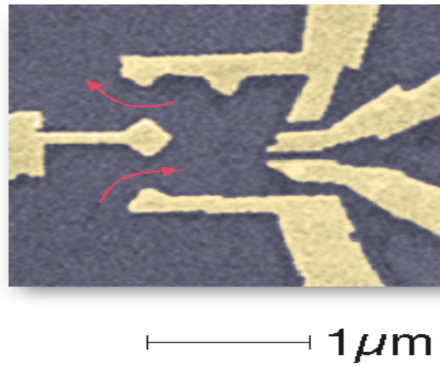
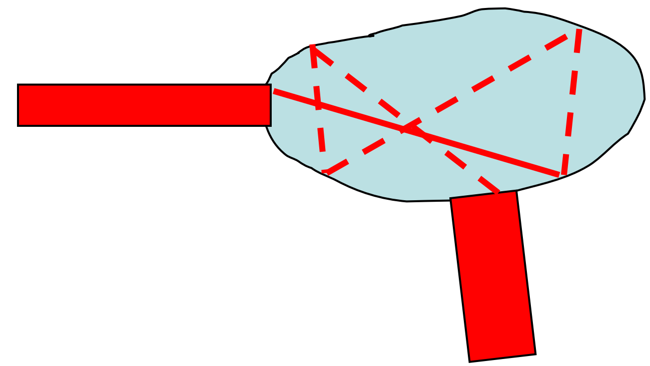
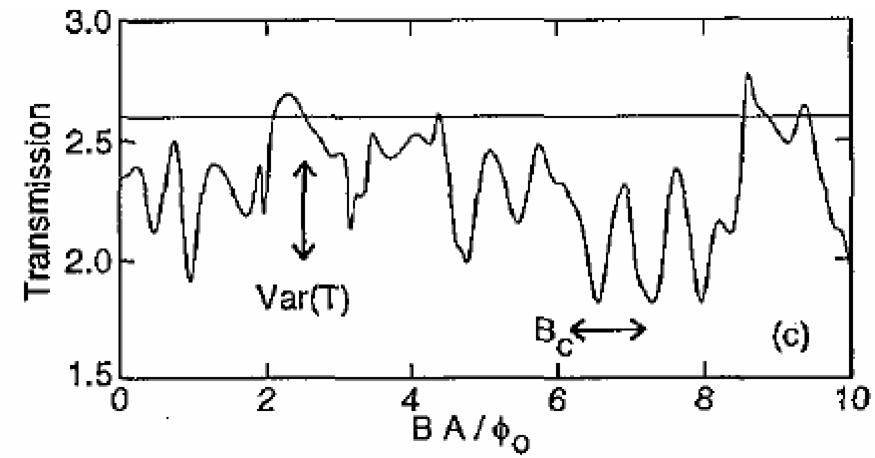
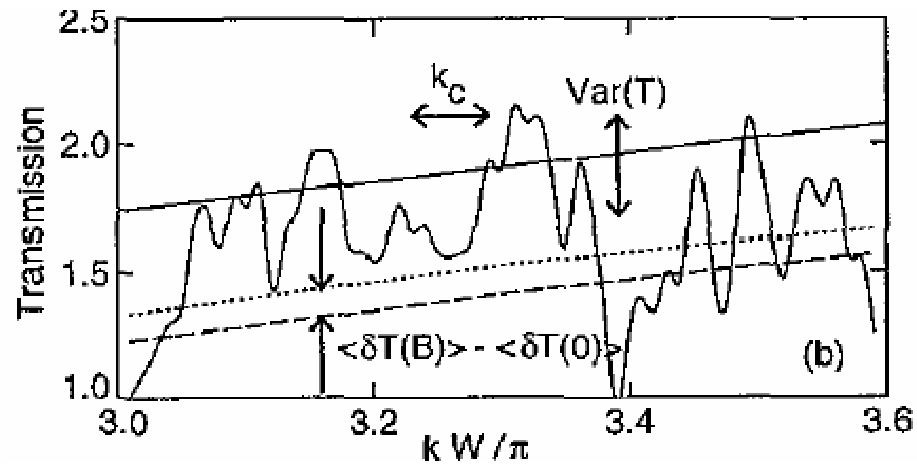
Lecture 4 - Boulder CM School - A. Douglas Stone
Semiclassical theory of ballistic junctions

$$G = \frac{e^2}{h} \sum_{a,b}^{N} |t_{ab}|^2$$









Want to calculate $<\Delta g(B)>$, Var(g), $C_g(\Delta E)$, $C_g(\Delta B)$ =>k_c (E_c), B_c - not just chaotic case

RMT: $<\Delta g(B)> = 1/4$, Var(g)= $1/(8\beta)$, lineshape but no dynamical scales, no prediction about regular or partially chaotic systems

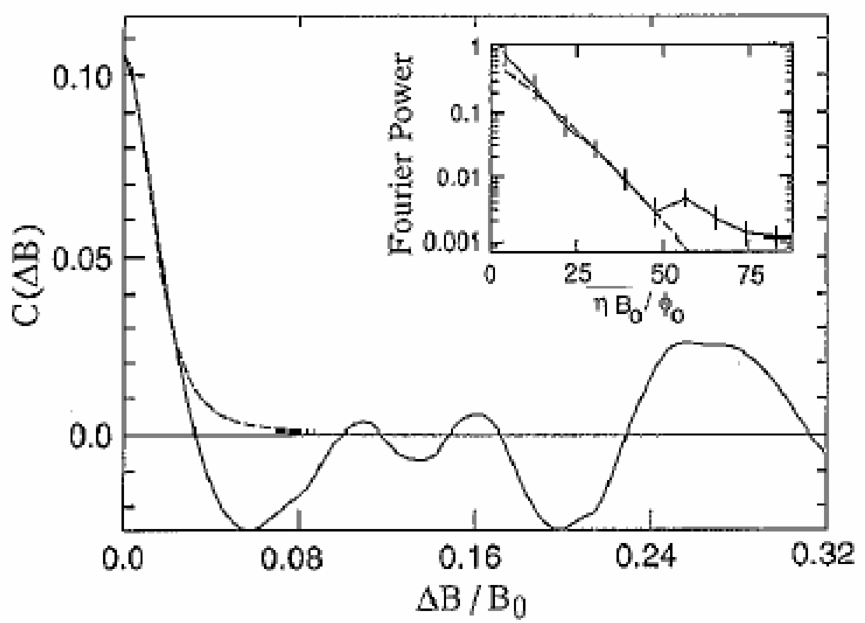
Conductance correlation functions

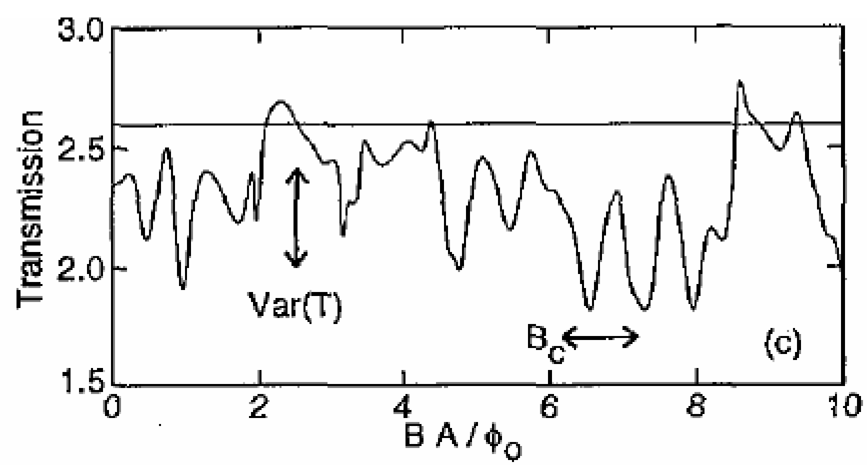
$$C_g(\Delta k) = < \delta g(k + \Delta k)\delta g(k) >_{avg}$$

$$C_g(\Delta B) = < \delta g(B + \Delta B)\delta g(B) >_{avg}$$

$$\delta g = g - < g >, \quad avg = \sum_{E_i, B_i, disorder} , \quad Var(g) = C_g(0)$$

Will give dynamical scales (Lee Stone PRL 1985)





Power spectra = FT of C_g

Why Semiclassical Approach?

- **What do we mean by drawing electron paths?**
- **Get the non-universal features**

$$G = \frac{e^2}{h} \sum_{a,b}^{N} |t_{ab}|^2$$

$$t_{ab} = -\frac{\sqrt{2\pi i\hbar}}{2W} \sum_{s(\bar{a},\bar{b})} \mathrm{sgn}(\bar{a})\, \mathrm{sgn}(\bar{b}) \sqrt{\tilde{D}_s}\, \exp\left(\frac{i}{\hbar}\tilde{S}_s(\bar{a},\bar{b},E) - i\frac{\pi}{2}\tilde{\mu}_s\right),$$

$$|t_{ab}|^2 = \sum A_s A_u e^{ik(L_s - L_u) + i(\phi_s - \phi_u)}$$

$$< \exp[ikL_s + i\phi_s - ikL_u - i\phi_u] >= \delta_{su}?$$

$$< |t_{ab}|^2 >\approx \sum_s < (A_s^{ab})^2 >$$

Diagonal approximation (DA)

DA gives dynamical scales and lineshapes correctly, <u>not magnitudes</u> - but off-diagonal SC is possible and works

Dynamical Scales

$$\mathcal{C}(\Delta k) = t_{ab}(k)t^*_{ab}(k+\Delta k) = \sum_{s,u} A_s A_u e^{ikL_s + i\phi_s} e^{-i(k+\Delta k)L_u - i\phi_u}$$

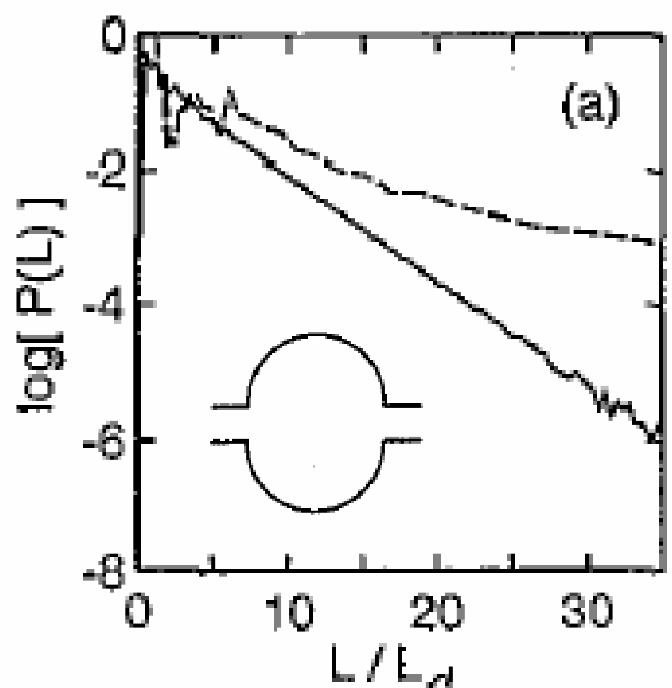
Look at diagonal terms, s=u:

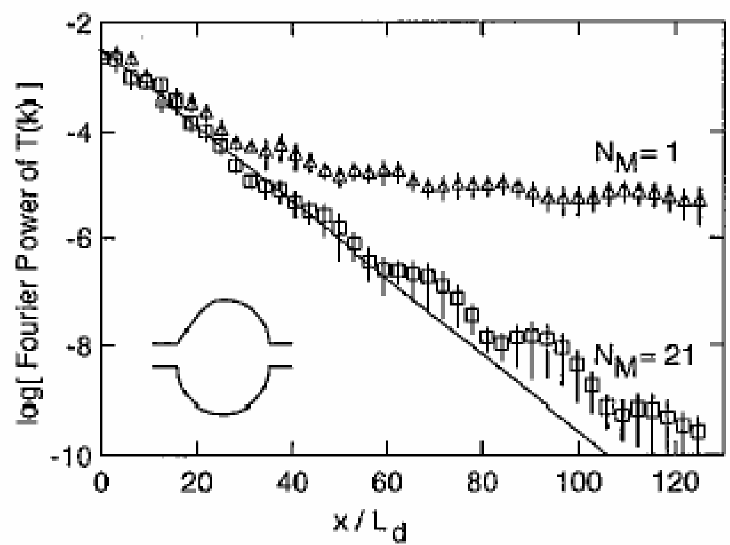
$$\sum_s A_s^2 e^{i\Delta k L_s} = \int_0^\infty dL e^{i\Delta k L} P(L)$$

$$P(L) = \gamma e^{-\gamma L} \qquad \gamma = \frac{2W}{\pi A} \qquad \mathcal{C}(\Delta k) = \frac{\gamma}{i\Delta k - \gamma}$$

$$C(\Delta k) = \ldots \Delta k)|^2 = \frac{\gamma^2}{\gamma^2 + \Delta k^2}$$

Lorentian correlation fcn with scale $\gamma = \gamma_c$



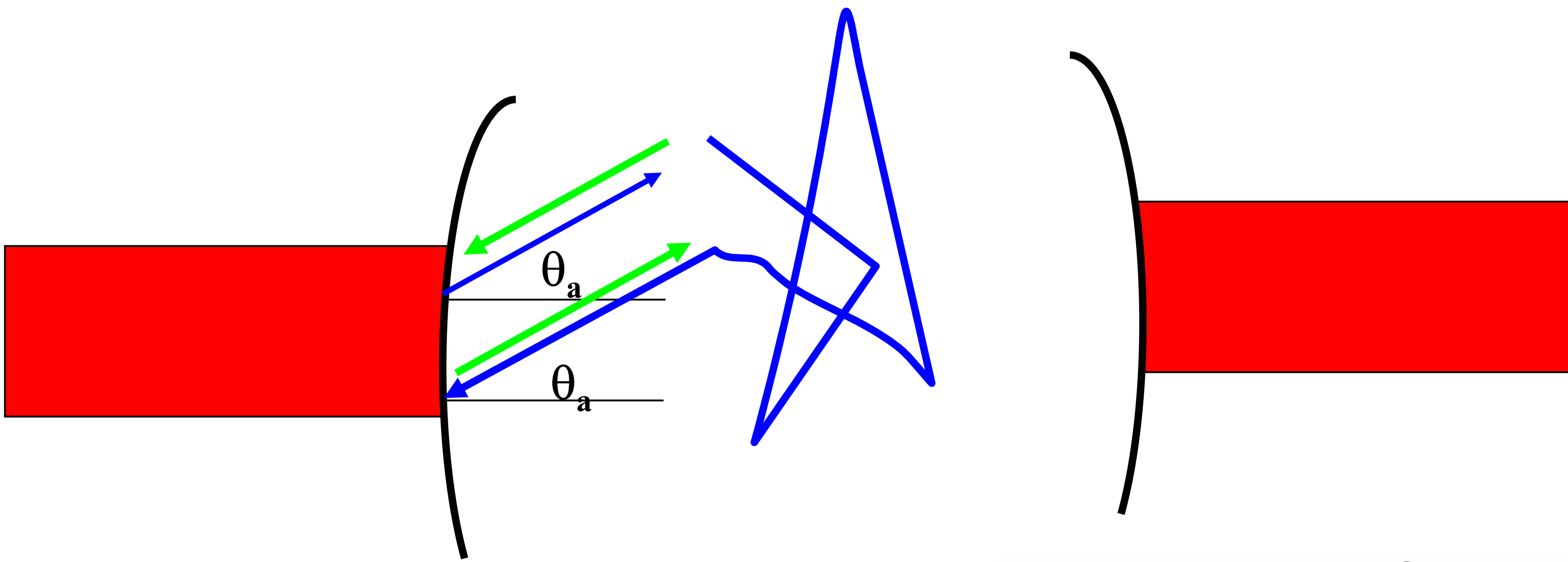


Dynamical scales in magnetic field

Instead of doing $C(\Delta B)$ using DA, do $\langle \delta G\,(B)\rangle_{WL}$ using generalized DA:

$$|r_{aa}|^2 = \sum_{s,u} A_s A_u e^{ikL_s + i\phi_s(B) - ikL_u - i\phi_u(B)}$$

In $|r_{aa}|^2$ $\exists$ exact TR pairs with $L_s = L_u$, will survive $\langle\ \rangle$



$$i\phi_s(B) - i\phi_u(B) = 2\Theta_s B/\phi_0 \quad \Theta_s \equiv (2\pi/B)\int \mathbf{A}\cdot\mathbf{dl}$$

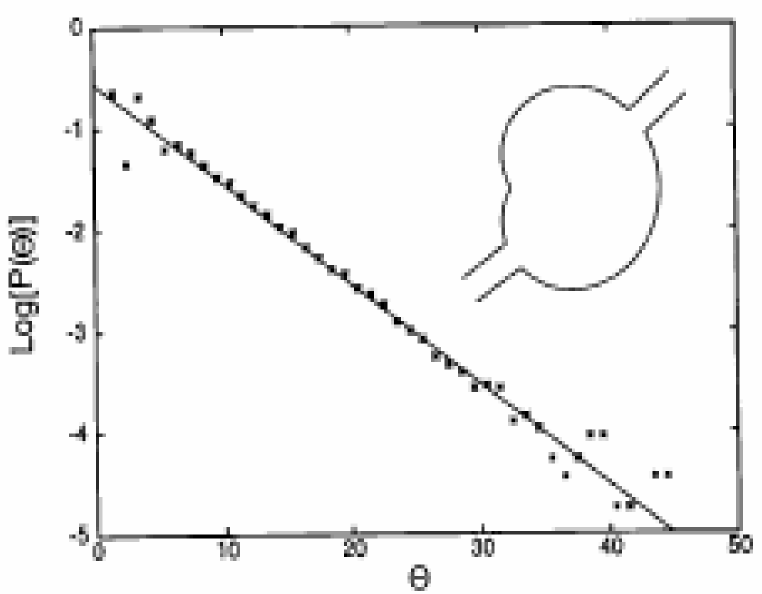
$$\delta R_D(B) \equiv \sum_a^N |r_{aa}|^2 \qquad \delta R_D(B) = \frac{1}{2}\int_{-1}^{1} d(\sin\theta) \sum_{s(\theta,\pm\theta)} \tilde{A}_s e^{i2\Theta_s B/\phi_0}.$$

$$\delta R_D(B) = \mathcal{R}\int_{-\infty}^{\infty} d\Theta P(\Theta)\exp[i2\Theta B/\phi_0],$$

See Les Houches Notes

$$P(\Theta) \propto \exp[-\alpha|\Theta|]$$

$$\boxed{\delta R_D(B) = \frac{\mathcal{R}}{1+(2B/\alpha_{cl}\phi_0)^2}}$$

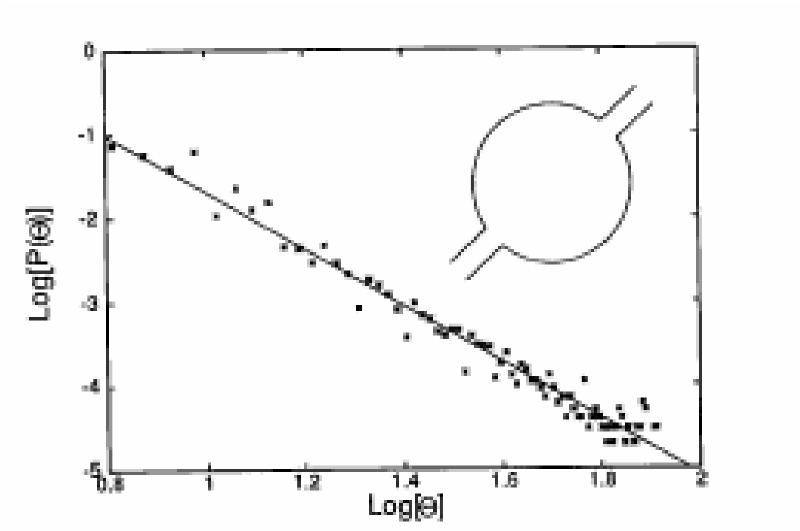


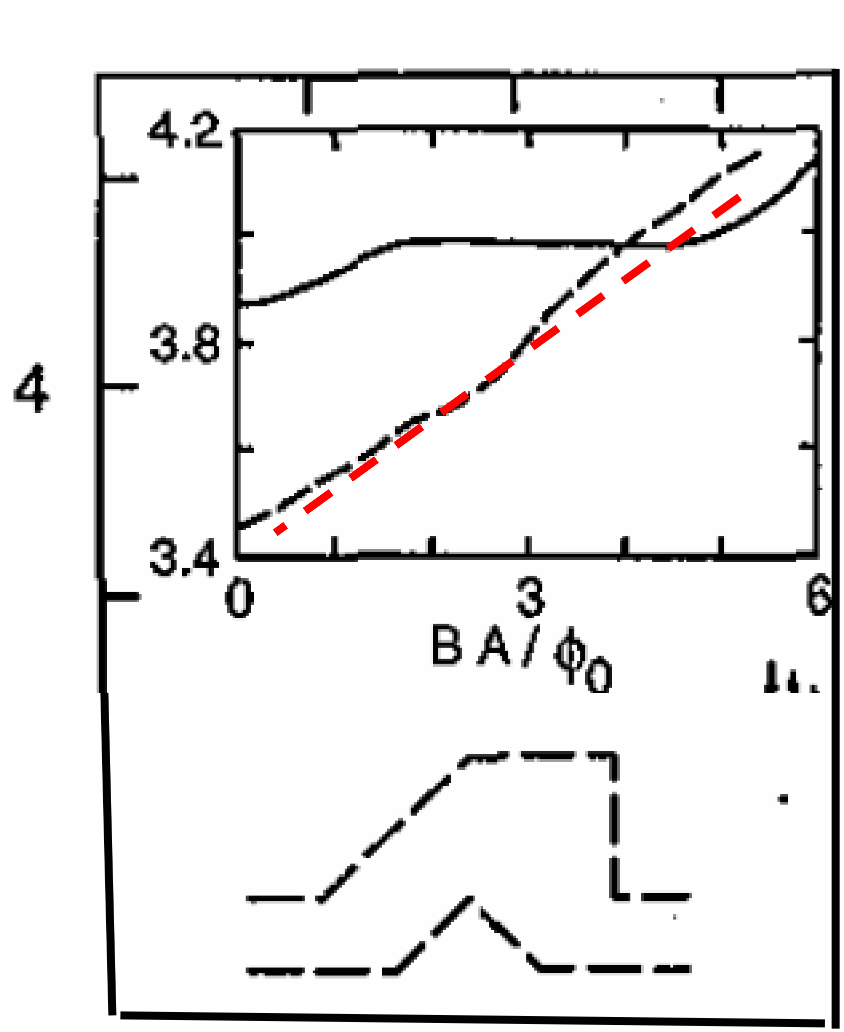
$R_D(0) = 2\, R_D\, (B >> \alpha\phi_0/2)$ - the coherent backscattering effect- gives WRONG δg_{WL}

Lorentzian WL lineshape agrees with RMT, but with no free parameter

Similar argument give C(ΔB) = (Lorentzian)^2

SC method can address <u>non-chaotic shapes</u>; all that changes is P(L) and P(Θ) - typically power law decays due to conserved quantities

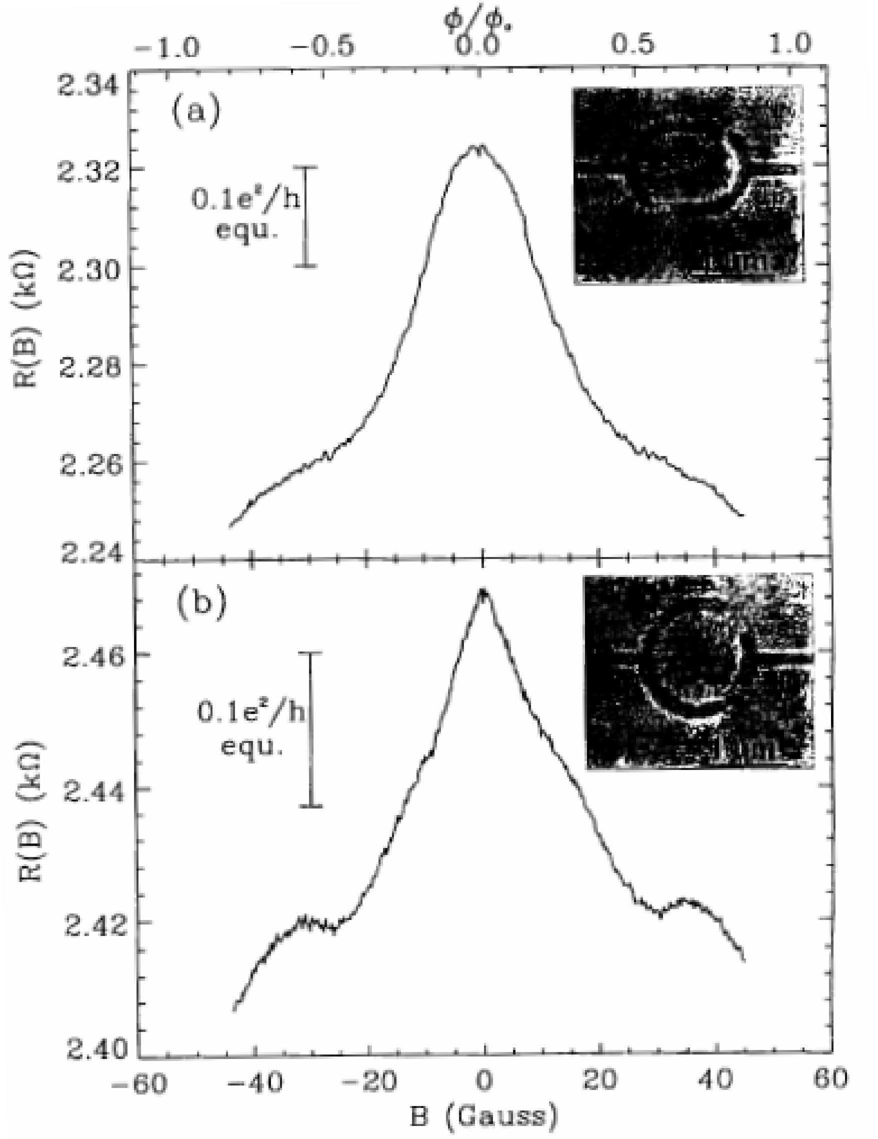




δR ∝FT{ P(Θ)}, leads to singularity δR as B → 0

Baranger et al. PRL 1993, linear WL lineshape predicted

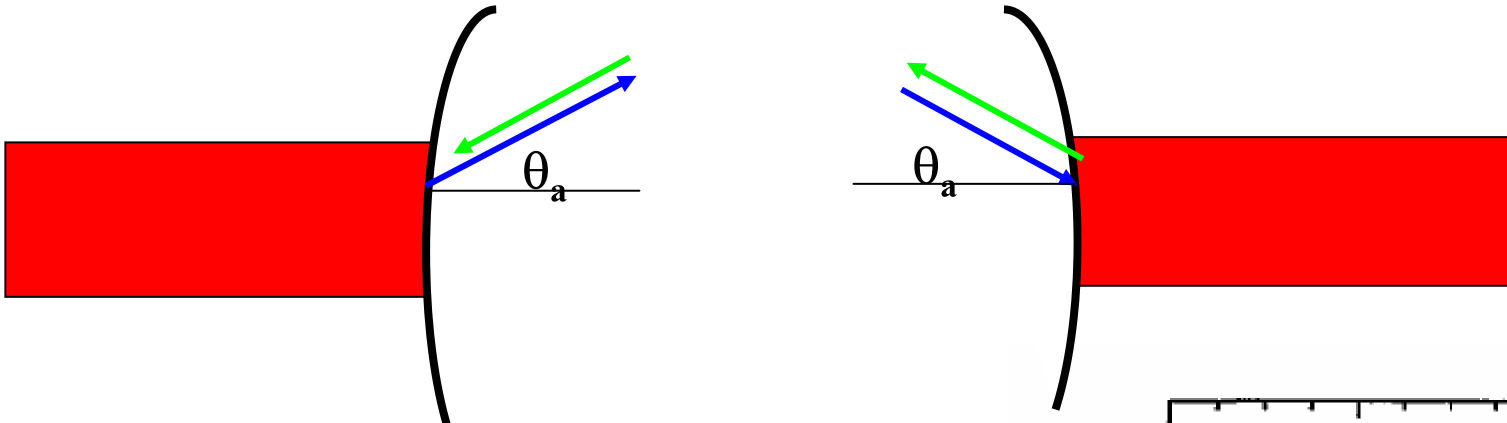
Chang et al., PRL 94; see also for CF Marcus et al., PRL 92



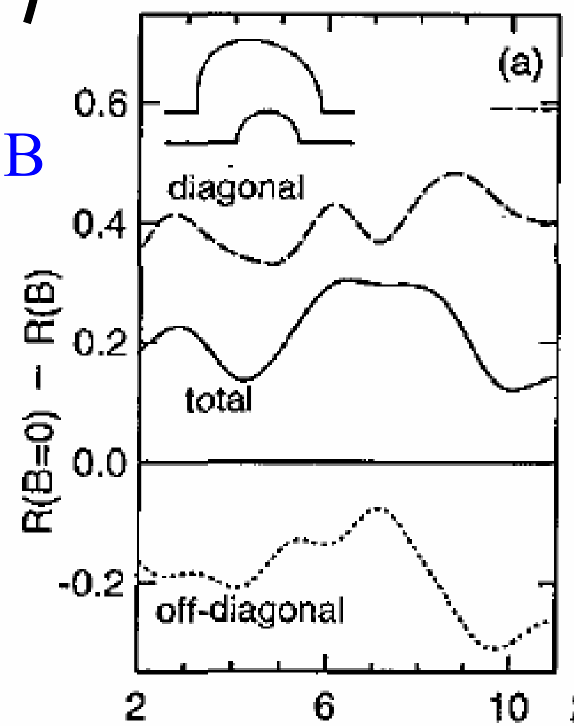
<u>Warning: Universal Hamiltonian is not universal!</u>

Problems with the generalized diagonal approximation:

- No semiclassical corrections to T due to TR symmetry

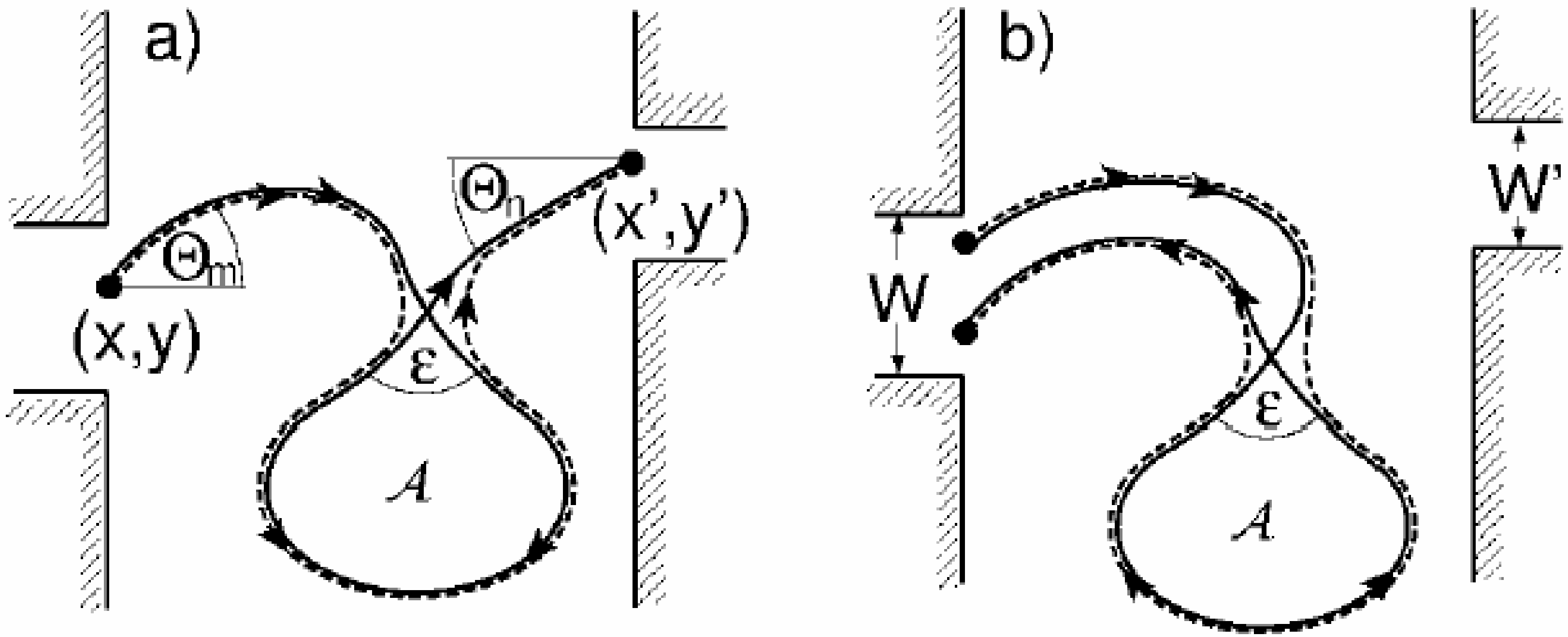


- $R + T = N$, $R_{OD} + R_D + T = N$; if $R_D \to R_D/2$ with B and T and R_{OD} unchanged => DA violates unitarity
- Numerically we do see change in T and R_{OD}
- WL and UCF diagrams for disordered case find correlations
- Apparently $< \exp[ik(L_s - L_u)]> \neq 0$ for $s \neq u$



Richter and Sieber, PRL 206801 (2002) and references therein

Proved the existence of such orbits in an ergodic system for sufficiently small crossing angle ε



$$\Delta S(\varepsilon) \approx \frac{p^2 \varepsilon^2}{2m\lambda}.$$

Can contribute when ΔS < h/2π, angles ε→0 in SC limit

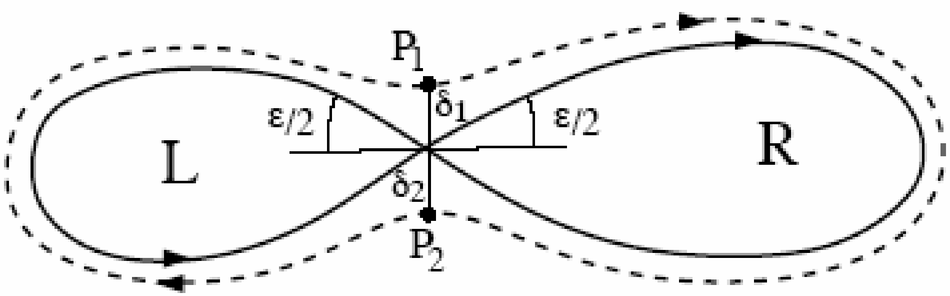
Related problem of spectral form factor K(τ); semiclassically determined by periodic orbits:

$$K(\tau) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\eta}{\bar{d}(E)} \left\langle d_{\mathrm{osc}}(E+\eta/2)\, d_{\mathrm{osc}}(E-\eta/2) \right\rangle_E e^{2\pi i \eta \tau \bar{d}(E)} ,$$

$$K^{\mathrm{GOE}}(\tau) = \begin{cases} 2\tau - \tau \log(1+2\tau) & \text{if } \tau < 1, \\ 2 - \tau \log \frac{2\tau+1}{2\tau-1} & \text{if } \tau > 1, \end{cases}$$

$$K(\tau) \approx \frac{1}{2\pi\hbar \bar{d}(E)} \sum_{\gamma,\gamma'} \left\langle A_\gamma A^*_{\gamma'}\, e^{i(S_\gamma - S_{\gamma'})/\hbar}\, \delta\left(T - \frac{T_\gamma + T_{\gamma'}}{2}\right)\right\rangle_E , \qquad \tau = T\Delta\varepsilon/h$$

Look for periodic orbits which differ but have very close actions.



$$\begin{pmatrix} \delta_2 \\ p(\gamma_2 + \varepsilon/2) \end{pmatrix} = R \begin{pmatrix} \delta_1 \\ p(\gamma_1 - \varepsilon/2) \end{pmatrix} , \qquad \begin{pmatrix} -\delta_2 \\ p(\gamma_2 - \varepsilon/2) \end{pmatrix} = L \begin{pmatrix} -\delta_1 \\ p(\gamma_1 + \varepsilon/2) \end{pmatrix}$$

$$\Delta S(\epsilon) \approx \frac{p\varepsilon}{2}(\delta_1 + \delta_2)\,. \quad \delta_1 + \delta_2 \propto \varepsilon,\ \Delta S \propto \varepsilon^2$$

$$\Rightarrow \quad \Delta S(\varepsilon) \approx \frac{p^2\varepsilon^2}{2m\lambda}.$$

$$K_{\text{off}}^{(2)}(\tau) \approx \frac{4}{2\pi\hbar\bar{d}(E)} \operatorname{Re} \int_0^\infty \mathrm{d}\varepsilon \sum_\gamma |A_\gamma|^2\, P(\varepsilon, T)\, \exp(i\Delta S(\varepsilon)/\hbar)\, \delta(T - T_\gamma)$$
$$\approx \frac{4}{2\pi\hbar\bar{d}(E)} \operatorname{Re} \int_0^\infty \mathrm{d}\varepsilon \int_0^\infty \mathrm{d}T'\, \rho(T') \frac{T'^2}{\exp(\lambda T')}\, P(\varepsilon, T)\, \exp(i\Delta S(\varepsilon)/\hbar)\, \delta(T - T')$$

Using form for $P(\varepsilon,T)$ they find $K_{OD} = -2\tau^2$, correct GOE result, needed to go to next order in ε to get it.

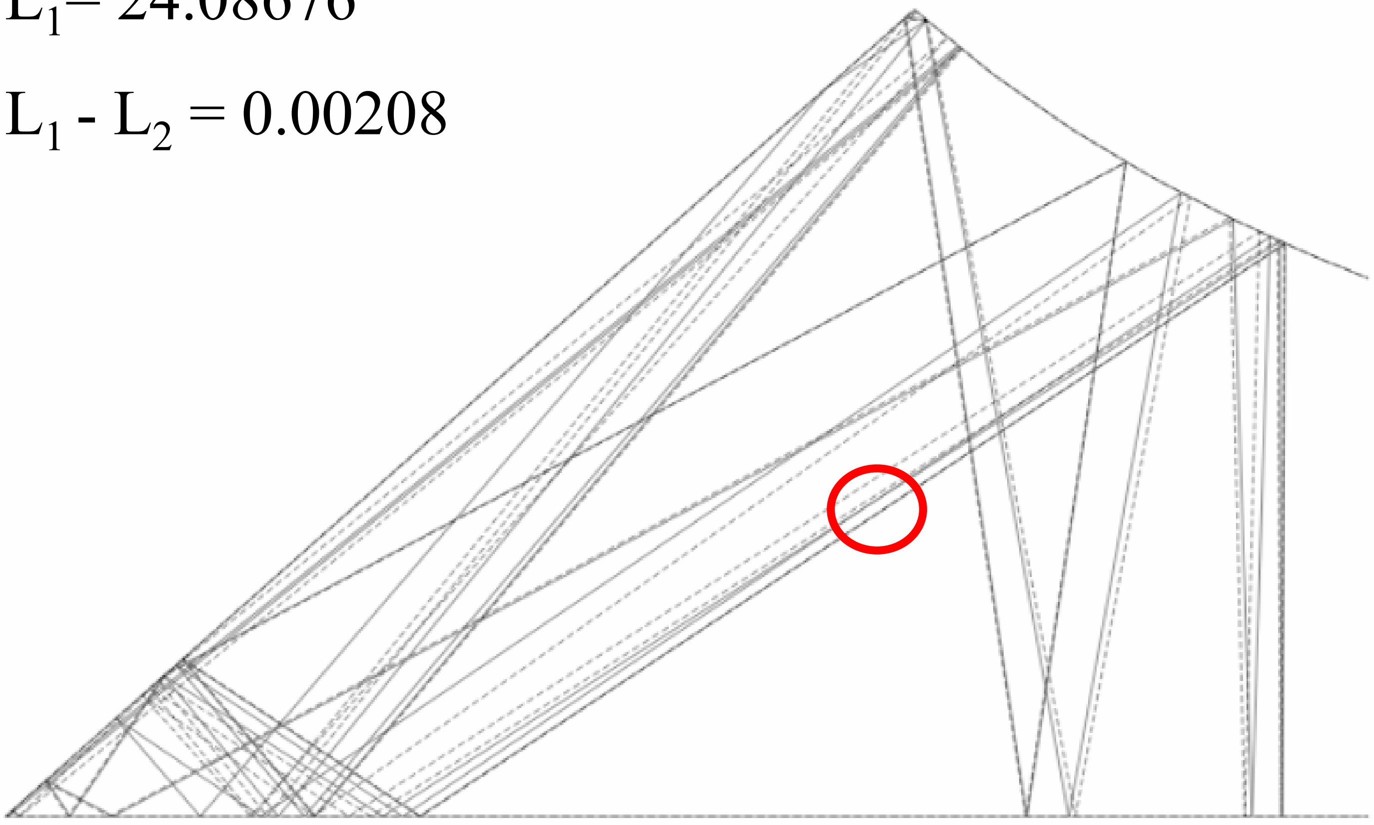
Are there such orbit pairs?

An example from the hyperbola billiard

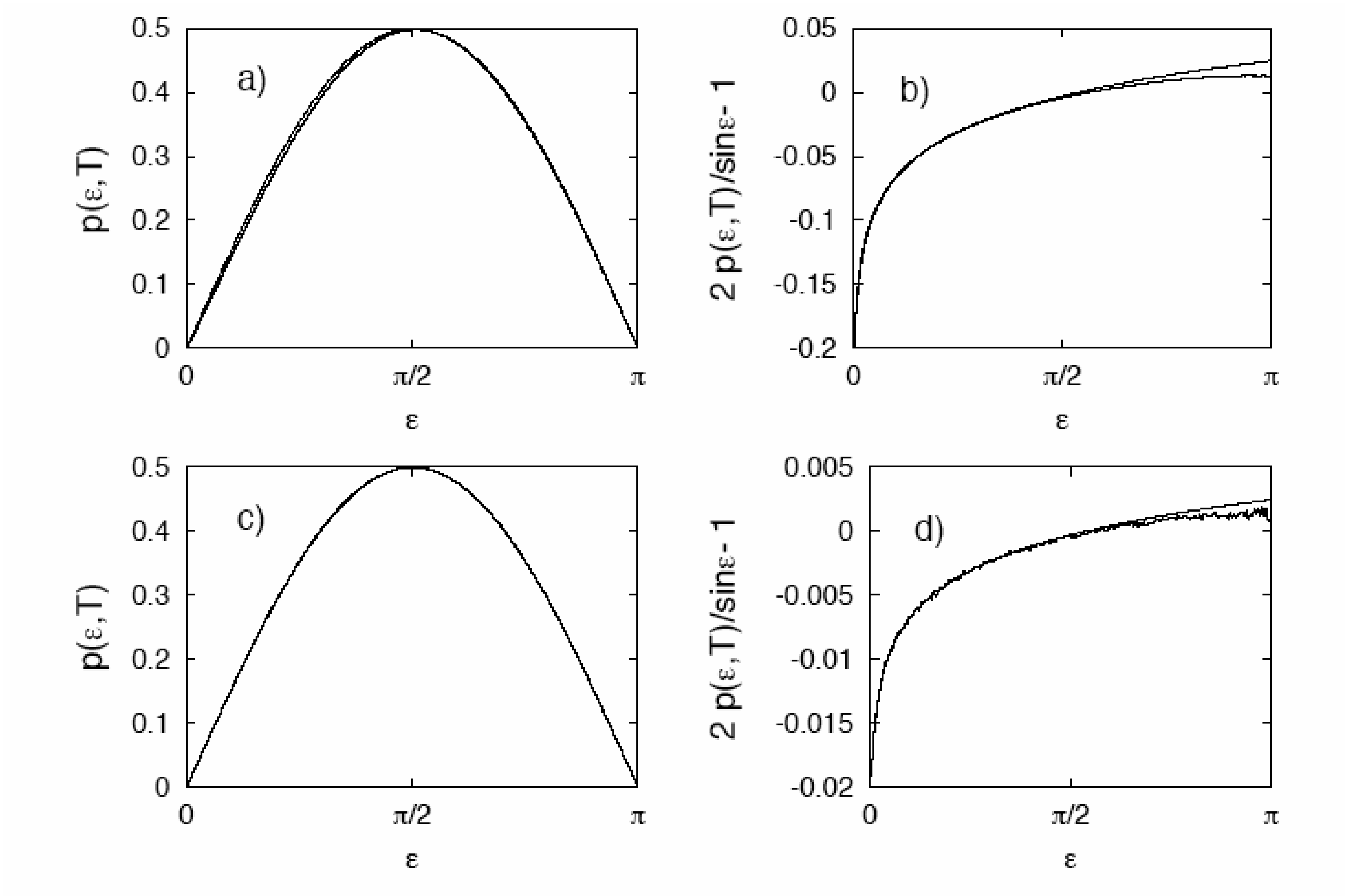
Where's Waldo?

$L_1 = 24.08676$

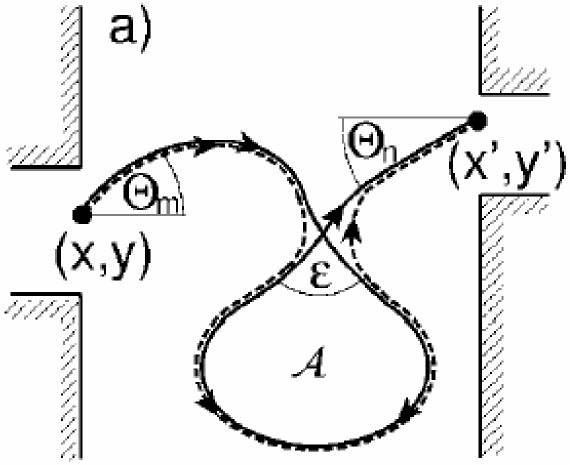
$L_1 - L_2 = 0.00208$



Did simulations of $P(\varepsilon,T)$ with 50×10^6 orbits



Back to the WL calculation:



$$|t_{nm}|^2_{\text{loop}} \simeq \frac{\pi\hbar}{ww'} \sum_{\gamma(\bar{n},\bar{m})} \frac{\delta(T-T_\gamma)}{|\cos\theta'_{\bar{n}} \cos\theta_{\bar{m}} M^\gamma_{21}|} I(T)$$

$$\simeq \frac{2\hbar}{mA} \int dT e^{-T/\tau} I(T)$$

$$I(T) = \text{Re} \int_0^\pi d\varepsilon P(\varepsilon;T) \exp\left(\frac{ip^2\varepsilon^2}{2\hbar m\lambda}\right) \quad = (\hbar/2mA)T$$

$$P(\varepsilon;T) \simeq 2mv^2 \int_{T_{\min}(\varepsilon)}^{T} dt(T-t)\sin(\varepsilon)\, p_{\text{erg}}, \qquad p_{\text{erg}} = 1/(2\pi mA)$$

$$c \simeq \varepsilon \exp[\lambda T_{\min}(\varepsilon)/2] \quad \Rightarrow \quad P(\varepsilon;T)d\varepsilon \sim \frac{T^2v^2}{\pi A}\frac{\sin\varepsilon}{2}\left[1-2\frac{T_{\min}(\varepsilon)}{T}\right]d\varepsilon$$

$$|t_{nm}|^2_{\text{loop}} \simeq -\left[\frac{\hbar}{mA}\right]^2 \int dT T e^{-T/\tau} = \frac{-1}{(2N)^2}$$

$$1/\tau = \frac{2vW}{\pi A}$$

$$\boxed{\Rightarrow \delta T_{loop} = -\frac{1}{4}}$$

Why do I like this so much?

A semiclassical explanation for UCF (finally!)

$$Var(g) = \sum_{ab}^{N} \sum_{cd}^{N} < \delta T_{ab} \delta T_{cd} >$$

Feng, Lee, Stone, 1988

$$< \delta T_{ab} \delta T_{cd} > \propto \frac{1}{N^4}$$

Need these off-diagonal correlations to get this

Thanks to long time collaborators:

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