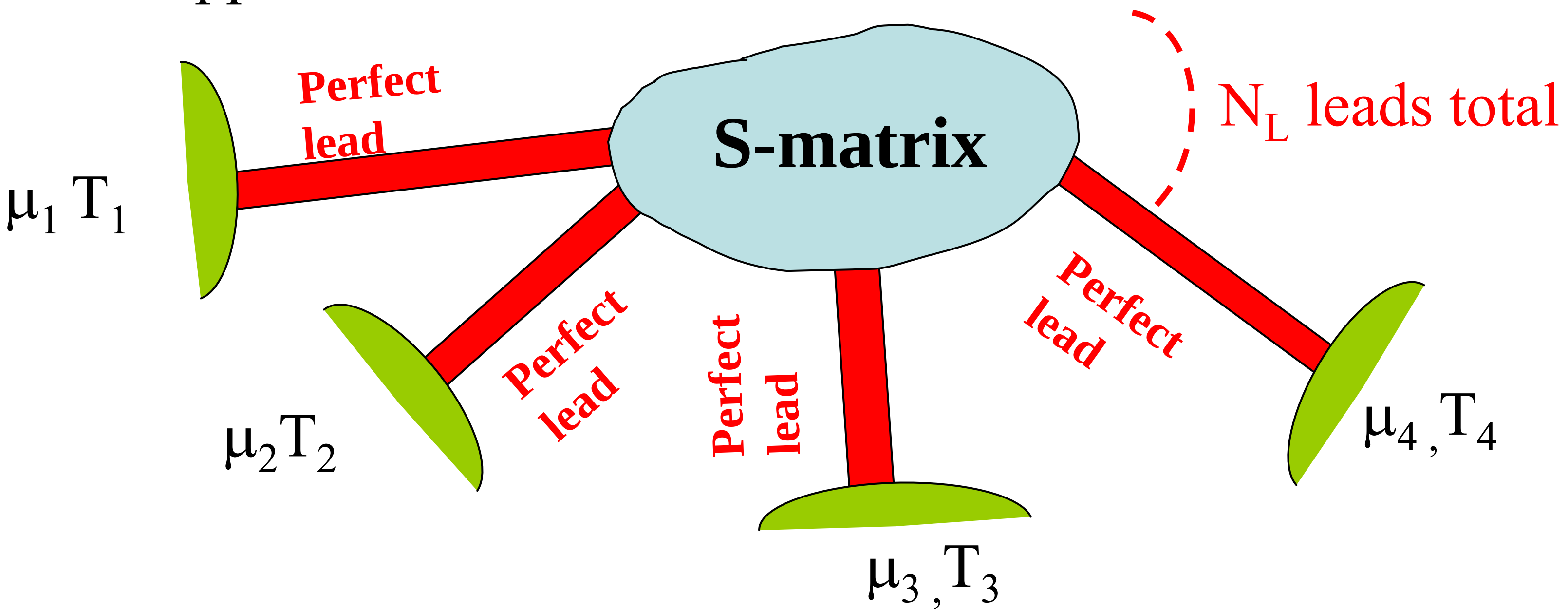
Transport theory of mesoscopic systems - random matrix approach to chaotic and disordered conductors



- Modeled by non-interacting Fermi gas of scattering states

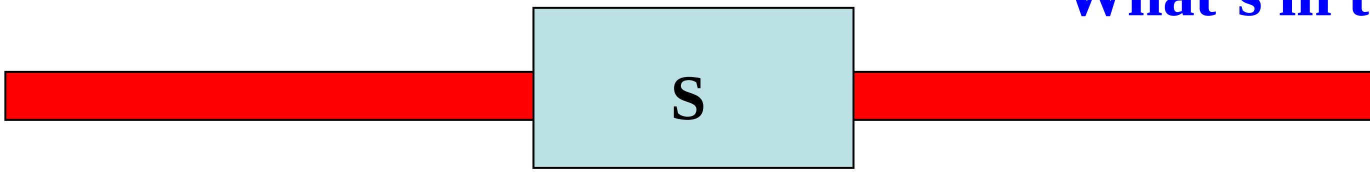
- Conductance and noise properties determined by S-matrix elements and their dependence on energy, magnetic field, gate voltage…

- 2 probe case: $G = (e^2/h)Tr\ \{\mathbf{tt}^\dagger\} = (e^2/h)\sum_a T_a$

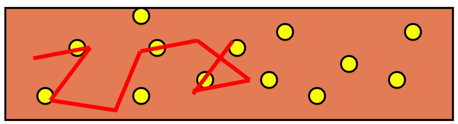
  $P = (2e^2/h)(\Delta\nu eV)\sum_a T_a(1 - T_a)$

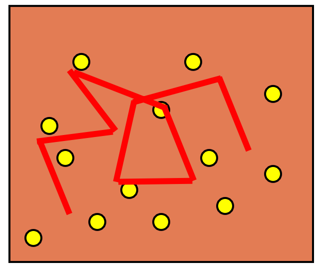
We will focus on 2-probe case

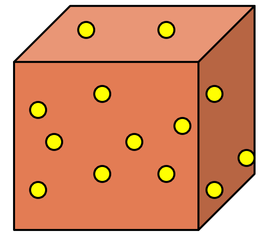
What's in the Box?



Disordered metal

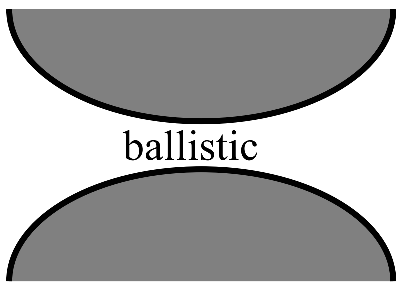






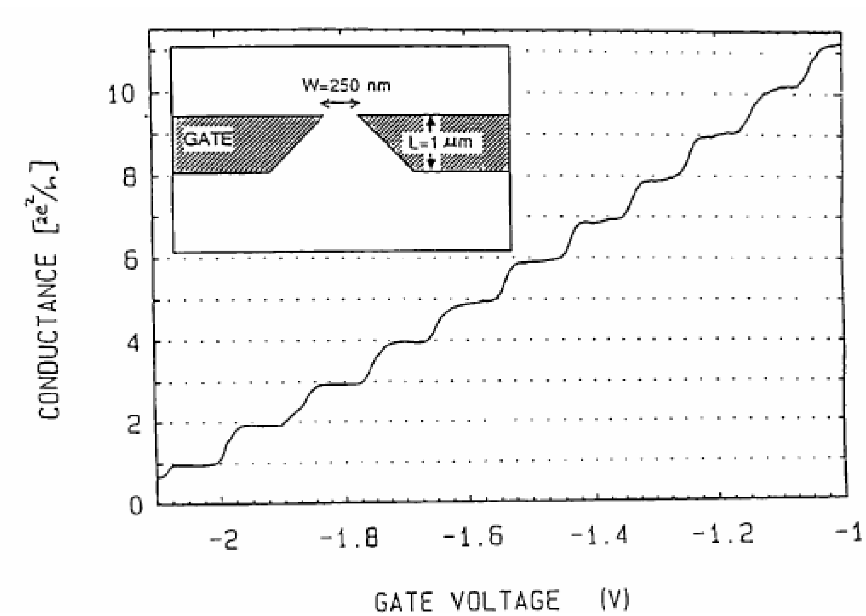
Weak localization
UCF , anomalous shot noise, anomalous thermopower…

Ballistic
point contact



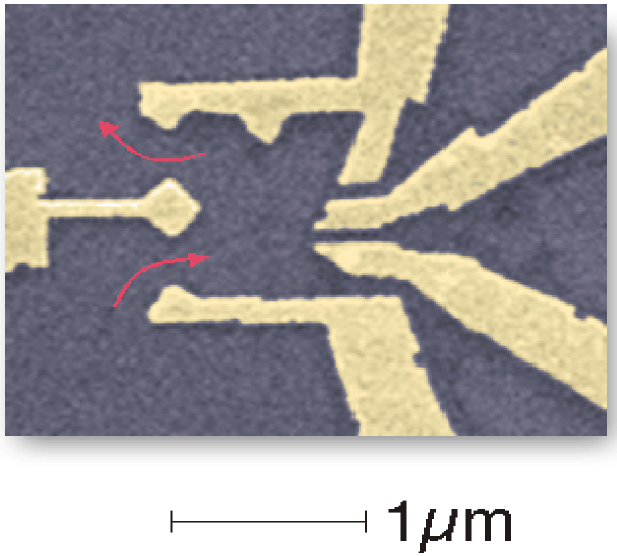
$$G = \frac{e^2}{h}\sum_{ab} T_{ab} = \frac{e^2}{h} N_{open}$$

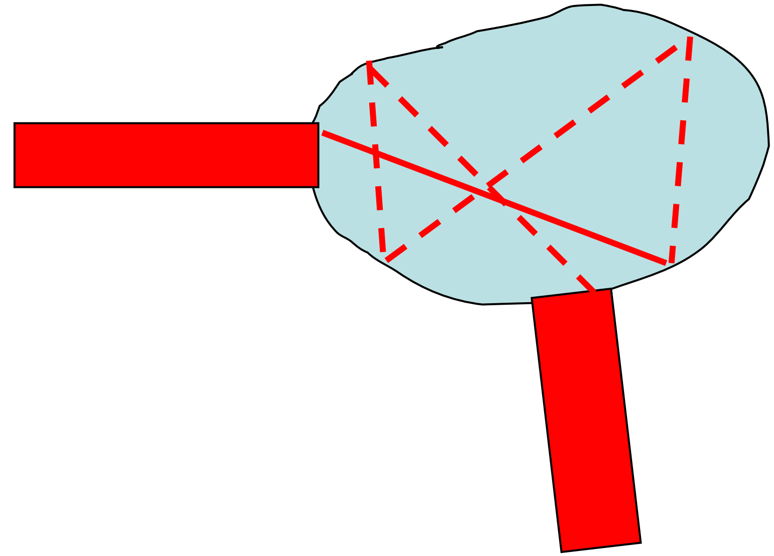
$$P_{shot} \propto T_n(1 - T_n) = 0$$



But not a result of adiabaticity! See Szafer and Stone PRL 1989

Ballistic
open
quantum
dot





Two approaches to understanding S-matrix:

• Semiclassical/statistical: dynamical/flexible (JBS 1990, 93,Marcus 1992)

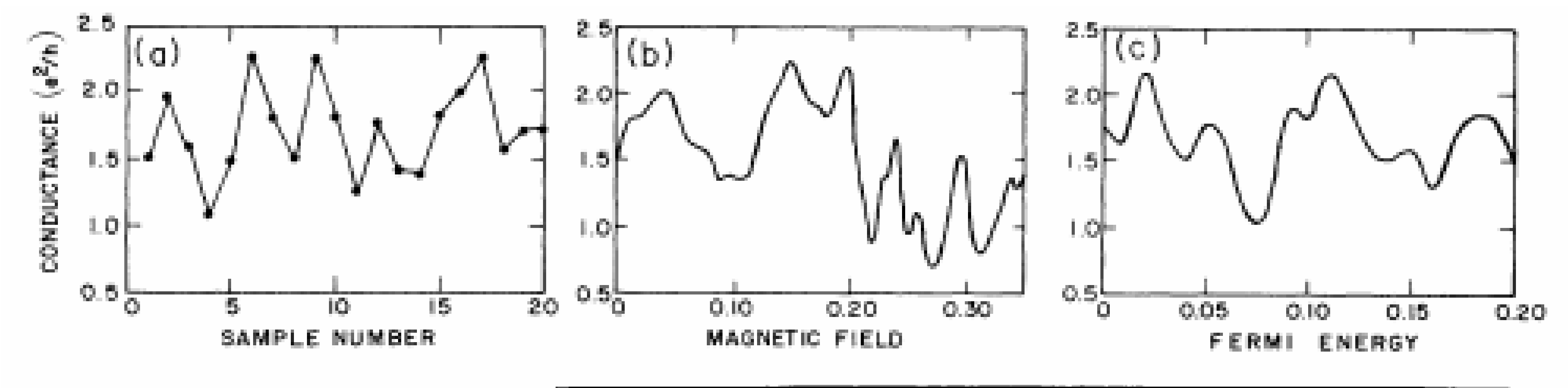
• Random matrix theory (hard chaos, purely statistical) (1994)
(Wigner, Dyson, Mehta…1950s - 1960s - Nuclear Physics; Quantum Chaos, BGS 1984; MesoPhys: Alt-Shkl. 1986, Imry 1986, Ballistic: Mello,Bar + Jalabert,Pich,Been 1994)

• Disordered Quasi-1D: Imry 1986, Muttalib et al. 1987, Dorokhov (1983) &MPK (1988), Beenakker (97) - relation to localization goes back to 1960's

• Belief: SC approach gave dynamical info, but not quantitative (see 1994 LH lectures); 2002 - Richter and Sieber solved problem!

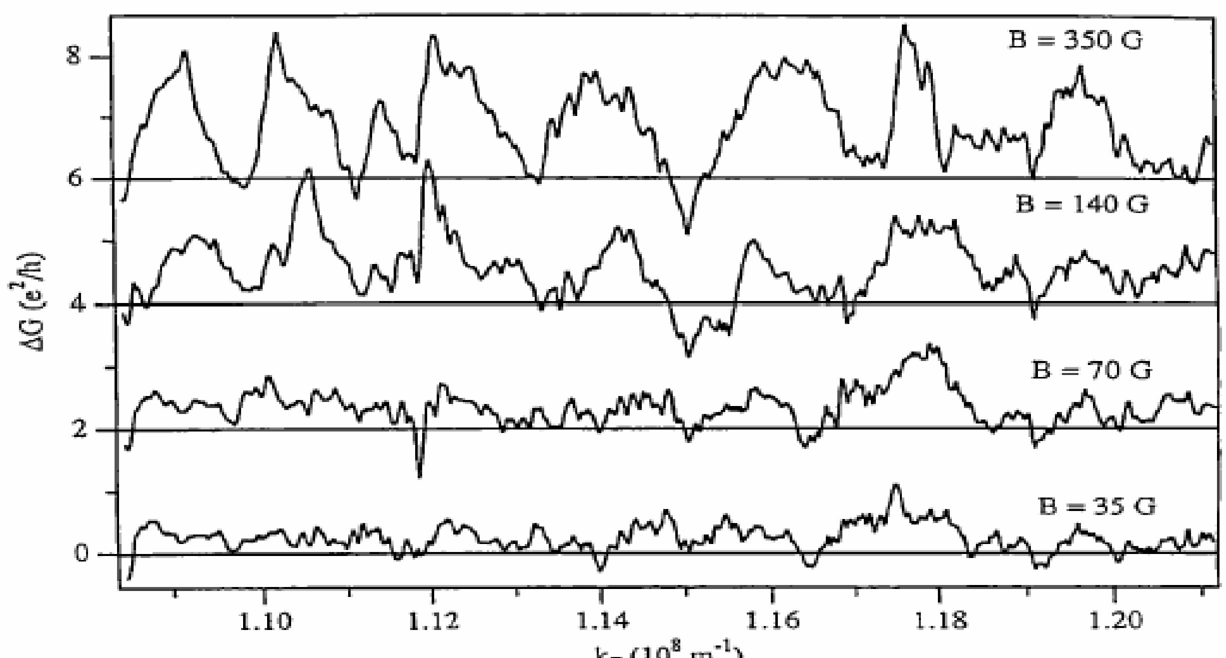
In both disordered and chaotic case it will be necessary to define ensembles and calculate averages over them - compare to exp't?

Ergodic hypothesis: (Lee and Stone, 1985)



Actual data from ballistic junction -

M. Keller and D. Prober 1995



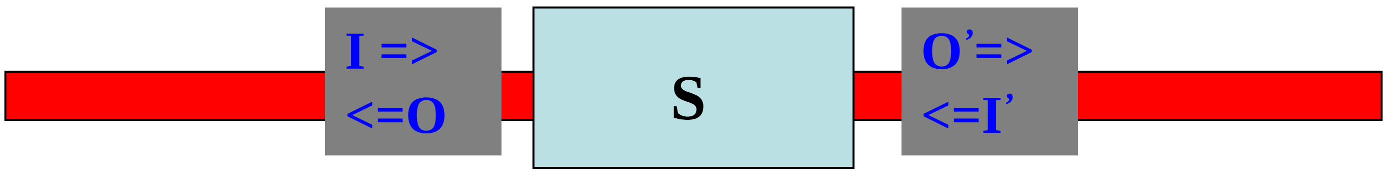
RMT of Ballistic Microstructures

Need $P(\{T_n\})$, then can calculate $\langle G \rangle$, Var(G), $\langle P_{shot} \rangle$ …

$\{T_n\}$ derived from S-matrix, need to define ensemble of S-matrices:

==> Most random distribution allowed by symmetry

$SS^\dagger = \mathbf{1}$ (no TR), $S = S^T$ (with TR symmetry)



$$S = \begin{bmatrix} \mathbf{r} & \mathbf{t} \\ \mathbf{t}' & \mathbf{r}' \end{bmatrix} \qquad \mathbf{S}\begin{bmatrix} \mathbf{I} \\ \mathbf{I}' \end{bmatrix} = \begin{bmatrix} \mathbf{O} \\ \mathbf{O}' \end{bmatrix}$$

S relates flux in to flux out, e-vectors and e-values not simply related to $\{T_n\}$

$$\mathbf{M}\begin{bmatrix} \mathbf{I} \\ \mathbf{O} \end{bmatrix} = \begin{bmatrix} \mathbf{O}' \\ \mathbf{I}' \end{bmatrix} \qquad \mathbf{M} = \begin{bmatrix} (\mathbf{t}^\dagger)^{-1} & \mathbf{r}'(\mathbf{t}')^{-1} \\ (\mathbf{t}')^{-1}\mathbf{r} & (\mathbf{t}')^{-1} \end{bmatrix}$$

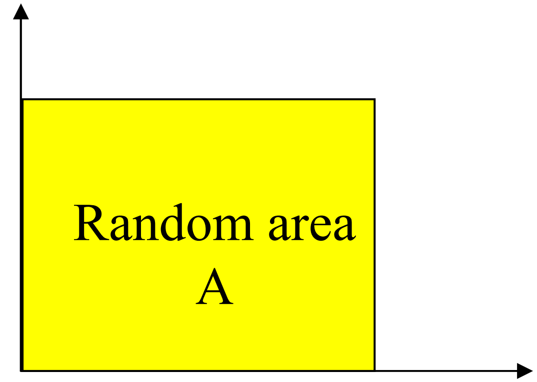
e-values of $MM^\dagger$ related to $\{T_n\}$, also M multiplicative - crucial for disordered case (later) - defined parameterization of S we need.

What does "most random" mean for an ensemble of matrices?

e.g 2D space: $P(x,y)dxdy$

Most random : $P(x,y)dxdy = dxdy/(\text{Area})$

$\mu(\mathbf{dr}) = dxdy/A$



Change variables: $r,\theta \rightarrow \mu(\mathbf{dr}) = r\,dr\,d\theta/A$

Where from?

$\mathbf{dr}\cdot\mathbf{dr} = dx^2+dy^2 = dr^2 + (r d\theta)^2 = \sum_{ij} g_{ij} q_i q_j \Rightarrow \mu(\mathbf{dr}) = (\det[\mathbf{g}])^{1/2}$

Need P(S), P(M), P(H)… must define space and metric for matrices

Dim. = # of ind. Parameters = $4N^2$, $N(2N+1)$; N channels, 2N x 2N matrices - what is metric? $dS^2 = \mathrm{Tr}\{\mathbf{dSdS^\dagger}\} \rightarrow \mathbf{g} \rightarrow \mu(\mathbf{dS}) = (\det[\mathbf{g}])^{1/2}$

Example: 2 x 2 real symmetric matrix (TR inv. TLS hamiltonian)

$$H = \begin{bmatrix} h_1 & h_3 \\ h_3 & h_2 \end{bmatrix} \qquad H + dH = \begin{bmatrix} h_1 + dh_1 & h_3 + dh_3 \\ h_3 + dh_3 & h_2 + dh_2 \end{bmatrix}$$

$$Tr\{\mathbf{dHdH^T}\} = dh_1^2 + dh_2^2 + 2dh_3^2$$

$$\longrightarrow \quad \mu(\mathbf{dH}) = dh_1 dh_2 2dh_3$$

More useful coordinate system: E-values + e-vectors

$$E_{1,2} = \frac{1}{\sqrt{2}}[\sigma \pm \sqrt{\Delta^2 + 2h_3^2}\,] \qquad \sigma = \frac{1}{\sqrt{2}}[h_1 + h_2] \quad \Delta = \frac{1}{\sqrt{2}}[h_1 - h_2]$$

$$dh_1 dh_2 2dh_3 = d\sigma d\Delta 2dh_3 = J(E_1, E_2, h_3) dE_1 dE_2 2dh_3$$

$$J(E_1, E_2, h_3) = J(E_1, E_2) = \begin{vmatrix} \partial\sigma/\partial E_1 & \partial\sigma/\partial E_2 \\ \partial\Delta/\partial E_1 & \partial\Delta/\partial E_2 \end{vmatrix}$$

$$\partial\Delta/\partial E_1 = \sqrt{(\Delta^2 + 2h_3^2)/2} = (E_1 - E_2)/2$$

$\mu(\mathbf{dH}) \propto (E_1 - E_2) dE_1 dE_2 dh_3$ **Eigenvalue repulsion, non-trivial metric**

$\mu(\mathbf{dH}) \propto (E_1 - E_2)^\beta dE_1 dE_2$ β = 1,2,4 for 3 symm. classes

Parameterize M, then S: “polar decomposition”

$$\mathbf{M} = \begin{bmatrix} \mathbf{u_1} & 0 \\ 0 & \mathbf{u_3} \end{bmatrix} \begin{bmatrix} \sqrt{(1+\lambda)} & \sqrt{\lambda} \\ \sqrt{\lambda} & \sqrt{(1+\lambda)} \end{bmatrix} \begin{bmatrix} \mathbf{u_2} & 0 \\ 0 & \mathbf{u_4} \end{bmatrix}$$

$\lambda = \mathrm{Diag}(\lambda_1, \lambda_2, \ldots \lambda_N)$, $\mathbf{u_i}$ are N x N unitary matrices

with TR: $u_3 = u_1^*$, $u_4 = u_2^*$.

$$[2+MM^\dagger+(MM^\dagger)^{-1}]^{-1}=\frac{1}{4}\begin{pmatrix} tt^\dagger & 0 \\ 0 & t'^\dagger t' \end{pmatrix}$$

$$T_a = \frac{1}{1+\lambda_a}, \quad R_a = \frac{\lambda_a}{1+\lambda_a}$$

Find $\mu(\mathbf{dM})$ in terms of $\{\lambda_a\}$, then $P(\{\lambda_a\}) \Rightarrow P(\{T_a\})$, avg.s of g

Can work directly with S

$$S=\begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}\begin{pmatrix} -\sqrt{1-\mathcal{T}} & \sqrt{\mathcal{T}} \\ \sqrt{\mathcal{T}} & \sqrt{1-\mathcal{T}} \end{pmatrix}\begin{pmatrix} U' & 0 \\ 0 & V' \end{pmatrix}.$$

$$\mathcal{T} = Diag(T_1, T_2 \ldots T_N) \quad d\mu(S)=J\prod_\alpha d\mu(U_\alpha)\prod_i dT_i$$

$$J(\{T_n\}) = \prod_{i<j} |T_i - T_j|^{\beta} \prod_k T_k^{-1+\beta/2}$$

$$P(\{T_n\}) = c \prod_{i<j} |T_i - T_j|^{\beta} \prod_k T_k^{-1+\beta/2} \exp[-\beta f(T_k)]$$

$$P(\{T_a\}) = C_\beta \exp[-\beta \sum_{a<b} \ln|T_a - T_b| + \sum_c V_\beta(T_c)]$$

Have the j.p.d, what do we need to do with it?

$$< g >=< \sum_a^N T_a >, \; < g^2 >=< \sum_{a,b}^N T_a T_b >, \; < P >=< \sum_a^N T_a(1 - T_a) >$$

WL UCF MesoNoise

$$< g >= \int \prod_{c=1}^N dT_c P(\{T_c\}) \sum_a T_a$$

$$= N \int \prod_{c=1}^N dT_c P(\{T_c\}) T_1 = \int_0^1 \rho(T) T$$

$$\rho(T) = \int \prod_{c=2}^N dT_c P(\{T_c\})$$

For Var(g) need

$$K(T, T') = \int \prod_{c=3}^N dT_c P(\{T_c\}) - \rho(T)\rho(T')$$

Need 1-pt and 2-pt correlation fcns of the jpd of $\{T_n\}$

Many methods to find these fcns and the two-pt corr. fcn is "universal" upon rescaling if only logarithmic correlations

Nice approach for β=2 is method of orthogonal polynomials

$$\prod_{a<b}^{N} |T_a - T_b|^2 = \begin{vmatrix} p_1(T_1) & p_1(T_2) & \dots & p_1(T_N) \\ p_2(T_1) & p_2(T_2) & \dots & p_N(T_N) \\ \dots & & & \\ p_N(T_1) & p_N(T_2) & \dots & p_N(T_N) \end{vmatrix}^2$$

$$\equiv < \Psi_N | \Psi_N >$$

p_n= orthog poly, choose Legendre, [0,1]

$$\rho(T) = < \Psi_N | \sum_a \delta(T - T_a) | \Psi_N > = \sum_{n=1}^{N} p_n^2(T)$$

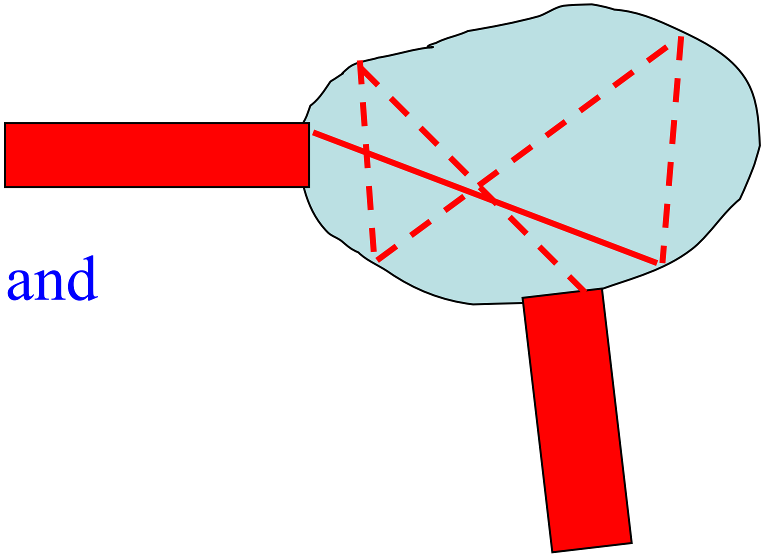
Use recursion relations, asymptotic form of p_n :

$$\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}$$

Same method gives K(T,T') in terms of $p_N p_{N-1}$

(normalized to N - so that G = (e^2/h) T

What do we expect for this system?



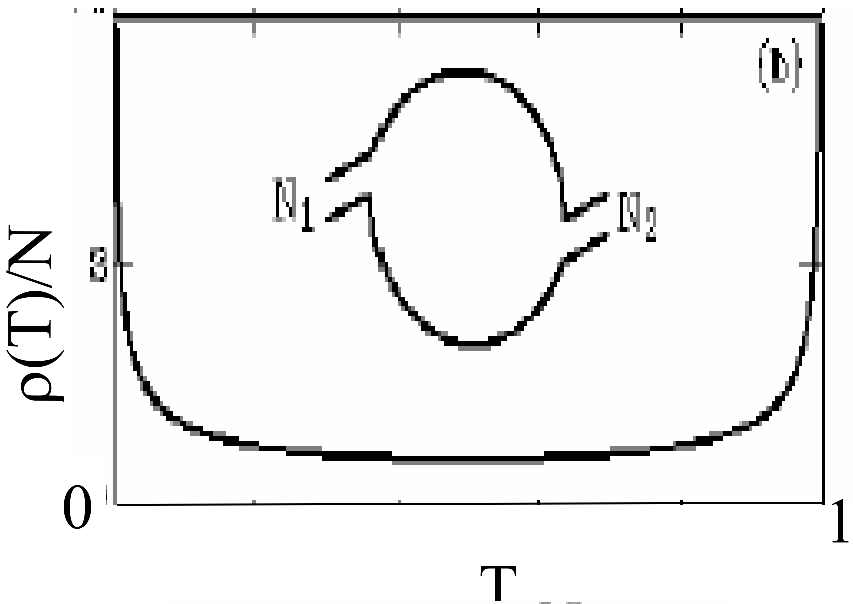
Classical symmetry betweeen reflection and transmission => <R> = <T> = N/2

$$< g >= \frac{N}{\pi}\int_0^1 dT\sqrt{\frac{T}{(1-T}} = \frac{N}{2}$$

Need to go to next order in N^{-1} to get WL effect

$$< P_{shot} > \propto \frac{N}{\pi}\int_0^1 dT\sqrt{T(1-T)}$$

$$= \frac{N}{8} \Rightarrow \frac{1}{4}P_{tunnel}$$



$$\rho(T) = \frac{N}{\pi\sqrt{T(1-T)}}$$

Due to symmetry between reflection and transmission can get order 1/N effects easily for the circular ensemble - don't need to distinguish T_{ab}, R_{ab} = S_{ab} just do averages over unitary group U(2N)

$$g = \sum_{a,b}^{N} |t_{ab}|^2 = \sum_{a,b}^{N} S_{ab} S^*_{ab}, \quad S \in U(2N)$$

$$< S_{ab} S^*_{cd} >_{CUE} = \int d\mu(S) S_{ab} S^*_{cd} = \frac{\delta_{ac}\delta_{bd}}{2N}$$

$$< S_{ab} S^*_{ab} >_{COE} = \int d\mu(U) [UU^\dagger]_{ab} = \frac{1+\delta_{ab}}{2N+1}$$

$$< g >_{CUE} = N^2 \cdot \frac{1}{2N} = \frac{N}{2}$$

$$< g >_{COE} = N^2 \cdot \frac{1}{2N+1} + N \cdot \frac{1}{2N+1} \approx \frac{N}{2} + \frac{1}{2} - \frac{1}{4}$$

Coherent backscattering ($+\frac{1}{2}$)

Off-diagonal correlations ($-\frac{1}{4}$)

δG_{WL} = (e^2/h)(1/4)

The Mystery

Similarly Var(g) = (1/8β)

All of these results are subtly different for a disordered/diffusive wire - will analyze next lecture