Phenomenological theory
and
symmetry aspects of
superconductivity

Generalized gap functions

Even parity, spin singlet:

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix} = i\sigma^y \psi(\vec{k})$$

Scalar wave function: $\psi(\vec{k})$ with $\psi(-\vec{k}) = \psi(\vec{k})$ $\quad |\Delta_{\vec{k}}|^2 = |\psi(\vec{k})|^2$

Odd parity, spin triplet:

$$\hat{\Delta}_{\vec{k}} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix} = i\vec{d}(\vec{k}) \cdot \vec{\sigma}\sigma^y$$

Vector wave function: $\vec{d}(\vec{k})$ with $\vec{d}(-\vec{k}) = -\vec{d}(\vec{k})$ $\quad |\Delta_{\vec{k}}|^2 = |\vec{d}(\vec{k})|^2$

Linearized gap equation and gap symmetry

Eigenvalue equation

$$-\lambda\psi(\vec{k}) = -N(0)\langle V_{\vec{k},\vec{k}'}\psi(\vec{k}')\rangle_{\vec{k}',FS}$$

$$-\lambda d_\mu(\vec{k}) = -N(0)\sum_\nu \langle V_{\vec{k},\vec{k}',\mu\nu} d_\nu(\vec{k}')\rangle_{\vec{k}',FS}$$

$$k_B T_c = 1.14\epsilon_c e^{-1/\lambda}$$

Solutions classified according to
the irreducible representations of symmetry group

representation Γ

example angular momentum l

degenerate solutions for $\lambda = N(0)V_\Gamma$

$\psi_{\Gamma m}(\vec{k})$

$\vec{d}_{\Gamma m}(\vec{k})$

$m = 1, \ldots, dim_\Gamma$

Symmetry operations

Symmetries of normal phase: $\mathcal{G} = G_o \times G_s \times K \times U(1)$ (G_o: orbital rotation, G_s: spin rotation, K: time reversal, $U(1)$: gauge)

symmetry operation	
orbital rotation	$\hat{g}_o c_{\vec{k}s} = c_{\hat{R}_o\vec{k}s}$ $\quad \hat{R}_o$ orbital rotation
spin rotation	$\hat{g}_s c_{\vec{k}s} = \sum_{s'} D_{ss'} c_{\vec{k}s'}$ $\quad \hat{D} = e^{i\vec{\theta}\cdot\hat{\vec{\sigma}}/2}$
time reversal	$\hat{K} c_{\vec{k}s} = \sum_{s'} (-i\sigma_y)_{ss'} c_{-\vec{k}s'}$
U(1) gauge	$\hat{\Phi} c_{\vec{k}s} = e^{i\phi/2} c_{-\vec{k}s'}$ $\quad \vec{A}' = \vec{A} + \frac{\hbar c}{2e}\vec{\nabla}\phi$

presence of strong spin-orbit coupling → spin and lattice rotation go together

Symmetry operations

Symmetries of normal phase: $\mathcal{G} = G_o \times G_s \times K \times U(1)$

symmetry operation	spin singlet	spin triplet
orbital rotation	$\hat{g}_o\psi(\vec{k}) = \psi(\hat{R}_o\vec{k})$	$\hat{g}_o\vec{d}(\vec{k}) = \vec{d}(\hat{R}_o\vec{k})$
spin rotation	$\hat{g}_s\psi(\vec{k}) = \psi(\vec{k})$	$\hat{g}_s\vec{d}(\vec{k}) = \hat{R}_s\vec{d}(\vec{k})$
time reversal	$\hat{K}\psi(\vec{k}) = \psi^*(\vec{k})$	$\hat{K}\vec{d}(\vec{k}) = \vec{d}^*(\vec{k})$
inversion	$\hat{I}\psi(\vec{k}) = \psi(\vec{k})$	$\hat{I}\vec{d}(\vec{k}) = -\vec{d}(\vec{k})$
U(1) gauge	$\hat{\Phi}\psi(\vec{k}) = e^{i\phi}\psi(\vec{k})$	$\hat{\Phi}\vec{d}(\vec{k}) = e^{i\phi}\vec{d}(\vec{k})$

presence of **strong spin-orbit coupling**

➡ spin and lattice rotation go together

spin triplet pairing: $g\vec{d}(\vec{k}) = \hat{R}_s\vec{d}(\hat{R}_o\vec{k})$ identical 3D rotations $\left\{ \begin{array}{l} \hat{R}_o \\ \hat{R}_s \end{array} \right.$

Symmetry operations

Symmetries of normal phase: $\mathcal{G} = G_o \times G_s \times K \times U(1)$

symmetry operation	spin singlet	spin triplet
orbital rotation	$\hat{g}_o\psi(\vec{k}) = \psi(\hat{R}_o\vec{k})$	$\hat{g}_o\vec{d}(\vec{k}) = \vec{d}(\hat{R}_o\vec{k})$
spin rotation	$\hat{g}_s\psi(\vec{k}) = \psi(\vec{k})$	$\hat{g}_s\vec{d}(\vec{k}) = \hat{R}_s\vec{d}(\vec{k})$
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inversion	$\hat{I}\psi(\vec{k}) = \psi(\vec{k})$	$\hat{I}\vec{d}(\vec{k}) = -\vec{d}(\vec{k})$
U(1) gauge	$\hat{\Phi}\psi(\vec{k}) = e^{i\phi}\psi(\vec{k})$	$\hat{\Phi}\vec{d}(\vec{k}) = e^{i\phi}\vec{d}(\vec{k})$

Possible broken symmetries:

U(1)-gauge	superconductivity
Orbital rotation	crystal structure
Spin rotation	magnetism
Time reversal	magnetism

Order parameter

relevant order parameter in represenation Γ with highest T_c (instability)

➡ *restrict to set (vector space basis)* $\left\{ \begin{array}{l} \{\psi_{\Gamma m}\} \\ \{\vec{d}_{\Gamma m}\} \end{array} \right.$

$$\psi(\vec{k}) = \sum_m \eta_m \psi_{\Gamma m}(\vec{k}) \qquad \vec{d}(\vec{k}) = \sum_m \eta_m \vec{d}_{\Gamma m}(\vec{k})$$

order parameter $\eta_m = \eta_m(\vec{r}, T)$ complex

trivial representation Γ ➡ *conventional pairing / SC phase*

e.g.: $l = 0$ *non-degenerate*

other representations Γ ➡ *unconventional pairing / SC phase*

e.g.: $l > 0$ *degeneracy* ≥ 1

Order parameter

relevant order parameter in represenation Γ with highest T_c (instability)

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order parameter $\eta_m = \eta_m(\vec{r}, T)$ complex

order parameters describe spontaneous symmetry breaking

➡ *Ginzburg-Landau theory*

Ginzburg-Landau theory for teneral order parameters

Landau's recipe: order parameters belong to irreducible representations of the normal state symmetry group

$$\psi(\vec{k}) = \sum_m \eta_m \psi_{\Gamma m}(\vec{k})$$

free energy functional as a scalar function of η_m { transform according to the representation

$$F[\eta_m] = \int d^3r \Big[a \sum_m |\eta_m|^2 + \sum_{m_1,\dots m_4} b_{m_1,\dots m_4} \eta^*_{m_1}\eta^*_{m_2}\eta_{m_3}\eta_{m_4} + \sum_{m_1,m_2}\sum_{n_1,n_2} K_{m_1 m_2 n_1 n_2} \left(D_{n_1}\eta_{m_1}\right)^* \left(D_{n_2}\eta_{m_2}\right) + \frac{1}{8\pi}\left(\vec{\nabla}\times\vec{A}\right)^2 \Big]$$

invariant under all symmetry operations of rotations, time reversal and *U(1)*-gauge

$a = a'(T - T_c)$, $b_m, K_{mm'nn'}$ real constants, $\vec{D} = \vec{\nabla} + i\frac{2e}{\hbar c}\vec{A}$ gradient: gauge-invariance

Ginzburg-Landau theory of conventional superconductor

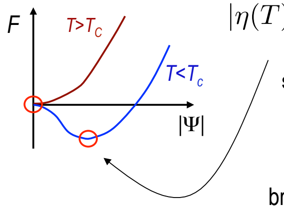
trivial representation $\psi(\vec{k}) = \eta(\vec{r},T)\psi_0(\vec{k})$ ← *Invariant under all symmetry operations*

GL free energy functional:

$$F[\eta,\vec{A}] = \int d^3r \left[a|\eta|^2 + b|\eta|^4 + K|\vec{D}\eta|^2 + \frac{\vec{\nabla}\times\vec{A}}{8\pi} \right]$$

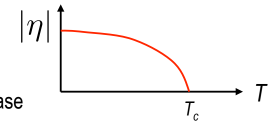
$a(T) = a'(T - T_c)$ $\quad a', b, K > 0 \quad \vec{D} = \vec{\nabla} + i\frac{2e}{\hbar c}\vec{A} \quad \vec{B} = \vec{\nabla}\times\vec{A}$

uniform phase:



$$|\eta(T)|^2 = \frac{a'(T_c - T)}{2b}$$

spontaneously chosen phase α



$$\eta = |\eta| e^{i\alpha}$$

broken *U(1)* gauge symmetry

Ginzburg-Landau theory of conventional superconductor

trivial representation $\psi(\vec{k}) = \eta(\vec{r},T)\psi_0(\vec{k})$ ← *Invariant under all symmetry operations*

GL free energy functional:

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variational GL equations for inhomogeneous order parameter:

- $\left\{ a + 2b|\eta|^2 - K\vec{D}^2 \right\}\eta = 0$ — *spatial structures: vortices, domain walls, boundary …*
- $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}$ — *magnetic properties* → *London equation*

supercurrent $\vec{J} = \frac{eK}{2\hbar i}\left\{ \eta^*(\vec{D}\eta) - \eta(\vec{D}\eta)^* \right\}$

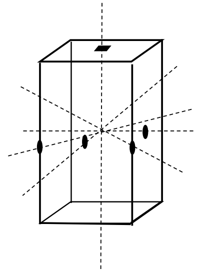
$$\vec{\nabla}^2\vec{B} = \lambda^{-2}\vec{B} \qquad \lambda^{-2} = \frac{32\pi e^2}{c^2}K|\eta|^2$$

Specific example:

Superconductor with tetragonal crystal structure

Example of a tetragonal crystal with spin orbit coupling

Point group: D_{4h}



D_{4h} contains inversion

→ even and odd representations

4 one-dim., 1 two-dim. representation

Character table for D_4

Γ	E	C_2	$2C_4$	$2C_2'$	$2C_2''$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

Example of a tetragonal crystal with spin orbit coupling

Point group: D_{4h}

4 one-dim., 1 two-dim. representation
even (g) / odd (u) parity

Γ	$\psi(\vec{k})$	Γ	$\vec{d}(\vec{k})$
A_{1g}	1	A_{1u}	$\hat{x}k_x + \hat{y}k_y$
A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	A_{2u}	$\hat{y}k_x - \hat{x}k_y$
B_{1g}	$k_x^2 - k_y^2$	B_{1u}	$\hat{x}k_x - \hat{y}k_y$
B_{2g}	$k_x k_y$	B_{2u}	$\hat{y}k_x + \hat{x}k_y$
E_g	$\{k_x k_z, k_y k_z\}$	E_u	$\{\hat{z}k_x, \hat{z}k_y\}$ $\{\hat{x}k_z, \hat{y}k_z\}$

Conventional: A_{1g}

Unconventional: everything else

only one representation is relevant for the superconducting phase transition

Ginzburg-Landau free energy functionals:

1-dimensional representations: $\psi(\vec{k}) = \eta\, \psi_0(\vec{k})$, $\vec{d}(\vec{k}) = \eta\, \vec{d}_0(\vec{k})$

$$F[\eta, \vec{A}] = \int d^3r \left[a|\eta|^2 + b|\eta|^4 + K|\vec{D}\eta|^2 + \frac{1}{8\pi}(\vec{\nabla}\times\vec{A})^2 \right]$$

like conventional SC

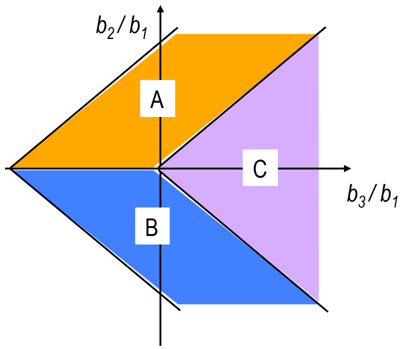
2-dimensional represenations: $\psi = \eta_x \psi_x + \eta_y \psi_y$, $\vec{d} = \eta_x \vec{d}_x + \eta_y \vec{d}_y$

$$F[\vec{\eta}, \vec{A}] = \int d^3r \left[a|\vec{\eta}|^2 + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\left\{\eta_x^{*2}\eta_y^2 + \eta_x^2\eta_y^{*2}\right\} + b_3|\eta_x|^2|\eta_y|^2 \right.$$
$$+ K_1(|D_x\eta_x|^2 + |D_y\eta_y|^2) + K_2(|D_x\eta_y|^2 + |D_y\eta_x|^2) + K_3(|D_z\eta_x|^2 + |D_z\eta_y|)$$
$$\left. + \{K_4(D_x\eta_x)^*(D_y\eta_y) + K_5(D_x\eta_y)^*(D_y\eta_x) + c.c.\} + \frac{1}{8\pi}(\vec{\nabla}\times\vec{A})^2 \right]$$

Possible homogeneous superconducting phases

Higher-dimensional order parameters are interesting $\vec{\eta} = (\eta_x, \eta_y)$

$$F[\vec{\eta}, \vec{A}] = \int d^3r \left[a|\vec{\eta}|^2 + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\left\{\eta_x^{*2}\eta_y^2 + \eta_x^2\eta_y^{*2}\right\} + b_3|\eta_x|^2|\eta_y|^2 \right]$$



phase	$\psi(\vec{k})$	$\vec{d}(\vec{k})$	broken symmetry
A	$(k_x \pm ik_y)k_z$	$\hat{z}(k_x \pm ik_y)$	$U(1), \mathcal{K}$
B	$(k_x \pm k_y)k_z$	$\hat{z}(k_x \pm k_y)$	$U(1), D_{4h} \to D_{2h}$
C	$k_x k_z, k_y k_z$	$\hat{z}k_x, \hat{z}k_y$	$U(1), D_{4h} \to D_{2h}$

$\mathcal{K}$ → magnetism

$D_{4h} \to D_{2h}$ → crystal deformation

Degeneracy: 2

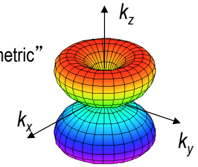
domain formation possible

Phases

A-phase: $\psi(\vec{k}) = \eta\, k_z(k_x + ik_y), \quad \eta\, k_z(k_x - ik_y)$

magnetic

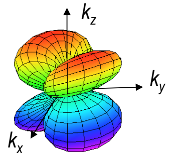
time reversal



B-phase: $\psi(\vec{k}) = \eta\, k_z(k_x + k_y), \quad \eta\, k_z(k_x - k_y)$

nematic

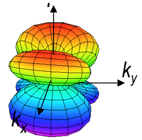
C_4



C-phase: $\psi(\vec{k}) = \eta\, k_z k_x, \quad \eta\, k_z k_y$

nematic

C_4



Weak-coupling condensation energy:

$$E_{kond} = -\frac{1}{2}\langle N_0(\vec{k})|\Delta_{\vec{k}}|^2\rangle_{\vec{k},FS}$$

less nodes → more stable

➡ A-phase more stable than B/C-phase

Anisotropy

B- and C-phase violated crystal symmetry: *tetragonal* → *orthorhombic*

➡ spontaneous crystal deformation *Tiny !*

Diamagnetic screening: supercurrents $\vec{j} = -c\dfrac{\partial F}{\partial \vec{A}}$

$$j_x = 8\pi e\left[K_1\eta_x^* D_x\eta_x + K_2\eta_y^* D_x\eta_y + K_3\eta_x^* D_y\eta_y + K_4\eta_y^* D_y\eta_x + c.c.\right]$$

$$j_y = 8\pi e\left[K_1\eta_y^* D_y\eta_y + K_2\eta_x^* D_y\eta_x + K_3\eta_y^* D_x\eta_x + K_4\eta_x^* D_x\eta_y + c.c.\right]$$

$$j_z = 8\pi e K_5\{\eta_x^* D_z\eta_x + \eta_y^* D_z\eta_y + c.c.\}\ .$$

tensorial London equation: $\nabla^2\vec{B} = \hat{\Lambda}\vec{B}$ Important for vortex lattice structure!

$$\hat{\Lambda}_A = \begin{pmatrix} \lambda^{-2} & 0 & 0 \\ 0 & \lambda^{-2} & 0 \\ 0 & 0 & \lambda_z^{-2} \end{pmatrix} \qquad \hat{\Lambda}_B = \begin{pmatrix} \lambda^{-2} & \tilde{\lambda}^{-2} & 0 \\ \tilde{\lambda}^{-2} & \lambda^{-2} & 0 \\ 0 & 0 & \lambda_z^{-2} \end{pmatrix} \quad \hat{\Lambda}_C = \begin{pmatrix} \lambda^{-2} & 0 & 0 \\ 0 & \lambda'^{-2} & 0 \\ 0 & 0 & \lambda_z^{-2} \end{pmatrix}$$

tetragonal (A); orthorhombic (B, C)

Volovik-Gor'kov classification

- *orbital angular momentum:* $\vec{L}_{\vec{k}} = i\hbar d_\nu^*(\vec{k})\{\vec{k}\times\vec{\nabla}_{\vec{k}}\}d_\nu(\vec{k})$

$\langle\vec{L}_{\vec{k}}\rangle_{\vec{k}} \neq 0$ *"ferromagnetic"* *(chiral)*

e.g. $(d_{x^2-y^2} + id_{xy})$ *-wave*

$\langle\vec{L}_{\vec{k}}\rangle_{\vec{k}} = 0$ *"antiferromagnetic"*

e.g. $(d_{x^2-y^2} + is)$ *-wave*

- *spin:* $\vec{S}_{\vec{k}} = i\hbar\vec{d}^*(\vec{k})\times\vec{d}(\vec{k})$

non-unitary states, e.g. A_1-phase of superfluid ^{3}He in a magnetic field

analogous Volovik-Gor'kov classification

Volovik-Gor'kov classification

- *orbital angular momentum:* $\vec{L}_{\vec{k}} = i\hbar d_\nu^*(\vec{k})\{\vec{k}\times\vec{\nabla}_{\vec{k}}\}d_\nu(\vec{k})$

Ferromagnetic or chiral phase A:

For pairing state within representation Γ

decomposition of $\Gamma\otimes\Gamma$ includes pseudovector representation

e.g.: *chiral p-wave* $\vec{d}(\vec{k}) = \hat{z}(k_x + ik_y)$ in E_u

$$E_u\otimes E_u = A_{1g}\oplus A_{2g}\oplus B_{1g}\oplus B_{2g}$$

A_{2g} → m_z

in a magnetic field

analogous Volovik-Gor'kov classification

Orbital angular momentum and magnetic moment

chiral p-wave phase: $\vec{d}(\vec{k}) = \eta_0 \hat{z}(k_x \pm i k_y) = \hat{z}\vec{\eta}\cdot\vec{k}$

with $\vec{\eta} = (\eta_x, \eta_y) = \eta_0(1, \pm i)$

orbital angular momentum:

$$\vec{L}_{\vec{k}} = |\eta_0|^2 \hbar \begin{pmatrix} k_z(k_x \mp i k_y) \\ -i k_z (k_x \mp i k_y) \\ \pm(k_x^2 + k_y^2) \end{pmatrix} \rightarrow \langle \vec{L}_{\vec{k}} \rangle_{\vec{k}} \parallel \hat{z}$$

$$\langle \vec{L}_{\vec{k}} \rangle_{\vec{k}} \propto i(\vec{\eta}^* \times \vec{\eta})$$

electron charge: $\langle \vec{L}_{\vec{k}} \rangle_{\vec{k}} \propto \vec{M}$ magnetic moment

Effective magnetic moment

Cooper pairs with angular momentum $L_z = \pm 1$

→ magnetic moment of superconductor $\vec{M} = \hat{z}\mu_B \frac{n_s}{2}$??

$\vec{d} = \hat{z}(k_x + i k_y) = \hat{z} k e^{i\theta}$

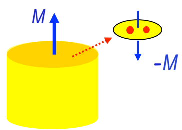
symmetry transformations:
- *rotation* $\vec{d} \rightarrow \vec{d} \times e^{i\phi}$
- *U(1)-gauge* $\vec{d} \rightarrow \vec{d} \times e^{i\phi}$

conserved "charge": $L_z - N/2$

N: electron charge

Volovik

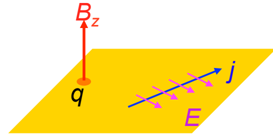
Moment



anomalous electromagnetism

$$\rho \approx \frac{e^2}{hc} B_z$$

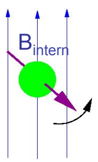
$$\vec{j} \approx \frac{e^2}{h}(\vec{E} \times \hat{z})$$



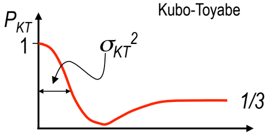
Goryo & Ishikawa
Volovik & Yakovenko

Intrinsic magnetism

μSR zero-field relaxation



depolarization in random internal field:

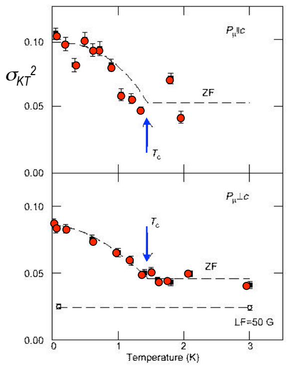


polarization of spin of trapped muon

$$P_{KT}(t) = \frac{1}{3}\left[1 + 2\left(1 - \sigma_{KT}^2 t^2\right)\exp(-\sigma_{KT}^2 t^2/2)\right]$$

σ_{KT}^2 : 2nd moment of field distribution

Sr_2RuO_4



Luke, Uemura et al (1998)

Superconductivity generates magnetism *(0.1 - 1 Gauss)*

other SC showing intrinsic magnetism: $U_{1-x}Th_xBe_{13}$, Re_6Zr, $PrOs_4Sb_{12}$, UPt_3, SrPtAs, ...

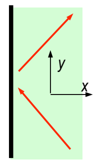
Surface states - spontaneous supercurrents for chiral phase

surface scattering detrimental interference effects:

E_u: $\vec{d}_x(\vec{k}) = \hat{z}k_x$

$\vec{d}_y(\vec{k}) = \hat{z}k_y$

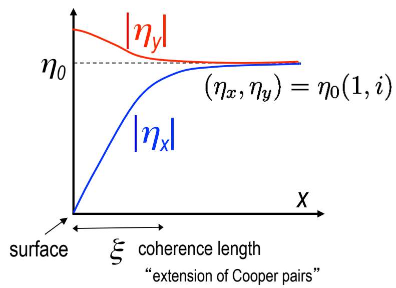
$\vec{d} = \eta_x(\vec{r})\vec{d}_x + \eta_y \vec{d}_y$



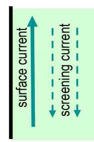
specular scattering $\begin{cases} k_x \rightarrow -k_x \\ k_y \rightarrow k_y \end{cases}$

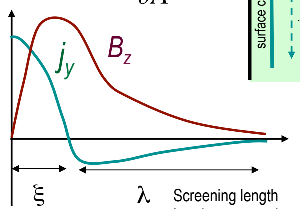
$\vec{d}_x \rightarrow -\vec{d}_x$ destructive

$\vec{d}_y \rightarrow +\vec{d}_y$ constructive



Supercurrent: $\vec{j} = -c\frac{\partial F}{\partial \vec{A}}$





Screening length
London penetration depth

Tests for the pairing symmetry

Example of a tetragonal crystal with spin orbit coupling

Point group: D_{4h}

4 one-dim., 1 two-dim. representation
even (g) / odd (u) parity

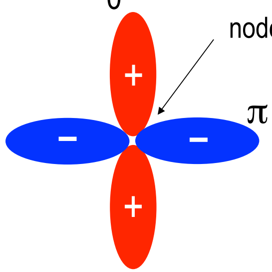
Γ	$\psi(\vec{k})$	Γ	$\vec{d}(\vec{k})$
A_{1g}	1	A_{1u}	$\hat{x}k_x + \hat{y}k_y$
A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	A_{2u}	$\hat{y}k_x - \hat{x}k_y$
B_{1g}	$k_x^2 - k_y^2$	B_{1u}	$\hat{x}k_x - \hat{y}k_y$
B_{2g}	$k_x k_y$	B_{2u}	$\hat{y}k_x + \hat{x}k_y$
E_g	$\{k_x k_z, k_y k_z\}$	E_u	$\{\hat{z}k_x, \hat{z}k_y\}$ $\{\hat{x}k_z, \hat{y}k_z\}$

cuprate high-T_c superconductor: B_{1g} $\psi_{B_{1g}}(\vec{k}) = k_x^2 - k_y^2$

$d_{x^2-y^2}$ - wave pairing

Internal phase structure

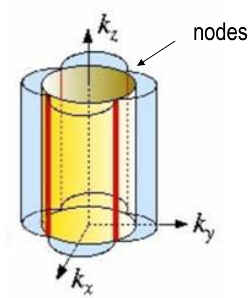
$$\psi(\vec{k}) = \Delta_0 (k_x^2 - k_y^2)$$



different phase in different directions

$$C_4\, \psi(k) = -\, \psi(k) = e^{i\pi}\, \psi(k)$$

Gap structure $|\Delta_{\vec{k}}| = |\psi(\vec{k})|$

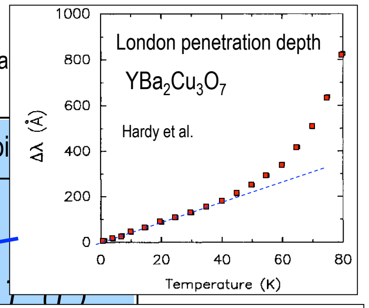


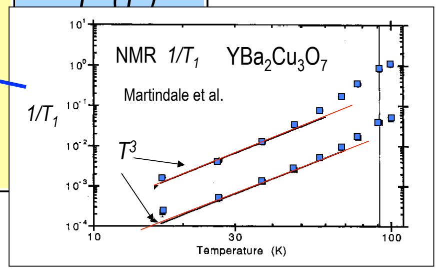
4 line nodes on basically cylindrical Fermi surface

Low-temperature properties

powerlaws in other quantities depending on ga

quantity	line nodes	poi
specific heat C(T)	T^2	
London penetration depth $\lambda(T)$	T (T^3)	
NMR $1/T_1$	T^3	
heat conductivity $\kappa(T)$	T^2	





high-temperature superconductors
with line nodes in the gap

Probing the phase structure

Phase probed by interference
using Josephson effect

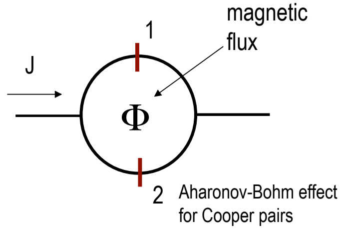
Supercurrent between SC1 and SC2

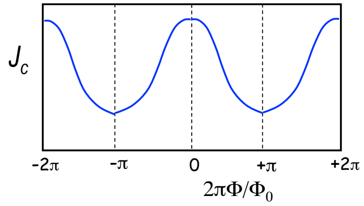
due to phase coherent Cooper pair tunneling

$$J = J_c \sin(\beta - \alpha)$$

Current determined by phase difference

SQUID (Superconducting QUantum Interference Device)





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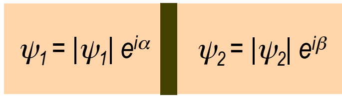

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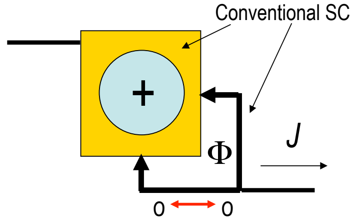

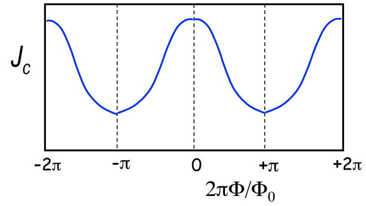

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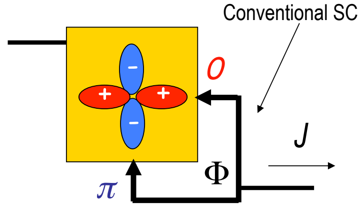
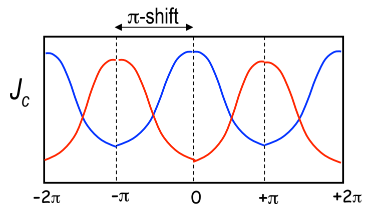
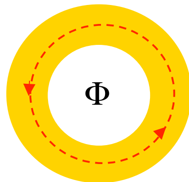




Wollman, van Harlingen et al. (1993), Brawner & Ott (1994), Mathai et al. (1995), Iguchi & Wen (1995)

Superconducting loops

Single-piece loop



Single valued order parameter

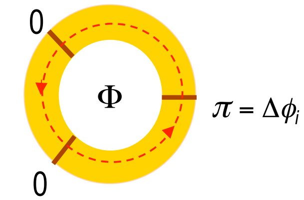
$\Psi = |\Psi|\, e^{i\phi}$ $\quad \phi \rightarrow \phi + 2\pi n$

$$J \propto \oint \left(\vec{A} - \frac{\Phi_0}{2\pi} \vec{\nabla}\phi \right) \cdot d\vec{s} = 0$$

$\Phi = \Phi_0 n$ $\quad n$: integer

unit-flux: $\Phi_0 = hc/2e$

Segmented loop



$$\oint \left(\vec{A} - \frac{\Phi_0}{2\pi} \vec{\nabla}\phi \right) \cdot d\vec{s} = \sum_i \Delta\phi_i$$

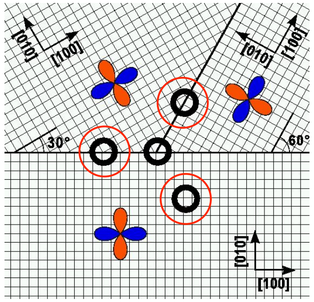
odd number of π−shifts $\Delta\phi_i$ $\quad \Phi = \Phi_0 \left(n + \frac{1}{2} \right)$

half-integer flux quantization

note: smallest flux $|\Phi| = \Phi_0 / 2$

Tsuei-Kirtley frustrated loops

Tri-cristall-configuration

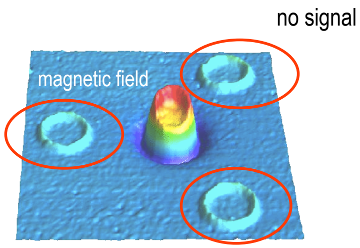


Tsuei, Kirtley et al. (1995)

Superconducting loop Ø = 60 μm
$YBa_2Cu_3O_7$ T_c = 92 K

even number of π-shifts

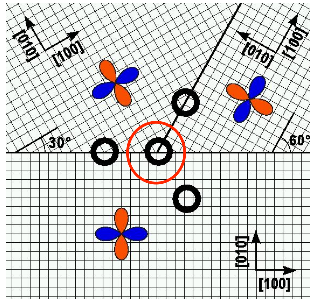
unfrustrated loops



SQUID-scanning-microscope

Tsuei-Kirtley frustrated loops

Tri-cristall-configuration



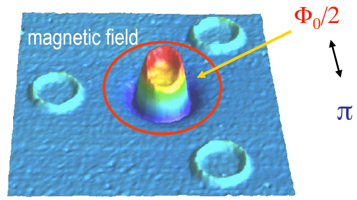
Tsuei, Kirtley et al. (1995)

Superconducting loop Ø = 60 μm
$YBa_2Cu_3O_7$ T_c = 92 K

odd number of π-shifts

frustrated loop

spontenous current and magnetic flux



SQUID-scanning-microscope