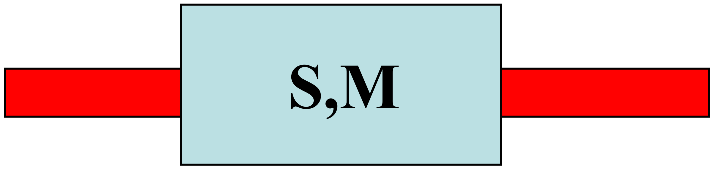
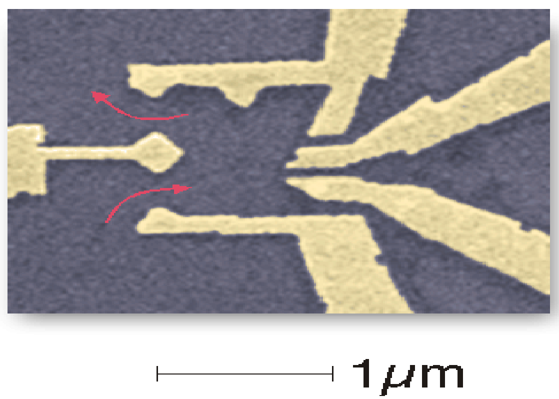
Lecture 3 - Boulder CM School - A. Douglas Stone

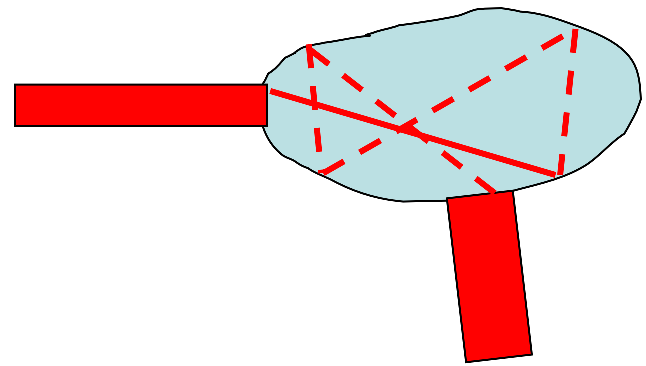


$$G = \frac{e^2}{h}\sum_{a,b}^{N}|t_{ab}|^2 = \sum_{n}^{N} T_n$$

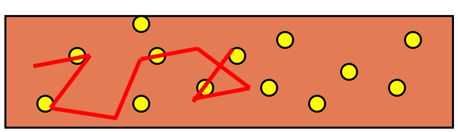
Need statistical ensemble of S & M Matrices, dμ(S), dμ(M)

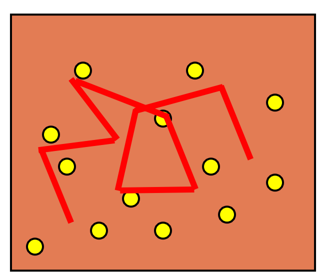
Open ballistic quantum dot

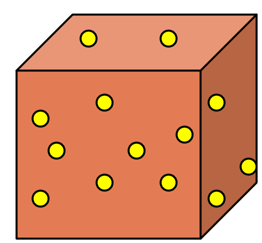




Disordered metal







Simplest possibility: completely uniform on the matrix space

- **This is possible for unitary S-matrix (circular ensembles),**
- **Not for pseudo-unitary M-matrix (non-compact space)**

Hypothesis: $P(S) = d\mu(S)/V$ for chaotic quantum dot

$dS^2 = \mathrm{Tr}\{\mathbf{dSdS^\dagger}\} = \sum_{ij} g_{ij}\, dq_i q_j \Rightarrow d\mu(\mathbf{S}) = (\det[\mathbf{g}])^{1/2} \prod_i dq_i$

Dyson $\left\{ \; d\mu(S) = \prod_{n<m} |\exp(i\phi_n) - \exp(i\phi_m)|^\beta \, d\mu(U) \prod_i d\phi_i \right.$

We need a parameterization in term of $\{T_n\}$

$$S = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \begin{pmatrix} -\sqrt{1-\mathcal{T}} & \sqrt{\mathcal{T}} \\ \sqrt{\mathcal{T}} & \sqrt{1-\mathcal{T}} \end{pmatrix} \begin{pmatrix} U' & 0 \\ 0 & V' \end{pmatrix}.$$

$\mathcal{T} = Diag(T_1, T_2 \ldots T_N)$ $\mathbf{S_{12}} = \mathbf{t} = V\sqrt{\tau}U' \Rightarrow \mathbf{tt}^\dagger = V\tau V^\dagger$

$$d\mu(S) = \prod_{n<m} |T_n - T_m|^{\beta} \prod_k T_k^{-1+\beta/2} \prod_\alpha d\mu(U_\alpha) \prod_i dT_i \,,$$

$$P(\{T_n\}) =$$

Coulomb Gas analogy

$$c_\beta \exp[-\beta\{\sum_{i<j} -\ln(|T_i - T_j| + \sum_k (1/\beta - 1/2)) \ln|T_k| + \delta V_\beta(T_k)\}]$$

Circular ensemble: take $\delta V_\beta = 0$, $P(\{T_n\})$ only from invariant measure; what to do with jpd?

$$< g > = < \sum_a^N T_a >, \; < g^2 > = < \sum_{a,b}^N T_a T_b >, \; < P > = < \sum_a^N T_a (1 - T_a) >$$

WL UCF MesoNoise

$$< g > = \int \prod_{c=1}^N dT_c P(\{T_c\}) \sum T_a$$

$$= N \int \prod_{c=1}^N dT_c P(\{T_c\}) T_1 = \int_0^1 \rho(T) T$$

$$\rho(T) = \int \prod_{c=2}^N dT_c P(\{T_c\})$$

For Var(g) need

$$K(T, T') = \int \prod_{c=3}^N dT_c P(\{T_c\}) - \rho(T)\rho(T')$$

Need 1-pt and 2-pt correlation fcns of the jpd of $\{T_n\}$

Many methods to find these fcns and the two-pt corr. fcn is "universal" upon rescaling if only logarithmic correlations

Nice approach for β=2 is method of orthogonal polynomials

$$\prod_{a<b}^{N} |T_a - T_b|^2 = \begin{vmatrix} p_1(T_1) & p_1(T_2) & \ldots & p_1(T_N) \\ p_2(T_1) & p_2(T_2) & \ldots & p_N(T_N) \\ \ldots & & & \\ p_N(T_1) & p_N(T_2) & \ldots & p_N(T_N) \end{vmatrix}^2$$

$$\equiv < \Psi_N | \Psi_N >$$

p_n= orthog poly, choose Legendre, [0,1]

$$\rho(T) = < \Psi_N | \sum_a \delta(T - T_a) | \Psi_N > = \sum_{n=1}^{N} p_n^2(T)$$

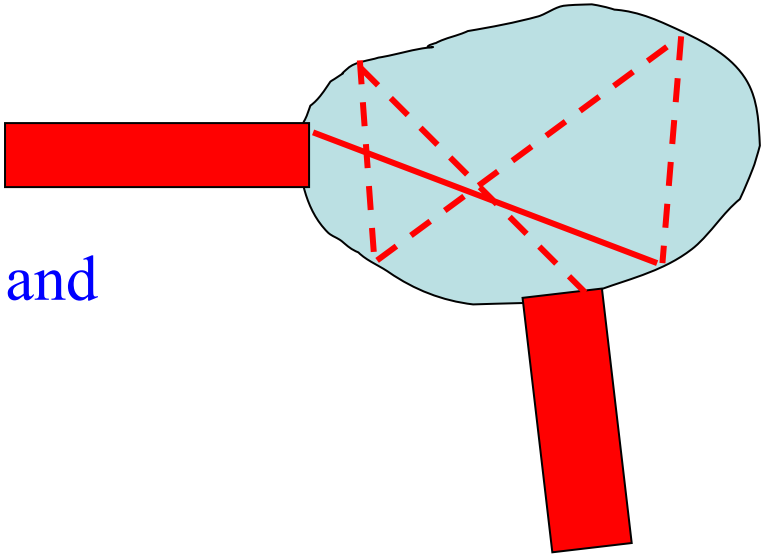
Use recursion relations, asymptotic form of p_n :

$$\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}$$

Same method gives K(T,T') in terms of $p_N p_{N-1}$

(normalized to N - so that G = (e^2/h) T

What do we expect for this system?



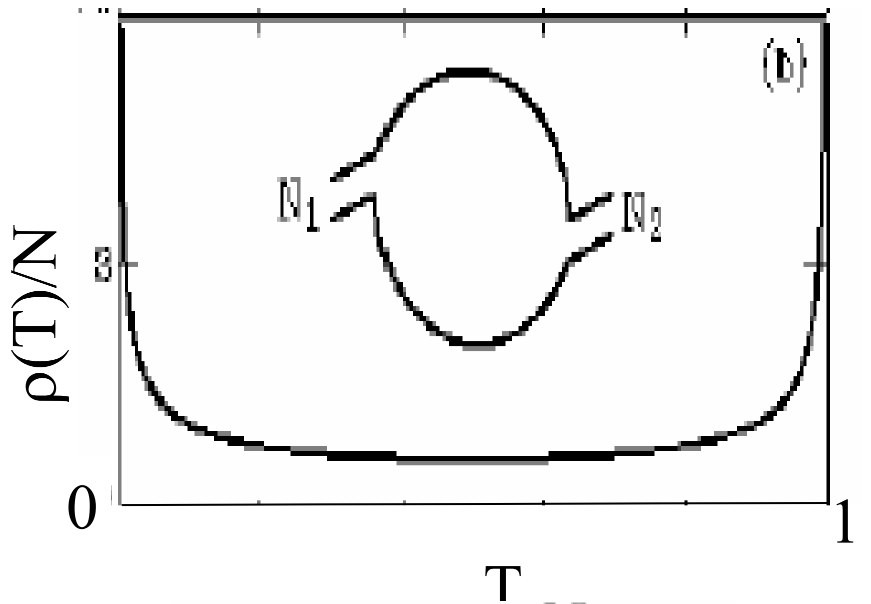
Classical symmetry betweeen reflection and transmission => <R> = <T> = N/2

$$< g >= \frac{N}{\pi}\int_0^1 dT\sqrt{\frac{T}{(1-T}} = \frac{N}{2}$$

Need to go to next order in N^{-1} to get WL effect

$$< P_{shot} > \propto \frac{N}{\pi}\int_0^1 dT\sqrt{T(1-T)}$$

$$= \frac{N}{8} \Rightarrow \frac{1}{4}P_{tunnel}$$



$$\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}$$

Can get order 1/N effects easily for the circular ensemble - do averages over unitary group U(2N)

$$g = \sum_{a,b}^{N} |t_{ab}|^2 = \sum_{a,b}^{N} S_{ab} S^*_{ab}, \quad S \in U(2N)$$

$Tr\{r\, r^\dagger\} = R$

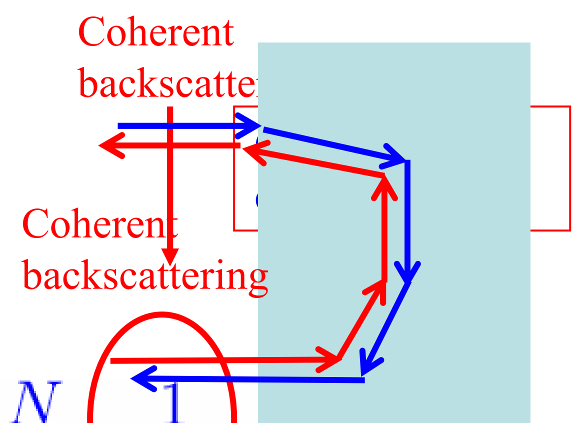
$S_{COE} = \boldsymbol{UU^T}$

$$< S_{ab} S^*_{cd} >_{CUE} = \int d\mu(S) S_{ab} S^*_{cd} = \frac{\delta_{ac}\delta_{bd}}{2N}$$

$$\boldsymbol{U} \in U(2N)$$

$$< g >_{CUE} = N^2 \cdot \frac{1}{2N} = \frac{N}{2}$$

$$< S_{ab} S^*_{ab} >_{COE} = \int d\mu(U) [UU^T]_{ab} = \frac{1 + \delta_{ab}}{2N+1}$$



$$< R >_{COE} = N^2 \cdot \frac{1}{2N+1} + N \cdot \frac{1}{2N+1} \approx \frac{N}{2} + \frac{1}{2} - \frac{1}{4}$$

$\delta G_{WL} = -(2e^2/h)(1/4)$

The Mystery

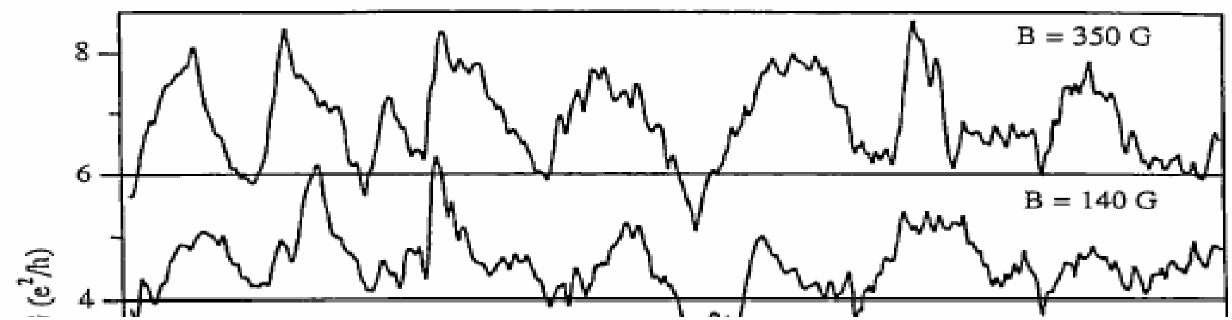
Similarly Var(g) = (1/8β)

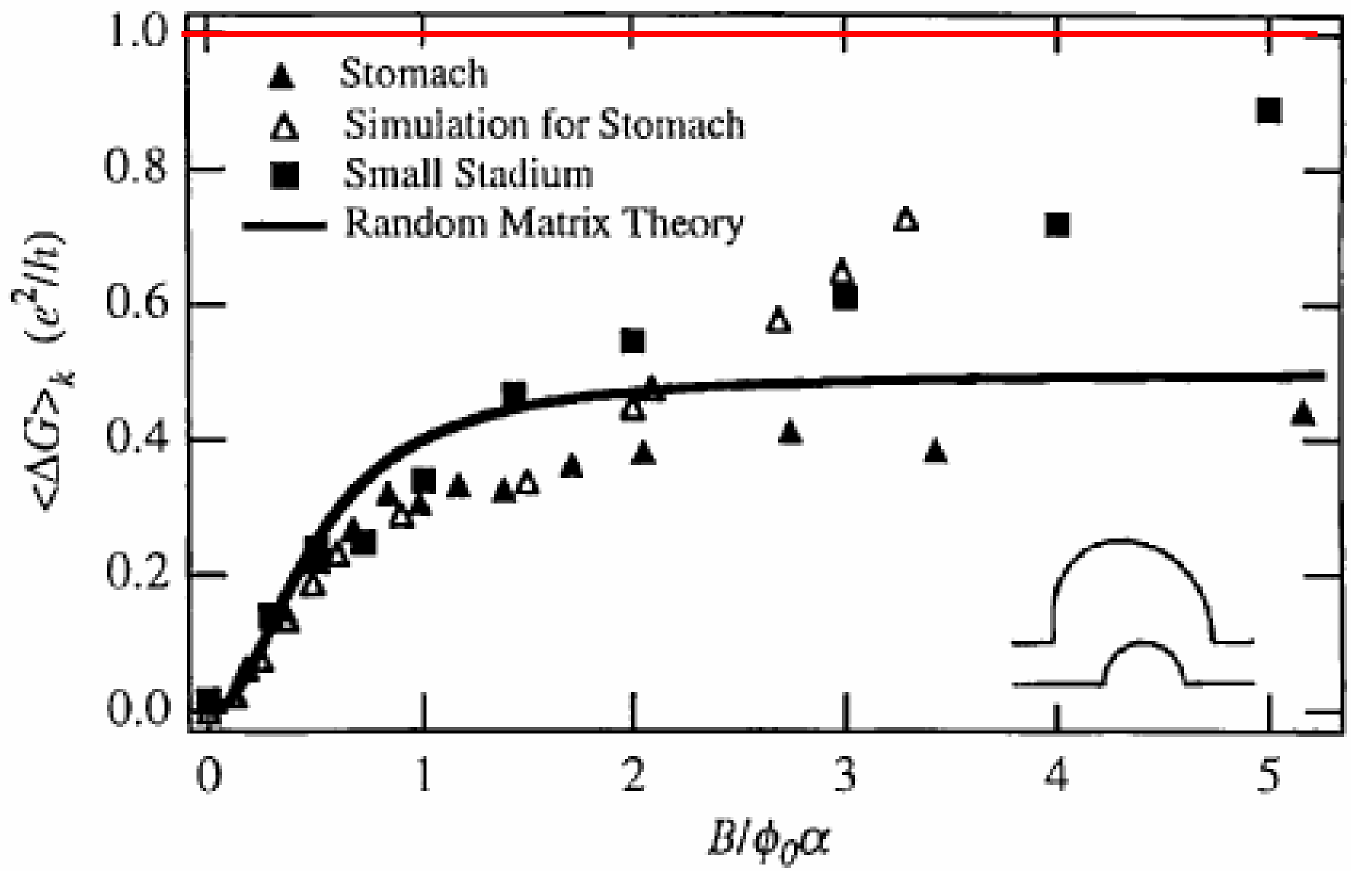
Agrees well with experiment and simulations

Actual data from ballistic junction -

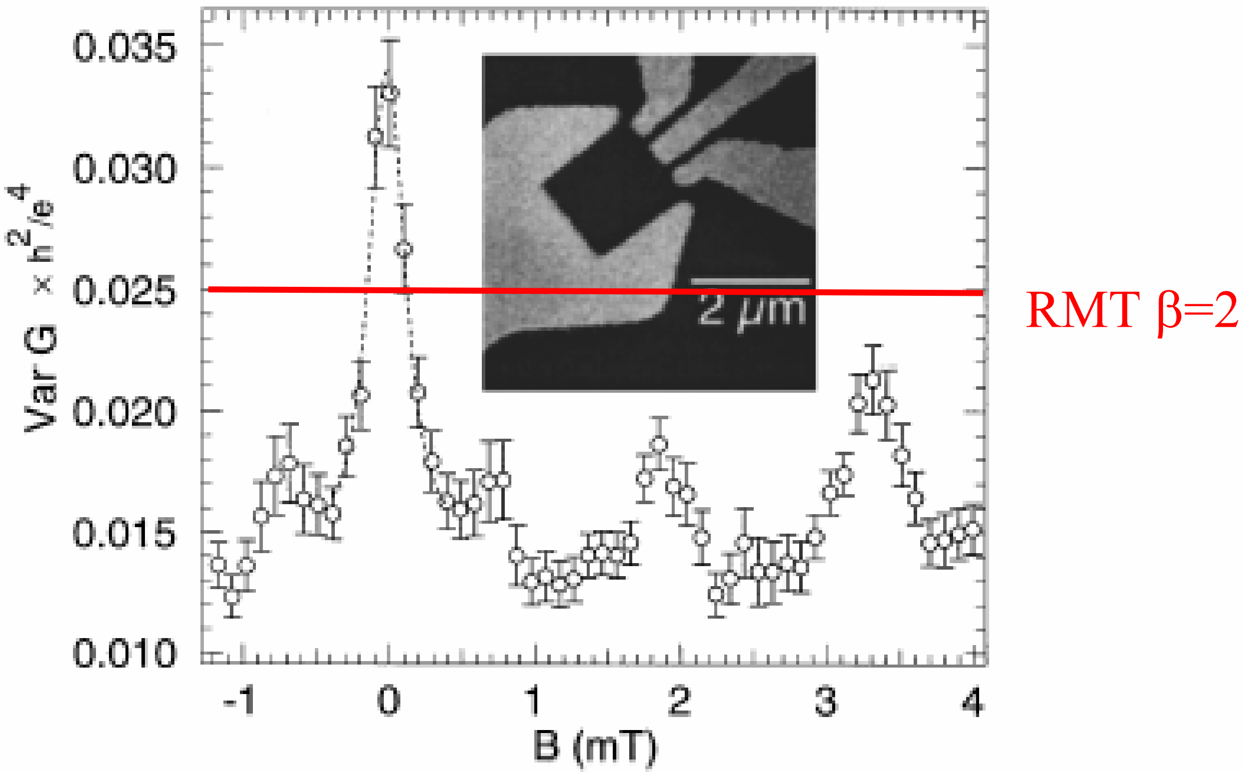
M

D.





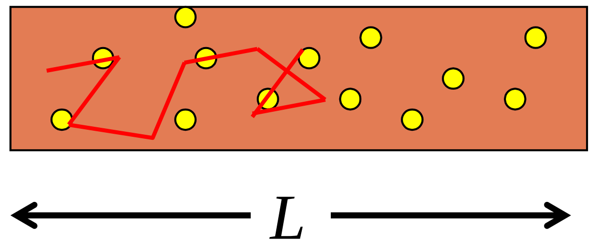
CB only



Variance of g for a quantum dot (Chan et al., 95),
Note factor of two reduction when $B \neq 0$ (β=2)

Disordered Wires

No $T \longleftrightarrow R$ symmetry

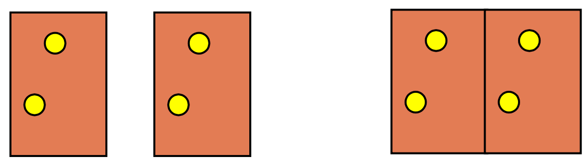


$\langle g \rangle = N(l/L)$, $l = mfp$

$\langle R \rangle \approx N$, $l << L$

Use M, not S

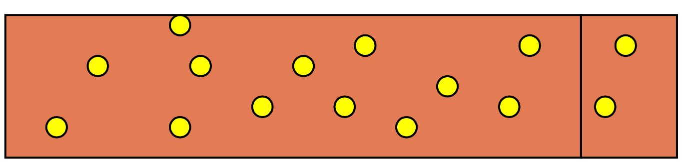
$$T_a = \frac{1}{1+\lambda_a}$$



M_1 M_2 $M = M_1 M_2$

Parameterize M(L) with polar decomposition

$$M = \begin{bmatrix} u_1 & 0 \\ 0 & u_3 \end{bmatrix} \begin{bmatrix} \sqrt{(1+\lambda)} & \sqrt{\lambda} \\ \sqrt{\lambda} & \sqrt{(1+\lambda)} \end{bmatrix} \begin{bmatrix} u_2 & 0 \\ 0 & u_4 \end{bmatrix}$$



M(L) dM(dL)

"isotropic"

$$P_{L+dL} = \int P_L(M)\, P_{dL}(dM)\, \delta(M_{L+dL} - M_L \bullet dM_{dL})$$

→ DMPK Equation

$$\boxed{l\frac{\partial P}{\partial L}=\frac{2}{\beta N+2-\beta}\sum_{n=1}^{N}\frac{\partial}{\partial\lambda_n}\lambda_n(1+\lambda_n)J\frac{\partial}{\partial\lambda_n}\frac{P}{J}}\qquad J=\prod_{i<j}^{N}|\lambda_i-\lambda_j|^{\beta}$$

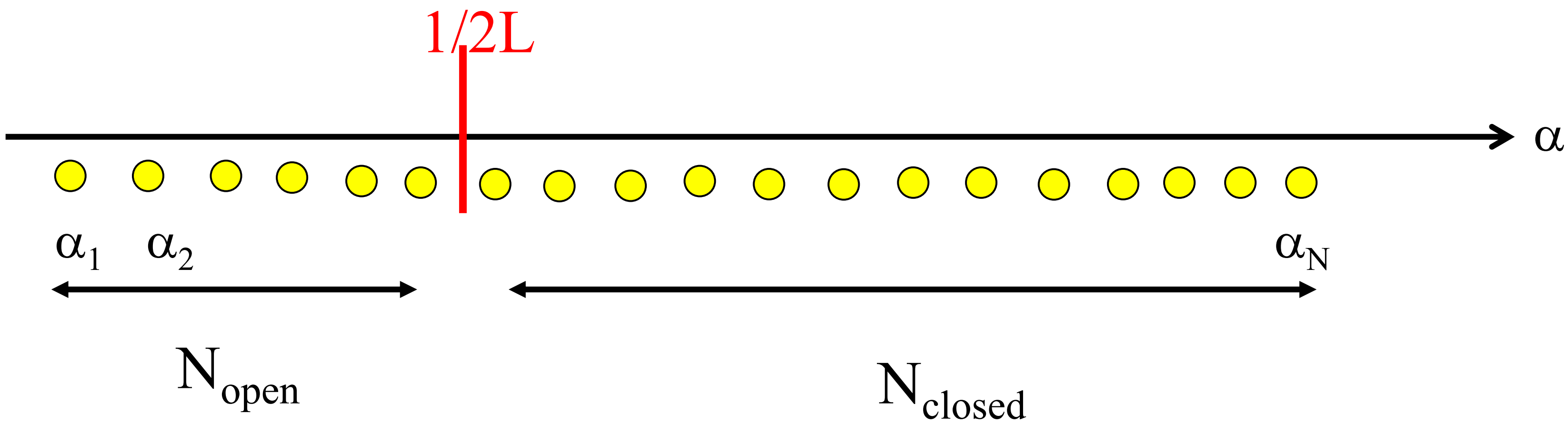
1D case solved early(1959), Mello (88,91), Beenakker (93), **RMP (97)**

Qualitative picture: Imry (86), open and closed channels

$$\lambda_n \equiv \sinh^2\nu_n,\quad T_n=\frac{2}{1+\cosh 2\nu_n}\Rightarrow g=\sum_n^N\frac{2}{1+\cosh 2\nu_n}$$

$$Thm: if M=\prod_{i=1}^{L}M_i,\quad \lim_{L\to\infty}\frac{\nu_n}{L}=\alpha_n$$ $\{\alpha_n\}$ = Inverse localization lengths

$$\Rightarrow \nu_n\approx\alpha_n L \qquad g=\sum_n^N\frac{2}{1+\cosh 2\alpha_n L}$$



$$g = \sum_{n}^{N} \frac{2}{1 + \cosh 2\alpha_n L}$$

$g \approx N_{open} = Nl/L$

$\Rightarrow \alpha_1 = 1/(2Nl)$

- When $L > 2Nl \Rightarrow g = \exp[-L/(Nl)]$ => quasi-1d localization, $\xi = Nl$.
- Fixed N (width) and increasing L always leads to localization.

$\mathrm{Var}(g) = \mathrm{Var}(N_{open}) \approx 1$ (spectral rigidity, $P(\{\alpha_n\}) \propto \prod_{open} |\alpha_n - \alpha_m|^{\beta}$)

UCF: $\mathrm{Var}(g) = 2/(15\beta)$, $\delta g_{WL} = -1/3$

Eigenvalue density

$$\rho(T)dT = \tilde{\rho}(\nu)d\nu$$

$$\rho(T) = \rho_0 \frac{d\nu}{dT}$$

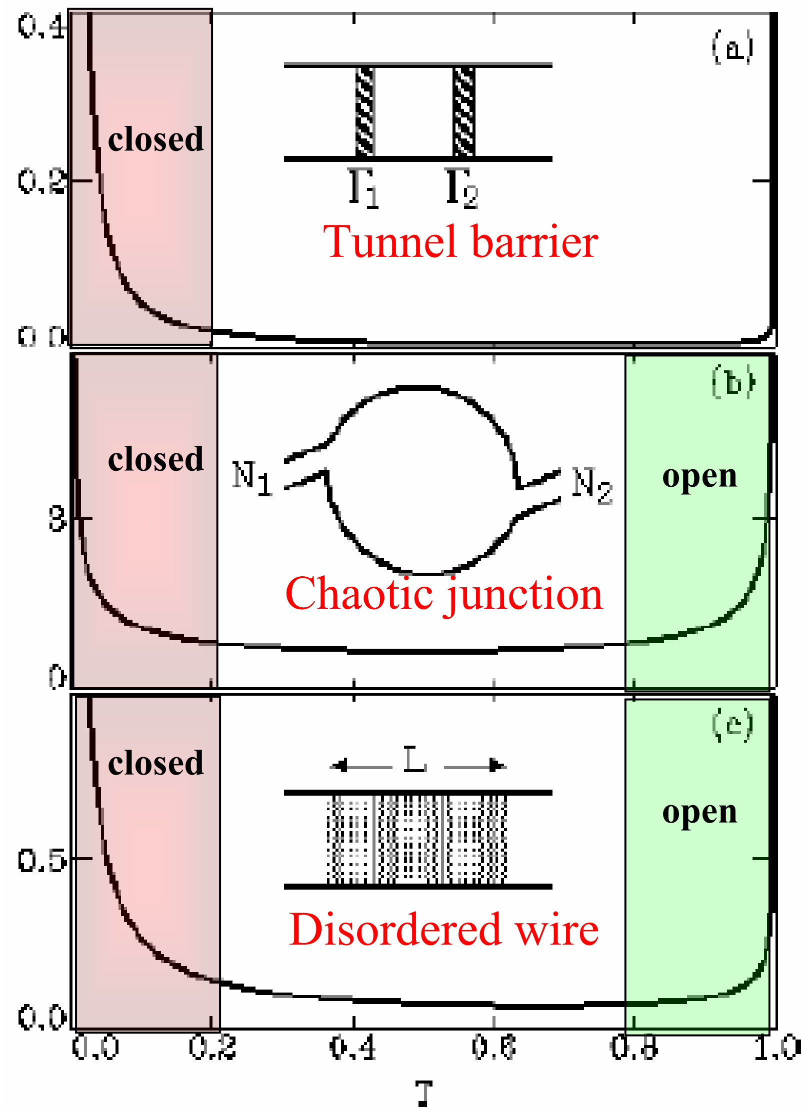
$$1/T = \cosh^2(\nu)$$

$$\boxed{\rho(T) = \frac{Nl}{2L}\frac{1}{T\sqrt{1-T}}}$$

disordered

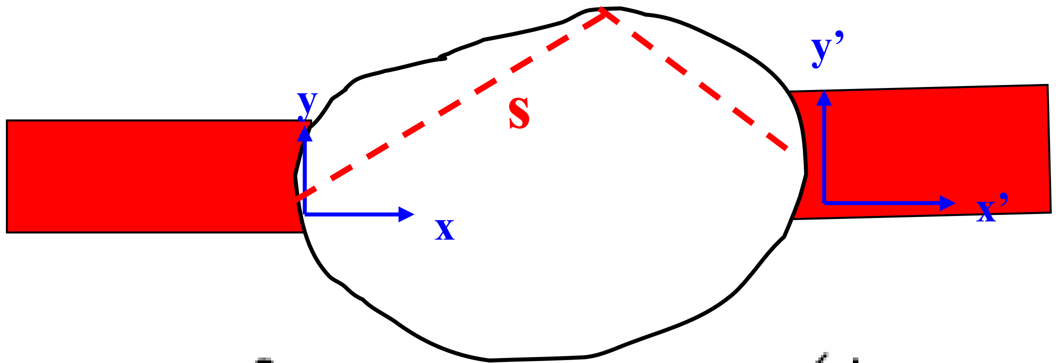
$$\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}$$

Ballistic/chaotic



Semiclassical Method for Ballistic Junctions

$$t_{ab} = -i\hbar(v_a v_b)^{1/2} \int dy' \int dy\, \phi_a^*(y')\, \phi_b(y)\, G(L, y'; 0, y; E).$$



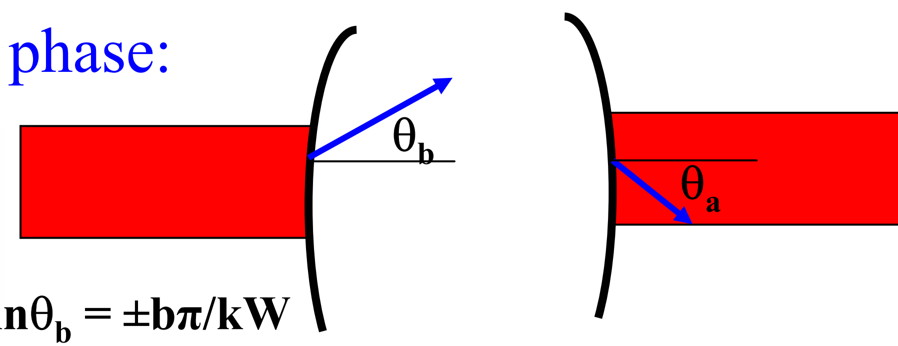
$$G^{\rm scl}(y'; y; E) = \frac{2\pi}{(2\pi i\hbar)^{(3/2)}} \sum_{s(y,y')} \sqrt{D_s} \exp\left(\frac{i}{\hbar} S_s(y', y, E_F) - i\frac{\pi}{2}\mu_s\right)$$

Obtained by stationary phase integration of FT of $G^{scl}(r,r',t)$;

$\int_0^T L\,dt \;\Rightarrow S_s(E) = \int_r^{r'} p\,dq = (h/2\pi)kL_s$ (for billiard)

Now do $\int\int dy\,dy'$ for t_{ab} by stationary phase:

$$\left(\frac{\partial S}{\partial y}\right)_{y'} = -p_y = -\frac{\bar{b}\hbar\pi}{W}, \quad \bar{b} = \pm b,$$



$$t_{ab} = -\frac{\sqrt{2\pi i\hbar}}{2W} \sum_{s(\bar{a},\bar{b})} \operatorname{sgn}(\bar{a}) \operatorname{sgn}(\bar{b}) \sqrt{\tilde{D}_s} \exp\left(\frac{i}{\hbar}\tilde{S}_s(\bar{a},\bar{b},E) - i\frac{\pi}{2}\tilde{\mu}_s\right),$$

$$\tilde{S}_s/\hbar - (\pi/2)\mu_s = kL_s + \phi_s(a,b)$$

$$|t_{ab}|^2 = \sum_{s,u} A_s A_u e^{ik(L_s - L_u) + i(\phi_s - \phi_u)}$$

g = N/2 comes from terms s=u, WL correction from s≠ u, but not simply from time-reversed pairs of path

$$|t_{ab}|^2 |t_{cd}|^2 = \sum_{s,u} \sum_{v,w} A_s A_u A_v A_w e^{ik(L_s + L_v - L_u - L_w) + i\Delta\phi_{abcd}}$$

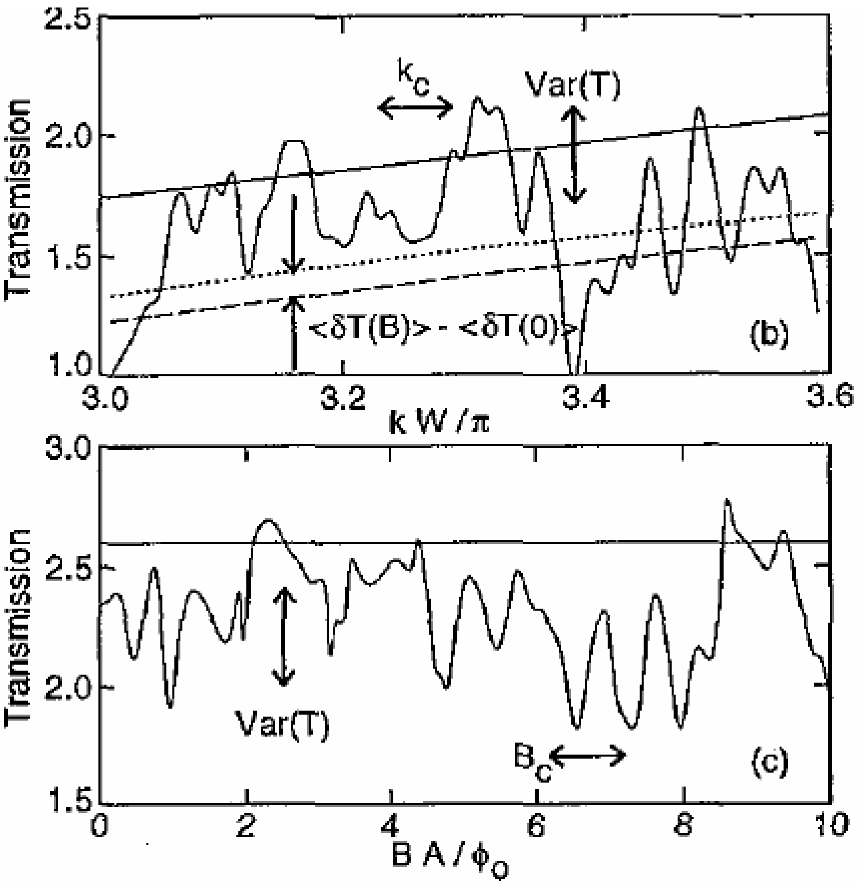
Conductance fluctuations come from random interference of paths, sensitive to B, k - Var(g) ≈ 1 comes from interference of all paths, require correlated actions for different paths

Fundamentally different from familiar speckle patterns

Can get dynamical scales just from diagonal terms

$$t_{ab}(k)t^*_{ab}(k+\Delta k) =$$

$$\sum_{s,u} A_s A_u e^{ikL_s + i\phi_s - i(k+\Delta k)L_u - i\phi_u}$$



Look at diagonal terms, s=u;

$$\sum_s A_s^2 e^{i\Delta k L_s}$$

Sum starts to decay when $\Delta k L_s \approx \pi$ => $k_c = <\pi/L> = \gamma_c$

For diffusive case $L_s = v_f t_D$ => $k_c = E_{th}/hv_f$

Similar analysis give $B_c = (h/e)/<A_{encl}>$

Quantitative approach next time.