July 4-29, 2005: Physics of Mesocopic Systems

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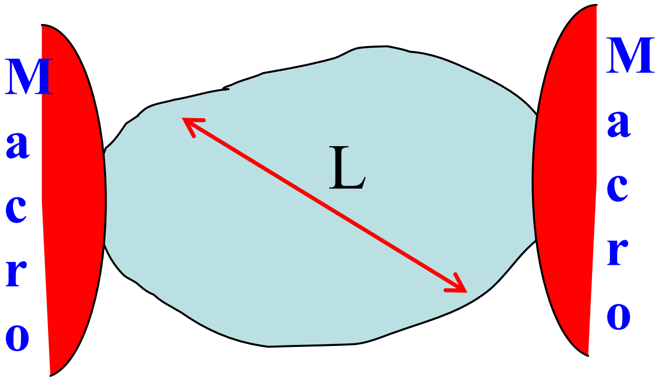
Transport theory of mesoscopic systems with applications to disordered and chaotic systems

Lecture 1; 7/6/05

- What is a mesoscopic system?

A system where certain size-dependent energy scales are greater than thermal energy scales, which are still much larger than 1-particle level spacings



- Not an absolute size, T-dependent size scale (1 μm typical)
- Not related to discreteness of energy levels (micro-system)

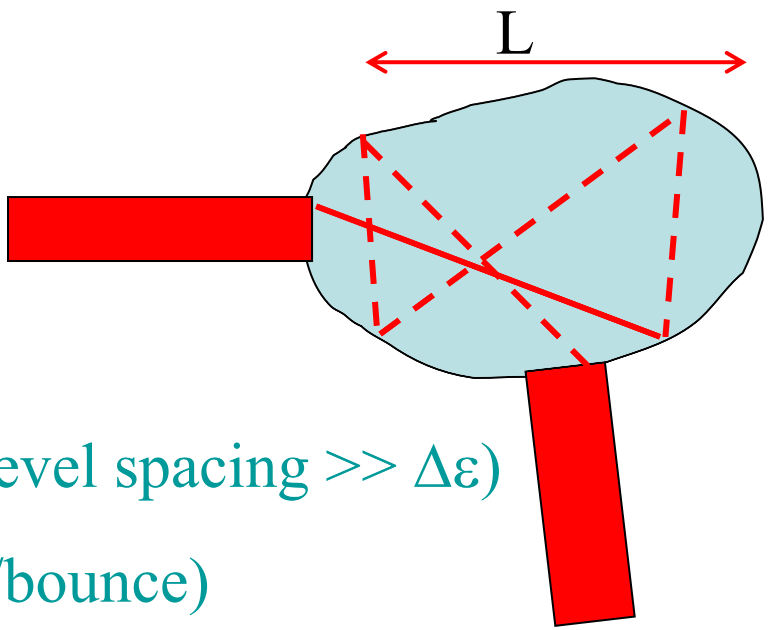
Quantum dot can be exception

- Second meaning: engineered micro and nanostructures

New regimes available

- Thermal energy/time scales?

k_BT, $h/\tau_\phi \propto (k_BT)^\alpha$



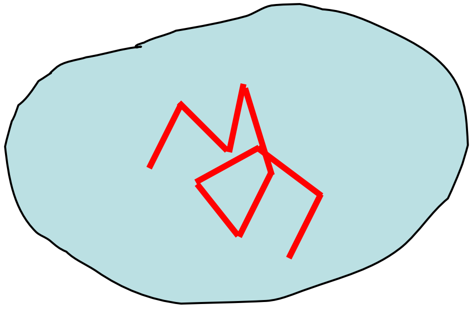
Transport energy/time scales:

Ballistic limit:

- $\tau_{erg} = L/v_f$ ($E_{erg} = hv_f/L$ = 1d level spacing $>> \Delta\varepsilon$)
- $\tau_{esc} = L/(v_f\, p_1)$ (p_1 = escape prob/bounce)

open system: $p_1 > 1/(k_fL)^{d-1} \approx N_c \Rightarrow E_{esc} \geq \Delta\varepsilon$

Diffusive limit: $l < L$

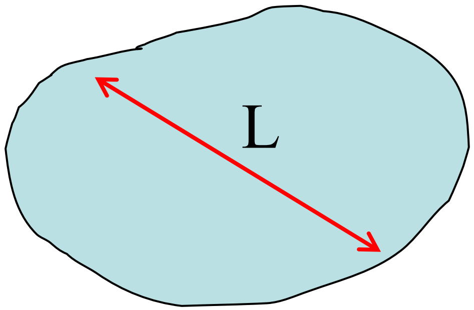


- $\tau_{erg} = L^2/D \Rightarrow E_{Th} = hD/L^2$ (Thouless energy)
- $\tau_{esc} = L^2/Dp_1 \Rightarrow E_{esc} = p_1E_{Th}$
  (often assume $p_1 \approx 1$, $E_{esc} \approx E_{th}$)

$g = E_{erg}/\Delta\varepsilon$ = dimensionless "conductance" - only the true conductance when $E_{esc} \approx E_{erg}$

Interaction energy/time scale

- Charging energy, $E_c = e^2/C$, $\tau_c = R_Q C$
  $C \propto 1/L^2$

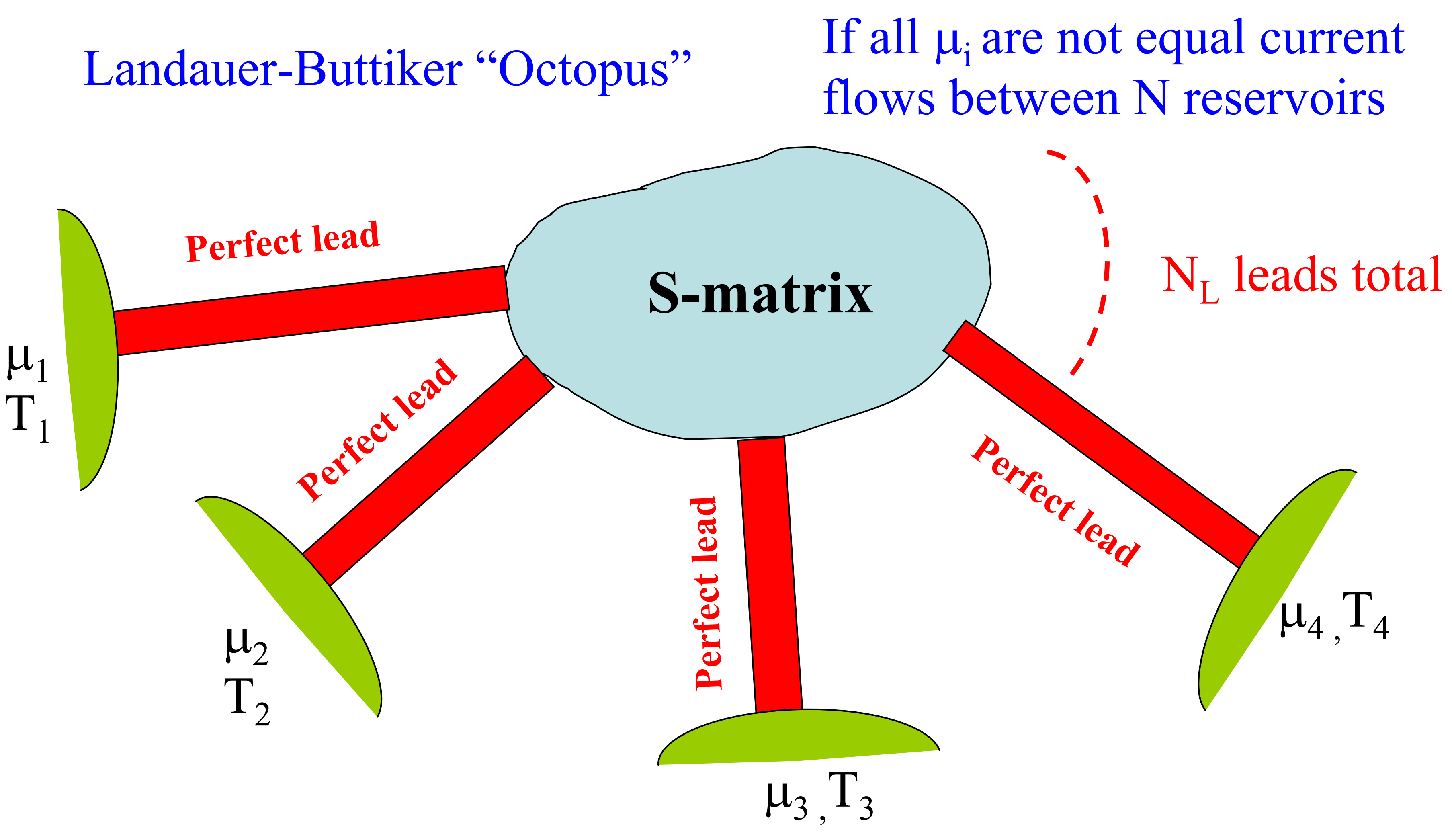


When E_{erg}, $E_c > kT$, h/τ_ϕ => characteristic mesoscopic phenomena:

Coherence, Fluctuations and Charging Effects

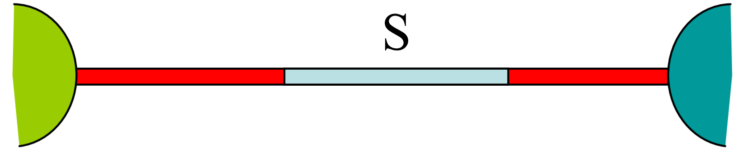
Will focus on coherence and fluctuations

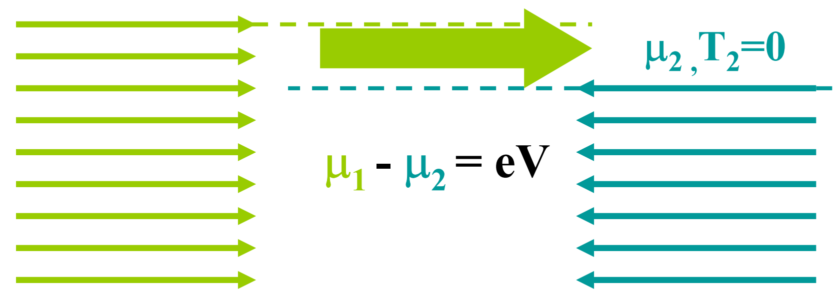
The Mesoscopic Fermi Gas



Mesoscopic system connected by perfect leads to phase-randomizing, thermal equilibrium non-interacting fermion reservoirs at μ_1, T_1, μ_2, T_2…

Landauer counting argument (1d):
(two reservoirs)





$$I = I_1 - I_2 = e\Delta N_1 v_f T$$
$$= e(\frac{dN_1^+}{d\epsilon} eV) v_f T$$
$$= e(\frac{1}{2\pi}\frac{dk}{d\epsilon} eV)\frac{1}{\hbar}\frac{d\epsilon}{dk} T \quad = (\frac{e^2}{h} T) V$$

$$G = \frac{e^2}{h} T(\epsilon_f)$$

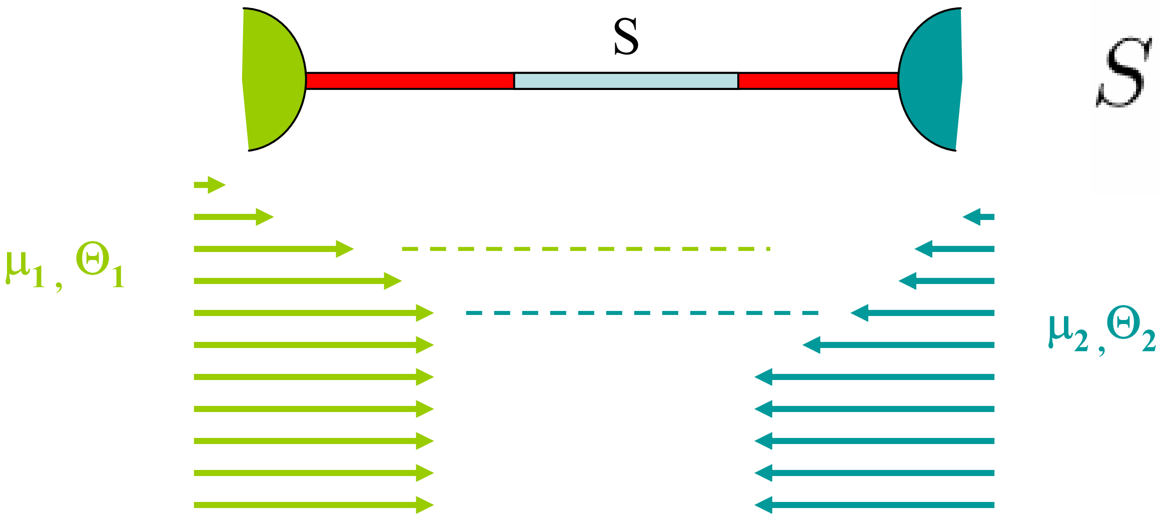
Two-probe Landauer Formula

$$G = \frac{e^2}{h} Tr\{\mathbf{t}(\epsilon_\mathbf{f})\mathbf{t}^\dagger(\epsilon_\mathbf{f})\}$$

(Two-probe, T=0)

Generalizations: $G = (e^2/h)$ T per incident degree of freedom, i.e. transverse channels, spin …
cancellation of velocity and DOS relies only on trans. invar. in leads

Two-probe, Temperature $\Theta \neq 0$



$$S = \begin{bmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{bmatrix}$$

$$|t_{21}|^2 = T_1, \ |t_{12}|^2 = T_2$$

Assume $\Theta_1 = \Theta_2 = \Theta$

$$\Delta N_1 v_f T(\epsilon_f) \rightarrow \int d\epsilon [n_1(\epsilon) v_1(\epsilon) T_1(\epsilon) f_1(\epsilon - \mu_1/\Theta_1) - n_2(\epsilon) v_2(\epsilon) T_2(\epsilon) f_2(\epsilon - \mu_2/\Theta_2)]$$

$$n_i v_i = \frac{1}{h}, \ \ T_1 = T_2, \ \ f_1(\mu_1 - \epsilon) - f(\mu_2 - \epsilon) \approx -df/d\epsilon|_{\epsilon_f} eV$$

$$\Rightarrow G = -e^2/h \int d\epsilon (df/d\epsilon) T(\epsilon)$$

Many-channel case:

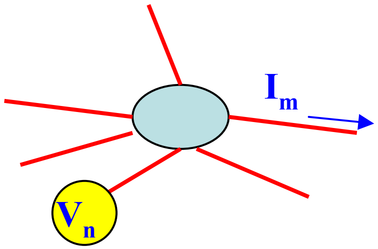
$r_{11} \to r_{11,ab}$, $t_{21} \to t_{21,ab}$

a,b = 1,2…N

$$(e^2/h) T_1(\epsilon) \rightarrow (e^2/h) Tr\{\mathbf{t_{21}}(\epsilon) \mathbf{t}^\dagger_{\mathbf{21}}(\epsilon)\}$$

If $\Theta_1 \neq \Theta_2$ can calculate thermoelectric coefficients in terms of S-matrix

Final Generalization: N_L leads



$$I_m = \frac{e^2}{h}\sum_{n=1}^{N_L}(T_{mn} - N\delta_{mn})V_n \equiv \sum_{n=1}^{N_L} G_{mn}V_n$$

- G_{mn} are conductance coefficients, necessary to describe 4-probe measurements, Hall resistance measurements

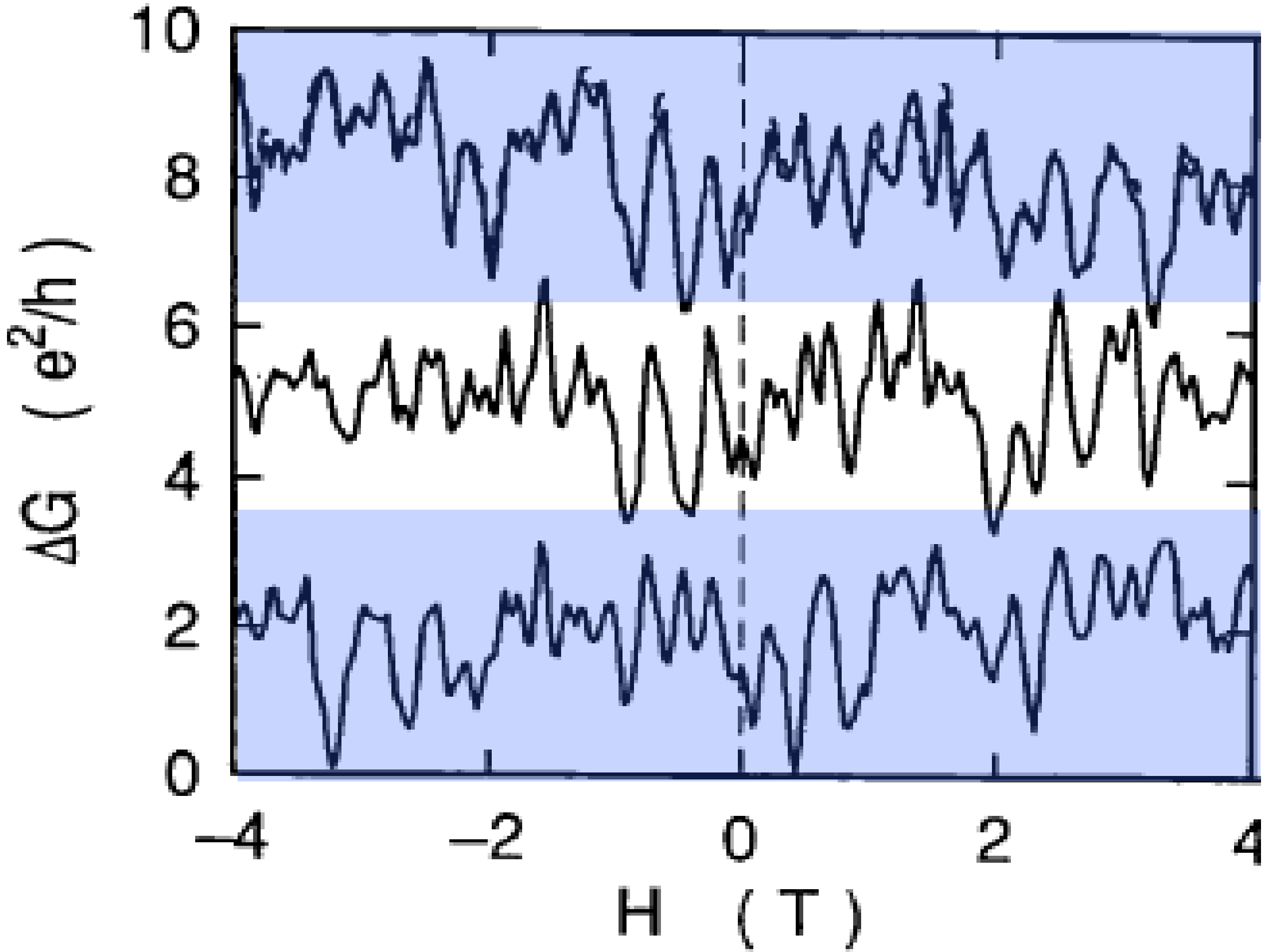
- Unitarity of S-matrix implies Kirchoff's Laws in general

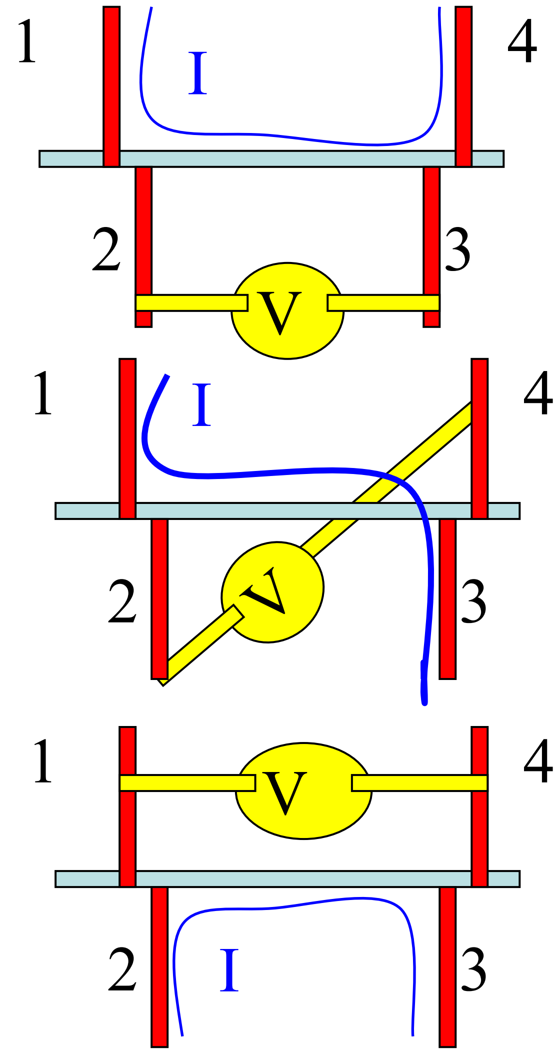
- $G_{mn} = G_{nm}$ only if B=0 or if only two probes, general TR symmetry of S-matrix implies $G_{mn}(B) = G_{nm}(-B)$ only.

- leads to van der Pauw reciprocity relations

- Properties of mesoscopic conductance: violates macrosymmetries, depends on measurement geometry, non-local (see Les Houches)

- If T_{mn} are integers then resistance is quantized to h/qe^2, q=integer



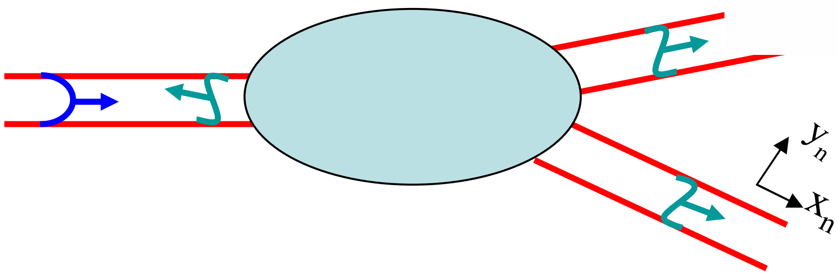


- Universal conductance fluctuations, sample-specific reproducible "noise" as fcn of B

- Looks like a longitudinal resistance measurement but $G(B) \neq G(-B)$

Making a quantum model for the LB counting argument

Non - interacting fermions at T=0, state is a Slater det or Fock state of single-particle fermion states - what are the sp states?



$$\psi_{m,a}(x,y) = \begin{array}{ll} e^{-ik_a x_m}\phi_a(y) + \sum_b^N r_{mm,ba} e^{ik_b x_m}\phi_b(y) & (x \in x_m) \\ \sum_b^N t_{nm,ba} e^{ik_b x_n}\phi_b(y) & (x \in x_n) \end{array}$$

$\phi_a(y)$ = channel wavefunction, normalized to unit flux => unitary S-matrix, ψ = orthonormal basis

- Linear response: fill up these states to common ε_f and calculate current response to linear order in potential $\{V_n\}$ imposed on leads - see Les Houches Notes
- Mesoscopic fermi gas: fill each scattering state to appropriate μ_n and calculate the currents I_m which flow in this states - see Buttiker, PRB, **46**, 12485 (1992)

$$|\Psi_{meso}> = \prod_{\alpha}^{occ.} c_\alpha^\dagger |0> \qquad \hat{J}(x,y) = \sum_{\alpha,\beta}^{\infty} J_{\alpha\beta}(x,y) c_\alpha^\dagger c_\beta \qquad \hat{I}_m = \int dy_m \hat{x}_m \cdot \hat{J}(x_m, y_m)$$

$$\alpha = (m, a, k_a) \qquad J_{\alpha\beta}(x,y) = \frac{e\hbar}{2mi}[\psi_\alpha^* \nabla \psi_\beta - \psi_\beta \nabla \psi_\alpha^*] \qquad 1D : \hat{I}_m = \hat{J}(x_m)$$

$$|\Psi_{meso}> = \prod_{m=1}^{N_L} \prod_{a=1}^{N} \prod_{k_m}^{k_f^{(m,a)}} c_{m,a,k_m}^\dagger |0> \qquad \frac{\hbar^2 (k_f^{m.a})^2}{2M} = \mu_m - \epsilon_a$$

Drop channel indices, treat as 1D

Expectation Value at T=0 and T≠ 0

$$< c^{\dagger}_{m,k_m} c_{n,k_n} >= \begin{cases} \delta_{mn}\delta_{k_n,k_m}\theta(\epsilon_{k_n}-\mu_n) \\ \delta_{mn}\delta_{k_n,k_m}f(\epsilon_{k_n}-\mu_n,T) \end{cases}$$

Expectation values < c†c c†c>, < c†c c†c c†c >, given by Wick's Thm, can calculate correlations and fluctuations in term of S-matrix; note presence of μ_n, this is not an equilibrium state.

$$I_m =< \hat{I}_m >=< \Psi|\hat{J}_m(x_m)|\Psi >= \sum_n^{N_L} \underbrace{\sum_{k_n}^{occ} < n,k_n|J(x_m)|n,k_n >}_{I_{mn}}$$

$$I_{mn} = \frac{e\hbar}{2\pi M}\int_{k_0}^{k_f^{(n)}} dk_n k_n |t_{mn}|^2 = \frac{e}{h}\int_{\mu_0}^{\mu_n} d\epsilon |t_{mn}|^2(\epsilon) = \frac{e^2}{h}|t_{mn}|^2 V_n$$

$$\boxed{I_m = \sum_n^{N_L} I_{mn} = \frac{e^2}{h}\sum_n^{N_L} |t_{mn}|^2 V_n}$$

} LB equations!

Current noise in mesoscopic fermi gas

$$C_{mn}(t-t') \equiv < \Delta\hat{I}_m(t)\Delta\hat{I}_n(t') + \Delta\hat{I}_n(t')\Delta\hat{I}_m(t) >$$

$$\Delta\hat{I}(t) = \hat{I}(t) - < \hat{I}(t) >$$

$$= \int \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} < \Delta\hat{I}_m(\omega)\Delta\hat{I}_n(\omega') + \Delta\hat{I}_n(\omega')\Delta\hat{I}_m(\omega) > \exp[-i(\omega t + \omega' t')]$$

Noise power spectrum

$$< \Delta\hat{I}_m\Delta\hat{I}_n + \Delta\hat{I}_n\Delta\hat{I}_m > \equiv S_{mn}(\omega)\delta(\omega+\omega')$$

$$\hat{I}_m(t) = \sum_{n,k_n}\sum_{p,k_p} < n, k_n|J_m|p, k_p > c_n^\dagger(k_n)c_p(k_p)e^{i(\epsilon_n-\epsilon_p)t}$$

Need OD current matrix element

$$< n, k_n|J_m|p, k_p > = \frac{\hbar k_m e}{M}[S^*_{mn}(k_n)S_{mp}(k_p) - \delta_{nm}\delta_{mp}]$$

$$\equiv \frac{\hbar k_m e}{M}A^m_{np}(\epsilon_n, \epsilon_p)$$

$$\hat{I}_m(\omega) = (e/h)\sum_n\sum_p\int d\epsilon A^m_{np}(\epsilon, \epsilon+\omega)c_n^\dagger(\epsilon)c_p(\epsilon+\omega)$$

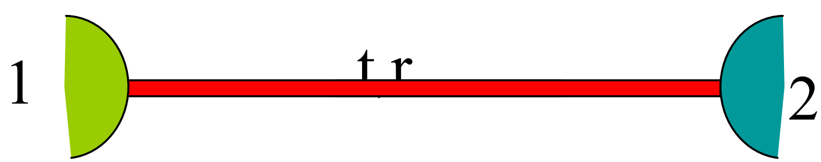
Simplify: 2-probe $< \Delta \hat{I}_m^2(\omega) > \equiv S(\omega)$

$$\sum_{npqr} < c_n^\dagger(\epsilon) c_p(\epsilon+\omega) c_q^\dagger(\epsilon') c_r(\epsilon'+\omega') >$$

“direct” contraction cancels with $<I_m>$, leaving “exchange”

$$\propto \delta_{nr}\delta_{pq}\delta(\epsilon-\epsilon'-\omega')\delta(\epsilon'-\epsilon-\omega) f_n(\epsilon)(1-f_p(\epsilon'))$$

$$P_m(\omega \to 0) = < S_m(\omega) >_{\Delta\nu} = \frac{2e^2}{h}\Delta\nu \sum_{np} \int d\epsilon A_{np}^m(\epsilon) A_{pn}^m(\epsilon) f_n(\epsilon)(1-f_p(\epsilon))$$



m =1 $A_{np}^m = S_{mn}^*(\epsilon_n) S_{mp}(\epsilon_p)$

T=0, $\mu_1 = \mu_2 + eV$, A_{12} A_{21} only

=> r^* t t^*r = RT=T(1-T)

$P_1(\omega \to 0) = (2e^2/h)\, T(1-T) eV\, \Delta\nu$

Mesoscopic shot noise

Many-channel: $A^1_{12}A^1_{21} = Tr\{\mathbf{r}^\dagger_{11}\mathbf{t}_{21}\mathbf{t}^\dagger_{21}\mathbf{r}_{11}\} = \sum_{\mathbf{a}=1}^{\mathbf{N}} \mathbf{T_a}(1-\mathbf{T_a})$

Weak transmission: $\frac{e^2}{h}\sum_{a=1}^{N} T_a(1-T_a) \approx \frac{e^2}{h}\sum_{a=1}^{N} T_a = G$

$P(\omega \to 0) = 2eGV\Delta\nu = 2eI\Delta\nu$ Tunneling shot noise

Temp $\Theta \neq 0$, V=0 => $f_n(1-f_n) \neq 0$,

$[2A^1_{12}A^1_{21} + A^1_{11}A^1_{11} + A^1_{22}A^1_{22}]f(1-f) = 2T -k_B\Theta\partial f/\partial\epsilon$

$4k_B\Theta\Delta\nu\int d\epsilon\frac{e^2}{h}T(-\partial f/\partial\epsilon) = 4k_B\Theta\Delta\nu G(T)$ } **Johnson Noise**

$$P(\omega\to 0,\Theta,V) = 2\Delta\nu\frac{e^2}{h}\sum_{a=1}^{N} 2k_B\Theta T_a^2 + T_a(1-T_a)eV\coth[eV/2k_B\Theta]$$

Cross-over function $\approx 2eI\Delta\nu\coth[eV/2k_B\Theta]$