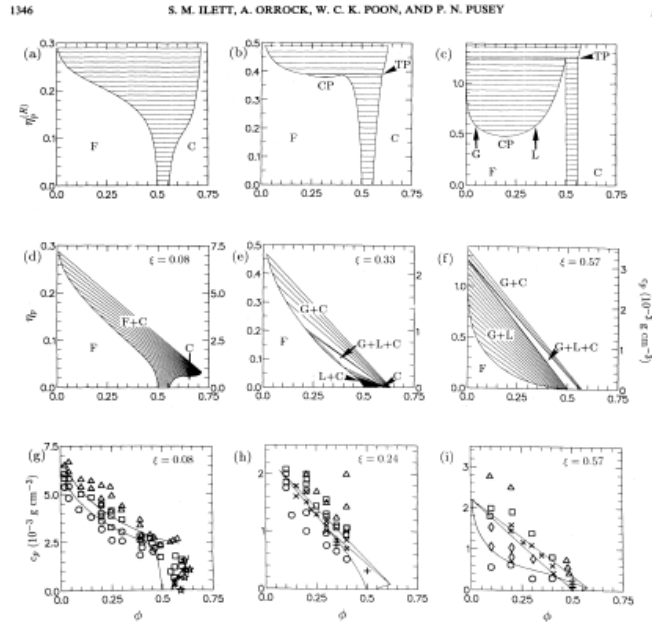
Depletion attraction



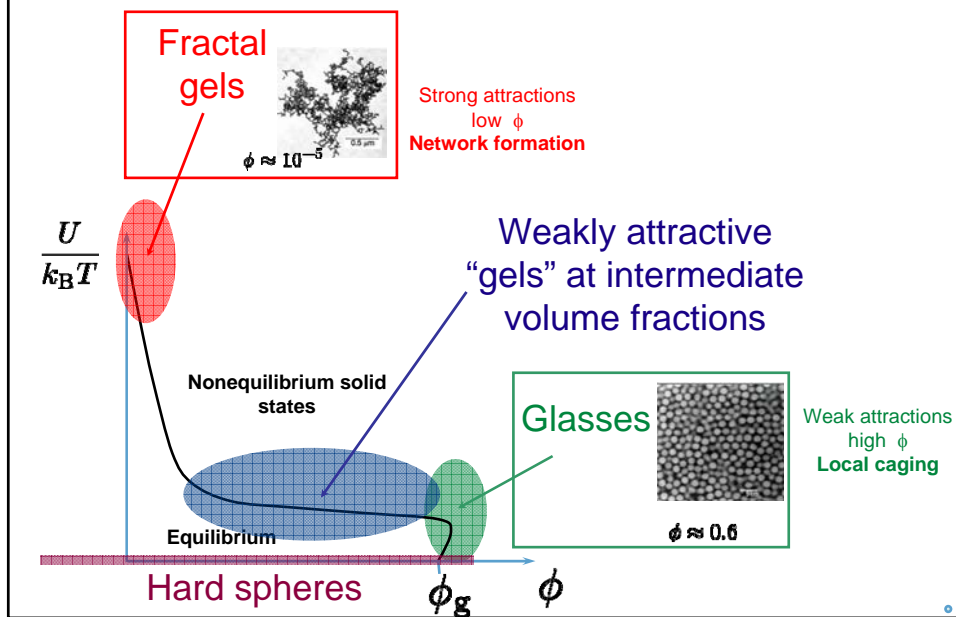
Polystyrene polymer, $R_g=37$ nm + PMMA spheres, $r_c=350$ nm

T. Dinsmore

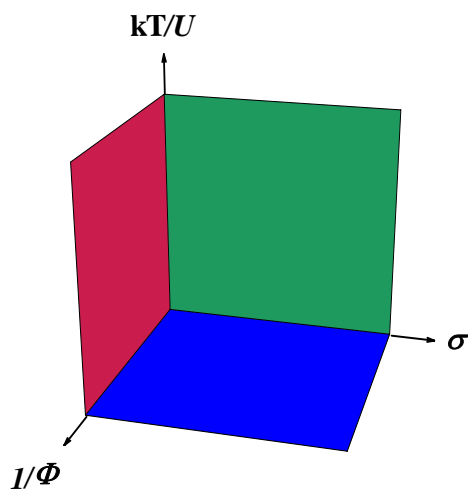
Phase diagram for attractive colloids



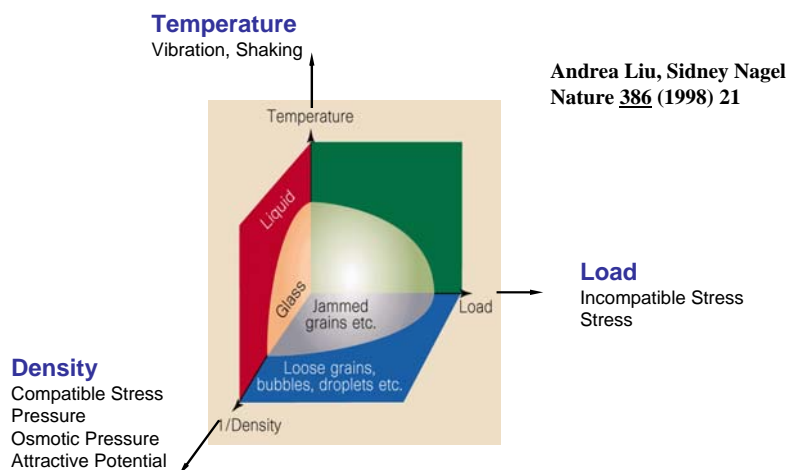
State diagram for colloidal particles



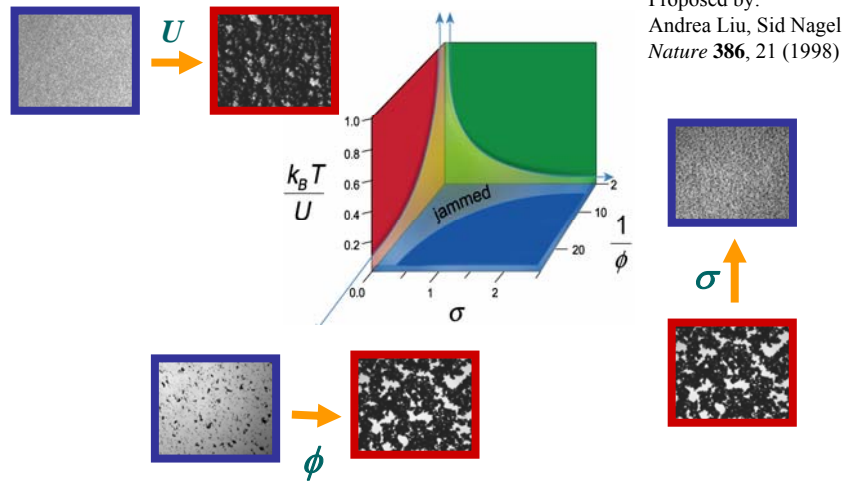
Jamming Transitions for Colloidal Systems with Attractive Interactions



Jamming Transition – Arrest of Motion



Jamming Phase Diagram for Attractive Systems



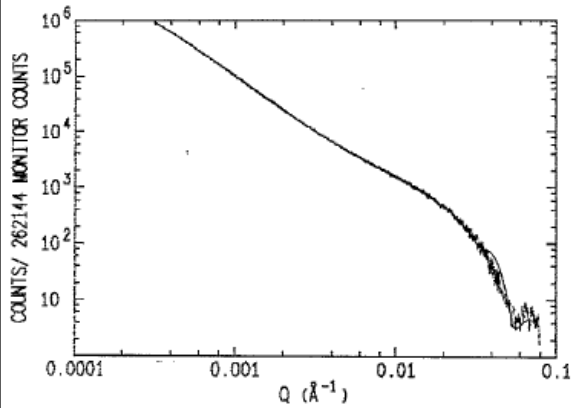
Proposed by:
Andrea Liu, Sid Nagel
Nature **386**, 21 (1998)

Trappe *et al*, *Nature* **411**, 772 (2001).

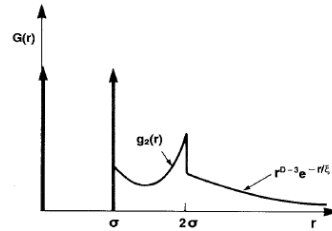
Experimental Techniques

- Light scattering
 - Static light scattering
 - Dynamic light scattering
 - Ultra-small angle dynamic light scattering
 - Diffusing-wave spectroscopy
- Microscopy
- Rheology

Static Light Scattering



X-ray scattering from gold aggregates



Model $g(r)$

P. Dimon, SofK Sinha

Static Scattering: Structure

Scattered field from single particle

$$E_m(q) = A_m e^{i\vec{q} \cdot \vec{r}_m} e^{-i\omega t}$$

Scattered intensity per particle
Phase factor
Frequency of light

Measure the scattered intensity from collection of particles

$$I(q) = \sum_{m,n} E_m E_n^* = \sum_{m,n} A_m A_n e^{i\vec{q} \cdot (\vec{r}_m - \vec{r}_n)} = P(q) S(q)$$

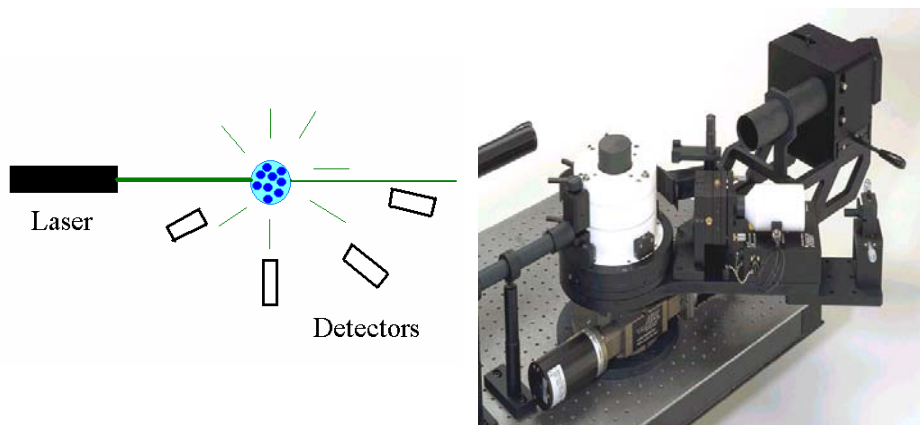
Form factor
Structure factor

- Measure q dependence of scattering
- Probes spatial Fourier Transform of density correlations

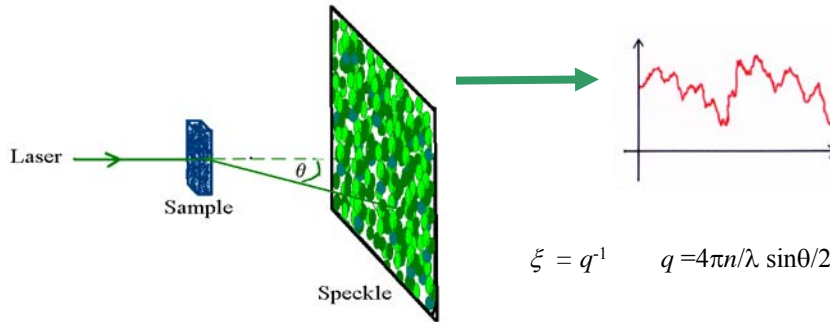
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Dynamic Light Scattering



Structure and Dynamics: Light Scattering

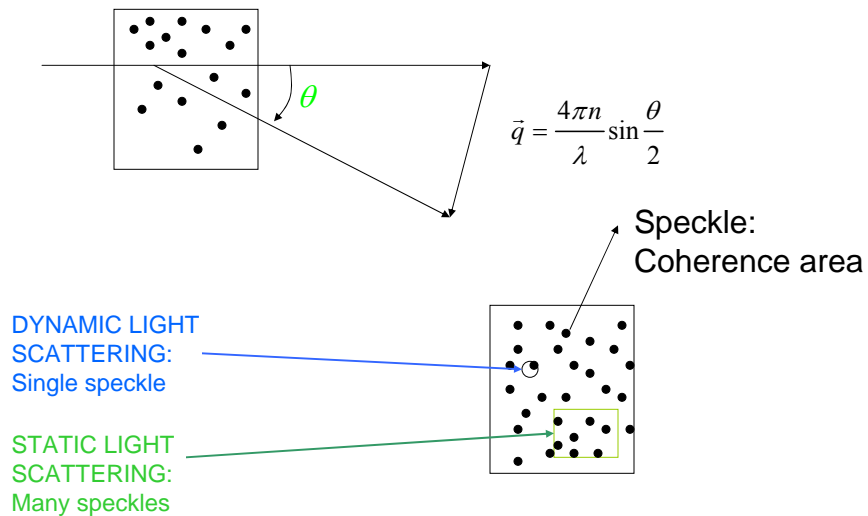


$$\xi = q^{-1} \quad q = 4\pi n/\lambda \sin\theta/2$$

- **SLS:** $\langle I \rangle$ vs. q \longrightarrow probe structure
- **DLS:** $\langle I(q,t)I(q,t+\tau) \rangle$ \longrightarrow probe dynamics
 $f(q,\tau)$

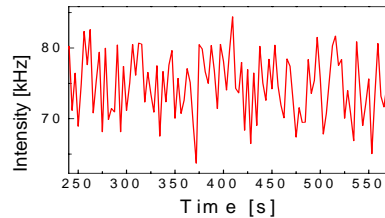
Light Scattering

Probes characteristic sizes of colloidal particles

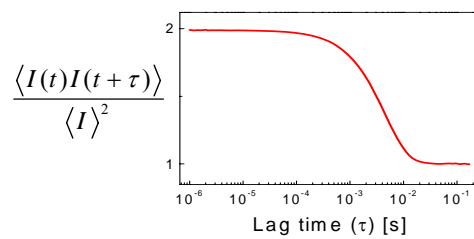


Dynamic Light Scattering

Measure temporal intensity fluctuations



Obtain an intensity autocorrelation function



Dynamic Light Scattering

Measure temporal correlation function of scattered light:
Intermediate structure factor

$$f(q, t) \sim \langle E(0)E(t) \rangle$$

$$\langle E(0)E(t) \rangle = \left\langle A^2 \sum_{m,n} e^{i\vec{q} \cdot \{\vec{r}_m(0) - \vec{r}_n(t)\}} \right\rangle \quad \text{Time average over all particles}$$

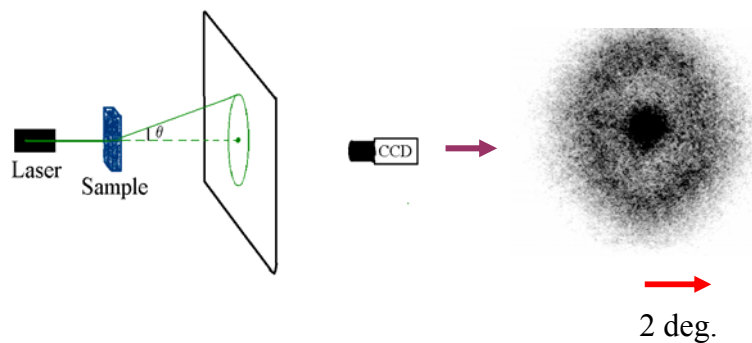
$$\sim e^{-q^2 \langle \Delta r^2(t) \rangle} \quad \begin{array}{l} \text{Correlations only between the same particles} \\ \text{Cumulant expansion: } \Delta r^2(t) \sim Dt \end{array}$$

$$\sim e^{-q^2 Dt} \quad \begin{array}{l} \text{Physics: How to change the phase of the field by } \pi \\ \text{Each particle must move by } \sim \lambda \end{array}$$

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Ultra Small Angle Light Scattering Probe Structure

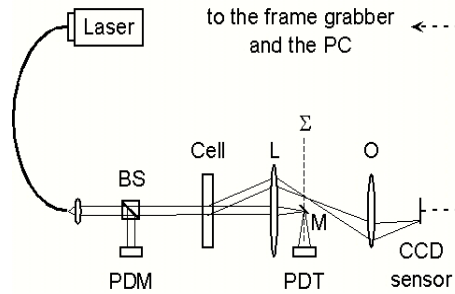


L. Cipelletti

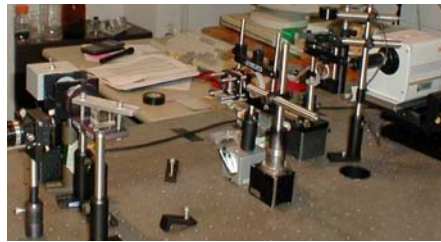
Experimental setup

Luca Cipelletti
Montpellier

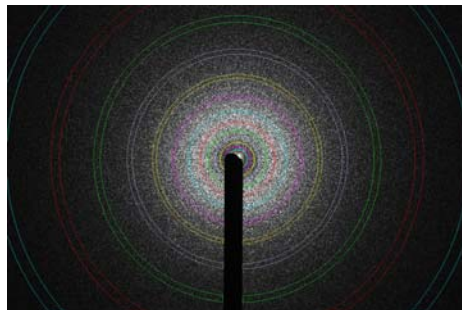
Suliana Manley
Harvard



0.07 deg to 5.0 deg



Multispeckle Detection



0.07 deg to 5.0 deg
 $100 \text{ cm}^{-1} < q < 7000 \text{ cm}^{-1}$

Average over constant q :

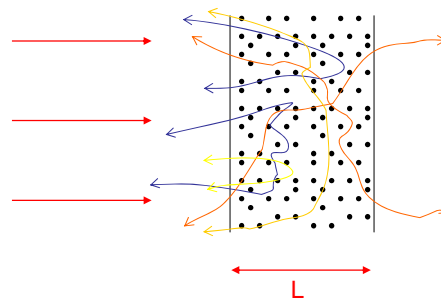
- non-ergodic samples
- avoid excessive time averaging

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Diffusing Wave Spectroscopy:
Very strong scattering

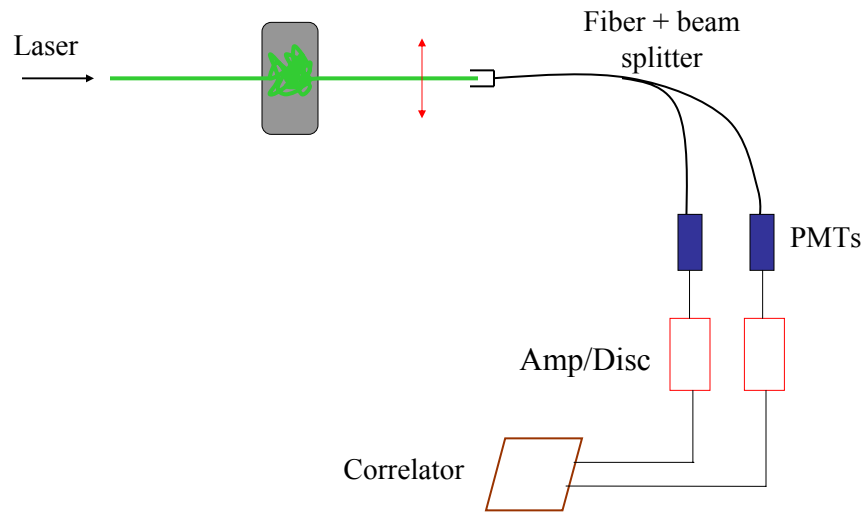
TRANSMISSION



MOST LIGHT IS SCATTERED BACK
↳ MILK IS WHITE!!

D. Pine, P. Chaikin, E. Herbolzheimer

Diffusing Wave Spectroscopy



TRANSPORT MEAN FREE PATH

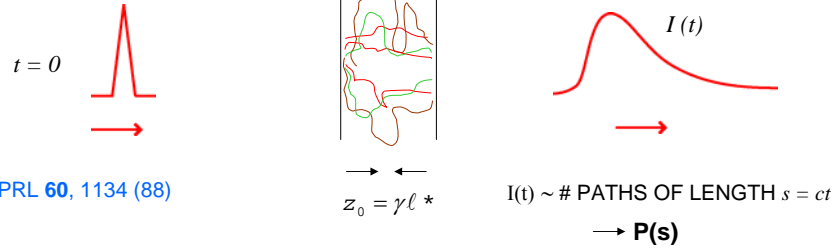


$$l^* = \sum_i l \langle \cos \theta \rangle^i$$

$$l^* = \frac{l}{1 - \langle \cos \theta \rangle} > l$$

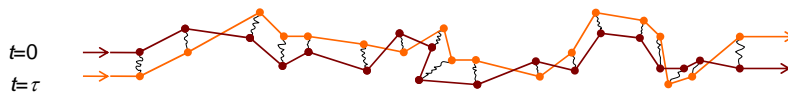
ANALOGY – PERSISTENCE LENGTH FOR SEMI-RIGID POLYMER

P(s): DIFFUSION EQUATION



SINGLE PATH

[MARET & WOLFE]



n SCATTERING EVENTS

$s = n\ell \rightarrow$ PATH LENGTH

$$g_1^n(\tau) = \left\langle e^{i \sum_{j=1}^n \vec{q} \cdot \Delta \vec{r}(\tau)} \right\rangle$$

$$= \left\langle e^{i \vec{q} \cdot \Delta \vec{r}(\tau)} \right\rangle_q^n$$

$$= e^{-\langle q^2 \rangle_q \left\langle \frac{\Delta r^2(\tau)}{6} \right\rangle_n}$$

$$= e^{-2k_0^2 \frac{\ell}{\ell^*} \left\langle \frac{\Delta r^2(\tau)}{6} \right\rangle_{\frac{s}{\ell}}}$$

PHASE CHANGE DUE TO TOTAL PATH LENGTH

STATISTICAL APPROACH

n LARGE → REPLACE BY AVERAGE
DON'T CONSERVE q AT
↙ EACH SCATTERING

→ APPROX. AT SHORT τ

$$\langle q^2 \rangle = 2k_0^2 \frac{\ell}{\ell^*}$$

$$s = n/\ell$$

$$g_1^n(\tau) = e^{-2 \left(\frac{\tau}{\tau_0} \right) \left(\frac{s}{\ell^*} \right)}$$

$$\tau_0 = \frac{1}{k_0^2 D}$$

CORRELATION FUNCTION:

MARET & WOLF

SUM OVER DISTRIBUTION OF LIGHT PATHS.

$$G_1(\tau) = \int_0^{\infty} P(s) e^{-2(\tau/\tau_0)(s/\ell^*)} ds$$

$P(s)$ DISTRIBUTION OF PATHS OF LENGTH s

→ DEPENDS ON GEOMETRY

→ DIFFUSION OF EQUATION FOR LIGHT

SINGLE SCATTERING DECAY

NUMBER OF (RANDOMIZING) SCATTERING EVENTS

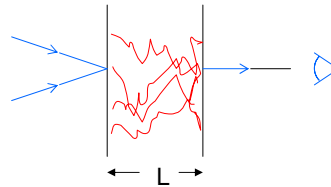
TRANSMISSION THROUGH A SLAB

FUNCTIONAL FORMS ARE KNOWN

-- USE MORE EXACT BOUNDARY CONDITIONS

-- J. de Phys. **51**, 2101 (1990)

POINT SOURCE:

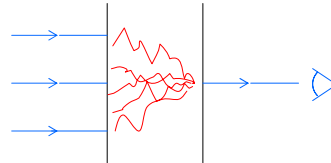


$$g_1(t) = \int_{\frac{L}{c^*} \sqrt{\frac{6t}{\pi_0}}}^{\infty} \left[A(s) \sinh s + e^{-s(1-\frac{4}{3} \ell^*/L)} \right] ds$$

with

$$A(s) = \frac{\left(\frac{2}{3} \frac{\ell^*}{L} s - 1\right) \left[\frac{2}{3} \frac{\ell^*}{L} e^{-\frac{4s\ell^*}{3L}} + \left(\sinh s + \frac{2}{3} \frac{\ell^*}{L} \cosh s\right) e^{-s(1-\frac{4}{3} \frac{\ell^*}{L})} \right]}{\left(\sinh s + \frac{2}{3} \frac{\ell^*}{L} s \cosh s\right)^2 - \left(\frac{2}{3} \frac{\ell^*}{L} s\right)^2}$$

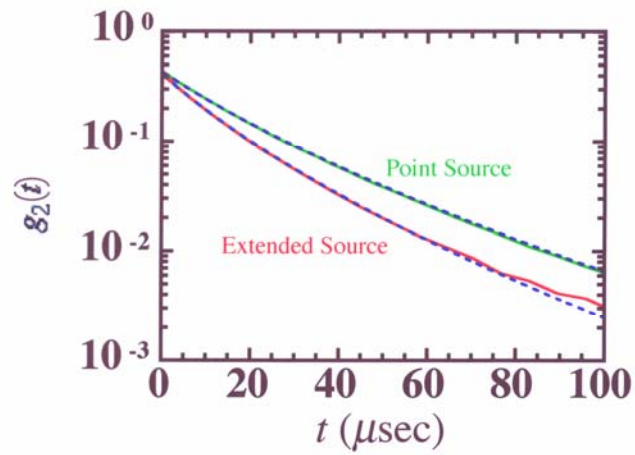
EXTENDED SOURCE



$$g_1(t) \approx \frac{\left(\frac{L}{\ell^*} + \frac{4}{3}\right) \sqrt{\frac{6t}{\tau_0}}}{\left(1 + \frac{8t}{3\tau_0}\right) \sinh\left[\frac{L}{\ell^*} \sqrt{\frac{6t}{\tau_0}} + \frac{4}{3} \sqrt{\frac{6t}{\tau_0}}\right] \cosh\left[\frac{L}{\ell^*} \sqrt{\frac{6t}{\tau_0}}\right]}$$

CHARACTERISTIC TIME SCALE: $\tau_0 \left(\frac{\ell^*}{L}\right)^2$

TRANSMISSION



$$d = 0.6 \mu\text{m}$$

$$\phi = 2\%$$

PHYSICS

- TOTAL PATH LENGTH CHANGES BY λ
- CONTRIBUTIONS FROM MANY SCATTERING EVENTS

SMALL τ → LONG PATH → MANY SCATTERERS

→ SMALL MOTION → FAST DECAY

LARGE τ → SHORT PATH → FEW SCATTERERS

→ LARGER MOTION → SLOWER DECAY

DWS PROBES MOTION ON SHORT LENGTH SCALES

PHASE OF PATH CHANGES WHEN PATH LENGTH CHANGES BY
~1 WAVELENGTH

$$\lambda \sim 5000 \text{ \AA}$$

BUT: LIGHT IS SCATTERED FROM MANY PARTICLES

$$\left(\text{Estimate: } \left(\frac{L}{\ell^*} \right)^2 \approx \left(\frac{10^3}{10} \right)^2 \approx 10^4 \right)$$

∴ MOTION OF EACH **INDIVIDUAL** PARTICLE CAN BE **MUCH LESS**

↙
→ CAN MEASURE PARTICLE MOTION ON SCALE OF
~ 5 Å

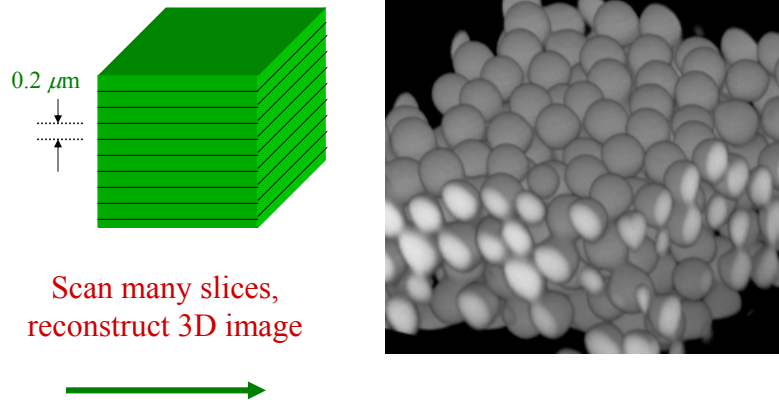
Experimental Techniques

- Light scattering
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The “PINK Monster”
Keep that door closed!!!!

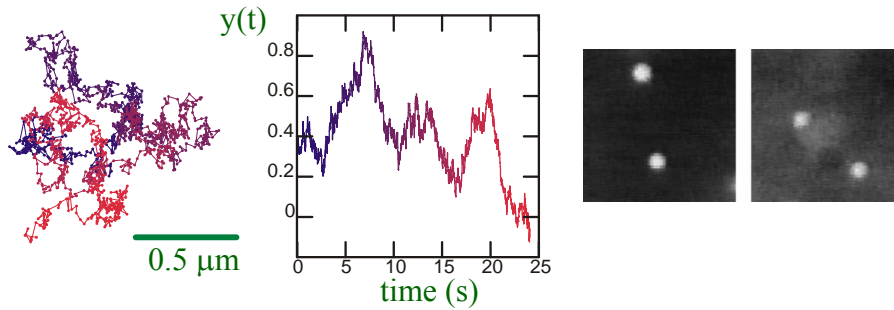


Confocal microscopy for 3D pictures



Brownian Motion

(2 μm particles, **dilute** sample)



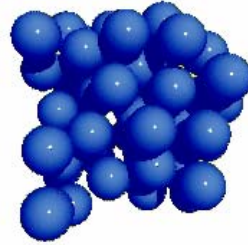
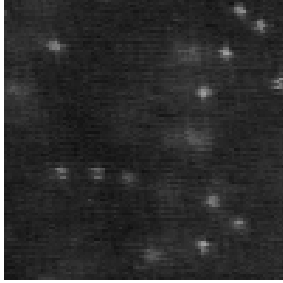
Leads to normal diffusion: $\langle \Delta x^2 \rangle = 2Dt$

$$D = \frac{k_B T}{6\pi\eta a}$$

Particle size a

viscosity η

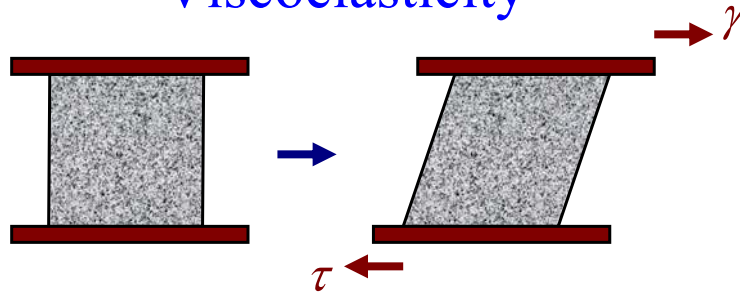
Brownian Motion in Real Time



Experimental Techniques

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Mechanical Properties of Soft Materials: Viscoelasticity



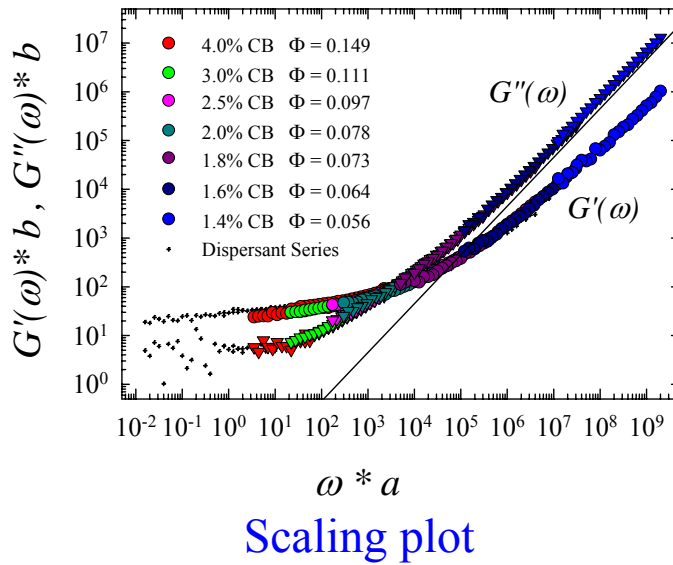
Solid: $\tau = G\gamma$
 Fluid: $\tau = \eta\dot{\gamma}$

$\gamma = \gamma_0 e^{i\omega t}$

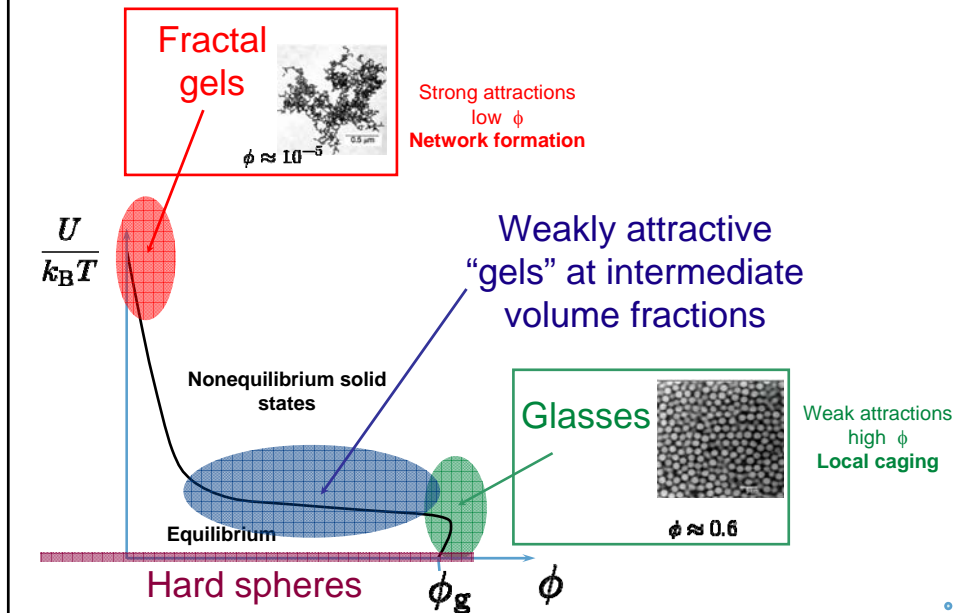
$\tau = [G'(\omega) + iG''(\omega)]\gamma$

Elastic Viscous

Rheology of soft materials



State diagram for colloidal particles



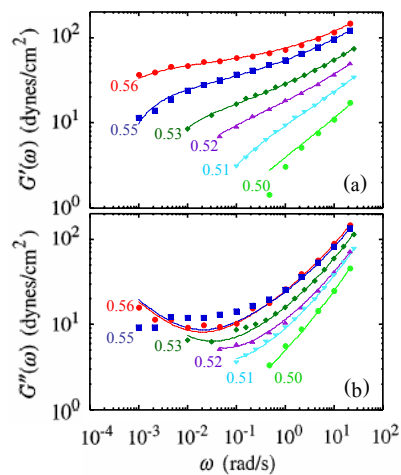
Storage and Loss Moduli of Hard Spheres

Storage

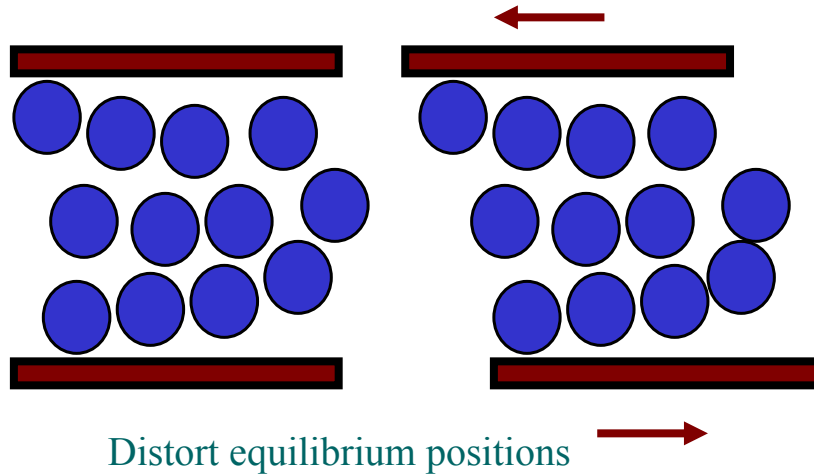
$$G'(\omega)$$

Loss

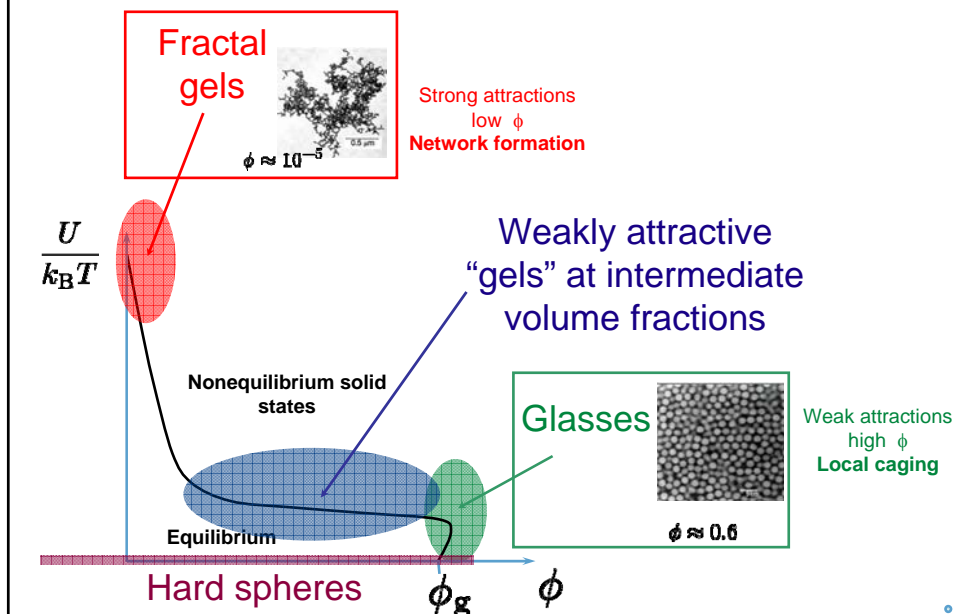
$$G''(\omega)$$



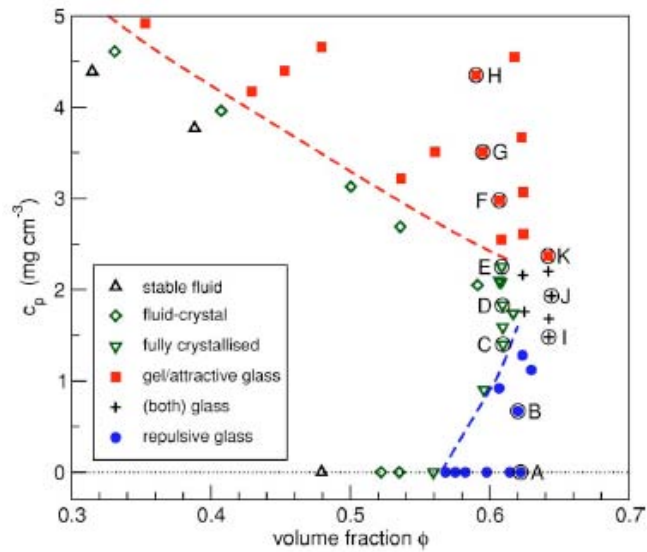
Origin of Elasticity



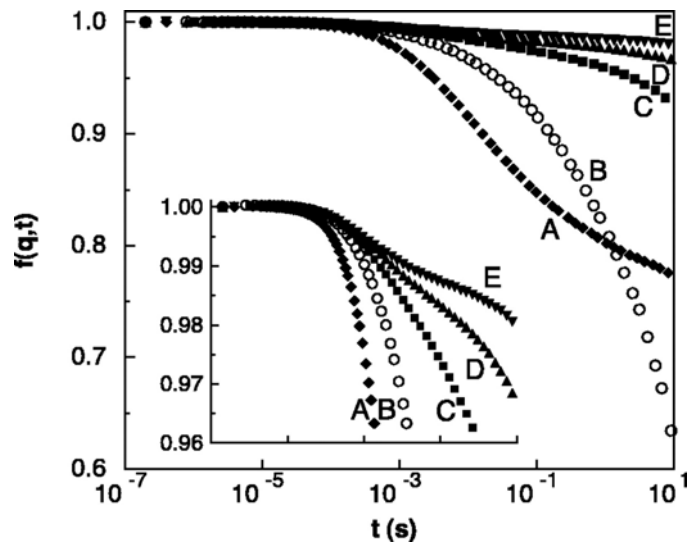
State diagram for colloidal particles



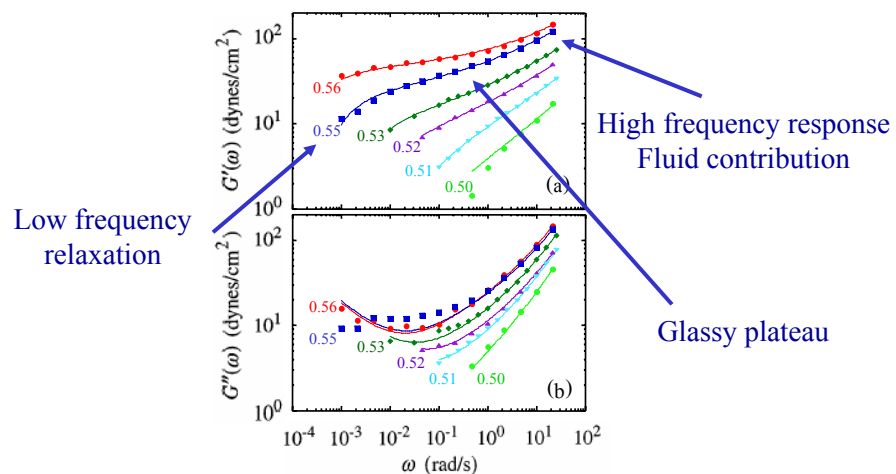
Attractive Glasses – Phase Behavior



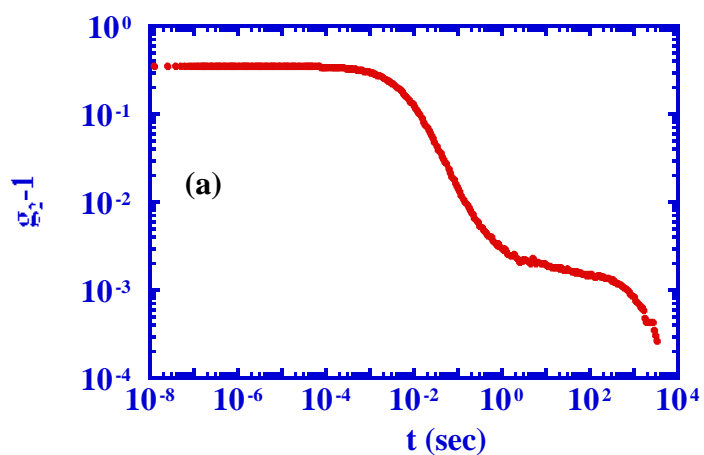
Attractive Glasses – Light Scattering



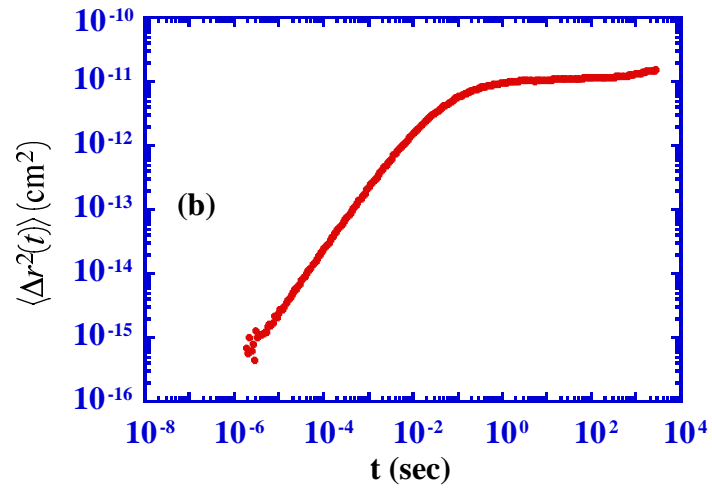
Viscoelasticity of Hard Spheres



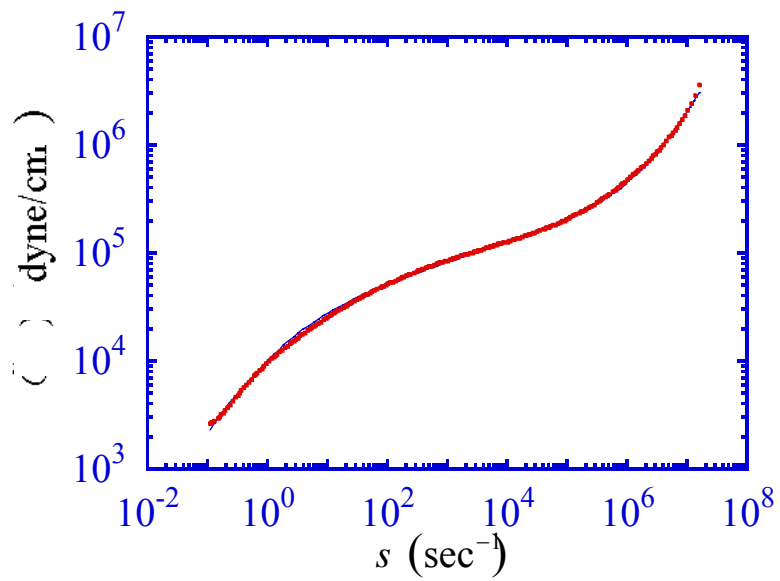
DWS correlation function of Hard Spheres



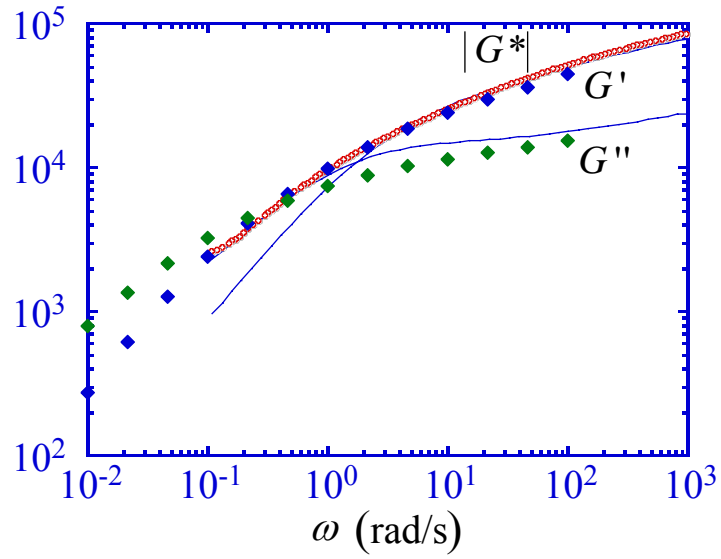
Mean Squared Displacement for Hard Spheres



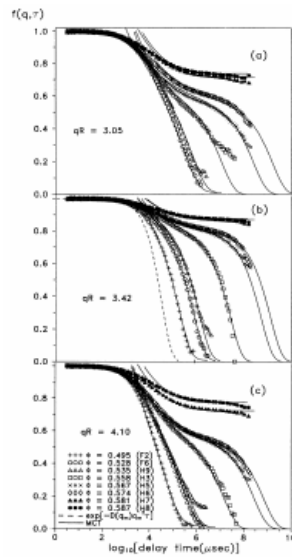
Frequency-Dependent Modulus for Hard Spheres



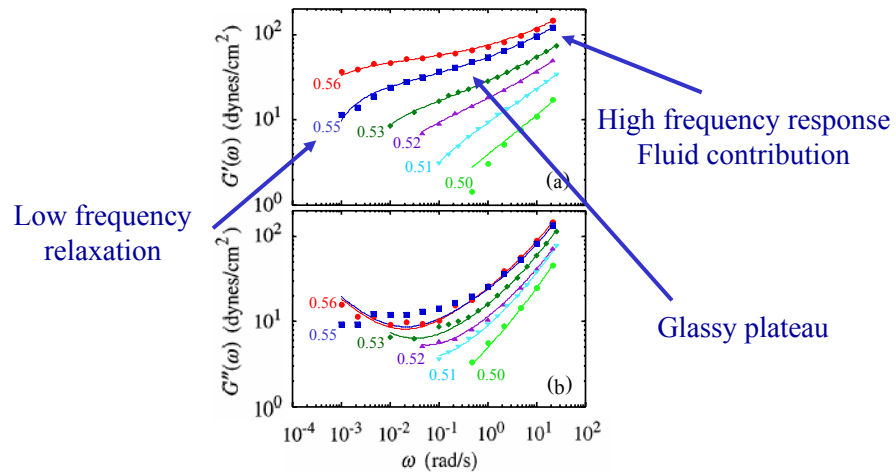
Frequency-Dependent Viscoelasticity for Hard Spheres



Correlation function of Hard Spheres

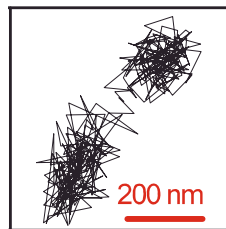
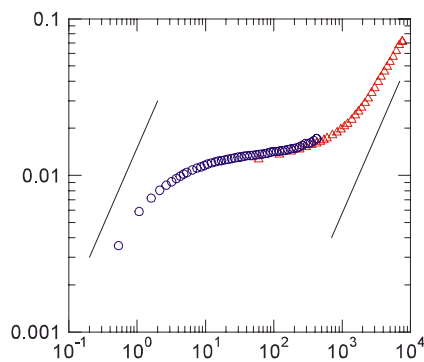


Viscoelasticity of Hard Spheres



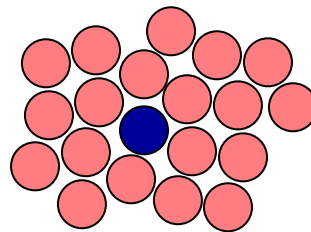
Mean-squared displacement

$\phi=0.53$ -- "supercooled fluid"



$\phi=0.56$, 100 min
(supercooled fluid)

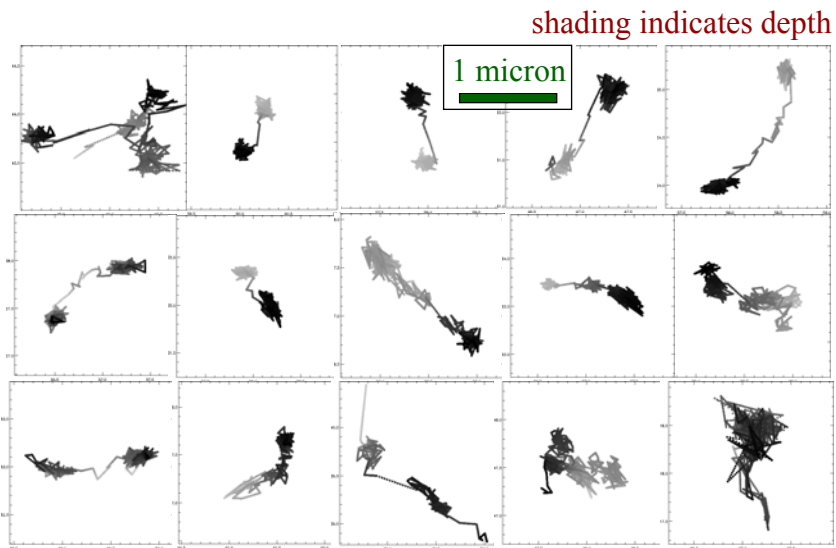
Cage trapping:



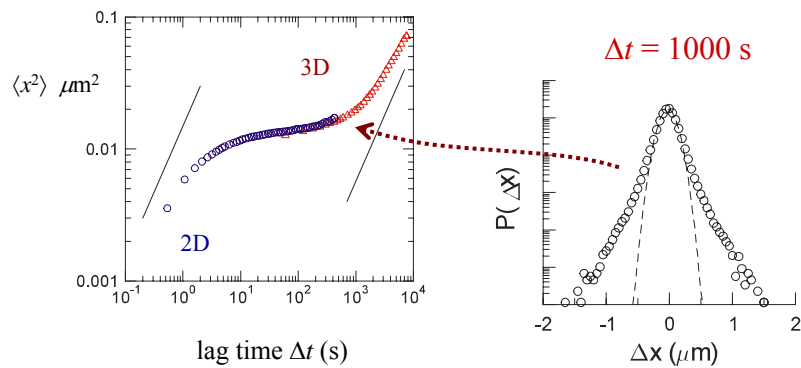
- Short times: particles stuck in "cages"
- Long times: cages rearrange

E. Weeks, J. Crocker

Trajectories of “fast” particles, $\phi=0.56$



Displacement distribution function

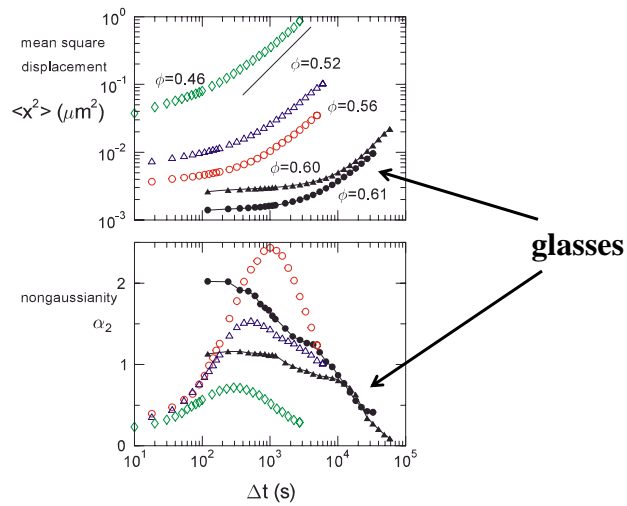


$\phi = 0.53$: “supercooled fluid”

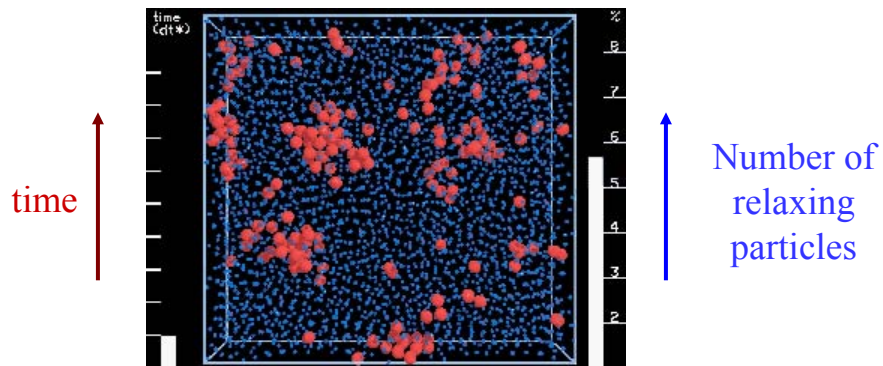
Nongaussian Parameter

$$\alpha_2 = \frac{\langle x^4 \rangle}{3\langle x^2 \rangle^2} - 1$$

How to pick Δt^* for glasses?

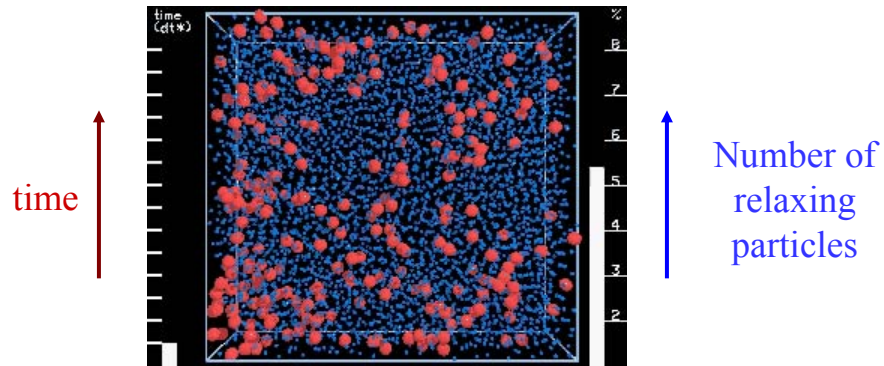


Structural Relaxations in a Supercooled Fluid



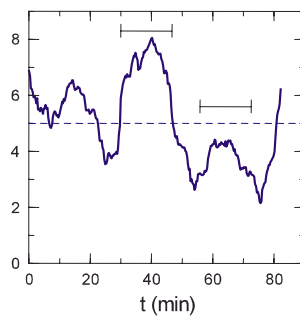
Relaxing particles are highly correlated spatially

Structural Relaxations in a Glass

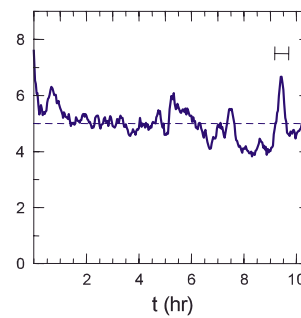


Relaxing particles are NOT correlated spatially

Fluctuations of fast particles

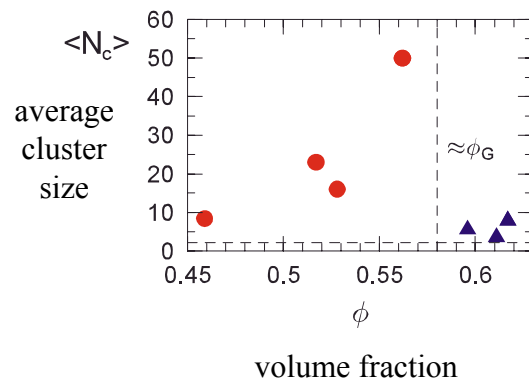


Supercooled fluid $\phi = 0.56$

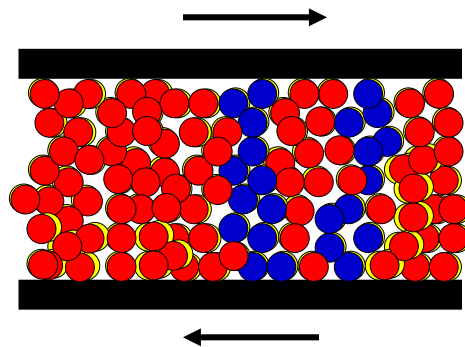


Glass $\phi = 0.61$

Cluster size grows as glass transition is approached



What is a Glass?

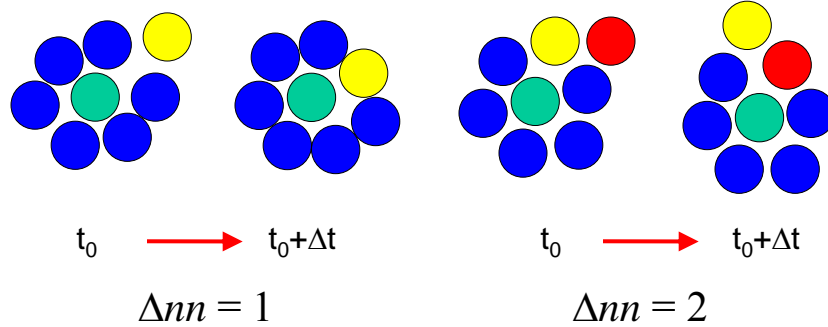


- Glass must have a low frequency shear modulus
- Must have force chains to transmit stress

J. Conrad, P Dhillon, D. Reichman

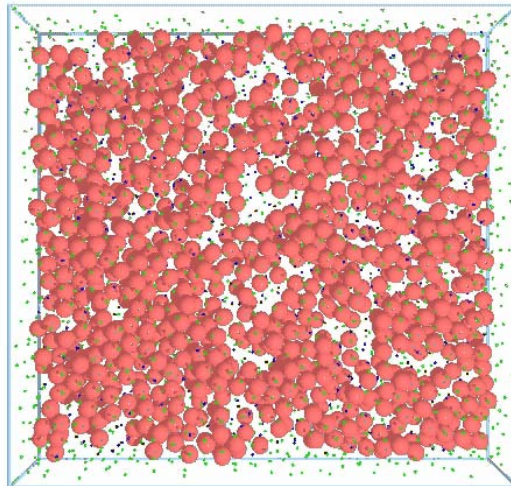
Topological Change: $\Delta nn(\Delta t)$

Identify nearest neighbors, calculate $\Delta nn(\Delta t)$

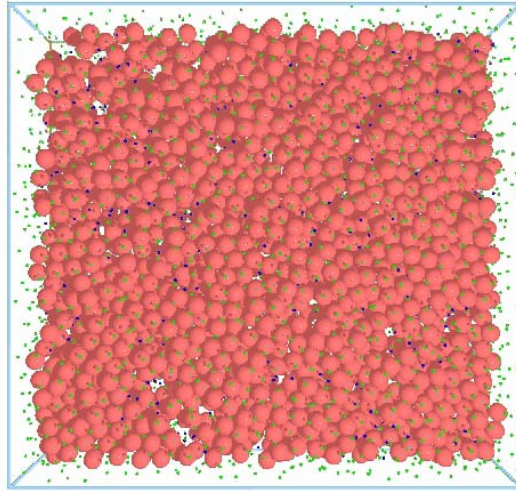


B. Doliwa and A. Heuer, J. Non-Cryst. Solids **307**, 32 (2002).

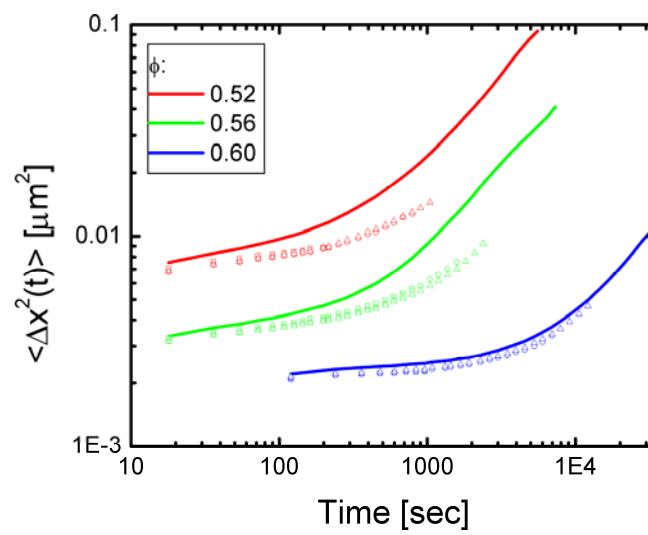
Static Particles ($\Delta nn = 0$) – $\phi = 0.52$



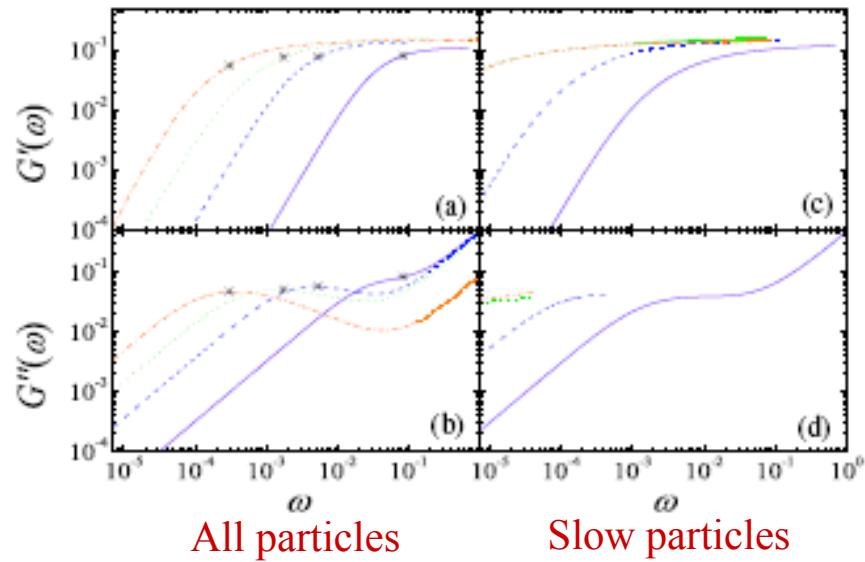
Static Particles ($\Delta nn = 0$) – $\phi = 0.60$



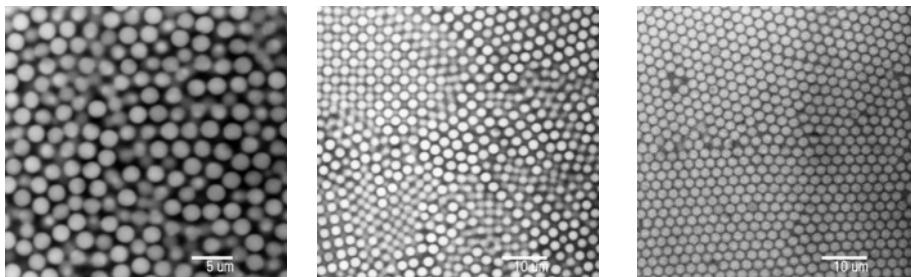
Static Particles Move Much Less



Rheology of slow particles



Intermezzo – nucleation of HS crystals

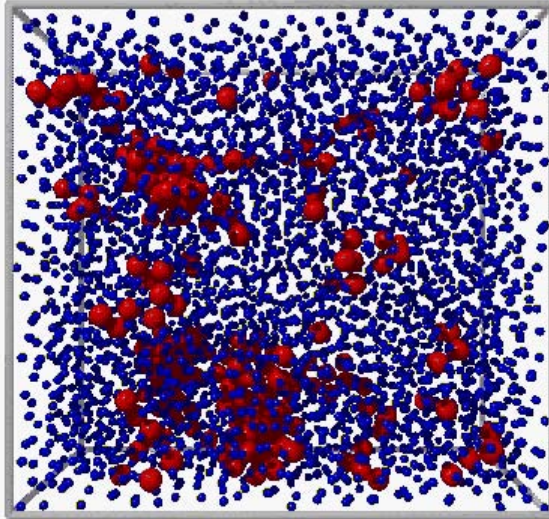


2.3 μm diameter PMMA spheres

Must identify incipient crystal nuclei

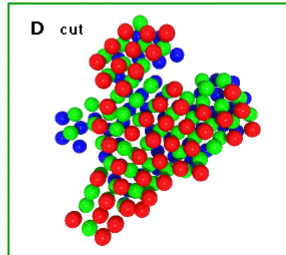
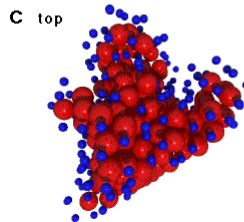
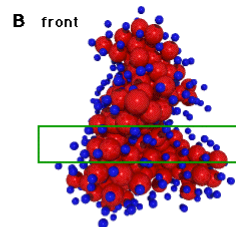
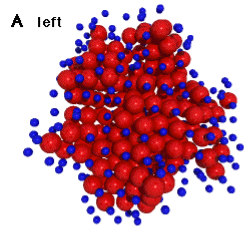
U. Gasser, E. Weeks, J. Crocker

Colloidal Crystallization



Crystal Nucleus Structure

$$R \sim R_c \quad \phi = 0.47$$



Finding Surface Tension

$$\Delta G = \gamma (4\pi r^2) - \Delta\mu \left(\frac{4}{3}\pi r^3\right)$$

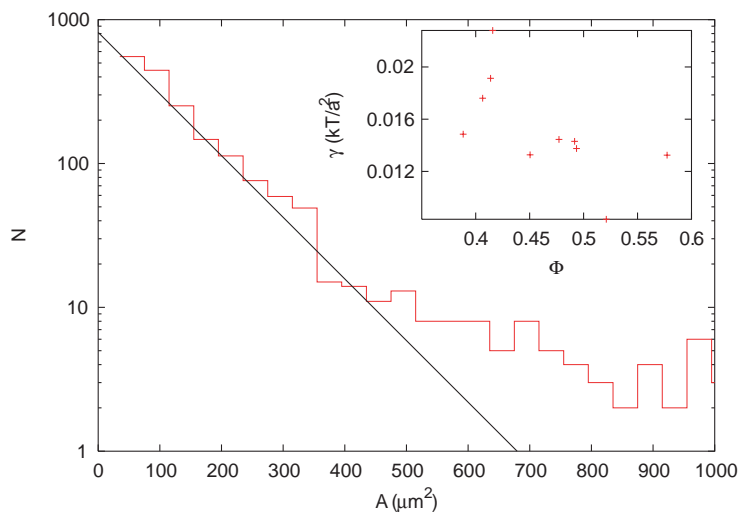
Surface energy

Chemical potential

$$P(r) \approx \exp\left(\frac{-\Delta G}{k_B T}\right) \approx \exp(-\gamma r^2)$$

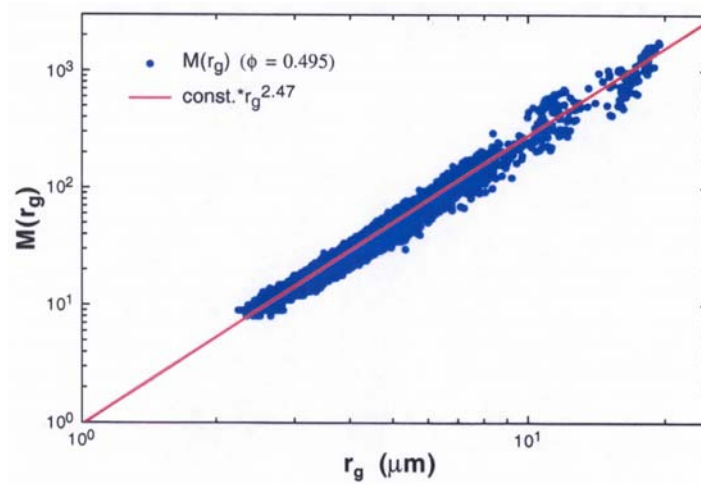
(for small r)

Measurement of Surface Tension

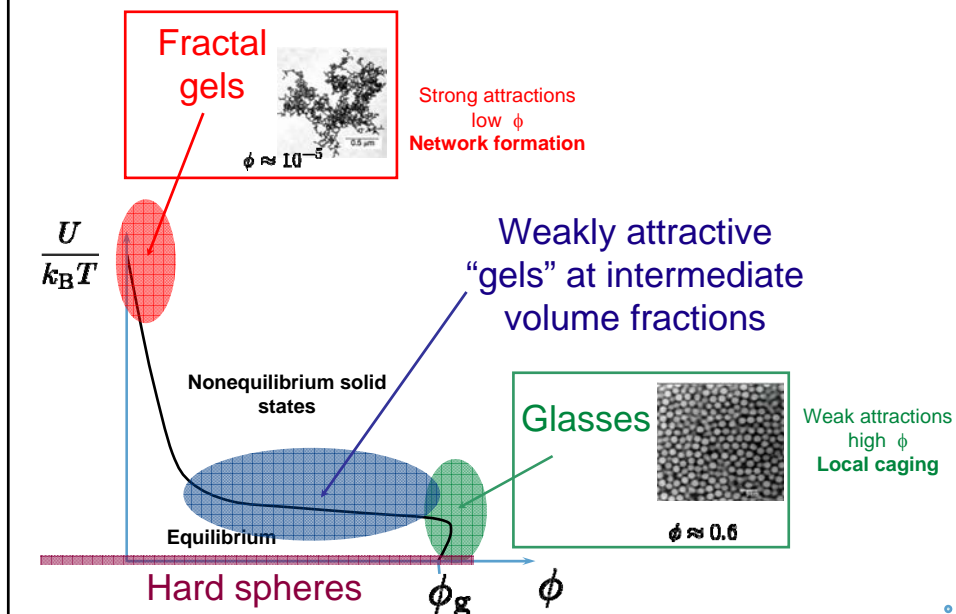


Surface tension is very low

Fractal Dimension of the Nuclei

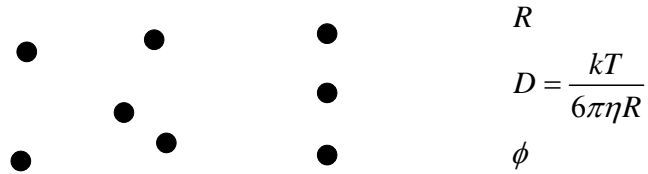


State diagram for colloidal particles



COLLOID AGGREGATION

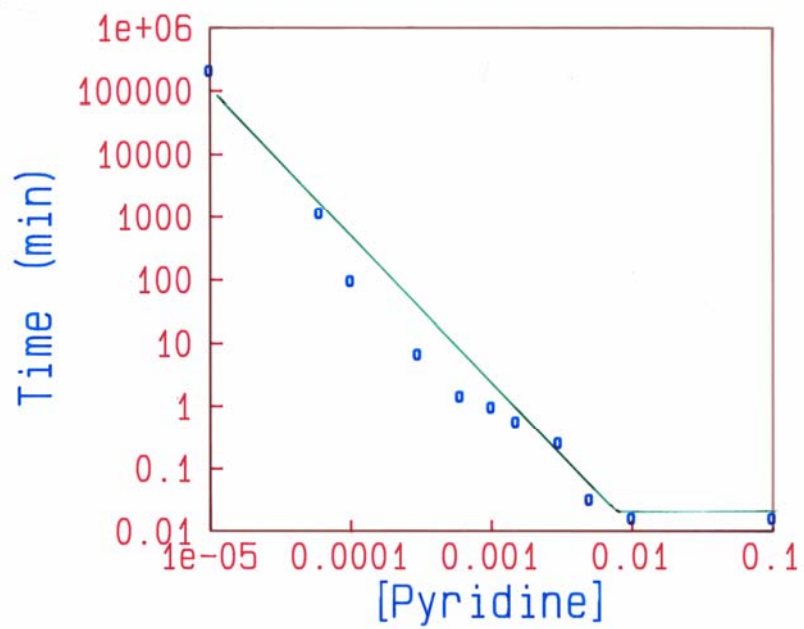
DILUTE, STABLE SUSPENSION

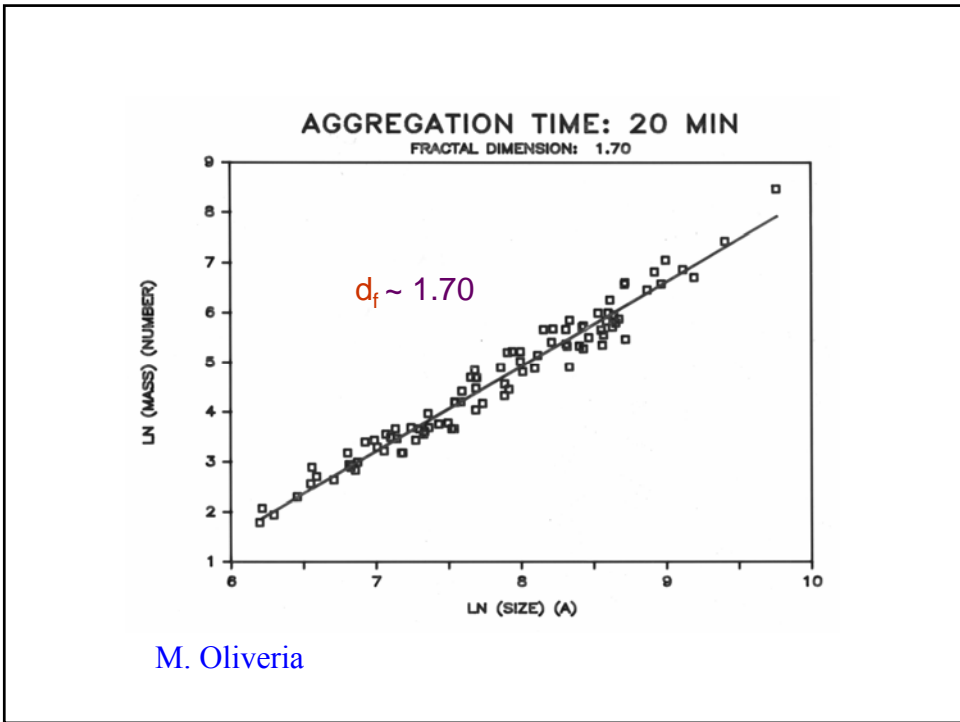
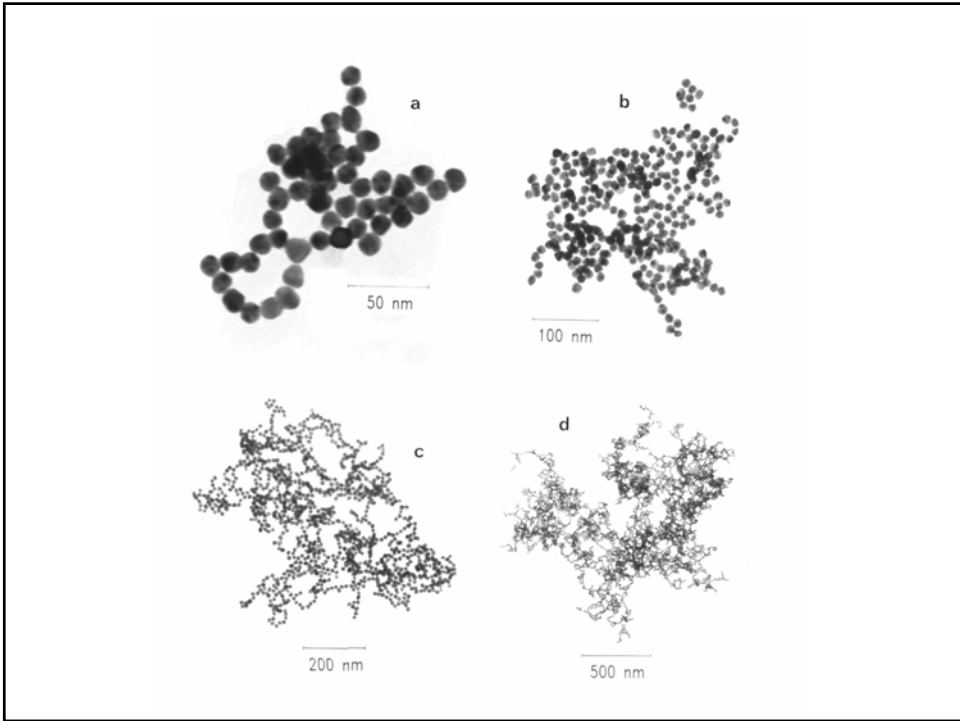


DESTABILIZE



Aggregation Time



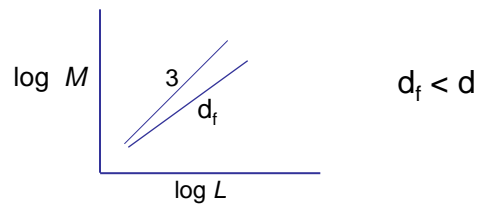


FRACTAL:

- SELF-SIMILAR
NO CHARACTERISTIC LENGTH SCALE

$$M \sim R^{d_f}$$

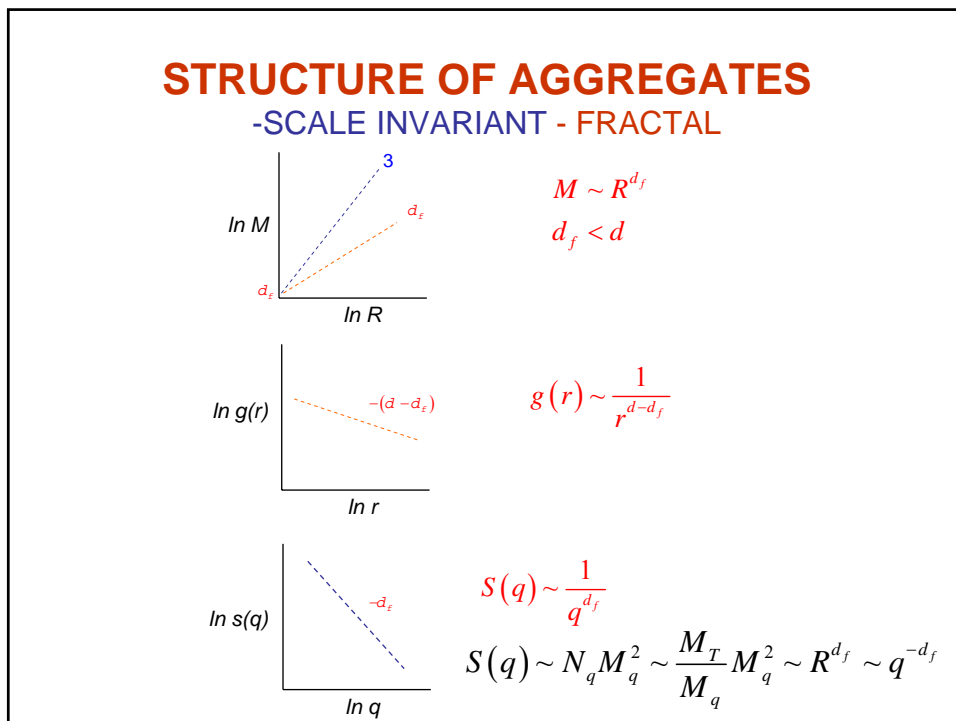
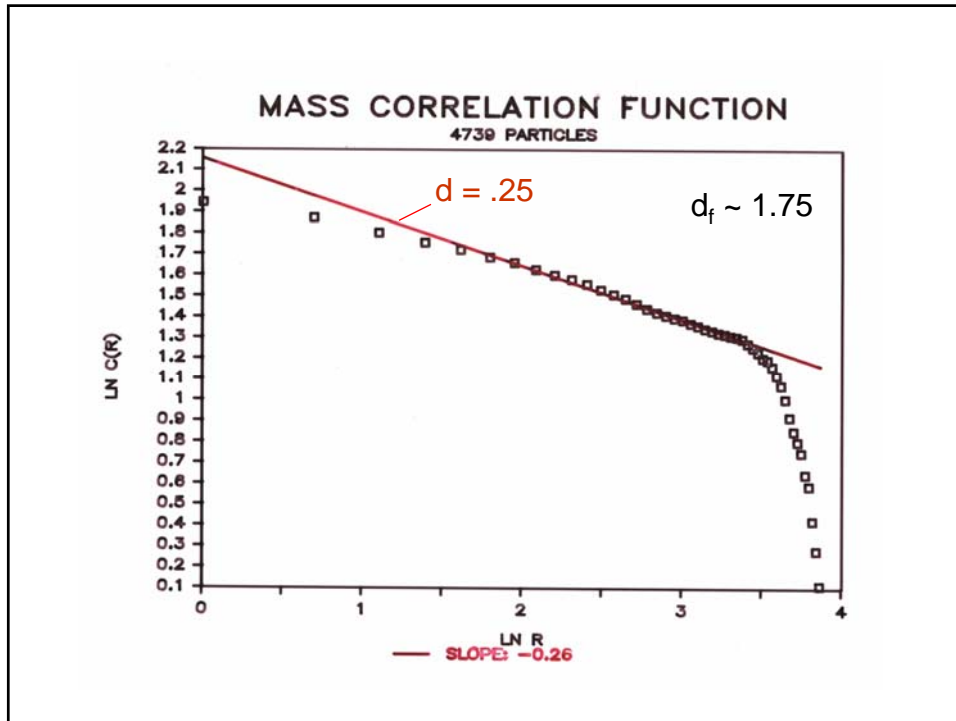
d_f : FRACTAL DIMENSION
NON-INTEGRAL



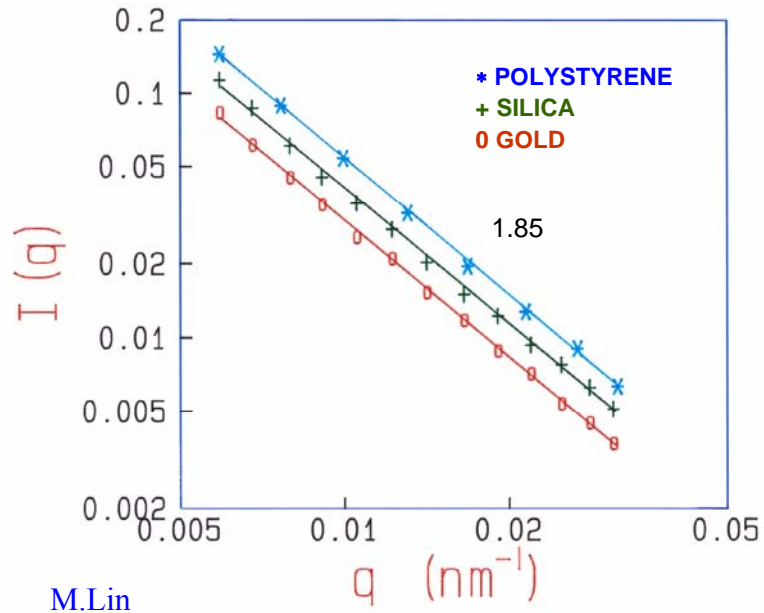
DENSITY: DECREASES WITH SIZE

$$\rho = \frac{M}{V} = \frac{L^{d_f}}{L^d} = L^{d_f-d}$$

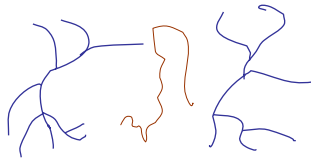




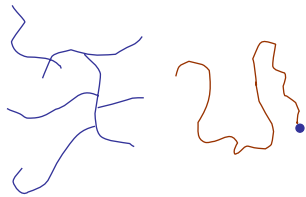
DIFFUSION-LIMITED COLLOID AGGREGATION



DIFFUSION-LIMITED CLUSTER AGGREGATION



$$d_f^1 + d_f^2 + d_t = 1.75 + 1.75 + 2 > 3.$$

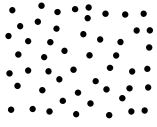





$$d_f^1 + d_f^2 + d_t = 1.75 + 0 + 2 > 3.$$

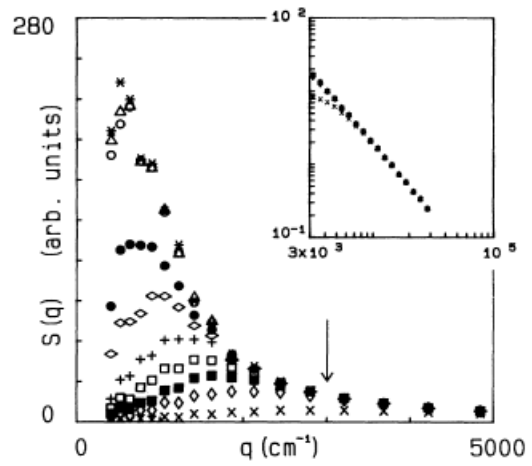
NO INTERPENETRATION
 BUT CLUSTERS STICK WITH OTHER CLUSTERS

$\therefore d_f \sim 1.8$ in 3-d.

DIFFUSION-LIMITED GELATION

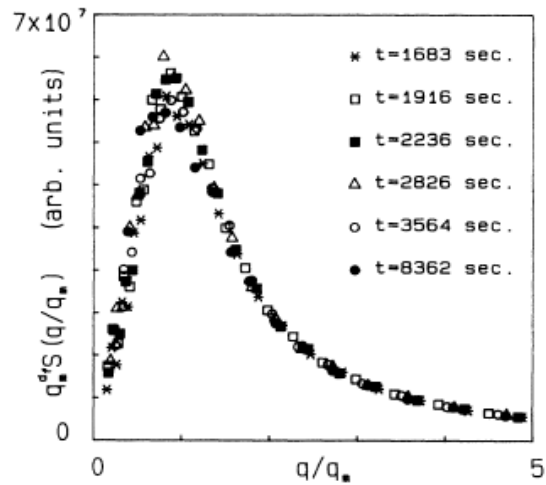
	MASS M_0	NUMBER N_0	VOLUME FRACTION ϕ_0
			
			
			
			
	$M_c = \left(\frac{R_c}{a} \right)^{d_f}$		
	$N_c = \frac{N_0}{M_c} = N_0 \left(\frac{R_c}{a} \right)^{-d_f}$		
	$\phi_c = \frac{V_c}{V} N_c = \frac{R_c^3}{V} N_0 \left(\frac{R_c}{a} \right)^{-d_f} = \phi_0 \left(\frac{R_c}{a} \right)^{3-d_f}$		
	$\phi_g = \phi_c = 1$		
	$R_c = a \phi_0^{1/(3-d_f)}$		

Static scattering from gelling colloid



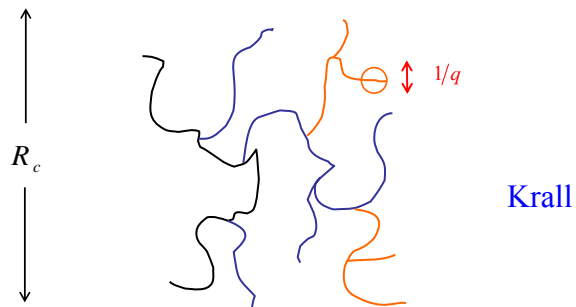
Carpineti and Giglio, PRL (1992)

Scaling of static scattering from gelling colloid



Carpinetti and Giglio, PRL (1992)

DYNAMICS OF A FRACTAL COLLOIDAL GEL

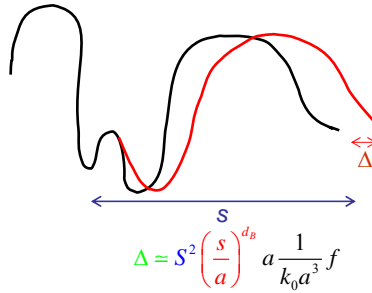


- CONSIDER BLOB OF SIZE $1/q = R_q$
- MOTION DUE TO FLUCTUATIONS OF ALL OTHER CONNECTED BLOBS
 - INDEPENDENT MODES
 - OVERDAMPED

LARGER BLOBS: LARGER DISPLACEMENT
SLOWER TIMESCALES

DYNAMICS "INSIDE" CLUSTER

KANTOR, WEBMAN; BALL



SIZE-DEPENDENT SPRING CONSTANT

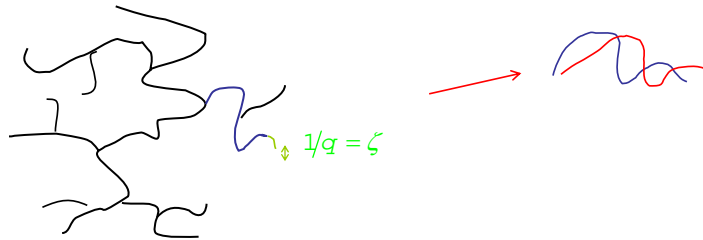
$$k(s) = k_0 \left(\frac{s}{a}\right)^{-\beta} \quad \beta = 2 + d_B \approx 3.1$$

$$k(s) = k_c \left(\frac{s}{R_c}\right)^{-\beta}$$

↑

AVERAGE CLUSTER SPRING CONSTANT

DYNAMICS OF SINGLE MODE, S



CONSTRAINED BROWNIAN MOTION $\langle \Delta r_c^2(t) \rangle_s = \frac{kT}{k(s)n(s)} [1 - e^{-t/\tau(s)}]$

1. SPRING CONSTANT:

$$k(s) = k_0 \left(\frac{s}{a}\right)^{-\beta}$$

SINGLE PARTICLE SPRING CONSTANT

SIZE DEPENDENT

KANTOR-WEBMAN

$$\beta = 2 + d_B$$

↑ "BOND" DIMENSION

2. TIME CONSTANT:

$$\tau(s) = \frac{6\pi\eta s}{k(s)}$$

VISCOUS DAMPING

3. MAXIMUM AMPLITUDE:

$$\delta_m^2 = \frac{kT}{k(s)n(s)}$$

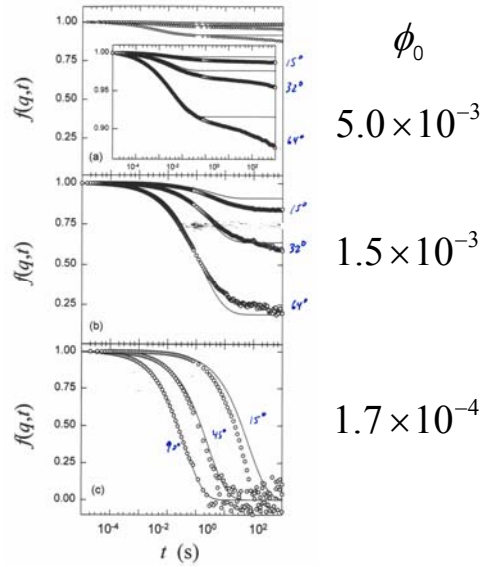
kT / NORMAL MODE

n(s): DENSITY OF LOCAL MODES

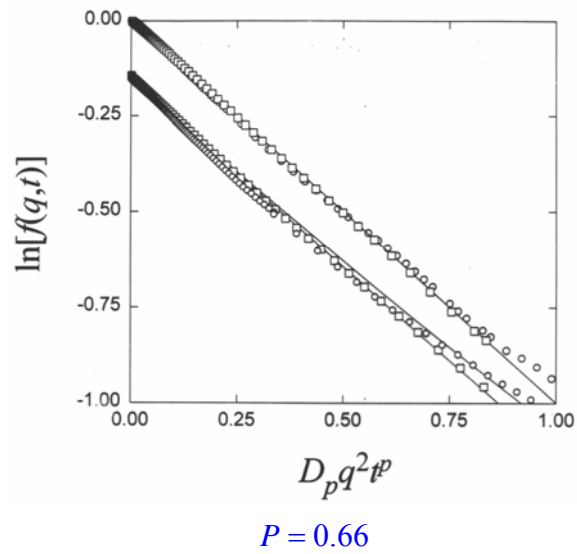
$$n(s) = N_c \left(\frac{a}{s}\right)^{d_f}$$

DYNAMIC LIGHT SCATTERING FROM COLLOIDAL GEL

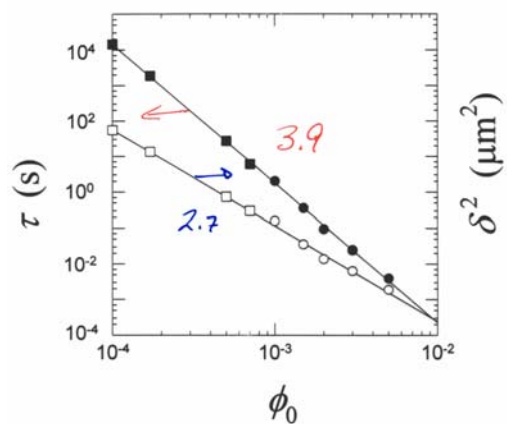
BOUYANCY-MATCHED POLYSTYRENE $a = 9.5 \text{ nm}$



SCALING STRETCHED EXPONENTIAL



VOLUME FRACTION DEPENDENCE LIGHT SCATTERING PROBE OF MODULUS



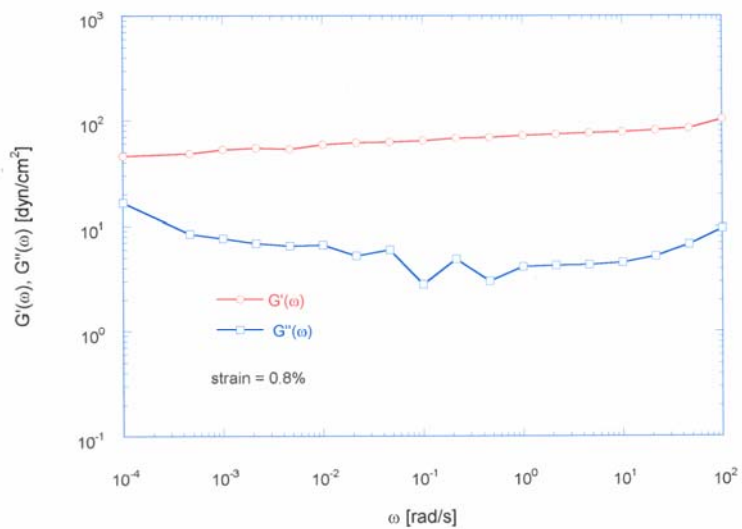
$$G \approx \frac{k_c}{R_c}$$

$$\text{BUT } \tau_c \sim \frac{R_c}{k_c}$$

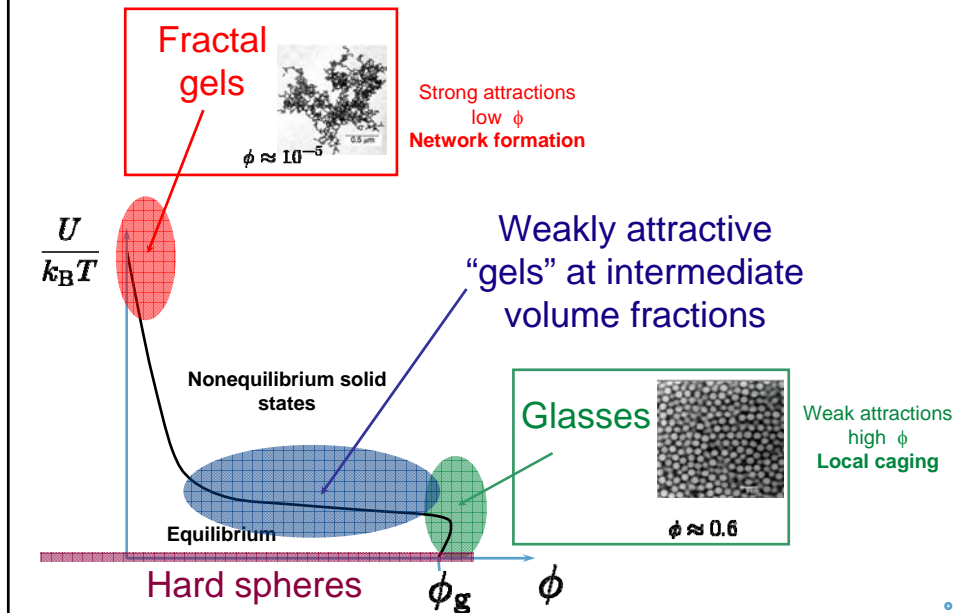
$$G \sim \phi^{3.9}$$

R_c ONLY LENGTH IN PROBLEM

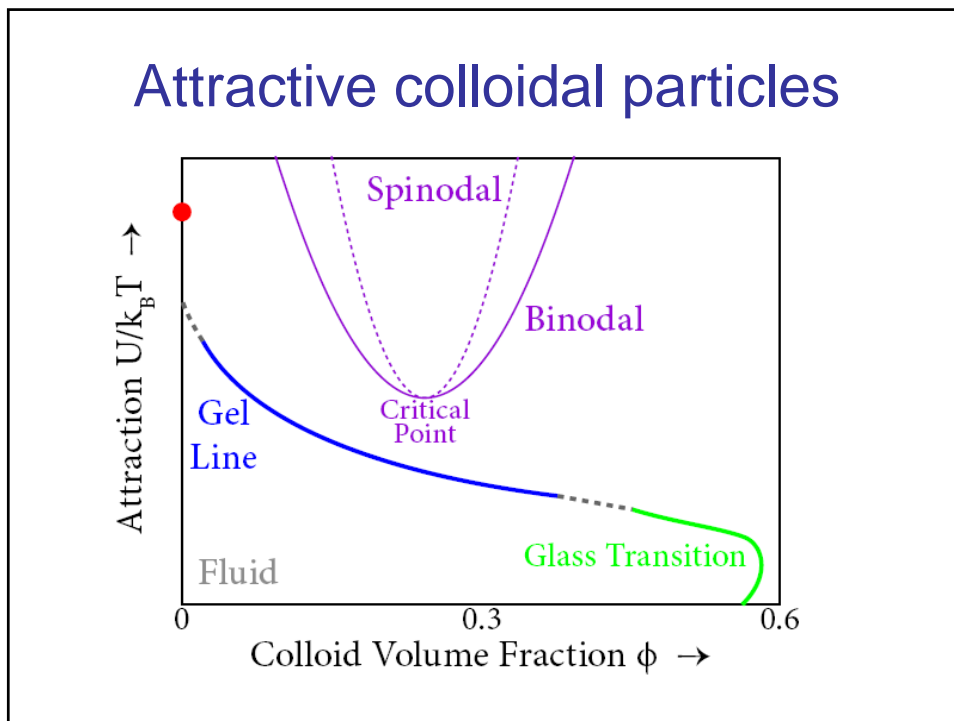
Polystyrene gel (19nm diameter particles)
 $\phi=0.0089$; 6mM MgCl_2



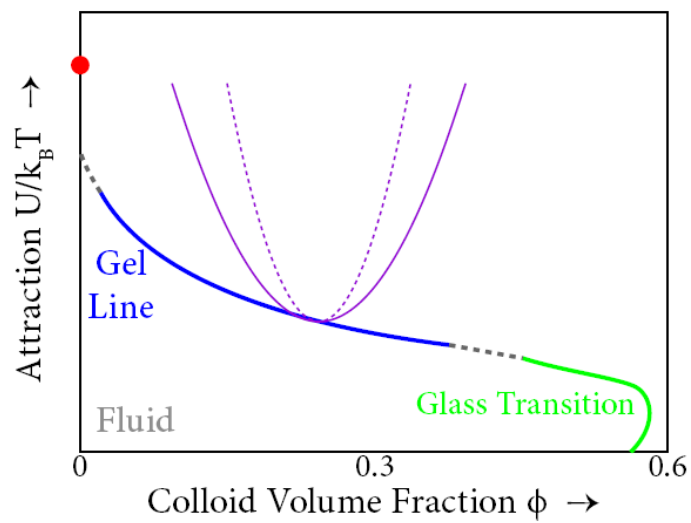
State diagram for colloidal particles



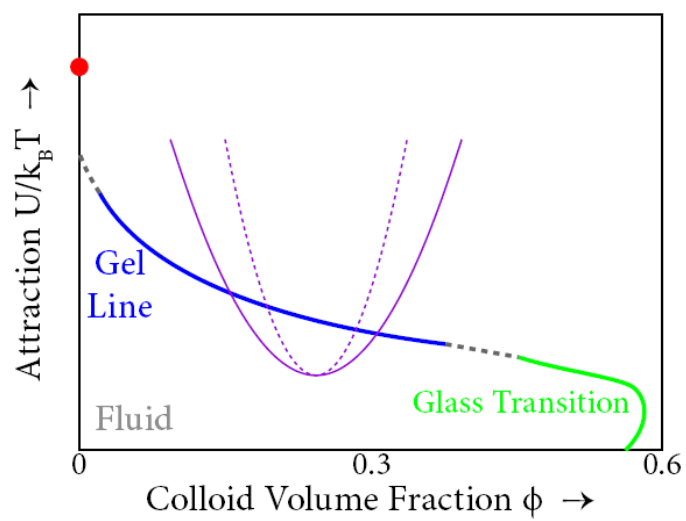
Attractive colloidal particles



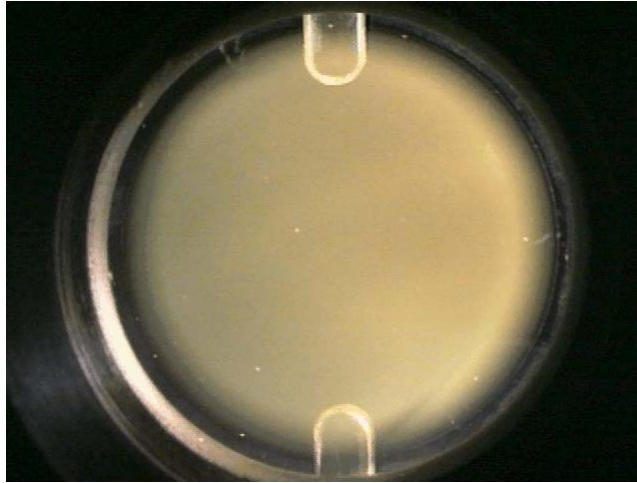
Attractive colloidal particles



Attractive colloidal particles



Spinodal Decomposition of Colloid Polymer



Col-Pol Critical Point: Sample Evolution



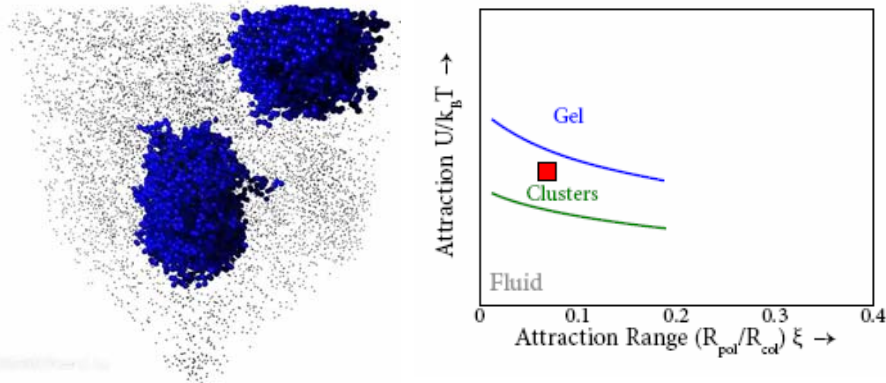
2. 3 hrs 50 min after mix

3. 13 hrs 24 min after mix

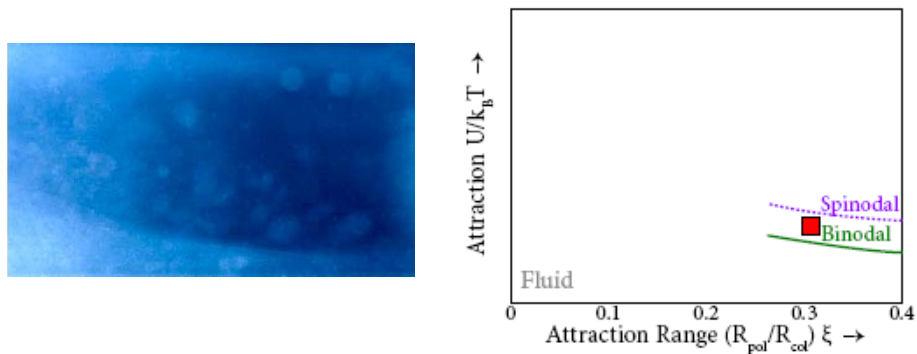
4. 35 Days, 2 hrs, 54 min after mix

Large scale phase separation - **Totally unprecedented**
Structure is **10,000** times larger

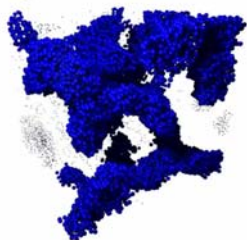
Stable clusters:
Binodal decomposition
Shorter-range interaction



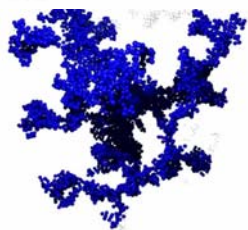
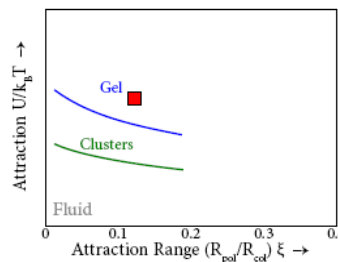
Stable clusters:
Binodal decomposition
Long-range interaction



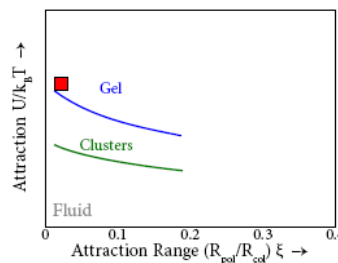
Structure depends on range



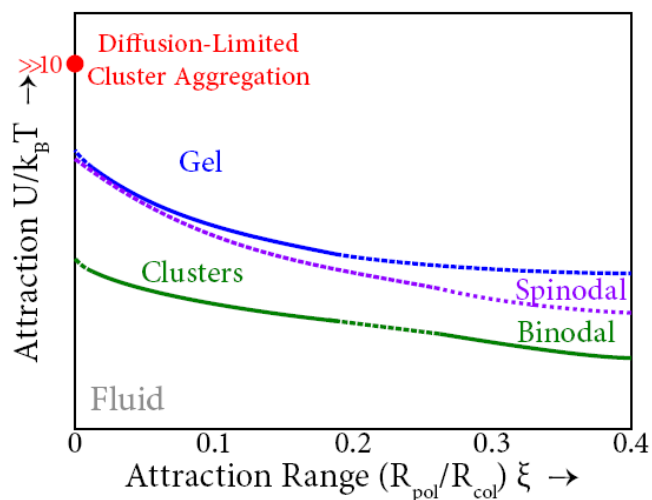
Long-range attraction



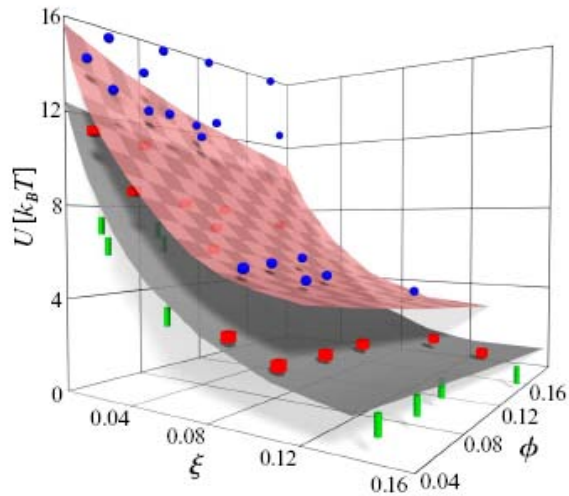
Short-range attraction



Unified Picture



Gel structure – long-range attraction



State diagram for colloidal particles

