Professor Shankar's lectures:

RG for Fermions, Part I (9:00-10:30 AM, Monday, June 30)

RG for Fermions, Part II (9:00-10:30 AM, Wednesday, July 2)

RG for Fermions, Part III (9:00-10:30 AM, Monday, July 7)

Would be based on the paper:

Renormalization-group approach to interacting fermions, R. Shankar, Review of Modern Physics, Vol **66**, No. 1, p129-191 (1994)

Renormalization-group approach to interacting fermions

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The stability or lack thereof of nonrelativistic fermionic systems to interactions is studied within the renormalization-group (RG) framework, in close analogy with the study of critical phenomena using ϕ^4 scalar field theory. A brief introduction to ϕ^4 theory in four dimensions and the path-integral formulation for fermions is given before turning to the problem at hand. As for the latter, the following procedure is used. First, the modes on either side of the Fermi surface within a cutoff Λ are chosen for study, in analogy with the modes near the origin in ϕ^4 theory, and a path integral is written to describe them. Next, an RG transformation that eliminates a part of these modes, but preserves the action of the noninteracting system, is identified. Finally the possible perturbations of this free-field fixed point are classified as relevant, irrelevant or marginal. A $d=1$ warmup calculation involving a system of fermions shows how, in contrast to mean-field theory, which predicts a charge-density wave for arbitrarily weak repulsion, and superconductivity for arbitrarily weak attraction, the renormalization-group approach correctly yields a scale-invariant system (Luttinger liquid) by taking into account both instabilities. Application of the renormalization group in $d=2$ and 3, for rotationally invariant Fermi surfaces, *automatically* leads to Landau's Fermi-liquid theory, which appears as a fixed point characterized by an effective mass and a Landau function F, with the only relevant perturbations being of the superconducting (BCS) type. The functional flow equations for the BCS couplings are derived and separated into an infinite number of flows, one for each angular momentum. It is shown that similar results hold for rotationally noninvariant (but time-reversal-invariant) Fermi surfaces also, with obvious loss of rotational invariance in the parametrization of the fixed-point interactions. A study of a nested Fermi surface shows an additional relevant flow leading to charge-density-wave formation. It is pointed out that, for small Λ/K_F, a $1/N$ expansion emerges, with $N=K_F/\Lambda$, which explains why one is able to solve the narrow-cutoff theory. The search for non-Fermi liquids in $d=2$ using the RG is discussed. Bringing a variety of phenomena (Landau theory, charge-density waves, BCS instability, nesting, etc.) under the one unifying principle of the RG not only allows us to better understand and unify them, but also paves the way for generalizations and extensions. The article is pedagogical in nature and is expected to be accessible to any serious graduate student. On the other hand, its survey of the vast literature is mostly limited to the RG approach.

CONTENTS

I. INTRODUCTION

This article is an expanded version of a short paper (Shankar, 1991) in which the application of the renormalization-group (RG) methods to interacting nonrelativistic fermions in more than one spatial dimension was considered. It contains more technical details than its predecessor and is much more pedagogical in tone. Several related topics are reviewed here so that readers with a variety of backgrounds may find the article accessible and self-contained. Consequently each reader is likely to run into some familiar topics. When this happens he or she should go through the section quickly to ensure that this is indeed the case and get used to the no-

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