

Trapped ion quantum computing, simulation, and sensing

John Bollinger, NIST, Boulder CO

Monday, July 2, 11:00 AM – Trapped ion quantum computing

Tuesday, July 3, 11:00 AM – Trapped ion quantum simulation

Thursday, July 5, 9:00 AM – Trapped ion quantum sensing

Trapped ion quantum computing, simulation, and sensing

John Bollinger, NIST, Boulder CO

Tuesday, July 3, 11:00 AM – Trapped ion quantum simulation

Quantum simulation \Leftrightarrow Analog quantum simulation

Ion trap quantum simulation references (incomplete):

review – R. Blatt and C.F. Roos, Nature Physics 8, 277 (2012); J. Bollinger et al.,

https://ws680.nist.gov/publication/get_pdf.cfm?pub_id=912704; C. Monroe, ..,

<http://iontrap.umd.edu/wp-content/uploads/2014/10/VarennaLecture2013.pdf>

Penning trap – Bohnet, .., Science 352, 1297 (2016); Gärttner, .., Nat. Phys., 13, 781 (2016)

linear rf trap - K. Kim, New J. Physics 13, 105003 (2011); P. Richerme, .., Nature 511, 198 (2014);

P. Jurcevic, .., Nature 511, 202 (2014); PJ. Smith, .., Nature Physics 12, 907 (2016);

Jurcevic, .., Phys. Rev. Lett. 119, 080501 (2017); J. Zhang, .., Nature 543, 217 (2017);

J. Zhang, .., Nature 551, 601 (2017).

Why quantum simulation?

- The 1st quantum revolution
(Bohr, Heisenberg,...) \Rightarrow understand single particle Q.M.
produced the transistor, laser, ...
- Many problems require much more than single particle treatments
 - quantum many-body systems, “More is different”
 - highly entangled states
 - phases of quantum matter
- quantum complexity ingredients:
qubits (spin-1/2)

$$|\uparrow\rangle \equiv |1\rangle \text{ and } |\downarrow\rangle \equiv |0\rangle$$

+superposition

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \quad ((\uparrow_1\rangle + |\downarrow_1\rangle) \otimes (\uparrow_2\rangle + |\downarrow_2\rangle) \dots$$

**N qubits,
 2^N states**

+entanglement

$$|\uparrow_1\rangle|\uparrow_2\rangle + |\downarrow_1\rangle|\downarrow_2\rangle$$

\Rightarrow quantum many problems can be “exponentially” difficult

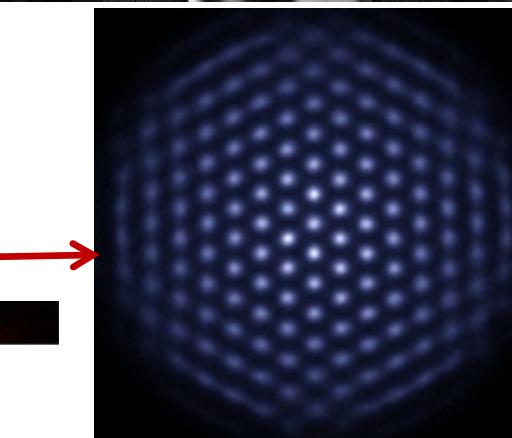
Motivation – understand quantum many-body physics

How are we to understand quantum many-body systems?

- quantum computation (if and when a Q. computer exists)
- develop novel approximate theoretical methods
- quantum simulation

- Feynman (1982) – use well controlled quantum system to emulate another system or model that is not understood

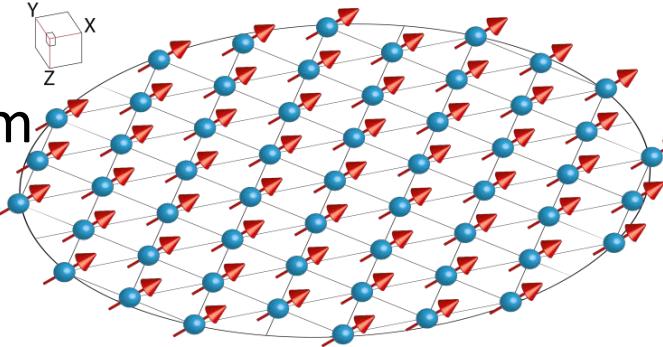
- platforms being used for quantum simulation:
 - neutral atoms and molecules in optical lattices
 - superconducting circuits
 - photons
 - trapped ions
 -
 -



Quantum Simulation in a Penning trap

- trapped ion crystals – good for simulating quantum magnetic interactions

(Ch Wunderlich et al., 2003;
Porras & Cirac, 2004)



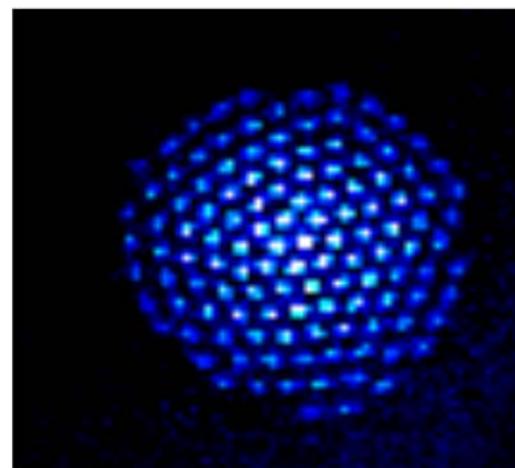
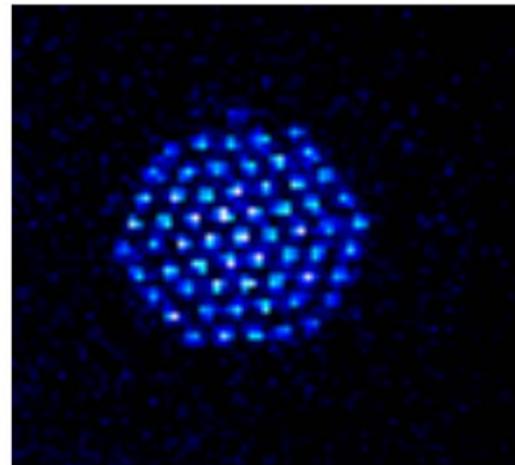
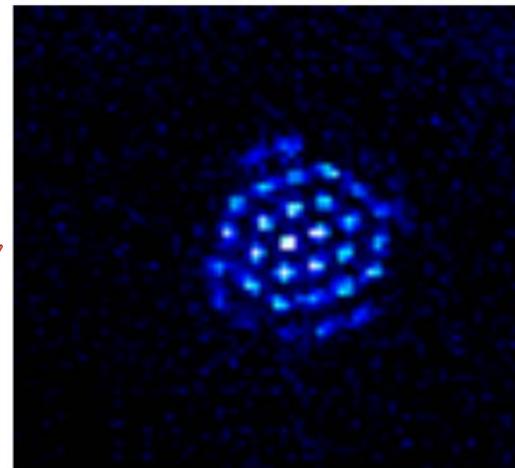
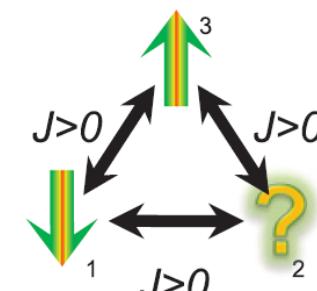
- exquisite quantum simulations with ion strings ($N < 20$) in linear rf traps (Blatt group, Monroe group, ...)



$$H_{\text{trans. Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z + B_{\perp} \sum_i \sigma_i^x$$

- Penning trap – platform for quantum simulation with large 2D ($N > 200$) ion crystals

- triangular lattice and antiferromagnetic interaction ($J_{i,j} > 0$) \Rightarrow frustration



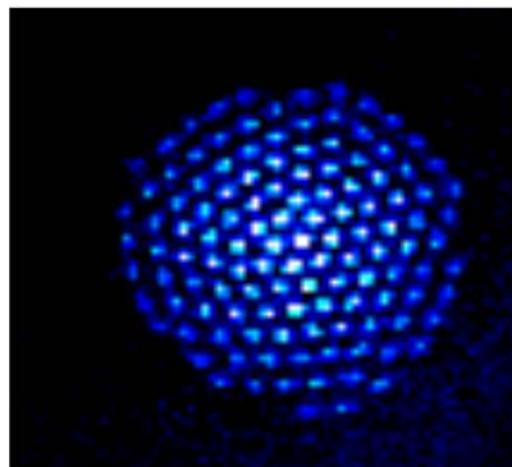
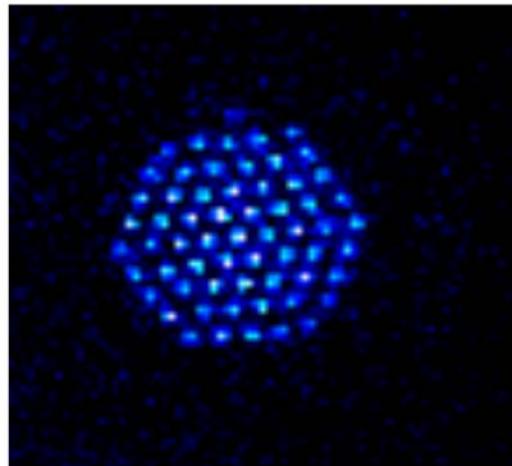
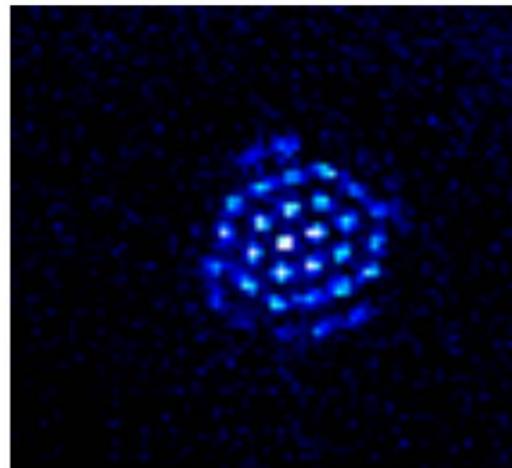
Quantum Simulation in a Penning trap

$$H_{\text{trans. Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z + B_{\perp} \sum_i \sigma_i^x$$

- Iconic model of quantum magnetism; calculation of ground states and dynamics very hard for frustrated systems

- quantum phase transitions
- quantum annealing protocols
- many-body localization
- spin-liquid behavior??

- benchmark quantum dynamics and entanglement
 - Bohnet et al., Science 2016, arXiv:1512.03756
 - Gaerttner et al., arXiv:1608.08938

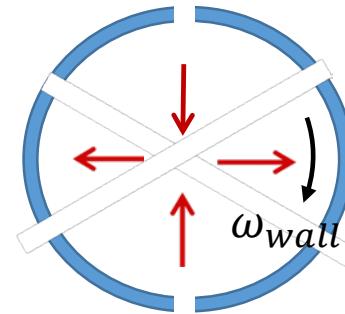


Trapped ion quantum simulation

1. Quantum simulation with 2d ion arrays in a Penning trap
2. Quantum simulation with 1d ion crystals in linear rf traps

1. Quantum simulation with 2d ion arrays in a Penning trap

- ion crystals in Penning traps

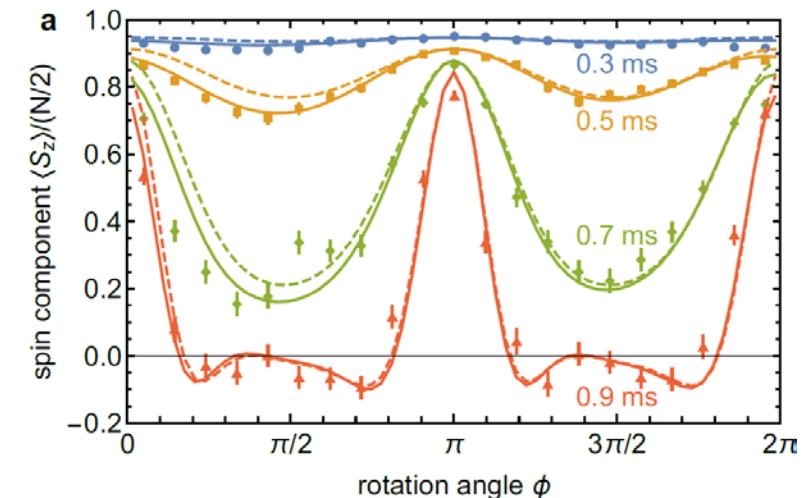


- - high magnetic field qubit
- modes

- engineering tunable Ising dynamics

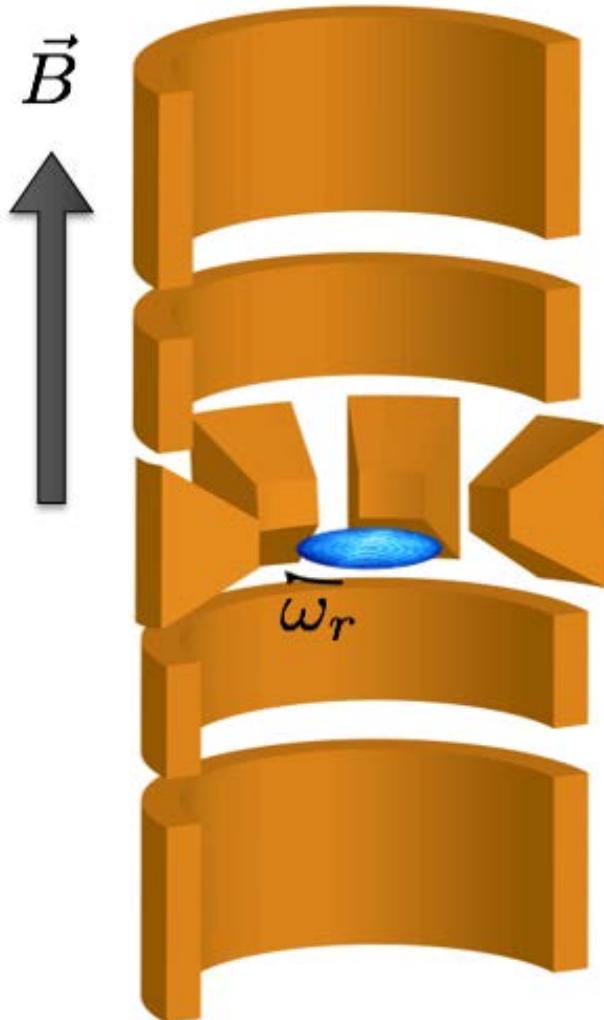
$$H_{Ising} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

- benchmark quantum dynamics, entanglement
 - spin squeezing
 - out-of-time correlations (OTOC)



Penning traps

Confinement through static electric and magnetic fields



g-factors,
atomic phys.

U of Wash., Mainz,
Harvard, Imperial
College, NIST, ..

mass
spectroscopy

U of Wash., Harvard,
ISOLDE/CERN, ...

ICR mass
spectroscopy

everywhere

cluster studies

Mainz/Griegswald

non-neutral
plasmas

UCSD, Berkeley,
Princeton, NIST, ..

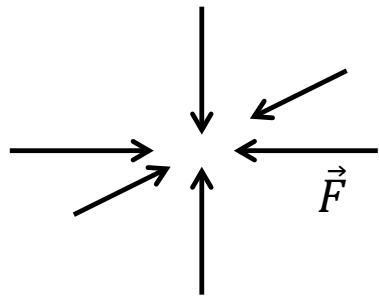
anti-matter

UCSD, Harvard,
CERN, Swansea, ...

quantum
information/simulation

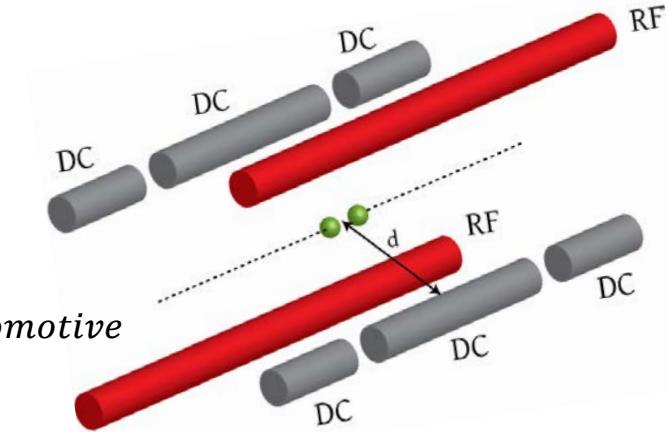
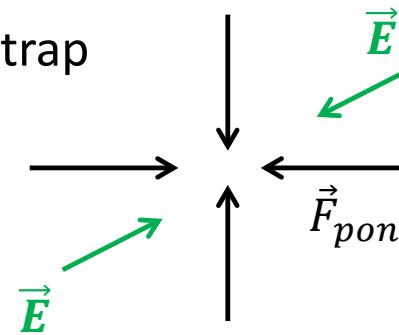
NIST, Imperial
College, Sydney,

linear rf trap vs Penning trap



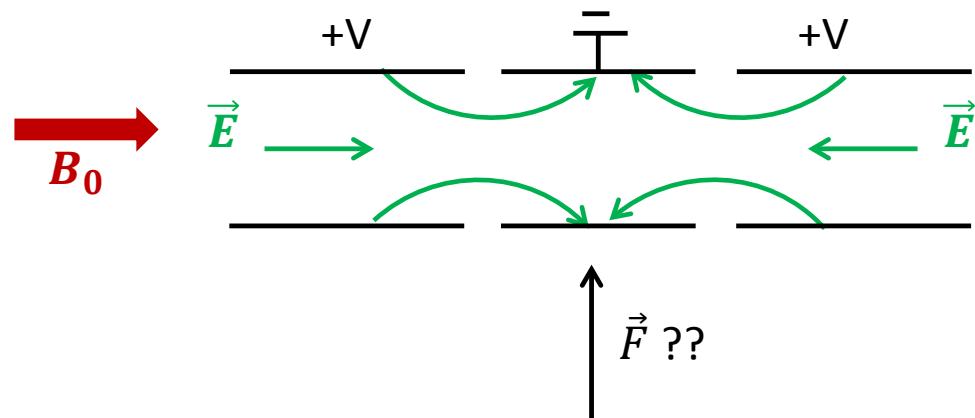
ideal ion trap
desire $\vec{V} \cdot \vec{F} \neq 0$

linear rf trap



confinement \leftrightarrow conservation of energy

Penning trap

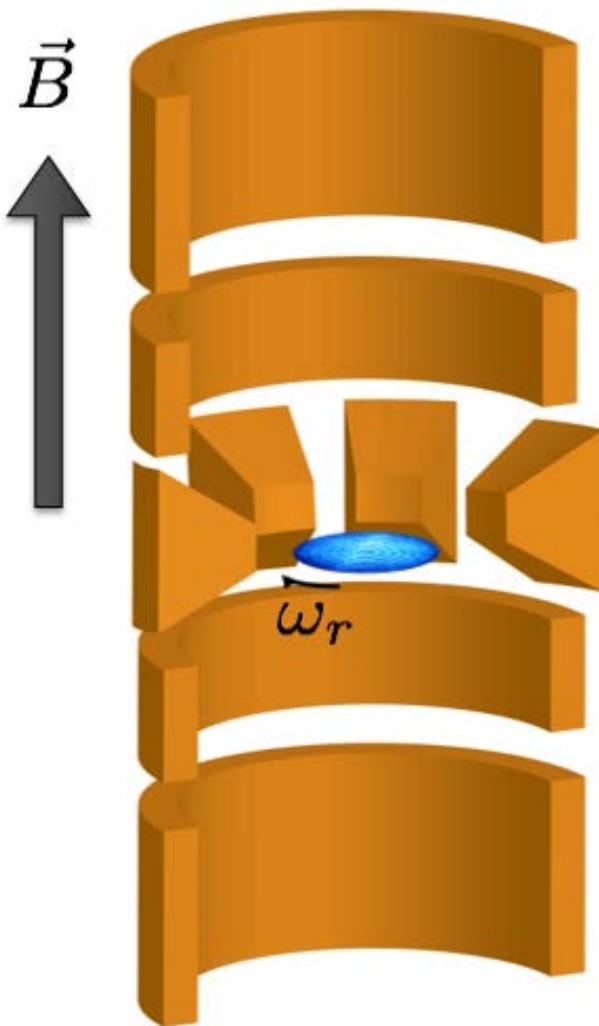


axial confinement \leftrightarrow conservation of energy

radial confinement \leftrightarrow conservation of angular momentum

Dubin and O'Neil, RMP 71 (1999)
 $\rho \sim \exp[-(H - \omega_r P_\theta)/(k_B T)]$

Penning trap: many particle confinement with static fields



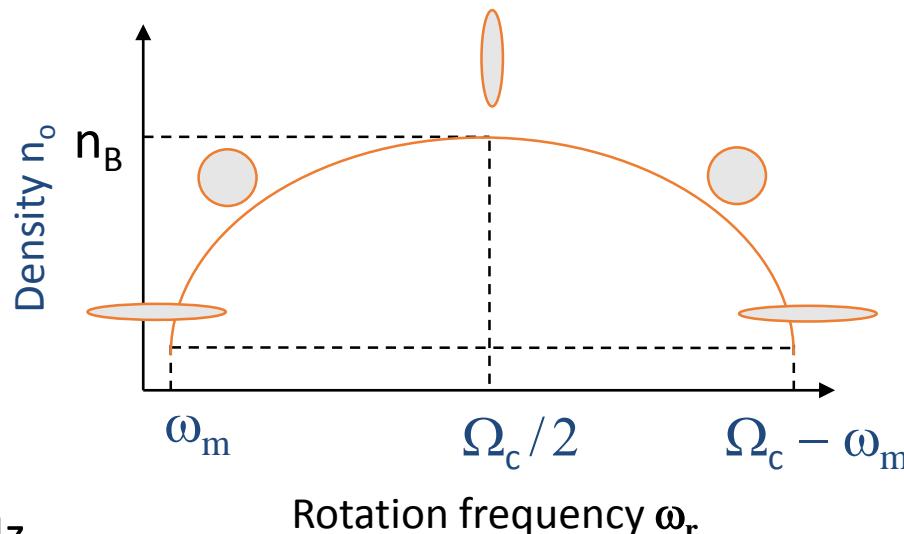
${}^9\text{Be}^+$, $B_0 = 4.5 \text{ T}$

$$\frac{\Omega_c}{2\pi} \sim 7.6 \text{ MHz}, \frac{\omega_z}{2\pi} \sim 1.6 \text{ MHz}, \frac{\omega_m}{2\pi} \sim 160 \text{ kHz}$$

- radial confinement due to rotation –
ion plasma rotates $v_\theta = \omega_r r$ due to $\mathbf{E} \times \mathbf{B}$ fields
Lorentz force from rotation is directed radially inward

rotating frame \Rightarrow

$$\varphi_{trap}(r, z) \approx \frac{1}{2} m \omega_z^2 \left(z^2 - \frac{r^2}{2} \right)$$
$$\varphi_{rot}(r, z) = \frac{1}{2} m \omega_z^2 \left(z^2 + \left(\frac{\omega_r (\Omega_c - \omega_r)}{\omega_z^2} - \frac{1}{2} \right) r^2 \right)$$

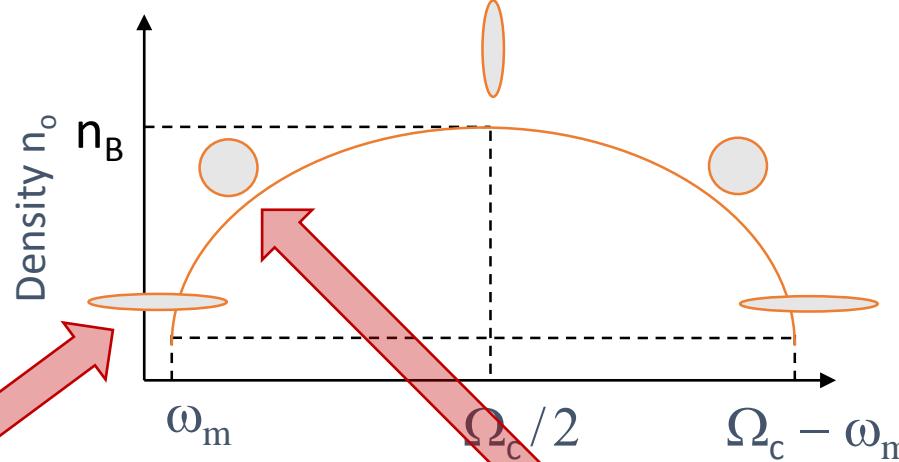


Ion crystals form as a result of minimizing Coulomb potential energy

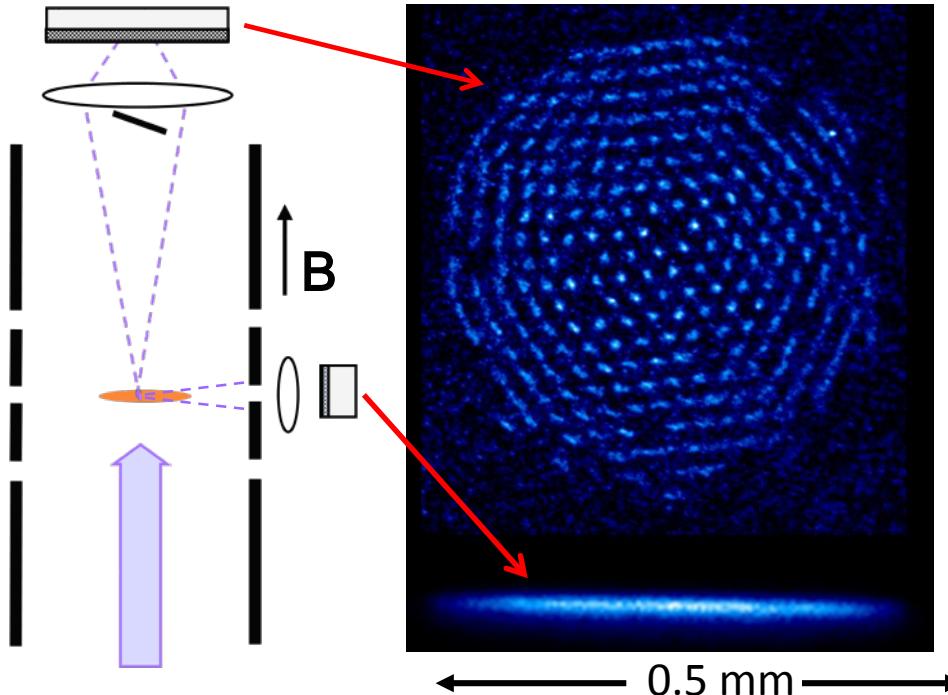
$T \rightarrow 0.4 \text{ mK}$ (Doppler laser cooling) $\Rightarrow q^2/a_{WS} \gg k_B T, 2a_{WS} \sim \text{ion spacing}$

type of crystal depends on
crystal size and shape

Mitchell et.al., Science (1998)



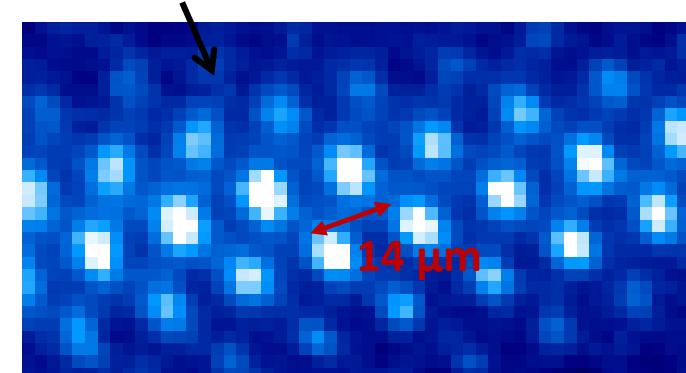
single planes



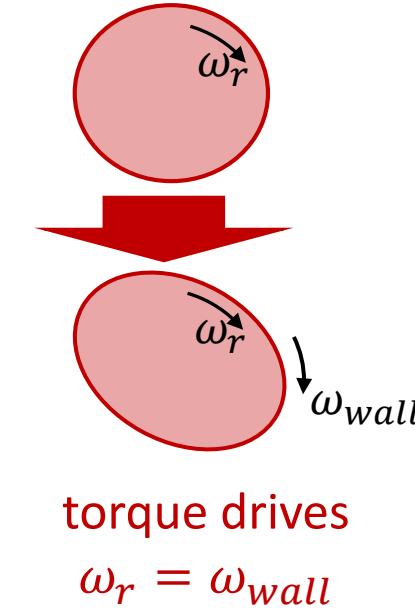
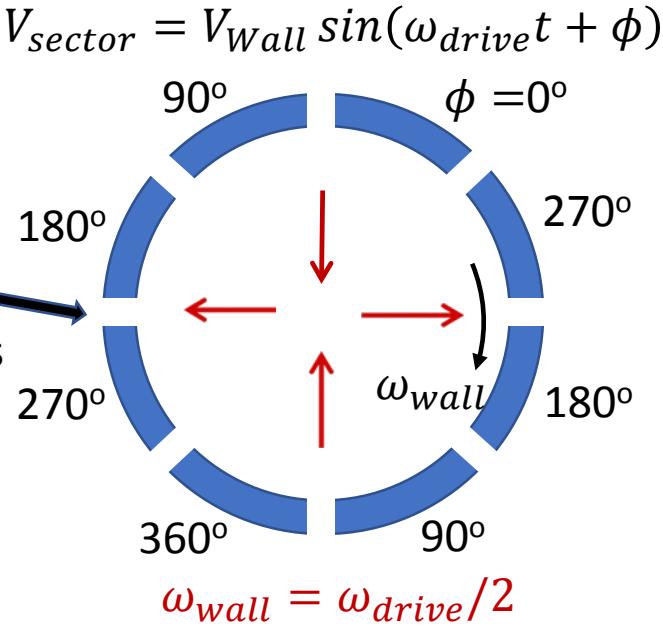
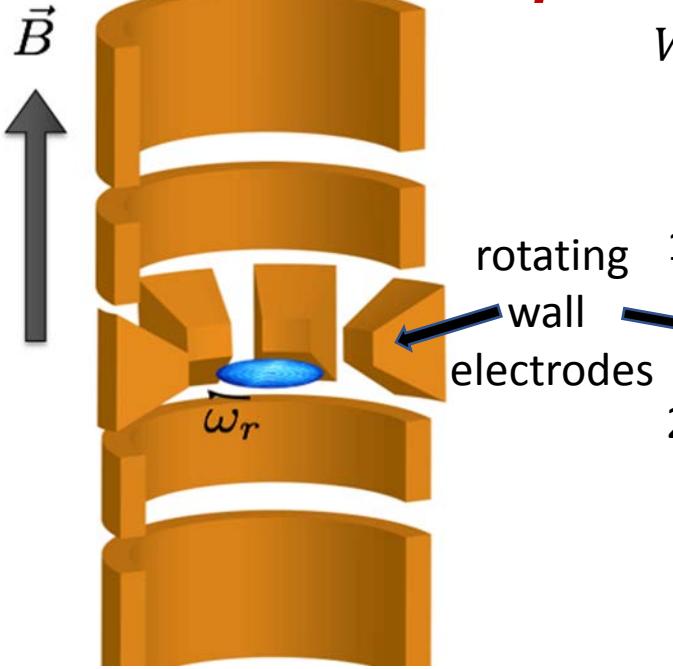
Rotation frequency ω_r

bcc crystals with $N > 100 \text{ k}$

observed with:
Bragg scattering
ion fluorescence imaging



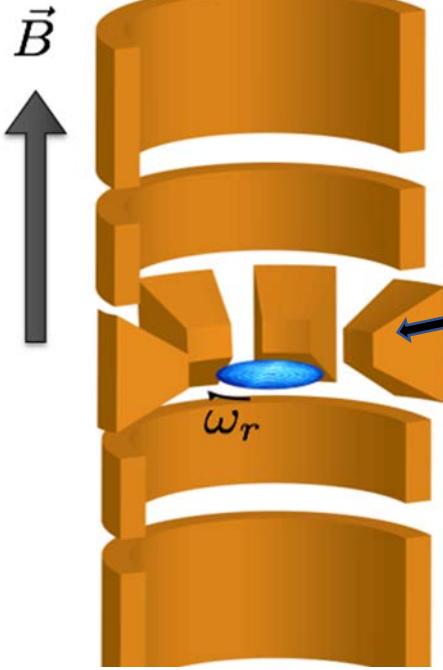
Precise ω_r control with a rotating electric field



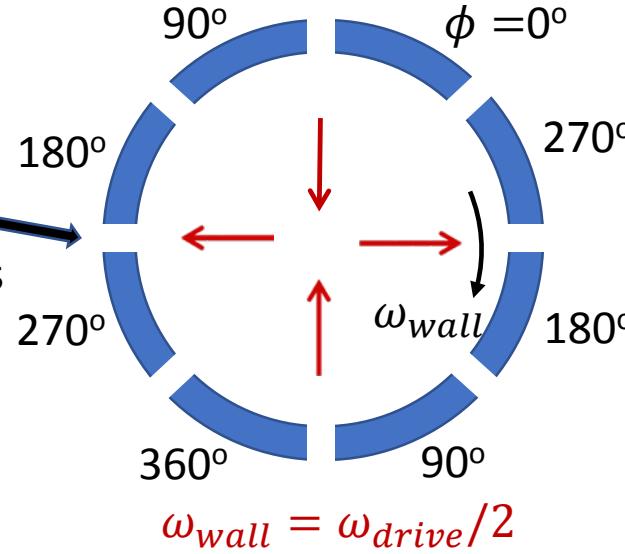
rotating wall potential simulation



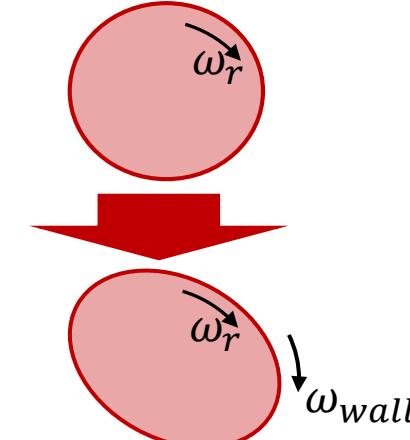
Precise ω_r control with a rotating electric field



$$V_{sector} = V_{Wall} \sin(\omega_{drive}t + \phi)$$

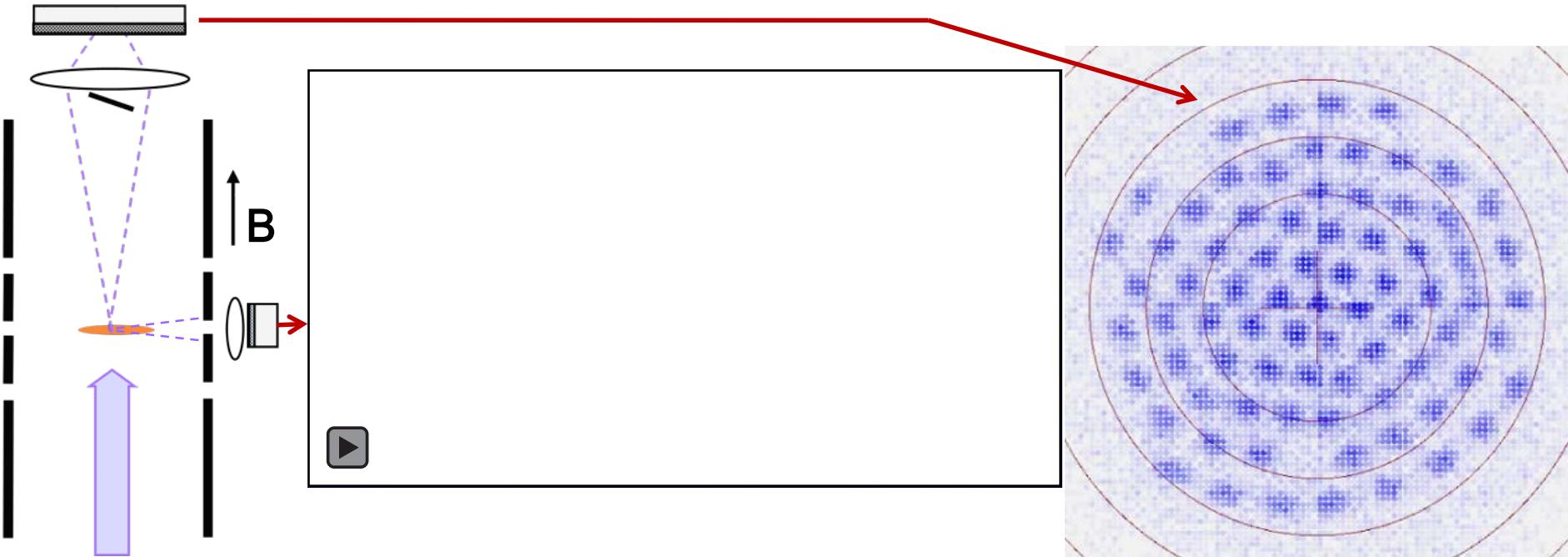


$$\omega_{wall} = \omega_{drive}/2$$

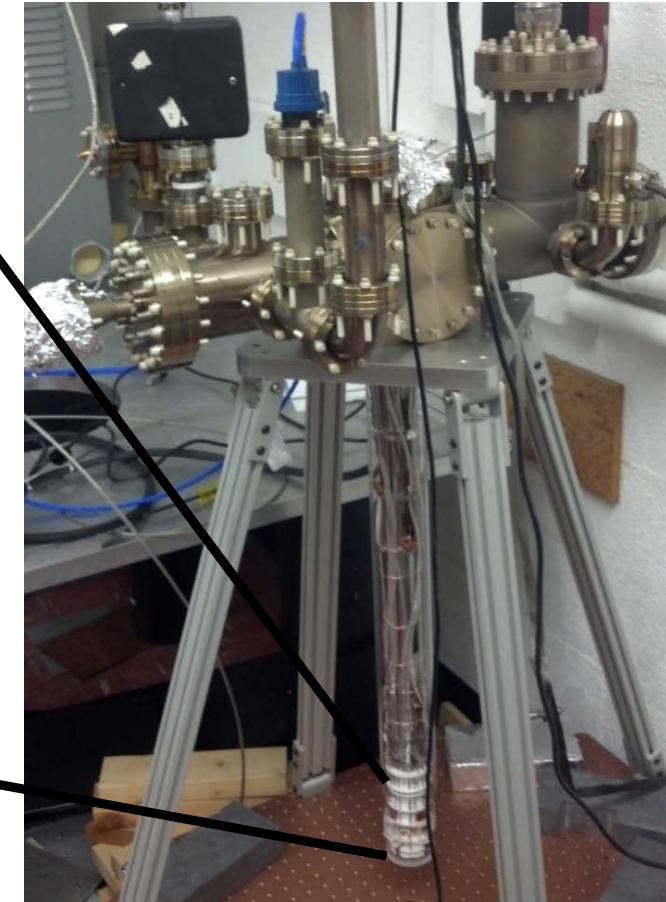
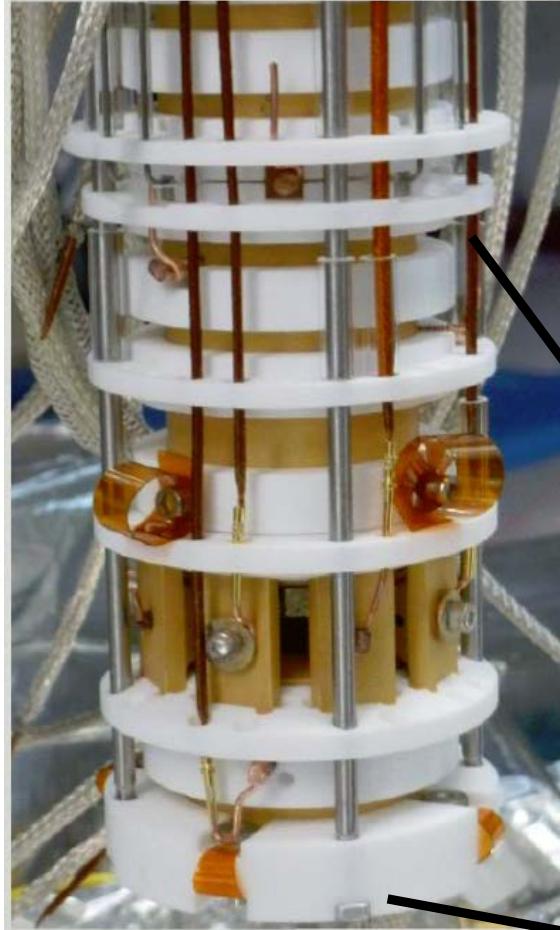
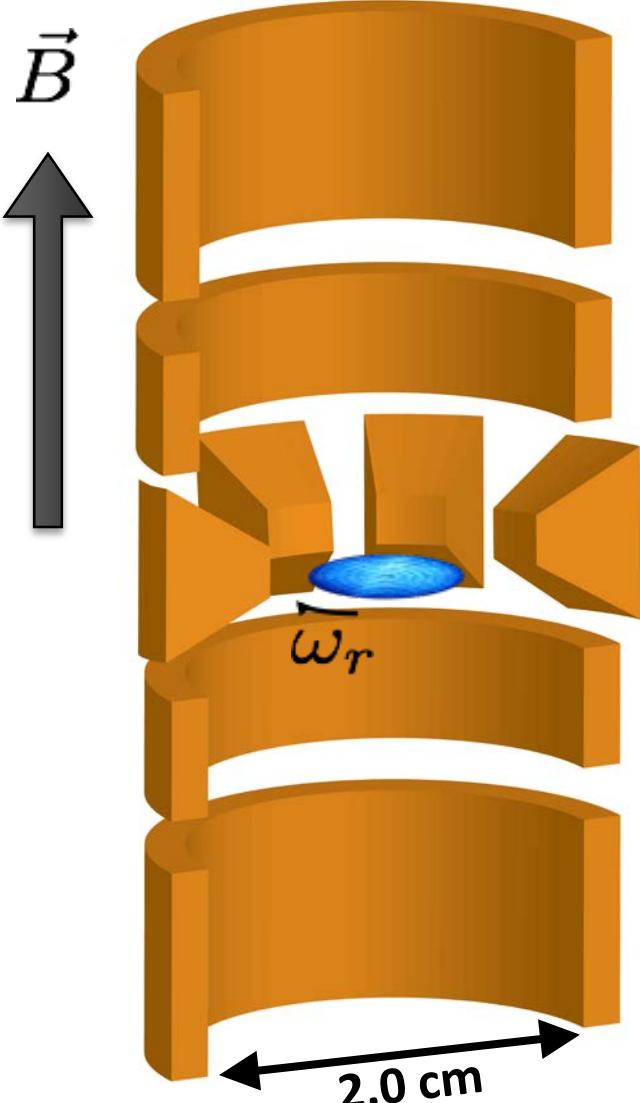


torque drives

$$\omega_r = \omega_{wall}$$

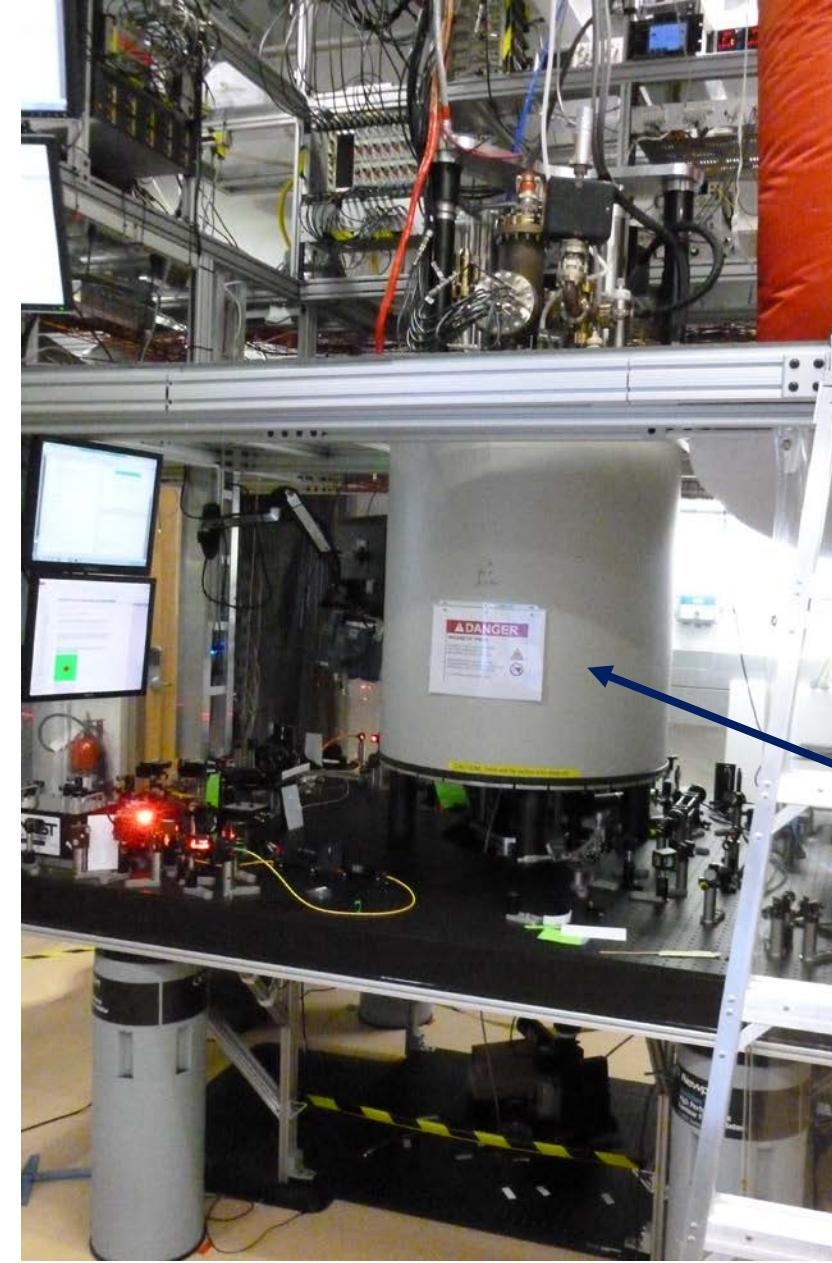
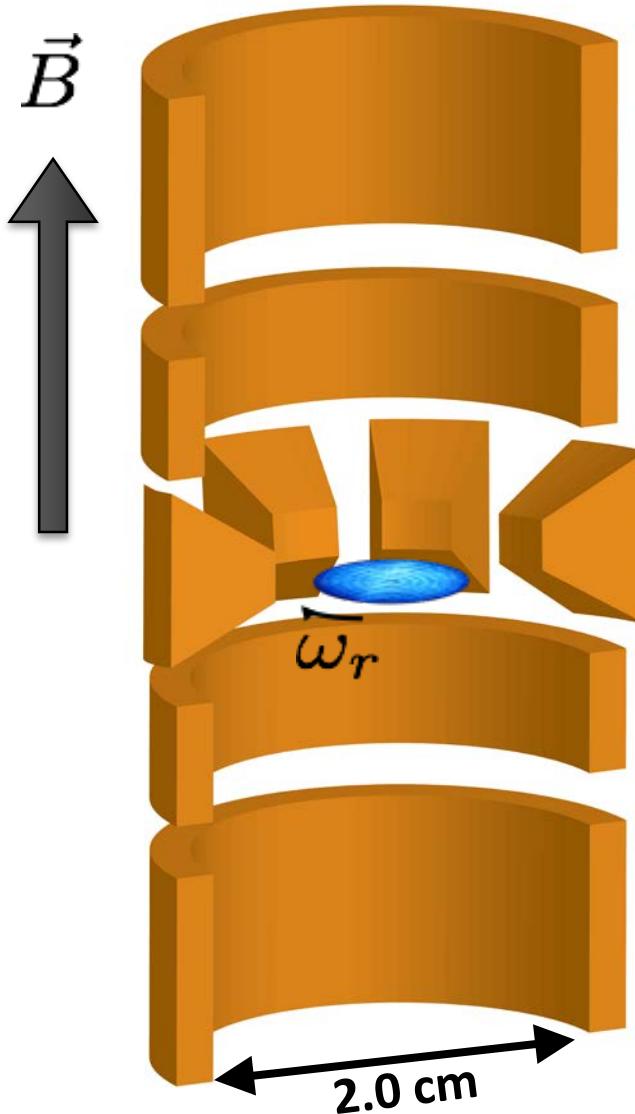


NIST Penning trap



${}^9\text{Be}^+$, $B_0 = 4.5 \text{ T}$, $\frac{\Omega_c}{2\pi} \sim 7.6 \text{ MHz}$, $\frac{\omega_z}{2\pi} \sim 1.6 \text{ MHz}$, $\frac{\omega_m}{2\pi} \sim 160 \text{ kHz}$

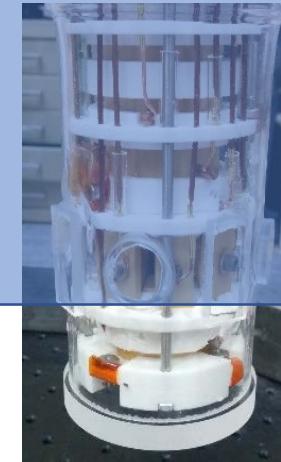
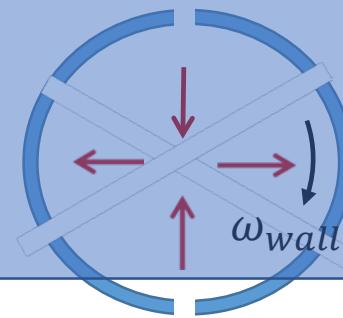
NIST Penning trap



4.5 Tesla
superconducting
solenoid

1. Quantum simulation with 2d ion arrays in a Penning trap

- ion crystals in Penning traps

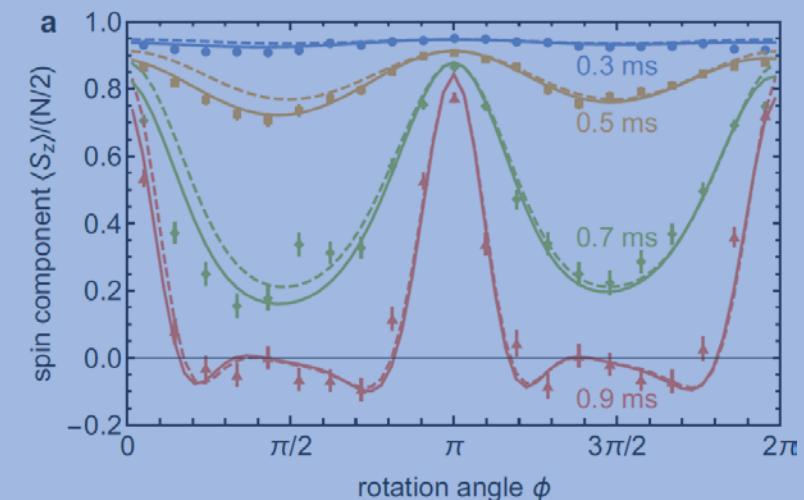


- - high magnetic field qubit
- modes

- engineering tunable Ising dynamics

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

- benchmark quantum dynamics, entanglement
 - spin squeezing
 - out-of-time correlations (OTOC)

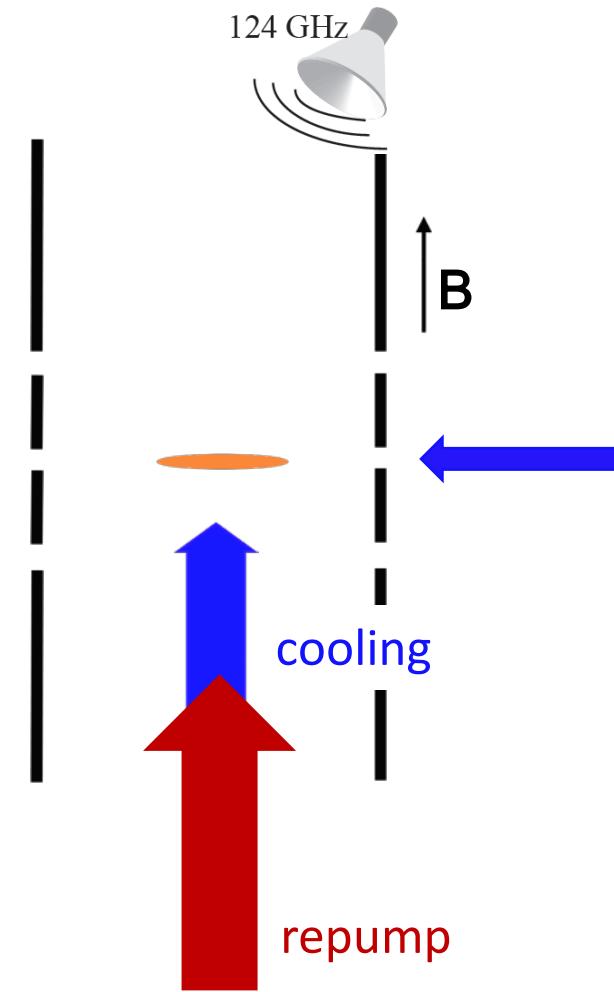
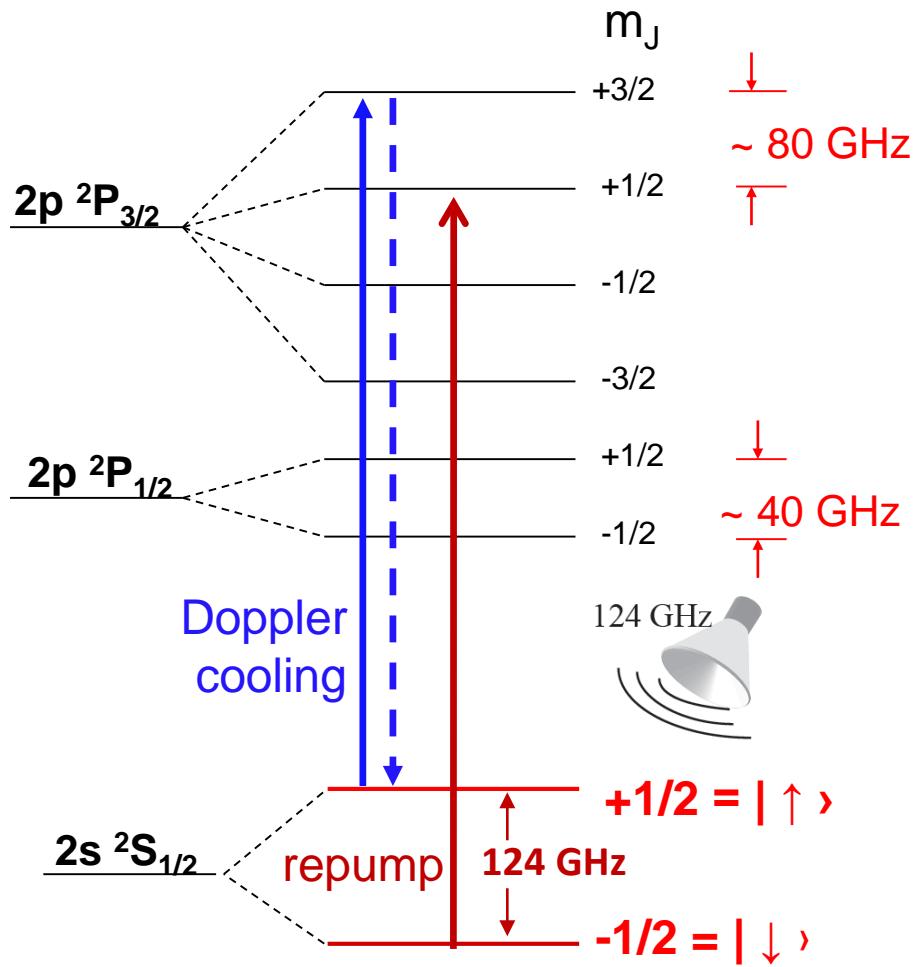


Be⁺ high magnetic field qubit

${}^9\text{Be}^+$, $B \sim 4.5 \text{ T}$, $\omega_0 / 2\pi \sim 124.1 \text{ GHz}$

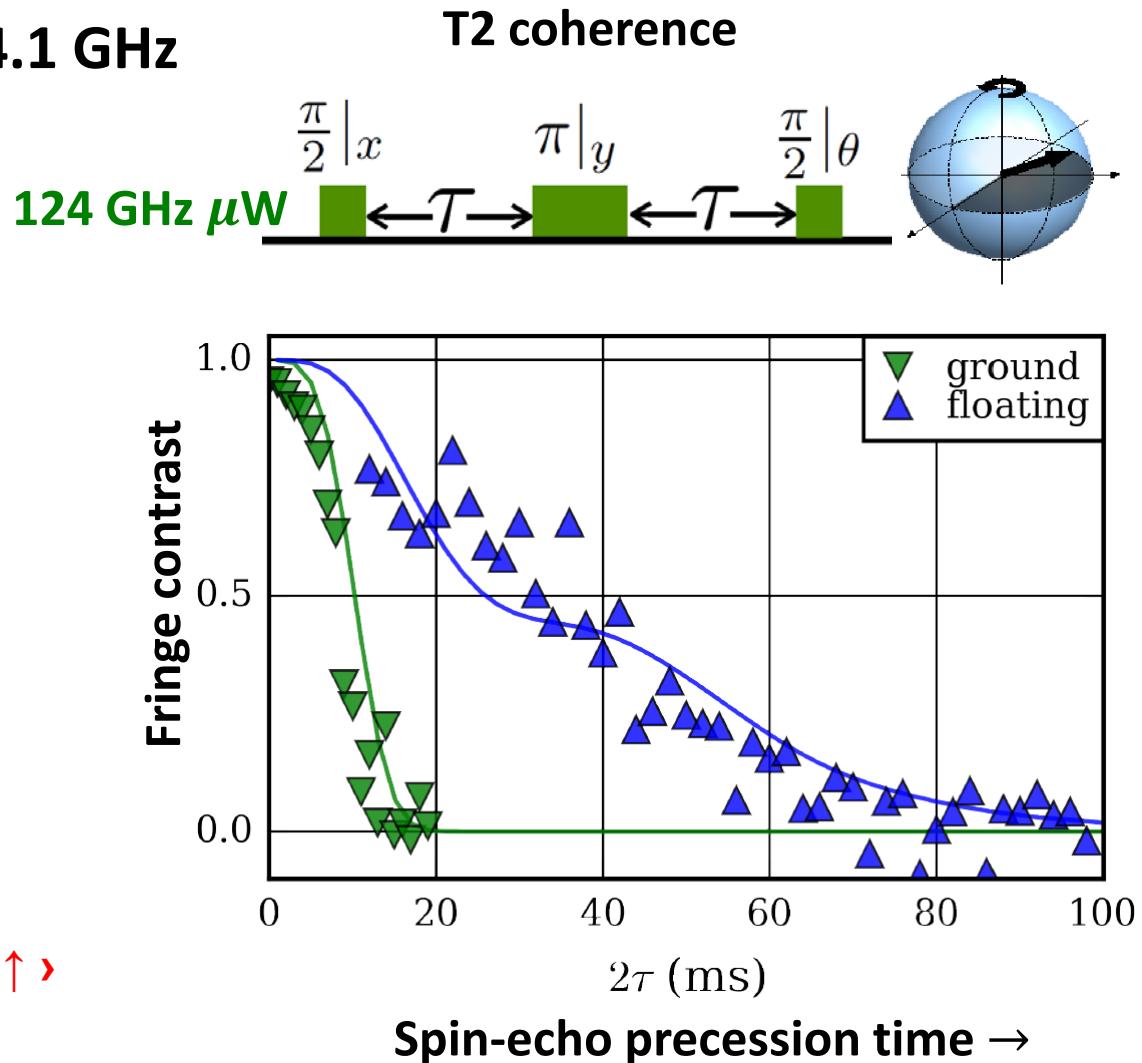
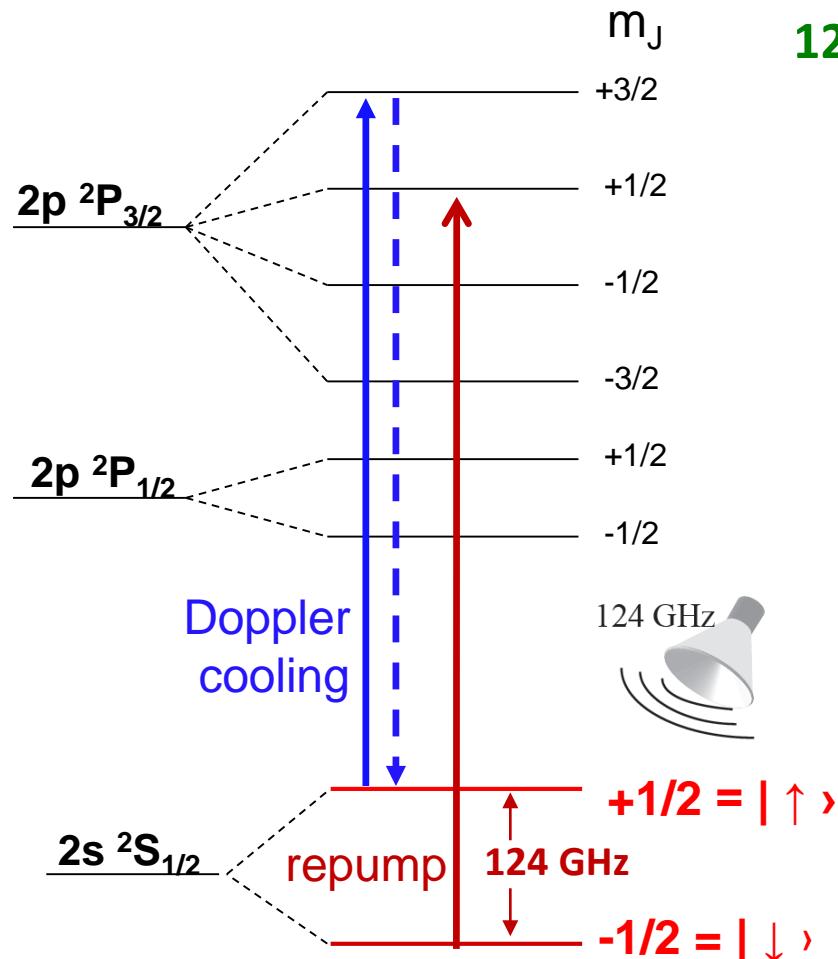
$$H_{\mu W} = \sum_i B_\perp \hat{\sigma}_i^x ,$$

$$B_\perp > 10 - 15 \text{ kHz}$$



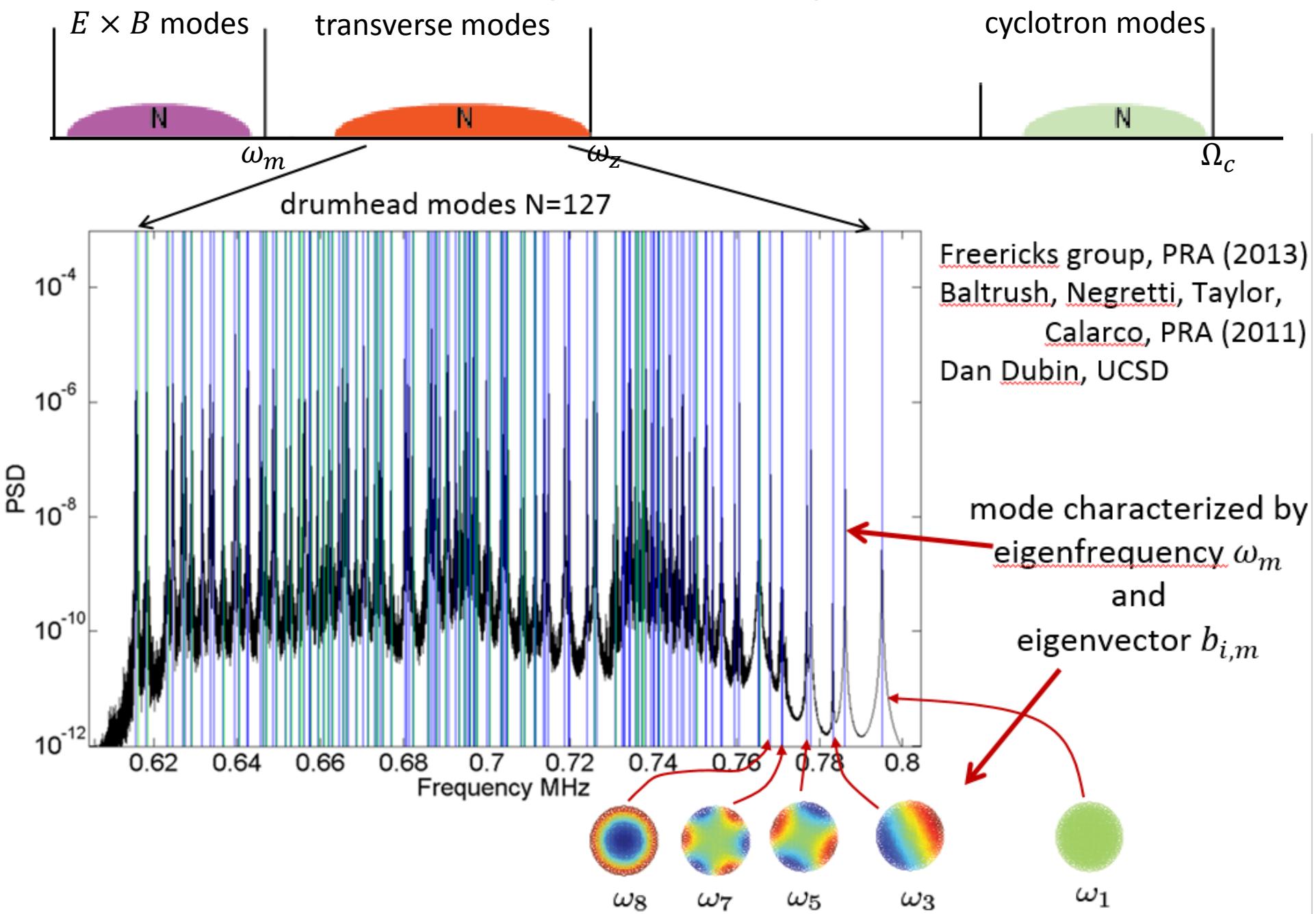
Be⁺ high magnetic field qubit

⁹Be⁺, B ~ 4.5 T, $\omega_0 / 2\pi \sim 124.1$ GHz

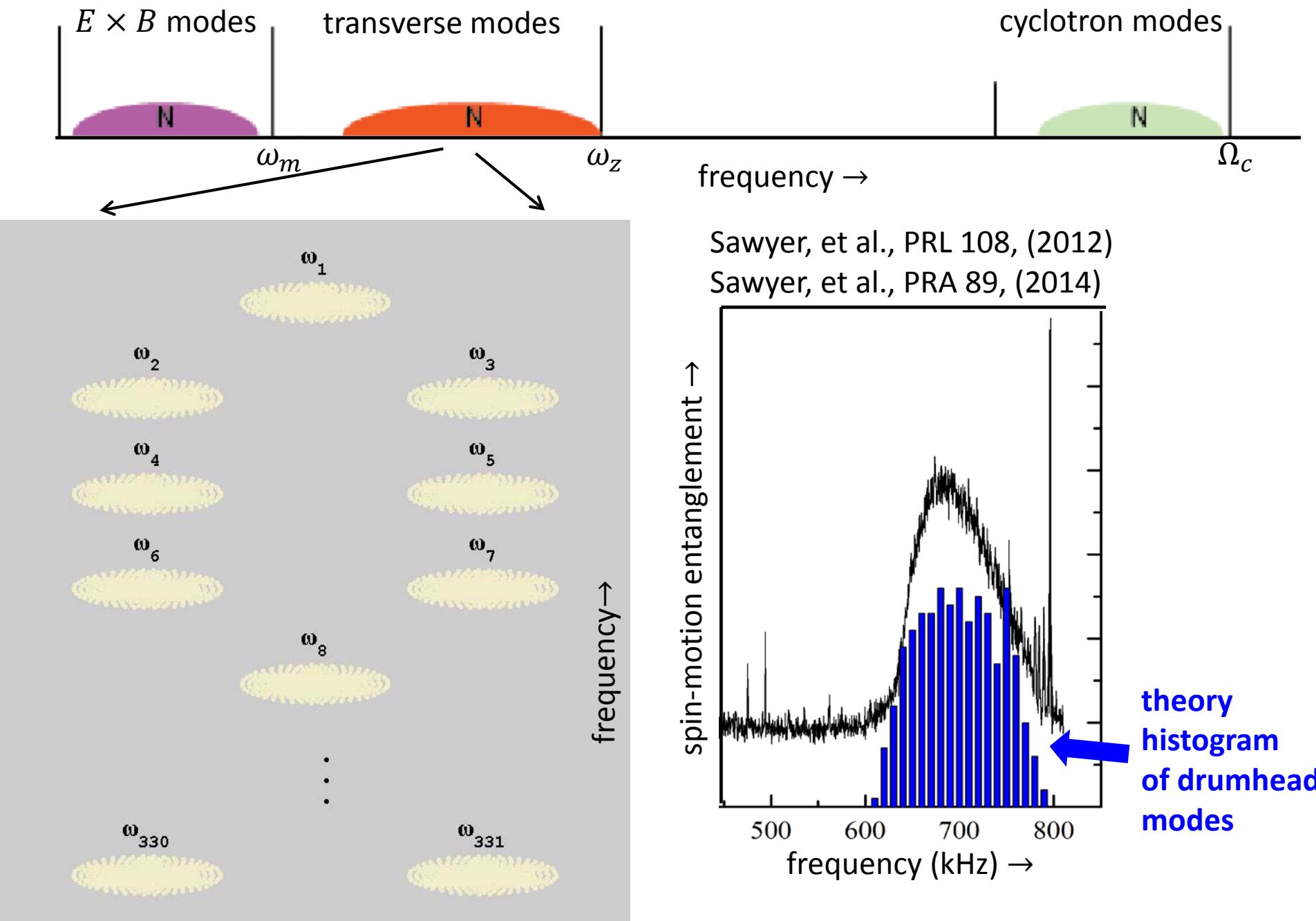


Britton et al., PRA (2016)
arXiv_1512.00801

Transverse (drumhead) modes

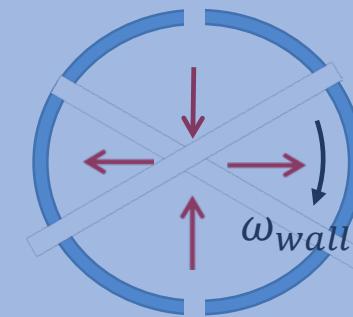


Transverse (drumhead) modes



1. Quantum simulation with 2d ion arrays in a Penning trap

- ion crystals in Penning traps

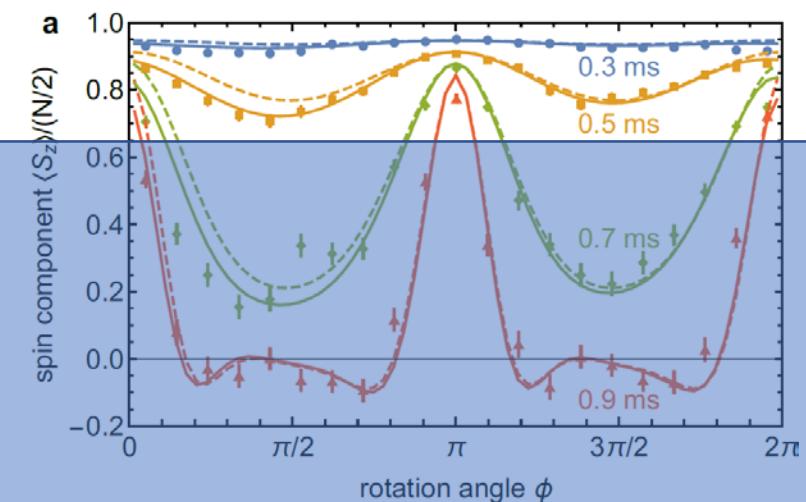


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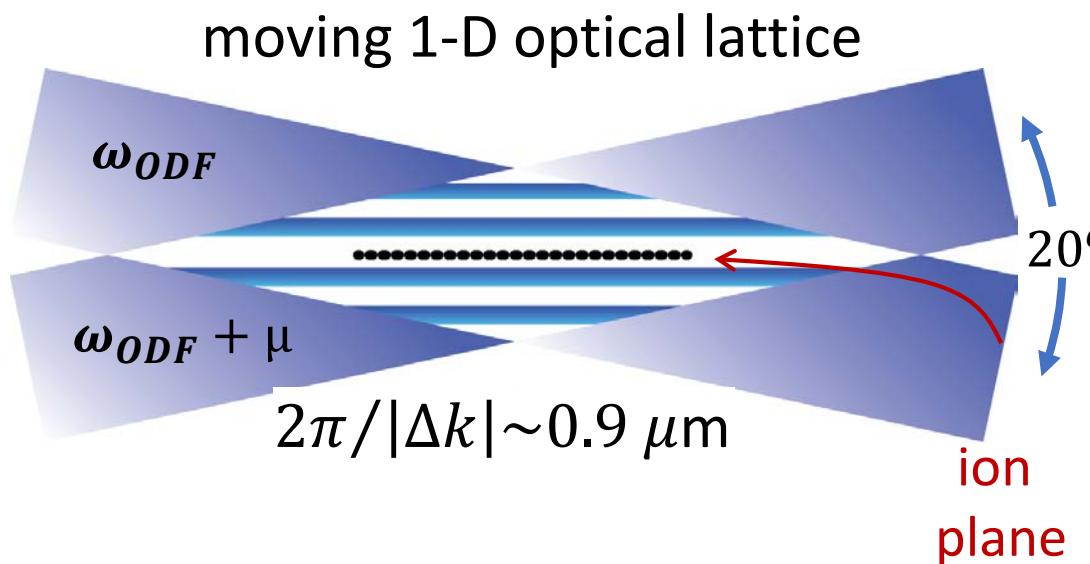
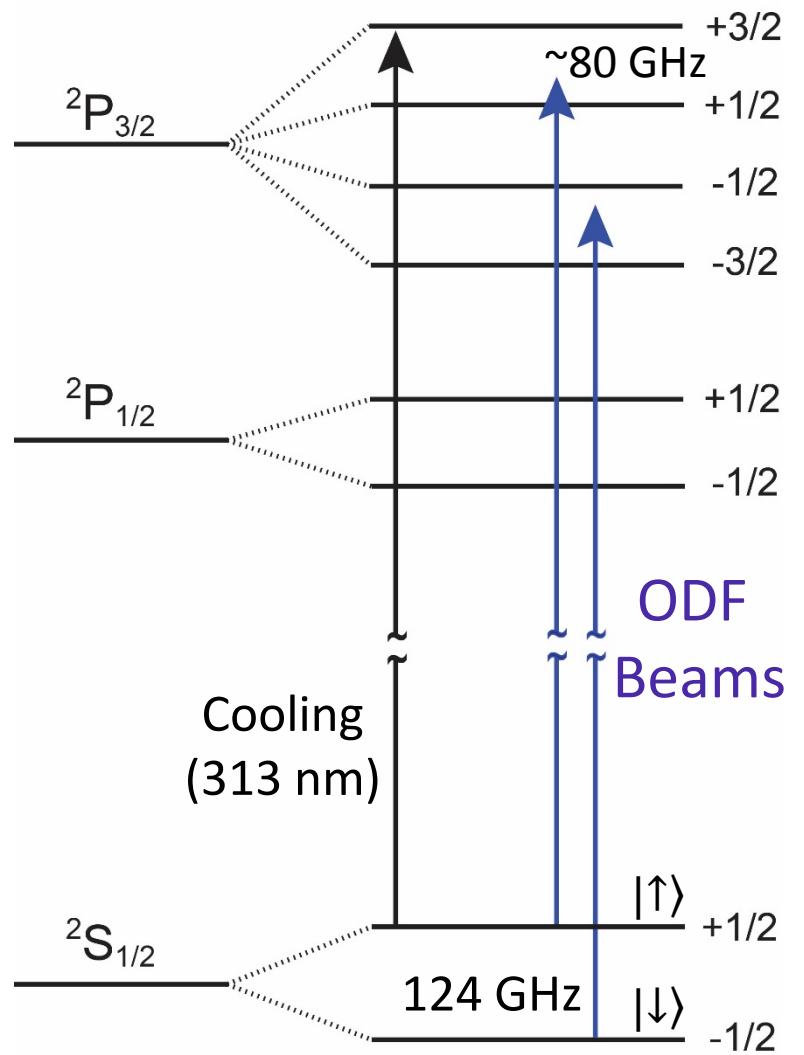
- engineering tunable Ising dynamics

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

- benchmark quantum dynamics, entanglement
 - spin squeezing
 - out-of-time correlations (OTOC)



Engineering quantum magnetic couplings with spin-dependent forces



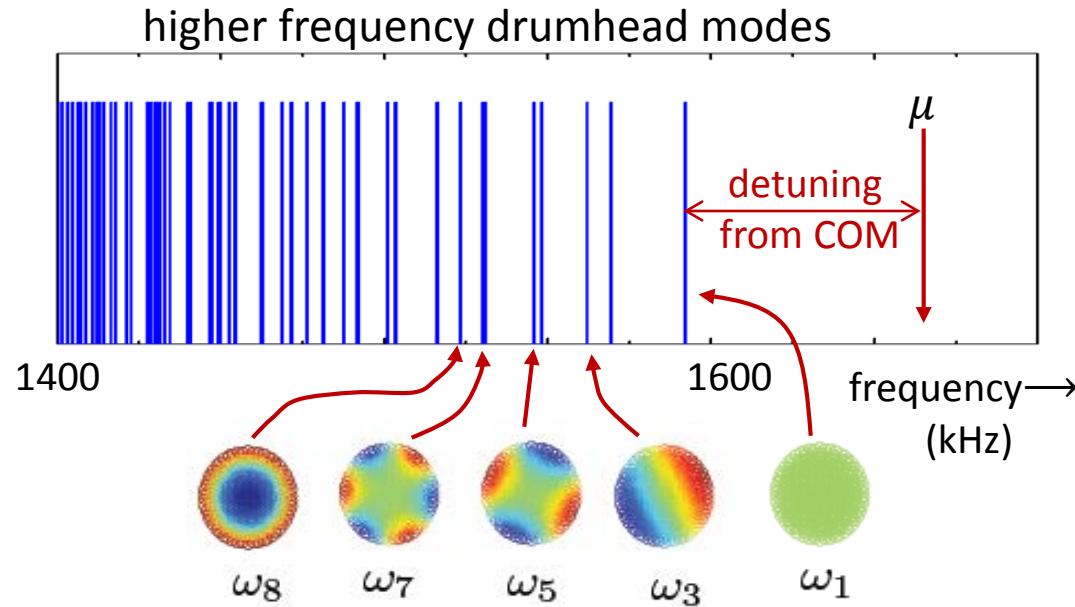
- $F_\uparrow(t) = -F_\downarrow(t)$
 $F_\uparrow(t) = F_0 \cos(\mu t)$
- alignment of 1D lattice and ion plane

Leibfried et al., Nature **422**, (2003) - quantum gates through spin-dependent forces
Sorensen and Molmer, PRL (1999) with small numbers of ion in rf traps

Engineering quantum magnetic couplings

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^N \hat{z}_j \cdot \hat{\sigma}_j^z = \\ \sum_{m=1}^N b_{jm} \sqrt{\frac{\hbar}{2M\omega_m}} (\hat{a}_m^\dagger e^{i\omega_m t} + \hat{a}_m e^{-i\omega_m t})$$

N drumhead eigenvalues ω_m and
eigenvector \vec{b}_m



$$\hat{U}_{ODF} = \hat{U}_{SP}(t) \cdot \hat{U}_{SS}(t)$$

Produces spin-phonon coupling

- useful metrology tool
- source of decoherence

dominant for large $|\mu - \omega_m|$

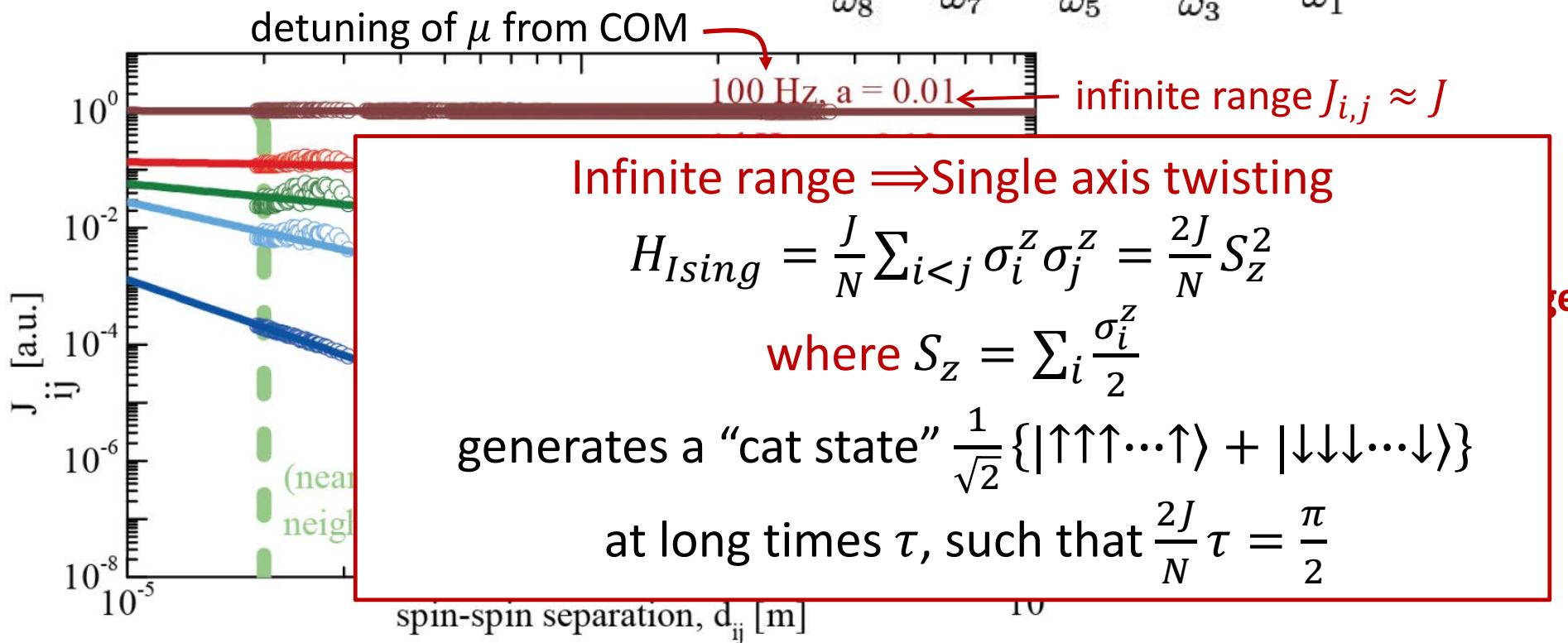
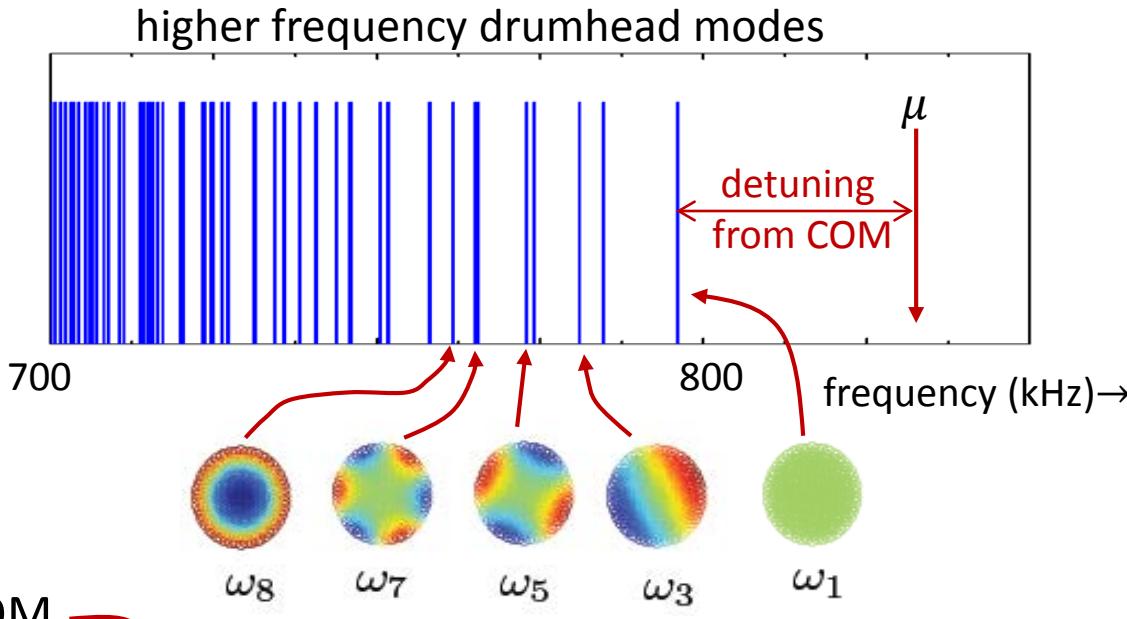
$$\hat{U}_{SS}(t) = \exp \left[-i \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z \right]$$

$$J_{i,j} \approx \frac{F_0^2 N}{2\hbar M} \sum_m \frac{b_{im} b_{jm}}{\mu^2 - \omega_m^2}$$

Ising coupling coefficients determined by transverse modes

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z,$$

$$J_{i,j} = \frac{F_0^2 N}{\hbar \cdot 2M} \sum_{m=1}^N \frac{b_{i,m} b_{j,m}}{\mu^2 - \omega_m^2}$$



Mean field dynamics: Simple spin precession

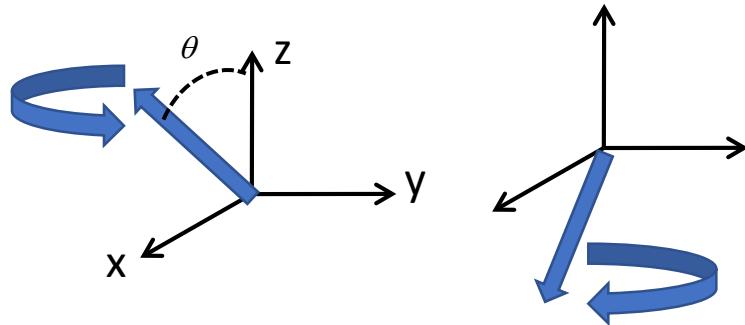
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mean field

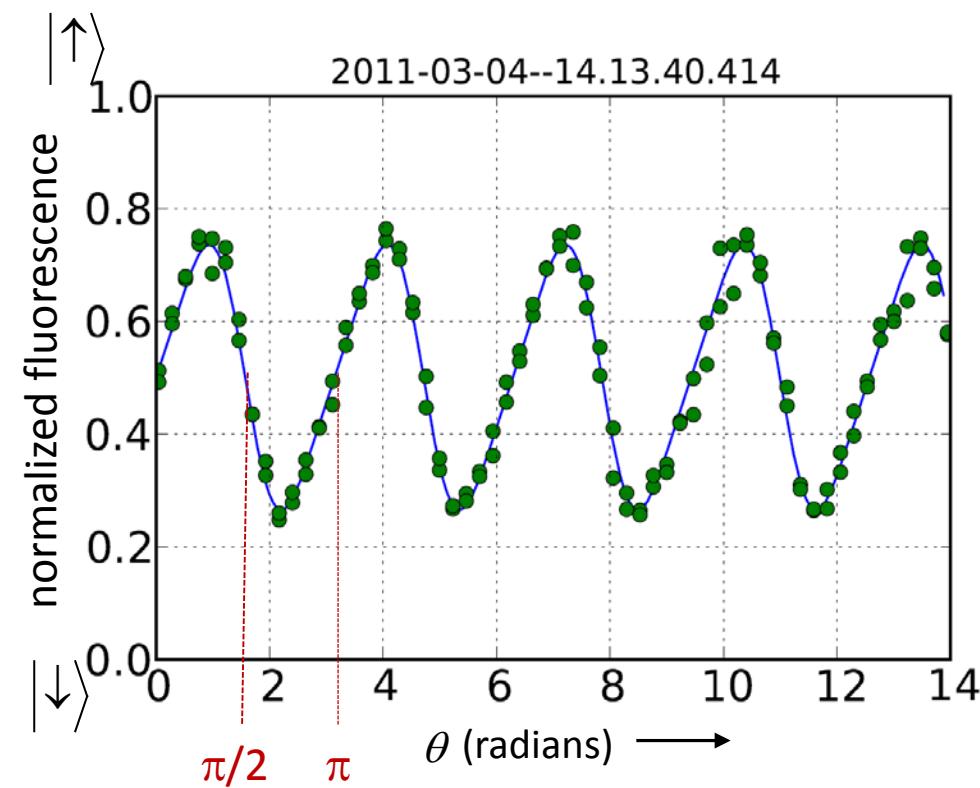
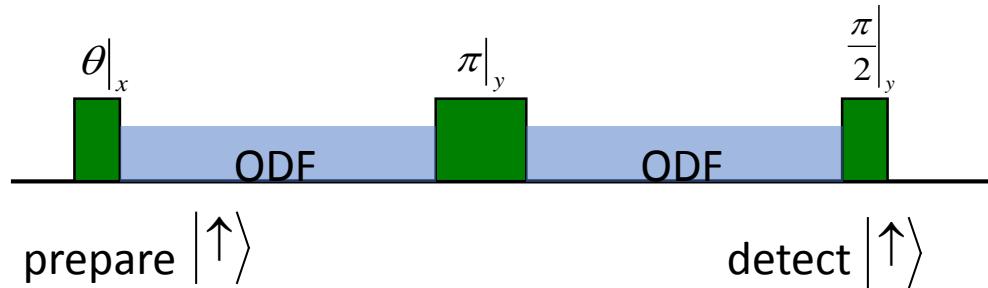
limit

$$H_{\text{Ising}}^{\text{MF}} = \sum_j \bar{B}_j \sigma_j^z / 2,$$

$$\bar{B}_j = \frac{2}{N} \sum_{i \neq j} J_{i,j} \langle \sigma_i^z \rangle = \frac{2}{N} \left(\sum_{i \neq j} J_{i,j} \right) \cos(\theta)$$



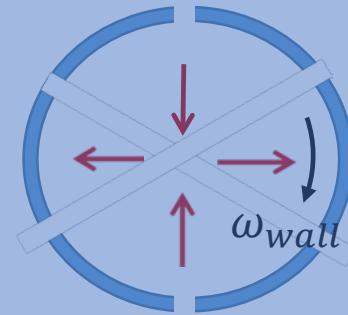
Measure $\bar{J} \equiv \frac{1}{N^2} \sum_{i \neq j} J_{i,j}$ with spin-echo



$$\bar{J}/(\hbar 2\pi) \approx 2 - 4 \text{ kHz}$$

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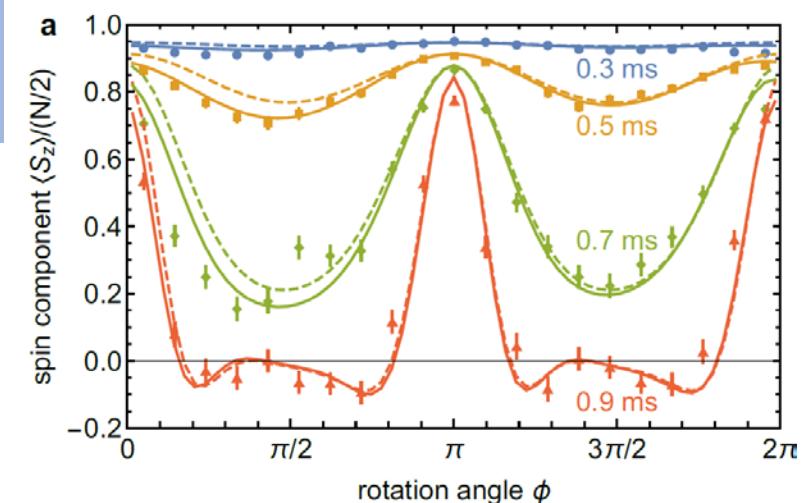


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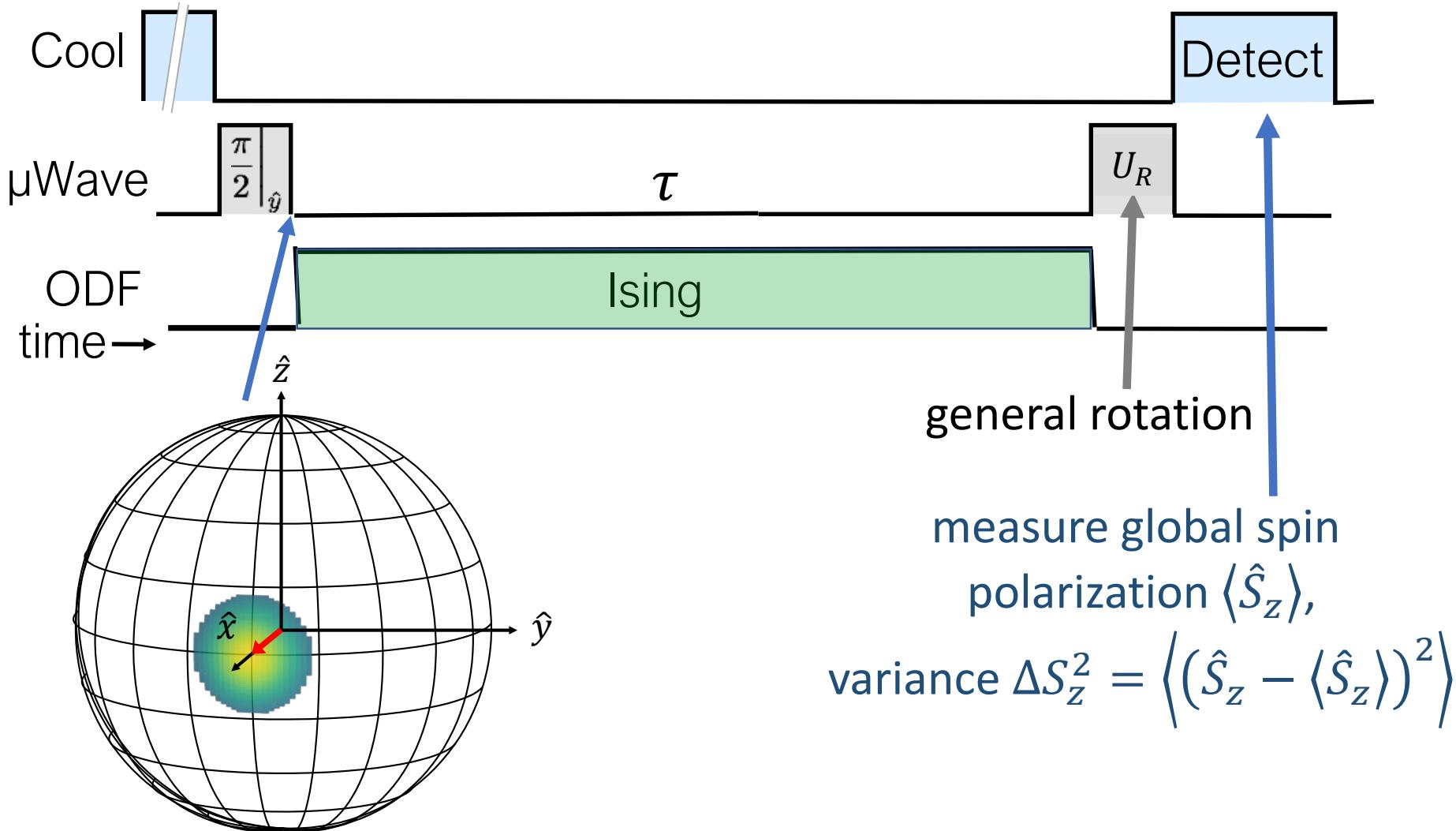
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 - out-of-time correlations (OTOC)



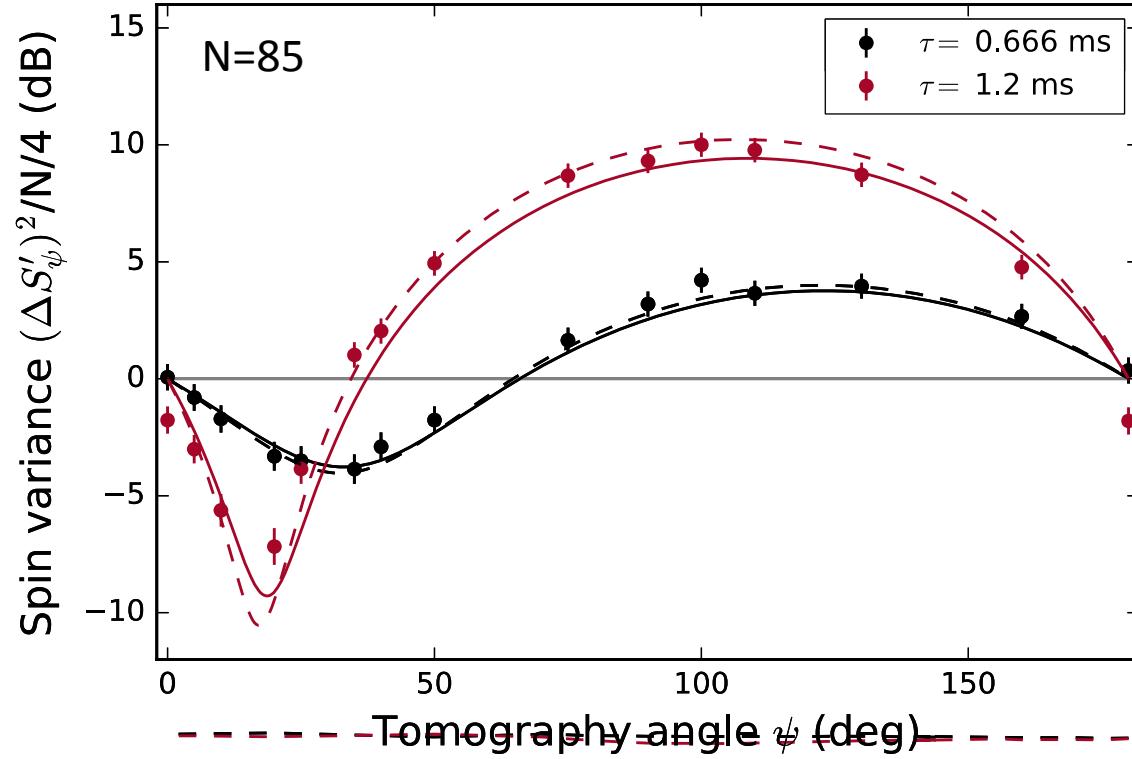
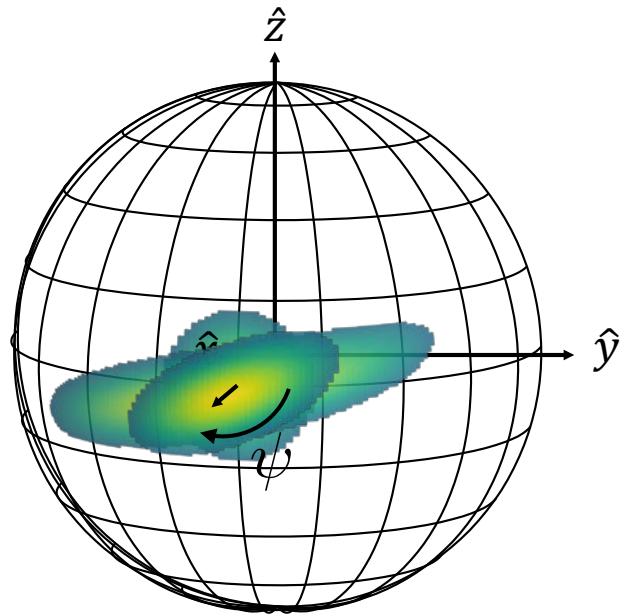
Benchmarking quantum dynamics

- employ infinite range interactions $H_{Ising} \approx \frac{2J}{N} S_z^2, S_z \equiv \sum_i \sigma_i^z / 2$
- prepare eigenstate of $H_{\perp} = \sum_i B_{\perp} \hat{\sigma}_i^x$, turn on H_{Ising}



Benchmarking quantum dynamics

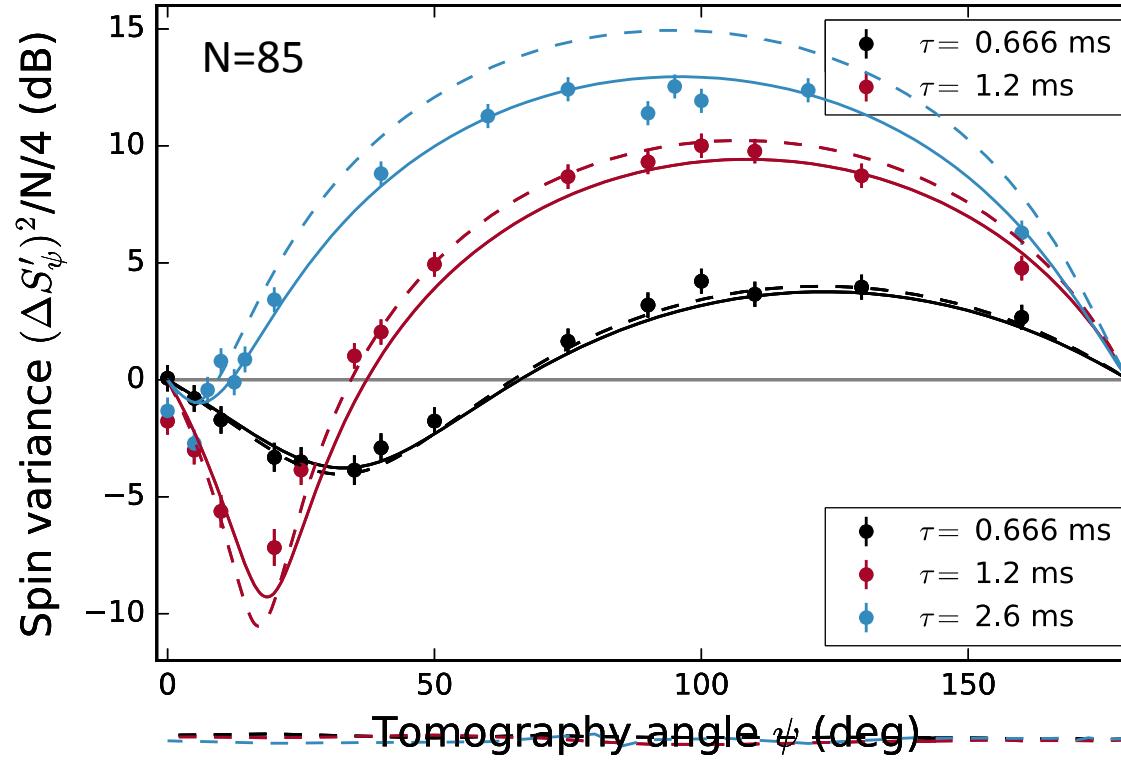
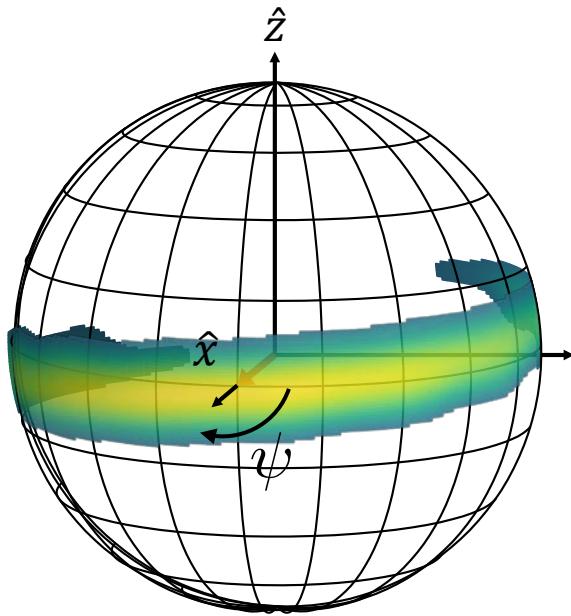
Bohnet *et al.*, *Science* 352, 1297 (2016)



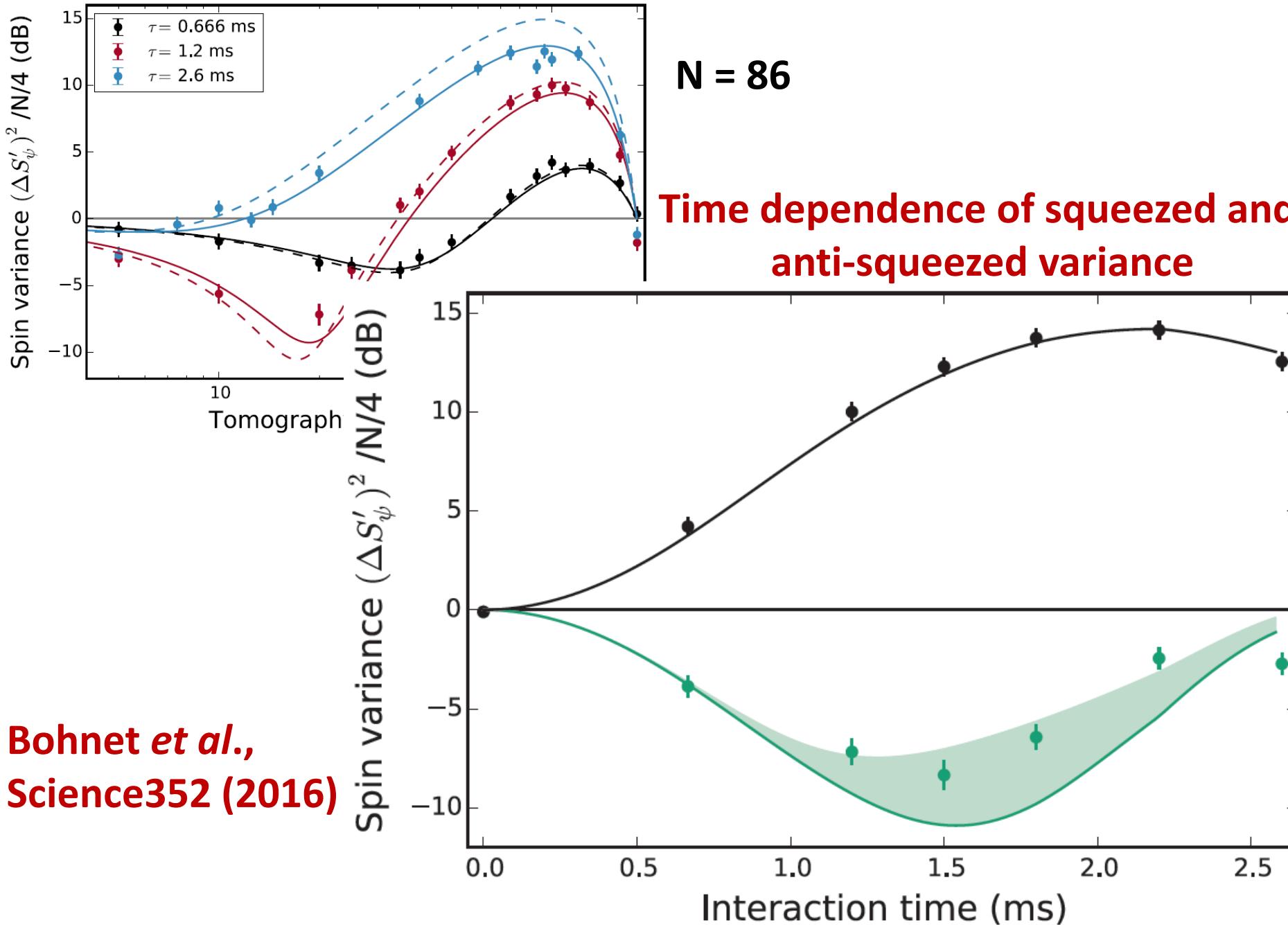
- Measurements of Ramsey squeezing parameter \Rightarrow
prove entanglement for $25 < N < 220$
- Largest inferred squeezing: -6.0 dB

Benchmarking quantum dynamics

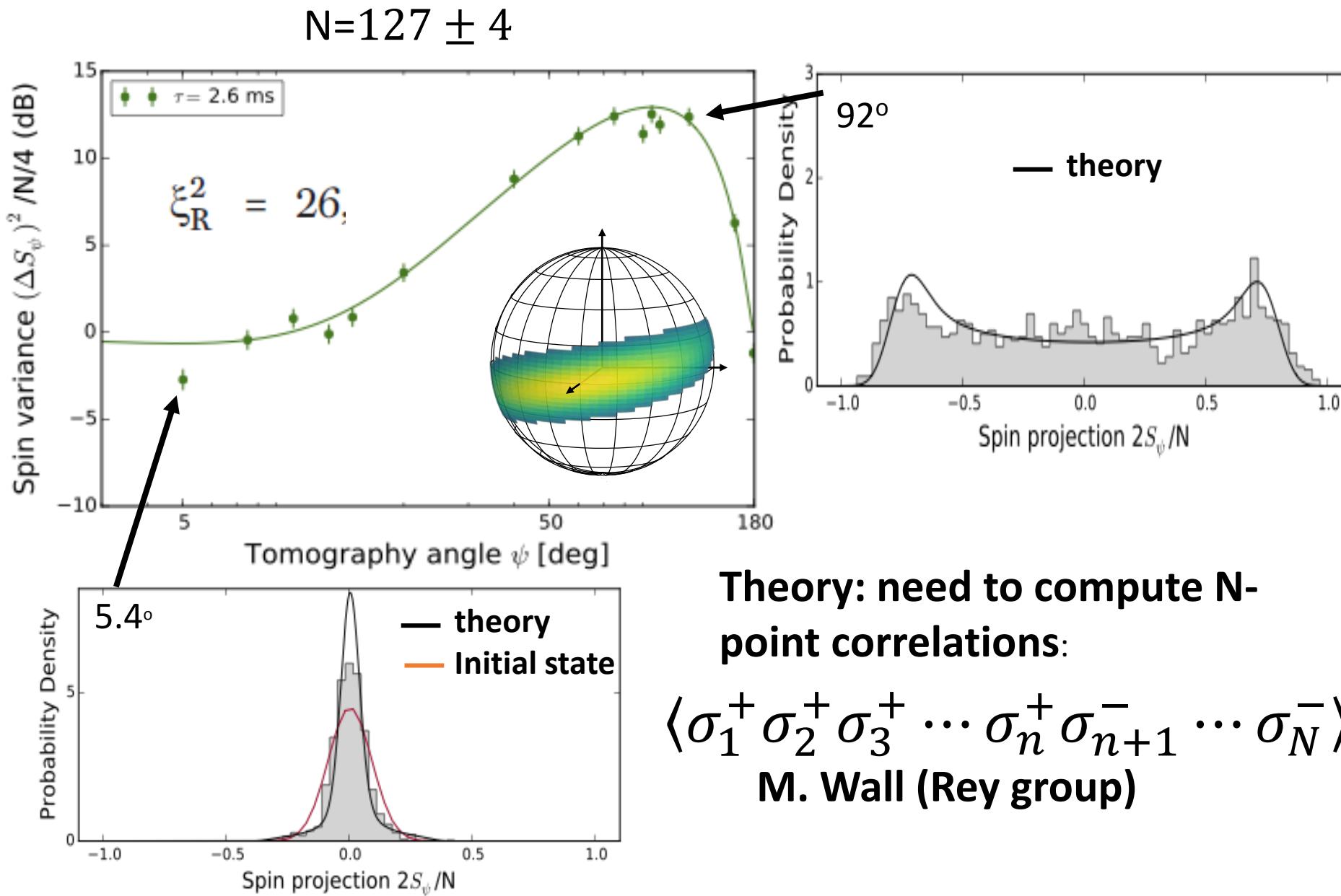
Bohnet *et al.*, *Science* 352, 1297 (2016)



Benchmarking quantum dynamics



Over-squeezed states: benchmark via full counting statistics



Out-of-time-order correlation functions

$$F(t) \equiv \langle \psi | W(t)^\dagger V^\dagger W(t) V | \psi \rangle \text{ where } W(t) = e^{iHt} W(0) e^{-iHt}, \\ [V, W(0)] = 0$$

$$\text{Re}[F(t)] = 1 - \langle |[W(t), V]|^2 \rangle / 2$$

⇒ **measures failure of initially commuting operators to commute at later times**

⇒ **quantifies spread or scrambling of quantum information across a system's degrees of freedom**

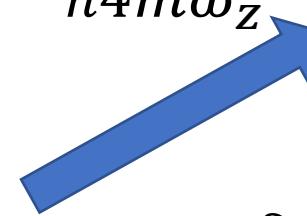
Swingle et al., arXiv:1602.06271; Shenker et al., arXiv:1306.0622; Kitaev (2014)

**Difficult to measure \Leftrightarrow possible with time-reversal of dynamics
time reversal is possible in many quantum simulators!**

Time reversal of the Ising dynamics

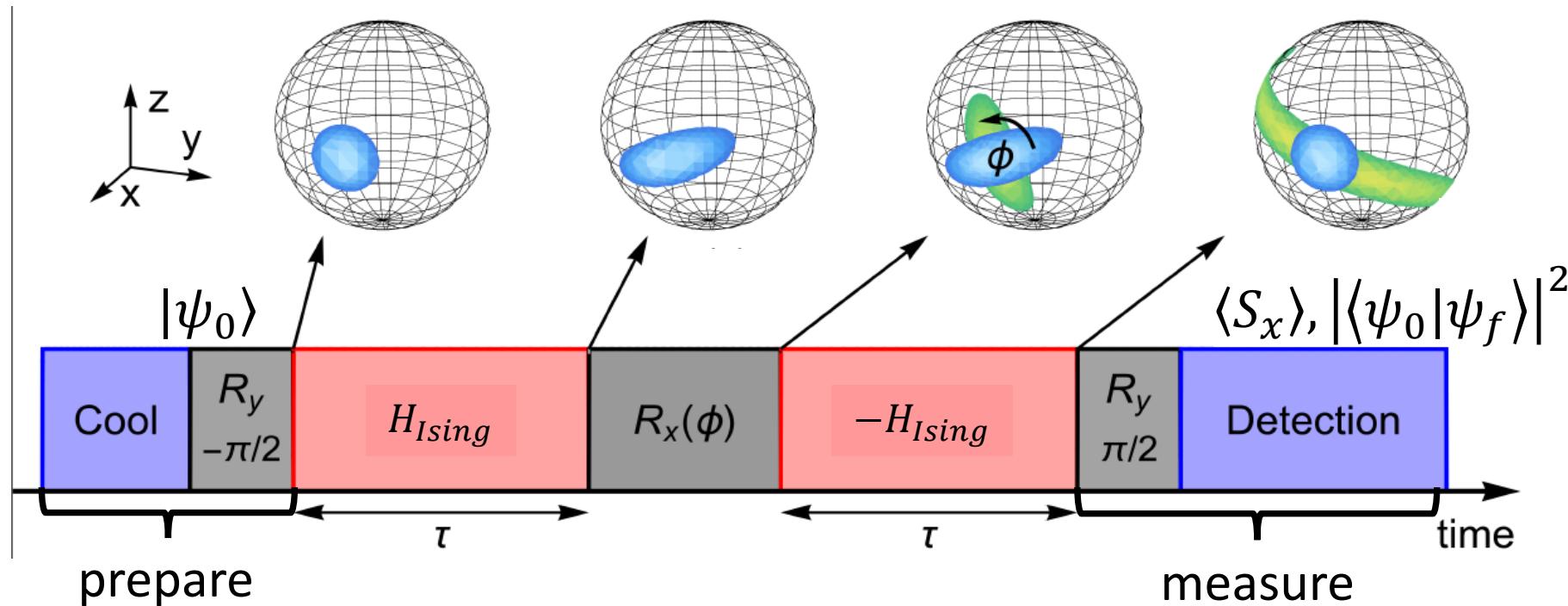
$$H_{Ising} = \frac{J}{N} \sum_{i < j} \hat{\sigma}_i^z \hat{\sigma}_j^z, \quad \frac{J}{N} \cong \frac{F_0^2}{\hbar 4m\omega_z} \cdot \frac{1}{\mu - \omega_z}$$

Change $\mu = \omega_z + \delta$ (antiferromagnetic)
to $\mu = \omega_z - \delta$ (ferromagnetic)

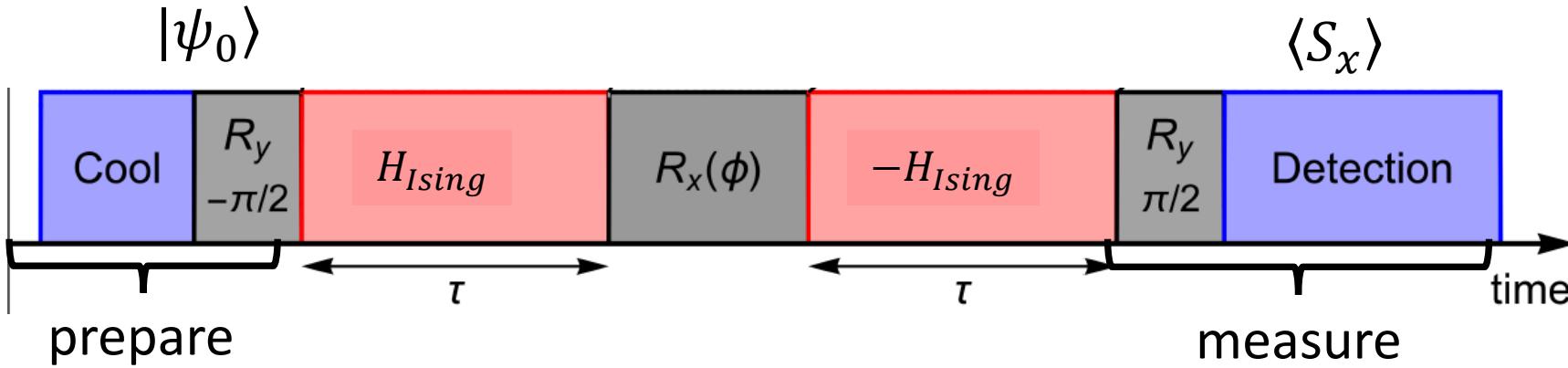


Multiple quantum coherence protocol

- Probe higher-order coherences and correlations (Pines group, 1985)



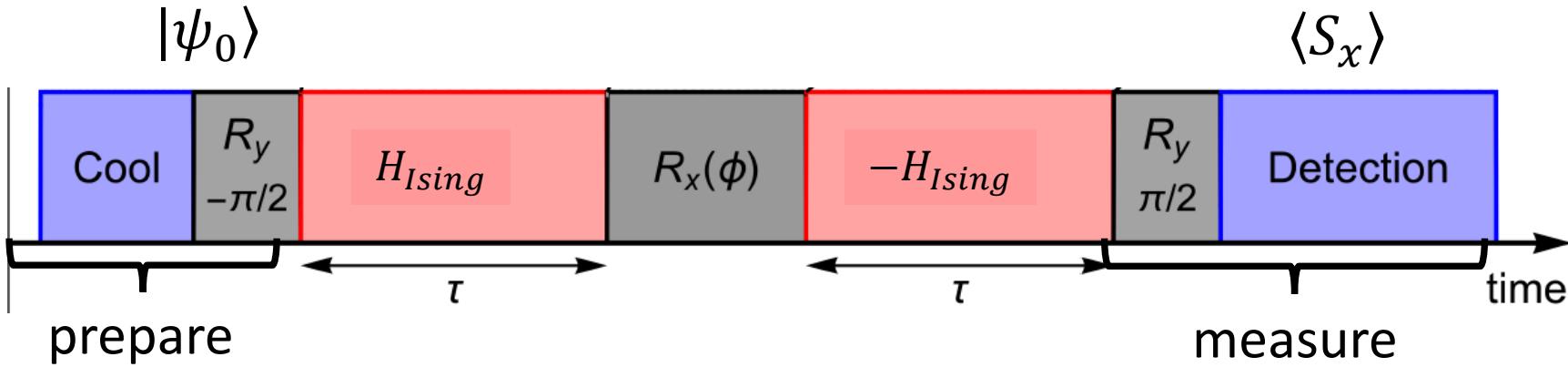
Multiple quantum coherence protocol



$$\begin{aligned}\langle S_x \rangle &= \langle \Psi_0 | e^{iH_{Ising}\tau} e^{i\phi S_x} e^{-iH_{Ising}\tau} S_x e^{iH_{Ising}\tau} e^{-i\phi S_x} e^{-iH_{Ising}\tau} |\Psi_0 \rangle \\ &= \frac{2}{N} \langle \Psi_0 | \underbrace{e^{iH_{Ising}\tau} W^\dagger}_{W^\dagger(t)} \underbrace{e^{-iH_{Ising}\tau} V^\dagger}_{V^\dagger(0)} \underbrace{e^{iH_{Ising}\tau} W}_{W(t)} \underbrace{e^{-iH_{Ising}\tau} V}_{V(0)} |\Psi_0 \rangle\end{aligned}$$

Out-of-time-order correlation (OTOC) function
⇒ quantifies spread or scrambling of quantum information across a system's degrees of freedom

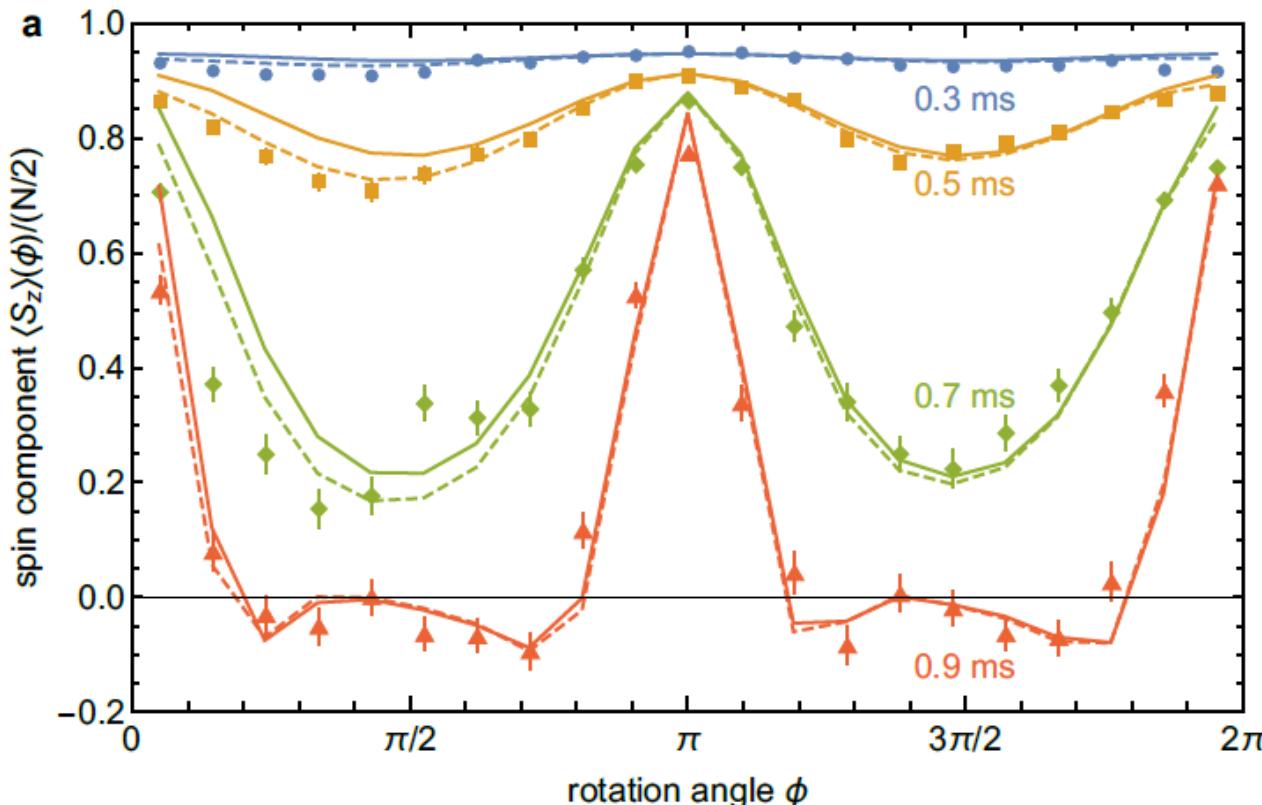
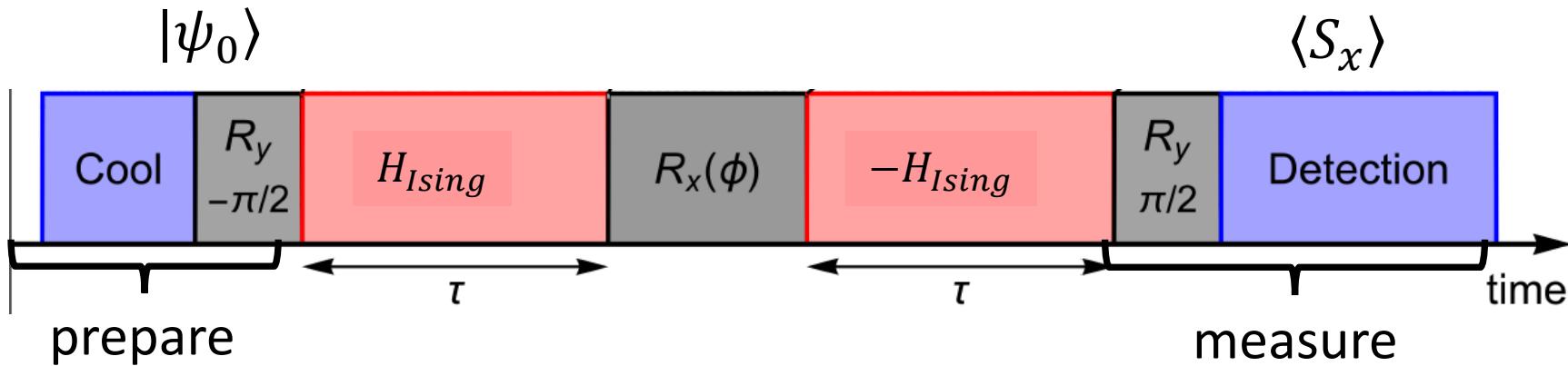
Multiple quantum coherence protocol



$$\begin{aligned}\langle S_x \rangle &= \langle \Psi_0 | e^{iH_{Ising}\tau} e^{i\phi S_x} e^{-iH_{Ising}\tau} S_x e^{iH_{Ising}\tau} e^{-i\phi S_x} e^{-iH_{Ising}\tau} |\Psi_0 \rangle \\ &= \sum_m \langle \Psi | C_m | \Psi \rangle e^{i\phi m} \quad C_m = \underbrace{\sigma_1^z \sigma_4^y \dots \sigma_k^z}_{\text{At least } m \text{ terms}} \equiv |\Psi\rangle\end{aligned}$$

m^{th} order Fourier coefficient $\langle \Psi | C_m | \Psi \rangle$ indicates
 $|\Psi\rangle$ has correlations of at least order m

MQC protocol – $\langle S_x \rangle$ measurement



$$H_{Ising} = J/N \sum_{i < j} \sigma_i^z \sigma_j^z$$

$$J \lesssim 5 \text{ kHz}$$

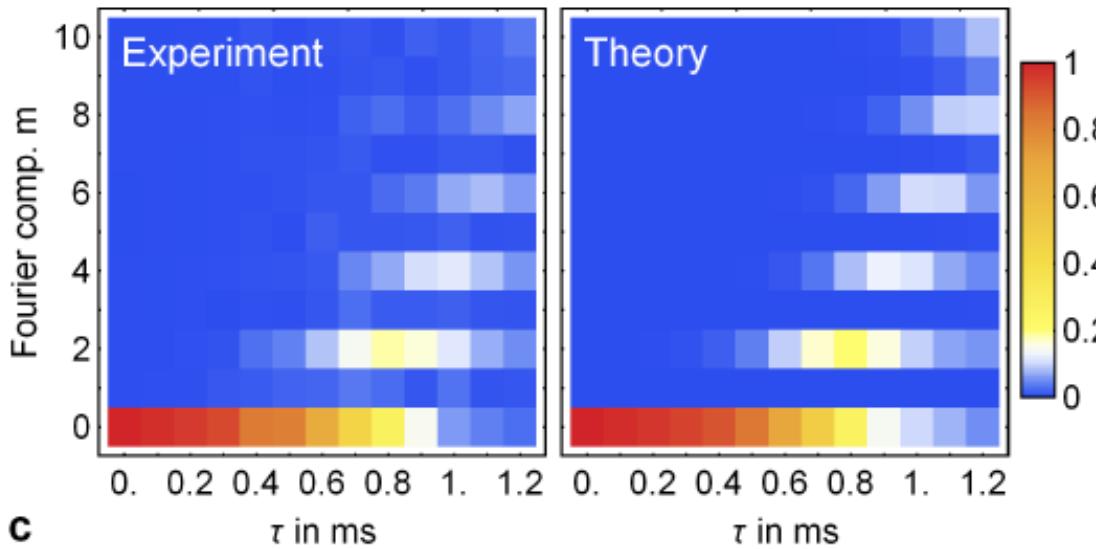
$$N = 111$$

$$\Gamma = 93 \text{ Hz}$$

[Gärttner, Bohnet et al.
Nature Physics 2017]

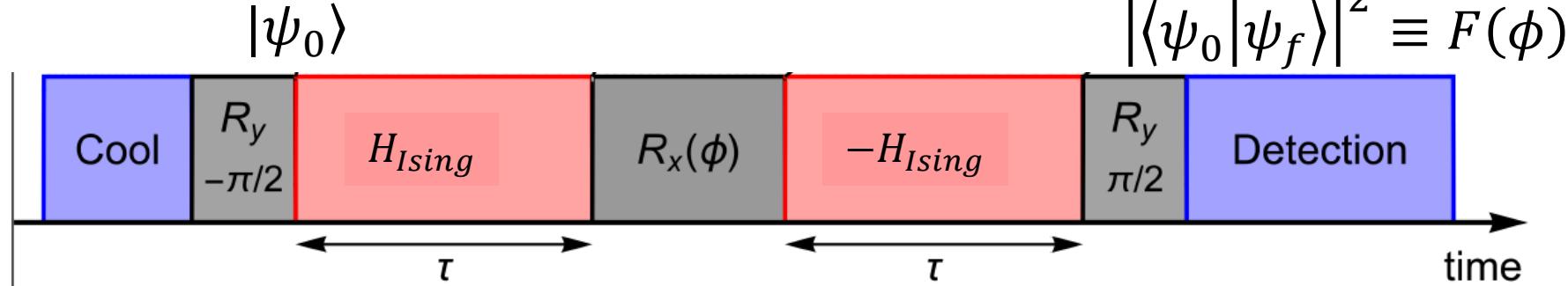
Fourier transform of magnetization

[Gärttner, Bohnet et al. Nature Physics 2017]

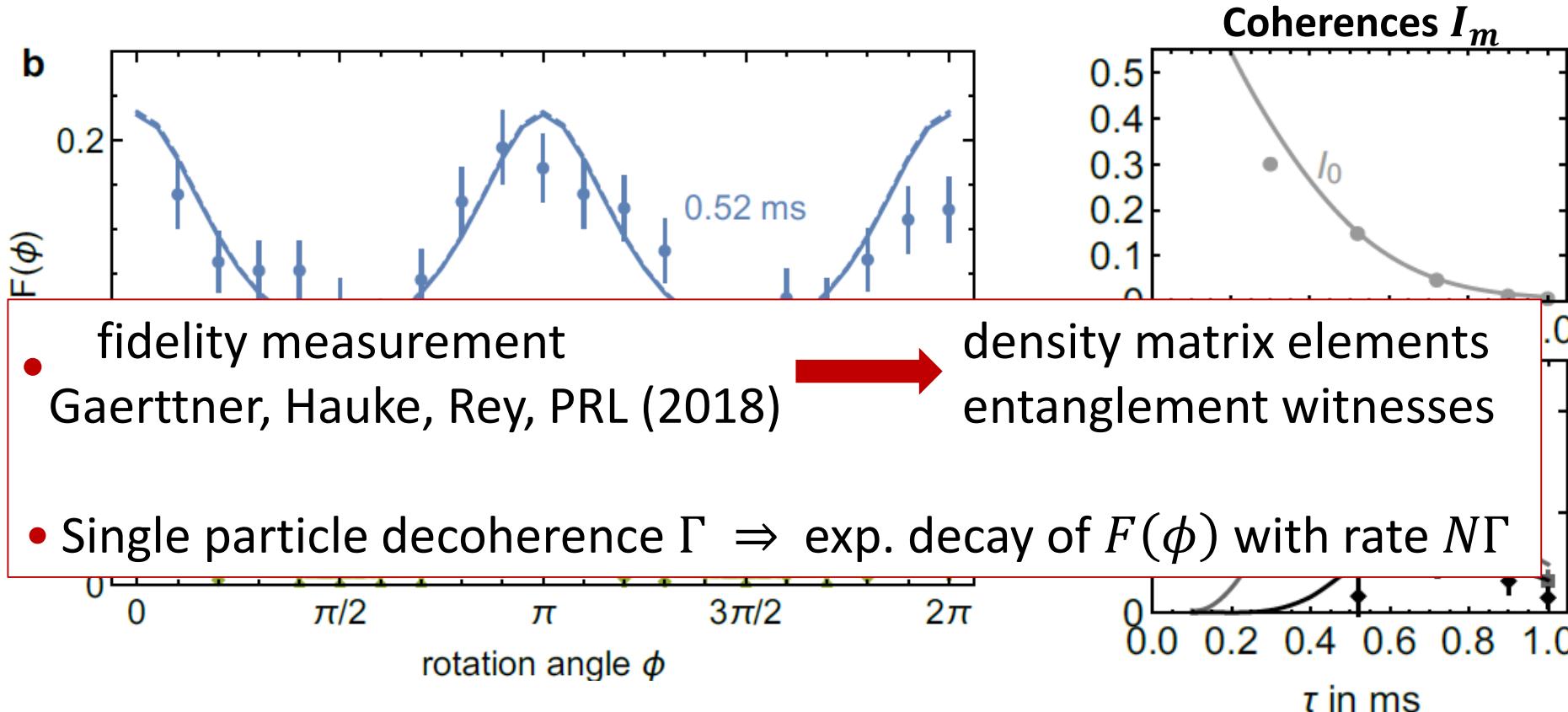


- Measure build-up of 8-body correlations
- Only global spin measurement
- Illustrates how OTOCs measure spread of quantum information

MQC protocol – $|\langle \psi_0 | \psi_f \rangle|^2$ fidelity measurement



- Measure $|\langle \psi | \downarrow \dots \downarrow \rangle|^2$ through photon count histograms



Trapped ion quantum simulation

2. Quantum simulation with 1d ion crystals in linear rf traps

- nice examples of non-equilibrium quantum simulations

- propagation of correlations in 1d crystals with long range interactions

"Non-local propagation of correlations in quantum systems with long-range interactions,"

P. Richerme, ..C. Monroe, Nature 511, 198 (2014).

"Quasiparticle engineering and entanglement propagation in a quantum many-body system"

P. Jurcevic, .., R. Blatt, C. F. Roos Nature 511, 202 (2014),

- dynamical phase transitions

"Direct observation of dynamical quantum phase transitions in an interacting many-body system", P. Jurcevic, .., R. Blatt, C. F. Roos, Phys. Rev. Lett. 119, 080501 (2017).

"Observation of a Many-Body Dynamical Phase Transition in a 53-Qubit Quantum Simulator,"
J. Zhang, .., C. Monroe, Nature 551, 601 (2017).

- non-equilibrium phases

"Observation of a Discrete Time Crystal," J. Zhang, P.W. Hess, A. Kyprianidis, P. Becker, Lee, J. Smith, G. Pagano, I.-D. Potirniche, A.C. Potter, A. Vishwanath, N.Y. Yao, C. Monroe, Nature 543, 217 (2017)

Observation of a discrete time crystal – Monroe group

Crystals in space are an example of the breaking of spatial translation symmetry

Could there be states of matter that break translation symmetry in time? (Wilczek, PRL 2012)

Forbidden for ground states and thermal equilibrium! (Bruno PRL (2013); Watanabe & Oshikawa PRL (2015))

What about non-equilibrium systems?

Yes for periodically driven Floquet systems
↔ possess a discrete time translation symmetry
↔ broken through a subharmonic response
↔ called a Discrete Time Crystal (DTC)

Properties of a Discrete Time Crystal:

1. Subharmonic oscillation stabilized by many-body interactions
2. Robust against perturbations (rigidity)
3. Infinite autocorrelation time

Observation of a discrete time crystal – Monroe group

10 $^{171}\text{Yb}^+$ ions in a linear rf trap
– 12.6 GHz hyperfine qubit

Floquet Hamiltonian:

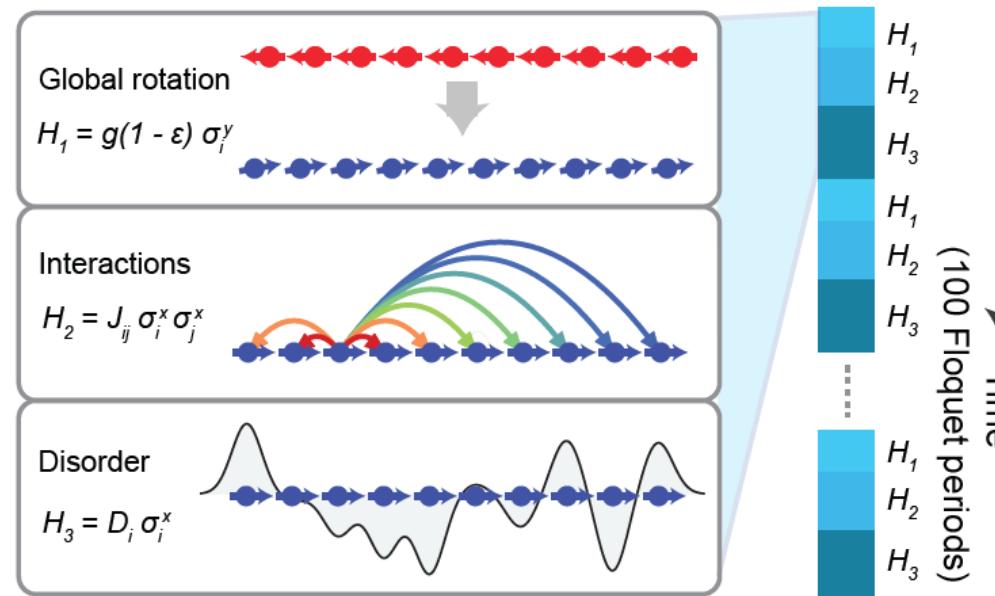
$$H = \begin{cases} H_1 = g(1 - \varepsilon) \sum_i \sigma_i^y, & \text{time } t_1 \\ H_2 = \sum_i J_{ij} \sigma_i^x \sigma_j^x, & \text{time } t_2 \\ H_3 = \sum_i D_i \sigma_i^x & \text{time } t_3 \end{cases}$$

Optically driven Raman transitions;
approx. π -pulse

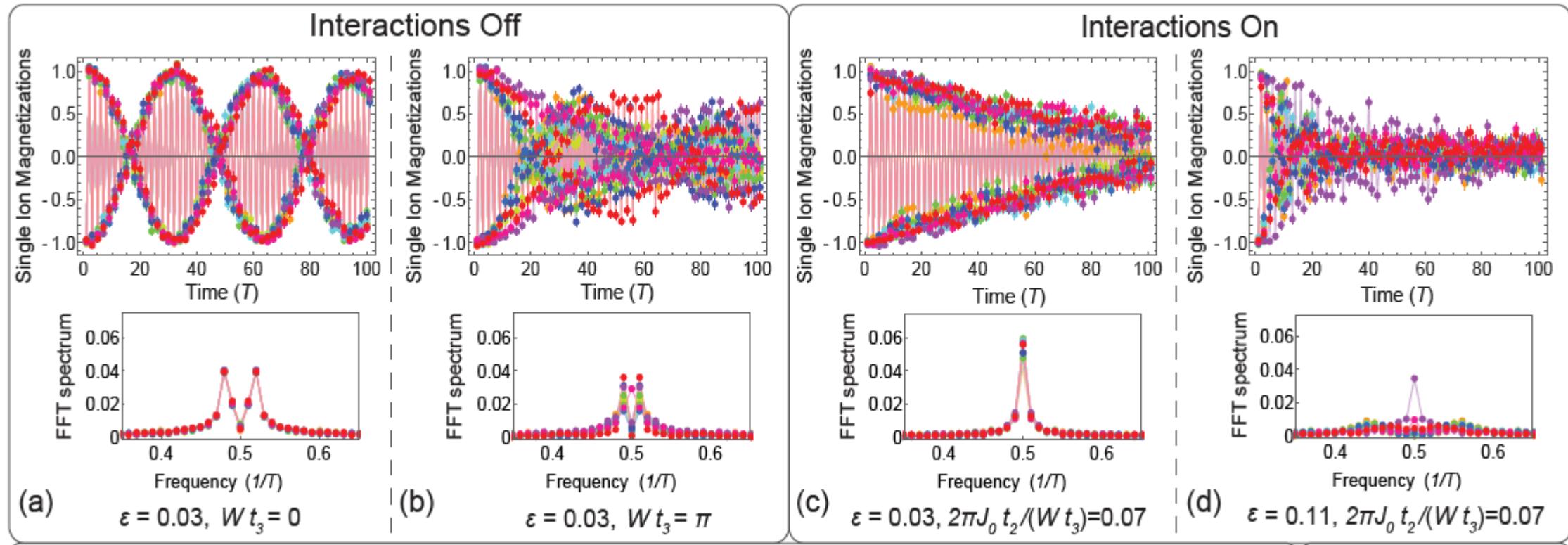
Globally applied MS interaction

Programmable disorder from
individual AC Stark shifts

Overall period
 $T = t_1 + t_2 + t_3$ repeated



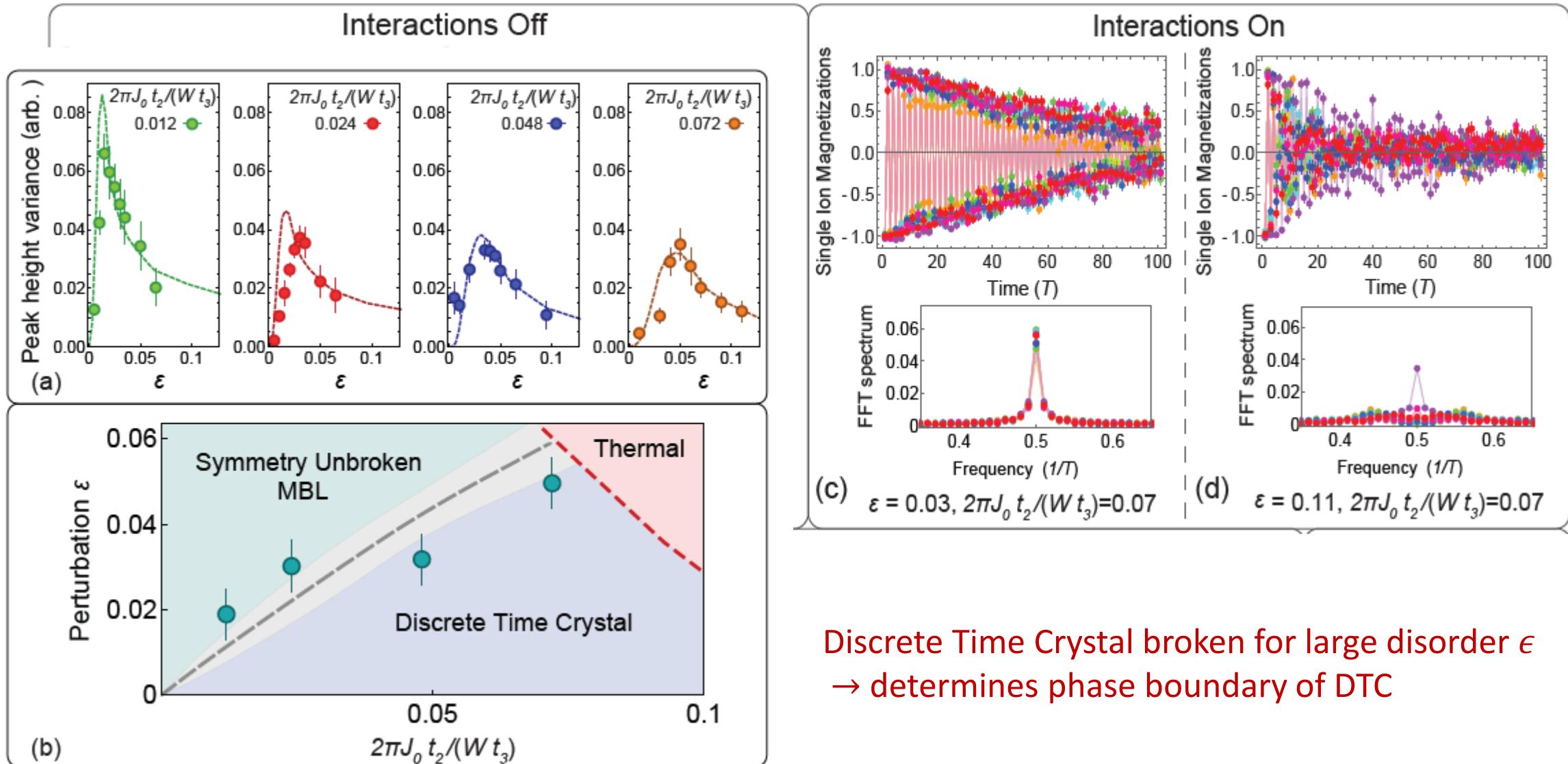
Observation of a discrete time crystal – Monroe group



$$T = t_1 + t_2 + t_3$$

$$H = \begin{cases} H_1 = g(1 - \varepsilon) \sum_i \sigma_i^y, & \text{time } t_1 \\ H_2 = \sum_i J_{ij} \sigma_i^x \sigma_j^x, & \text{time } t_2 \\ H_3 = \sum_i D_i \sigma_i^x & \text{time } t_3. \end{cases}$$

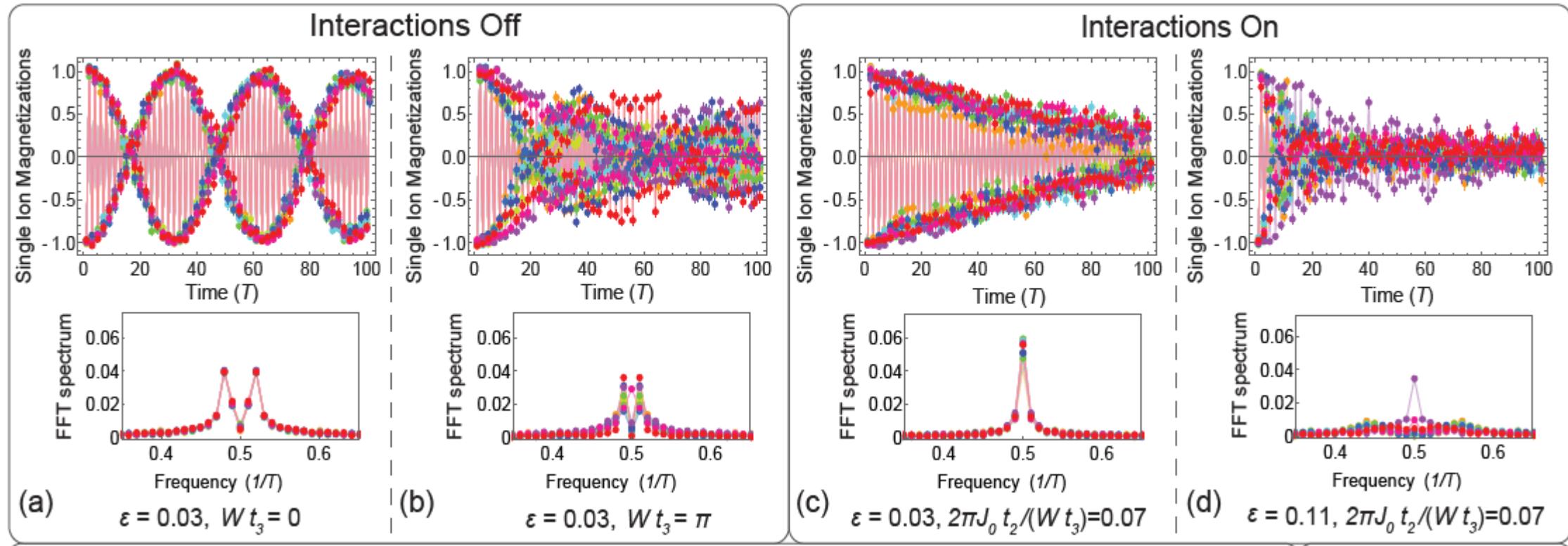
Observation of a discrete time crystal – Monroe group



Discrete Time Crystal broken for large disorder ϵ
 \rightarrow determines phase boundary of DTC

$$\mathcal{H}_3 = \sum_i \nu_i \sigma_i^- \text{ time } t_3.$$

Observation of a discrete time crystal – Monroe group



Properties of a Discrete Time Crystal:

1. Subharmonic oscillation stabilized by many-body interactions
2. Robust against perturbations (rigidity)
3. Infinite autocorrelation time ?

Desirable to increase the number of qubits !

Trapped ion quantum computing, simulation, and sensing

John Bollinger, NIST, Boulder CO

Monday, July 2, 11:00 AM – Trapped ion quantum computing

Tuesday, July 3, 11:00 AM – Trapped ion quantum simulation

Thursday, July 5, 9:00 AM – Trapped ion quantum sensing