Lecture 3: Unconventional quantum criticality

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Boulder School Lectures

The Mott transition

Tussle between kinetic and interaction energies most strikingly evident in vicinity of a 'Mott' transition.

System poised at the brink of transition between metal/ $\underbrace{\text{superconductor/superfluid}}_{\text{mobile particles}}$ and $\underbrace{\text{insulator}}_{\text{localized particles}}$.

The simple case: Bosons at integer filling

Review: Simple Mott transition of bosons

Bosons on a lattice at integer filling (say 1 boson/site on average)

Boson Hubbard model

$$H = -t \sum_{\langle ij \rangle} \left(b_i^\dagger b_j + h.c \right) + U \sum_i \frac{n_i(n_i - 1)}{2}$$

$t \gg U$: Superfluid state

$U \gg t$: Mott insulator with 1 boson per site

Approaching the transition

Approach from SF :

Condensate fraction Z

Superfluid density ρ_s (phase stiffness)

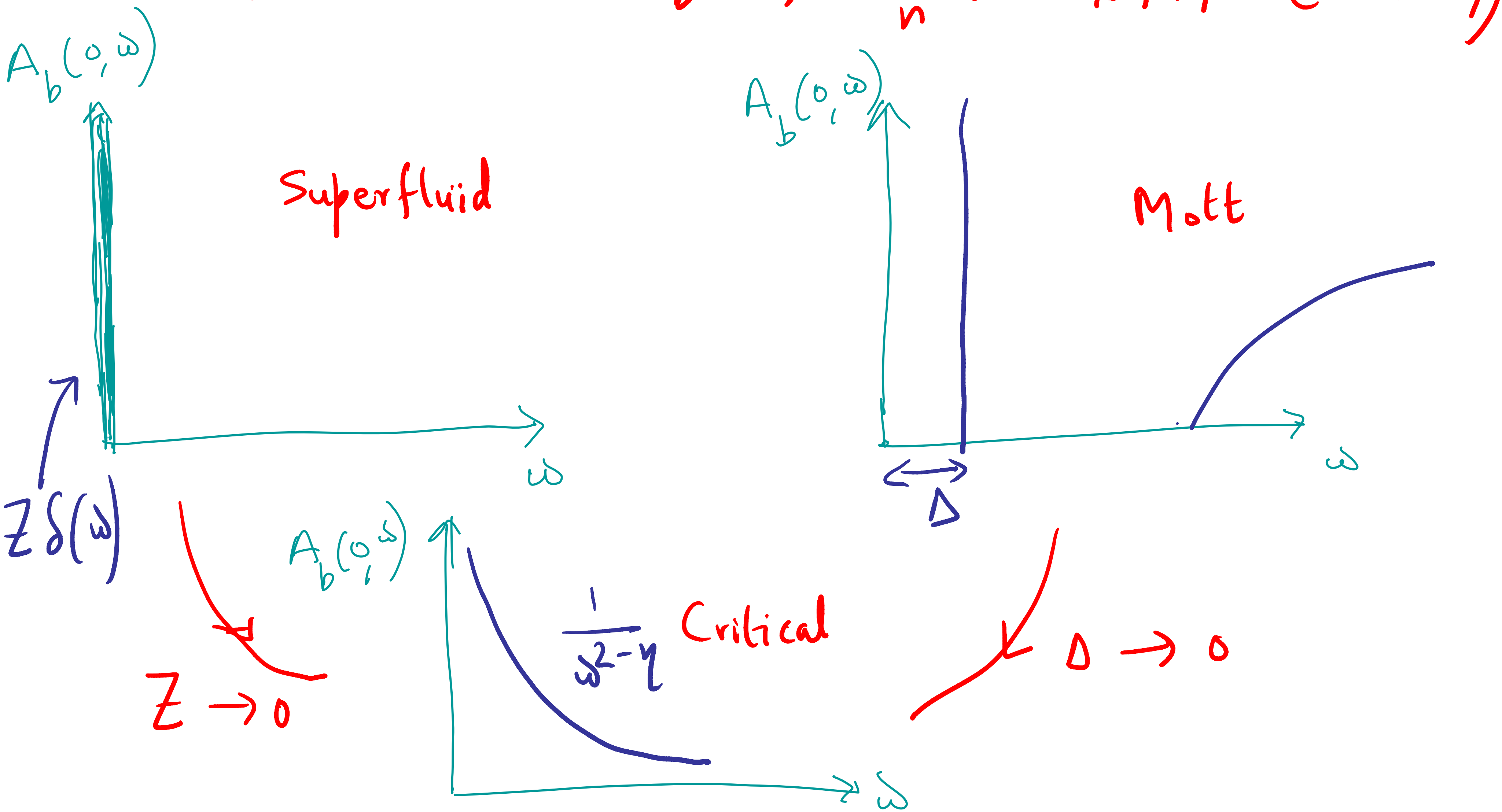
$\left.\begin{array}{l} Z \\ \rho_s \end{array}\right\} \searrow 0$

Approach from Mott :

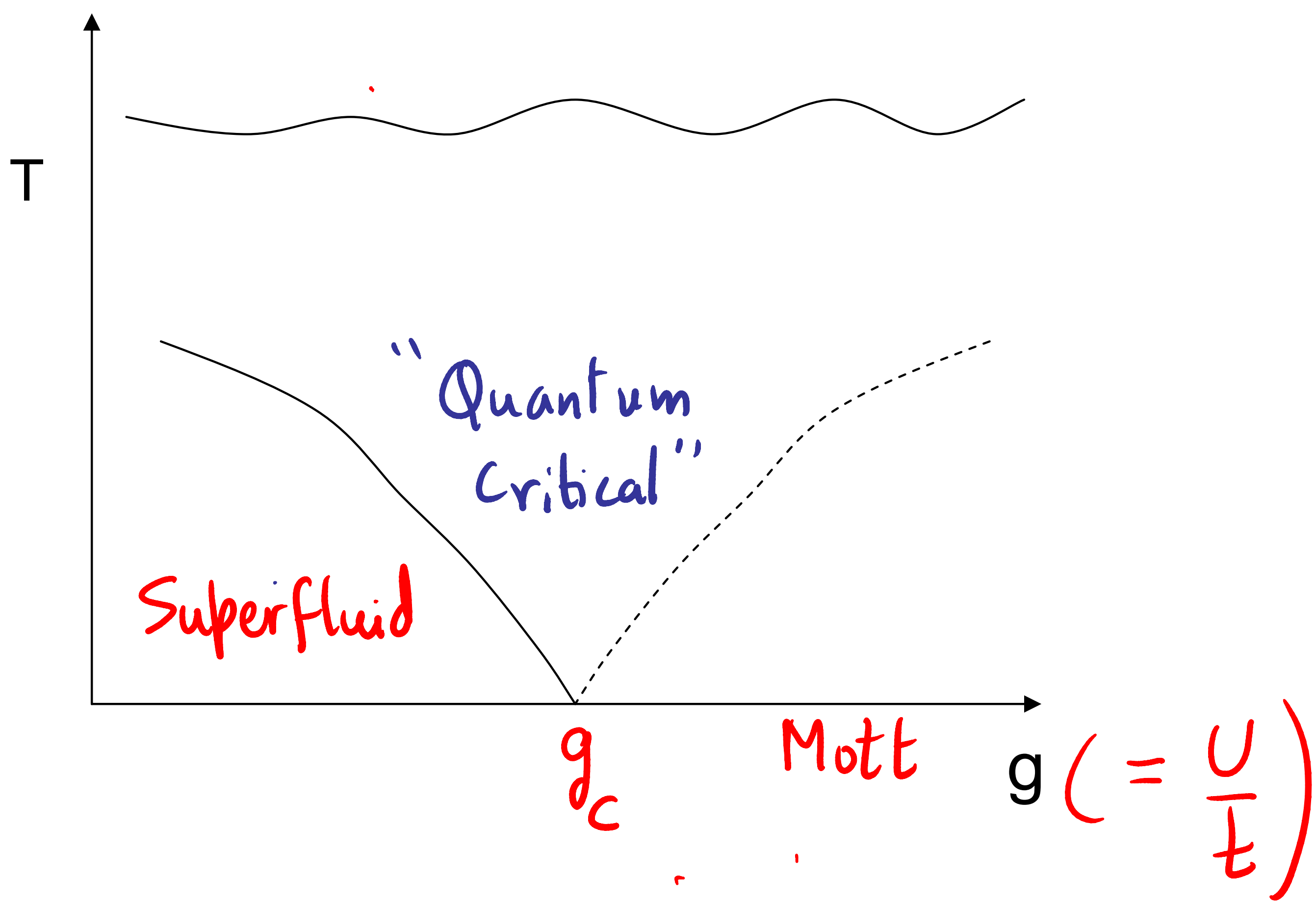
Energy gap to add a single boson $\Delta \searrow 0$

at $\vec{k} = 0$.

Boson spectral function $A_b(\vec{k},\omega) = \sum_n |\langle n| b^\dagger_{\vec{k}} |0\rangle|^2 \, \delta(\omega-(E_n-E_0))$



Finite temperature phase diagram



Landau-Ginzburg-Wilson theory

Field theory

$$S = \int d\tau\, d^2x \; |\partial_\tau \psi|^2 + |\nabla \psi|^2 + r|\psi|^2 + u|\psi|^4$$

$\psi \sim$ superfluid order parameter

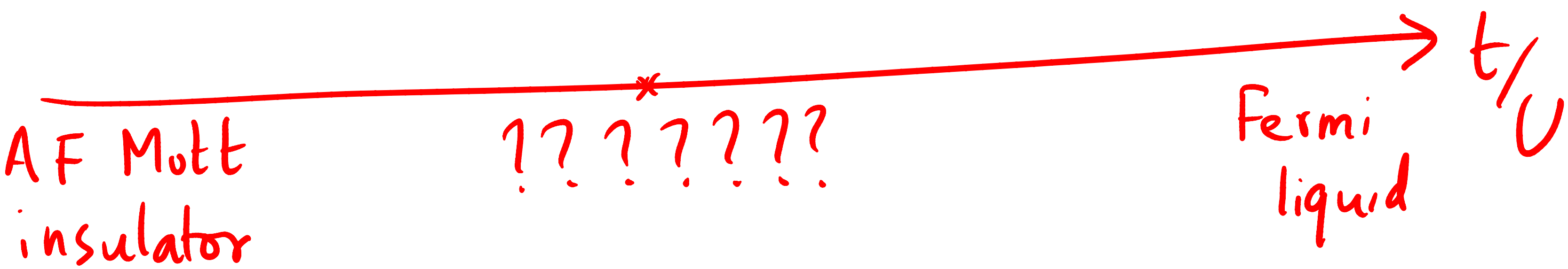
Study with conventional critical phenomena methods (ϵ-expansion, $1/N$ expansion, Monte Carlo, etc)

More difficult Mott and related quantum phase transitions

Questions

1. Electronic Mott transition in one band Hubbard model on non-bipartite lattice

$$H = -t \sum_{\langle ij \rangle} \left(c^\dagger_{i\alpha} c_{j\alpha} + h.c. \right) + U \sum_i \frac{n_i (n_i - 1)}{2} + \ldots\ldots$$



Can there be a direct 2^{nd} order transition between the Fermi liquid & the AF Mott insulator?

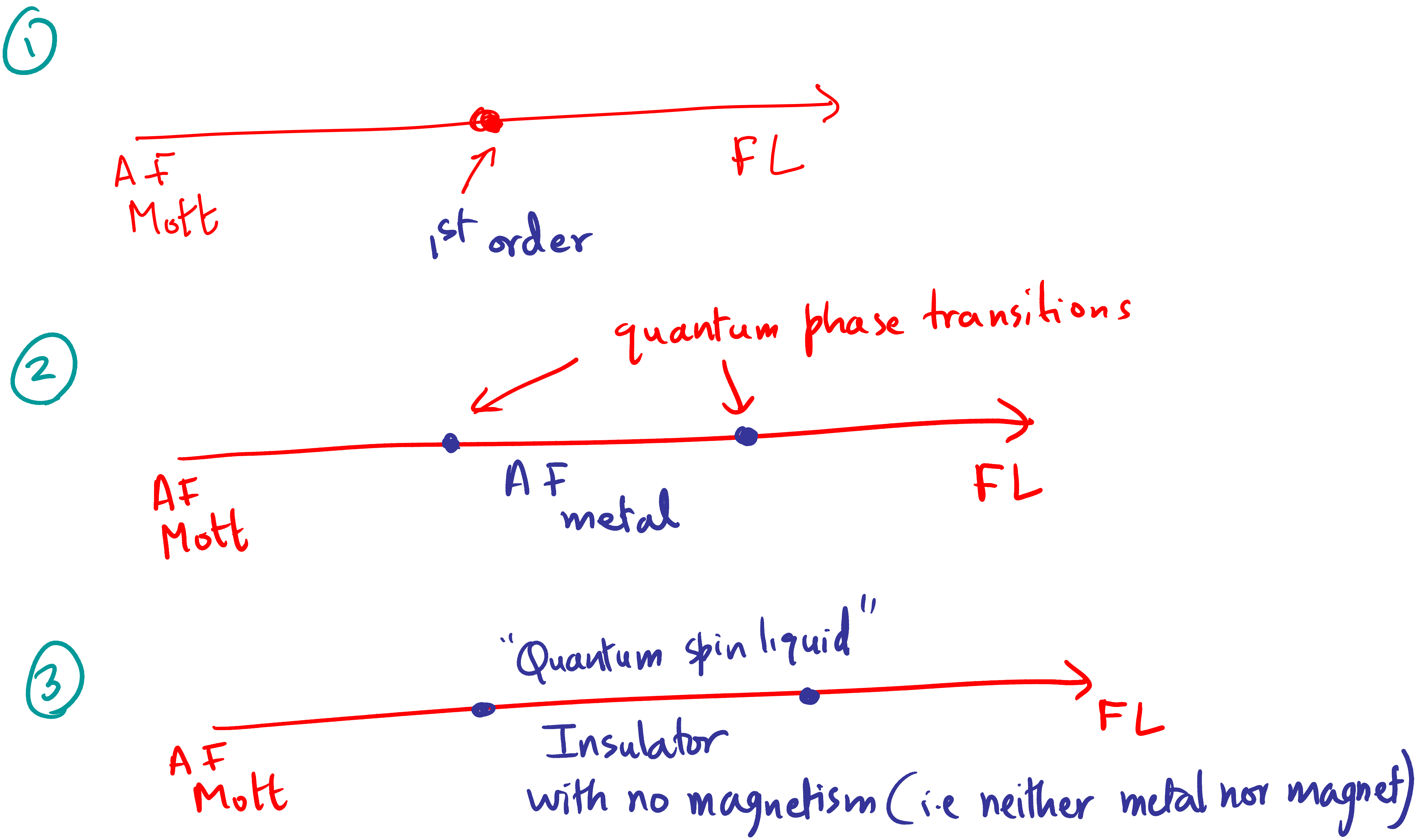
Why hard?

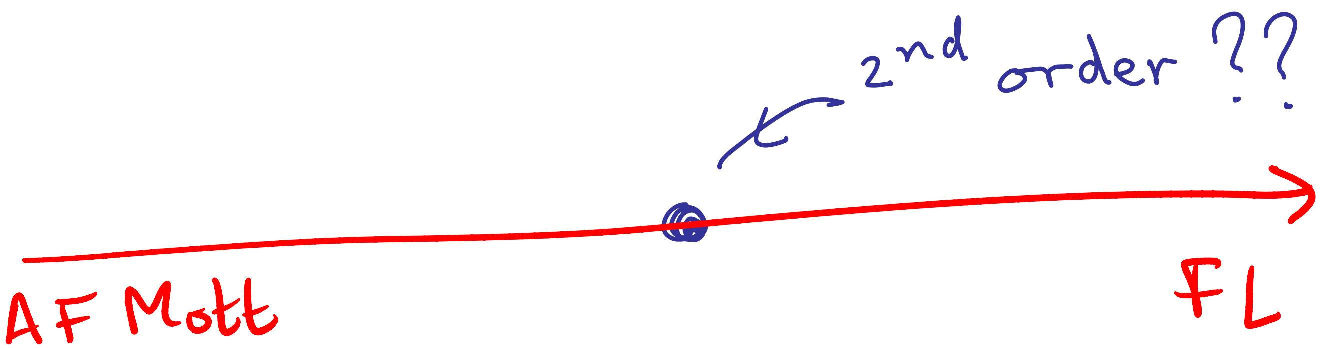
Fermi liquid : Sharp Fermi surface, no
broken spin symmetry

AF Mott insulator : No Fermi surface, but
broken spin symmetry.

How does the system evolve between these
2 states ?

Some simple possible T = 0 evolution between Fermi liquid and AF Mott insulator



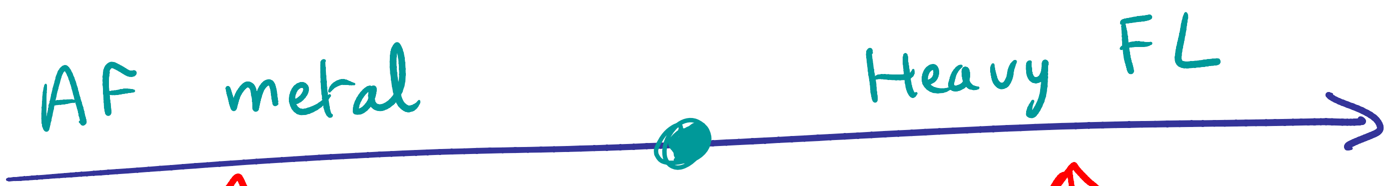


?? Fermi surface of metal disappearing at same critical point as appearance of magnetic ordering ??

IF possible, critical point <u>must</u> be very unusual !

Similar issue: Heavy electron critical points

Materials: $CeCu_{6-x}Au_x$, $YbRh_2Si_2$, "1-1-5", . . .



"Small" Fermi surface
(f-moments not part of Fermi sea)

Second order "non-Fermi" liquid critical point

Large Fermi surface
(f-moments dissolved in Fermi sea)

General questions

1. Can the disappearance of Fermi surface happen at the same critical point as the appearance of magnetic order?

2. How to understand quantum critical points where an entire Fermi surface disappears?

General questions

1. Can the disappearance of Fermi surface happen at the same critical point as the appearance of magnetic order?

Not known but similar issues arise in simpler problems in insulating magnets where they can be answered: theory of "deconfined criticality"

2. How to understand quantum critical points where an entire Fermi surface disappears?

Concept of 'critical Fermi surface'

New scaling hypotheses; calculational framework?

(TS, '08)

Discuss Question 2 first

- return to Question 1 later.

How might a Fermi surface disappear?

Can a Fermi surface disappear continuously through a 2^{nd} order transition?

One route – quasiparticle weight Z vanishes continuously and everywhere on Fermi surface!

(Brinckman-Rice 1973)

Concrete examples within slave boson theories of Mott transitions

Electronic structure at criticality: ``Critical Fermi surface"

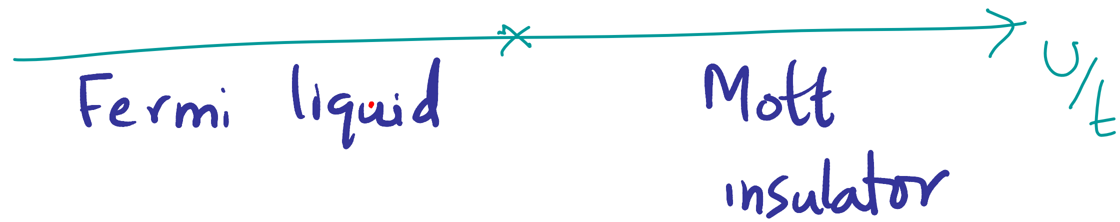
Crucial question: Nature of electronic excitations right at quantum critical point when $Z = 0$?

Claim: At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle.

"Critical Fermi surface".

Why a critical Fermi surface?

Mott transition example:



What is gap $\Delta(\vec{k})$ in electron spectral function $A(\vec{k},\omega)$?

Fermi liquid : $\Delta(\vec{k} \in FS) = 0$

Mott insulator : Sharp gap $\Delta(\vec{k}) \neq 0$ for all $\vec{k}$

Evolution of single particle gap

Approach from Mott

2^{nd} order transition to metal $\Rightarrow$ expect Mott gap

$\Delta(\vec{K})$ will close continuously

To match to Fermi surface in metal, $\Delta(\vec{K}) \to 0$
for all $\vec{K} \in FS$.

$\Rightarrow$ Fermi surface sharp at critical point.

But as $Z = 0$ no sharp quasiparticle.

$\Rightarrow$ Non-fermi liquid with sharp "critical" Fermi surface!

Why a critical Fermi surface?

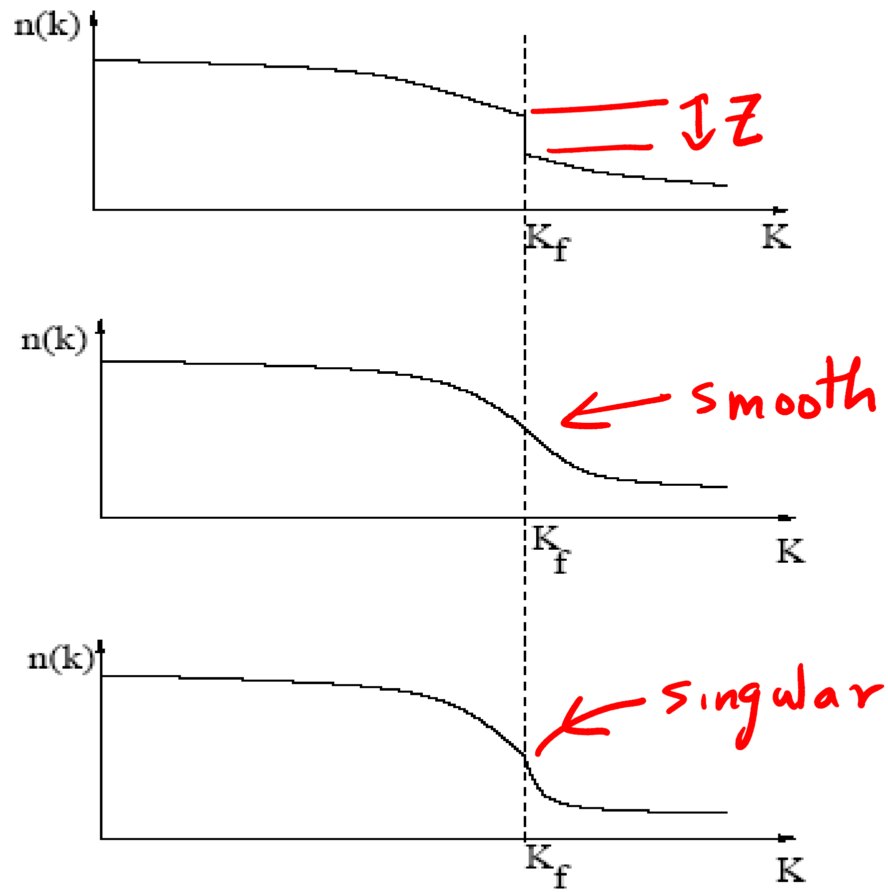
Evolution of momentum distribution

Metal with Fermi surface (a)

Phase where Fermi surface has disappeared (b)

Critical point (c)

$n(K)$ continuous at K_F but is singular.



Killing a Fermi surface

Disappearance of Fermi surface through a continuous transition

At critical point

(a) $Z = 0$

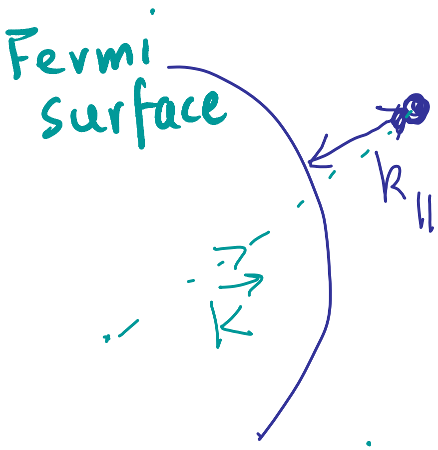
(b) Fermi surface sharp

(Similar argument for heavy fermion critical points, hiTc, Mott critical point, etc).

Scaling phenomenology at a quantum critical point with a critical Fermi surface?

Critical Fermi surface: scaling for single particle physics

Right at critical point expect universal scale invariant singularity in $A_c(\vec{K}, \omega)$ for small ω, $k_{\parallel}$



Scaling ansatz:

For every point θ on FS

$$A_c(\vec{K}, \omega, T) \sim \frac{1}{|\omega|^{\alpha/z}} F\left(\frac{\omega}{|k_{\parallel}|^{z}}, \frac{\omega}{T}\right)$$

New possibility: angle dependent exponents

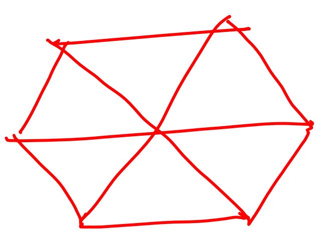
A priori must allow angle dependent exponents:

$$z = z(\theta), \quad \alpha = \alpha(\theta)$$

consistent with lattice symmetries.

Eg: Triangular lattice $\quad z(\theta + \pi/3) = z(\theta)$

$$\alpha(\theta + \pi/3) = \alpha(\theta)$$



Can expand $z(\theta) = \sum_n z_n \cos(6n\theta), \ldots$

Leaving the critical point

Expect scale invariant spectrum is cut off at $k_{\parallel} \sim \frac{1}{\xi}$, $\omega \sim \frac{1}{\xi^z}$ so that

$$A_c(\vec{k}, \omega) \sim \frac{1}{|\omega|^{\alpha/z}} F_1\left(\frac{\omega}{k_{\parallel}^z}, k_{\parallel}\xi\right)$$

Expect $\xi \sim |g-g_c|^{-\nu}$ but again a priori must let $\nu = \nu(\theta)$

Approach from the Fermi liquid

If Fermi liquid physics is part of scaling function

$$Z \sim |\delta g|^{\nu(z-\alpha)} \qquad \left(\Rightarrow z(\theta) \geqslant \alpha(\theta) \right)$$

$$v_F \sim |\delta g|^{\nu(1-z)}$$

$\Rightarrow$ Specific heat $C_v \sim T \int_{FS} \frac{1}{v_F} \sim T \int_{FS} |\delta g|^{-\nu(1-z)}$

If ν, z are θ-dependent, not a pristine power law

Asymptopia: Dominated by portion of FS with $\max(\nu(1-z))$

Specific heat singularity

In Fermi liquid $C_v \sim \int_{FS} T \xi^{z-1}$

$\Rightarrow$ Scaling ansatz : $C_v \sim \int_{FS} T^{1/z} Y(T\xi^z)$

At critical point expect $C_v \sim \int_{FS} T^{1/z}$

Again asymptopia dominated by portion of Fermi surface if z angle dependent.

Note: Spin susceptibility – different scaling models depending on fate of F_a^0 as $g \searrow g_c$.

Implications of angle dependent exponents

(i) Different properties dominated by different portions of Fermi surface

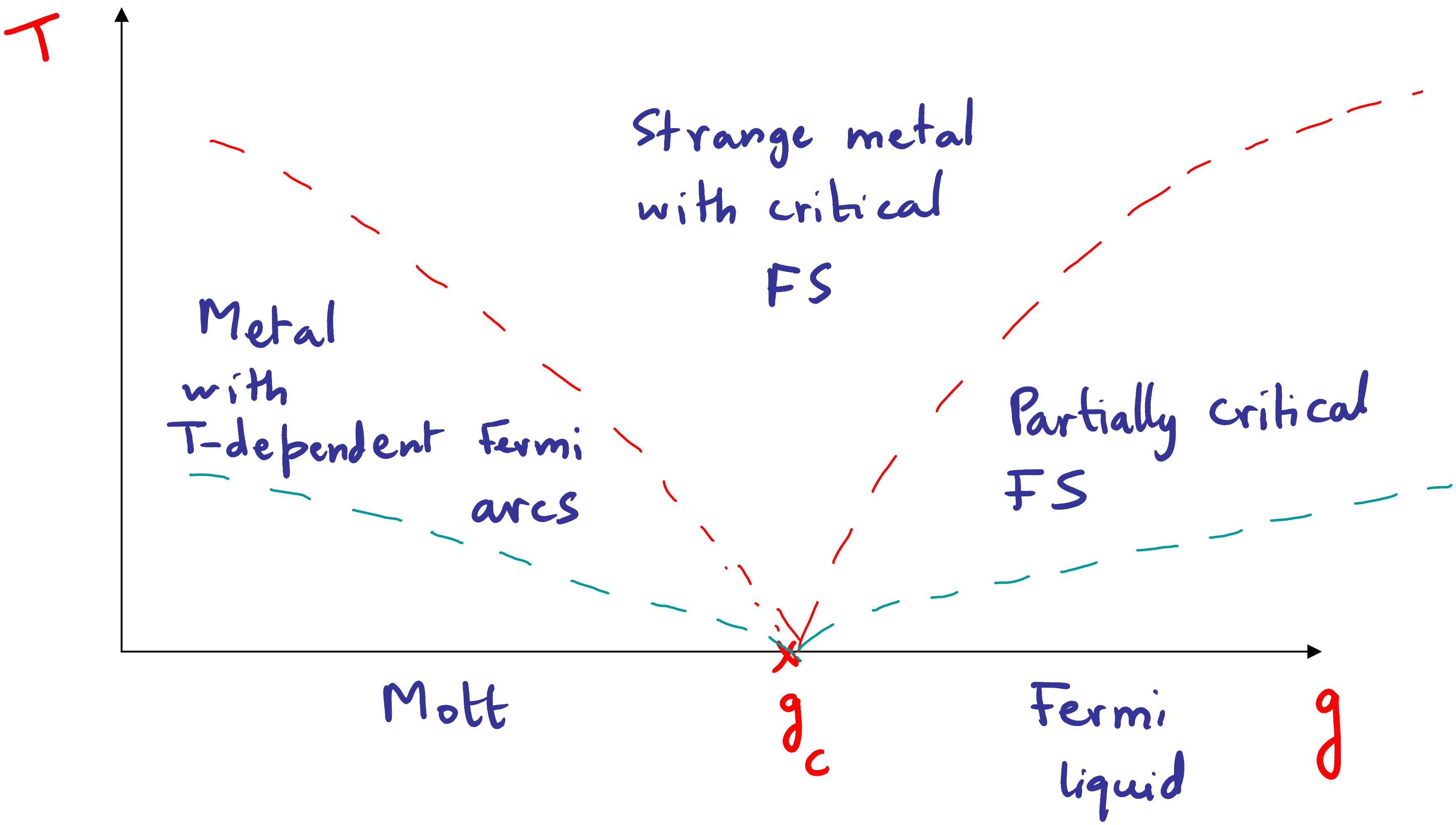
(ii) Different portions of Fermi surface will emerge out of criticality at different energy scales

Example: At Mott transition

Mott gap $\Delta(\theta) \sim |\delta g|^{z(\theta)\nu(\theta)}$

$\Rightarrow$ Finite-T x overs richer than usual

Finite T crossovers



Future: Calculational framework for critical Fermi surfaces

General questions

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Not known but similar issues arise in simpler problems in insulating magnets where they can be answered: theory of "deconfined criticality"

2. How to understand quantum critical points where an entire Fermi surface disappears?

Concept of 'critical Fermi surface'

New scaling hypotheses; calculational framework?

(TS, '08)

Can the disappearance of Fermi surface happen at the same critical point as the appearance of magnetic order?

Difficulty: Two different things seem to happen at the same time.

Study possibility of such phenomena in simpler systems.

Can ordered phases with two distinct broken symmetries have a direct second order transition?

General theoretical questions

- Fate of Landau-Ginzburg-Wilson ideas at <u>quantum</u> phase transitions?
- (More precise) Could Landau order parameters for the phases <u>distract</u> from the true critical behavior?

Study phase transitions in insulating quantum magnets

- Good theoretical laboratory for physics of phase transitions/competing orders.

(Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004)

Highlights

- Failure of Landau paradigm at (certain) quantum transitions

- Rough description: Emergence of `fractional' charge and gauge fields near quantum critical points between two CONVENTIONAL phases.

- ``Deconfined quantum criticality''

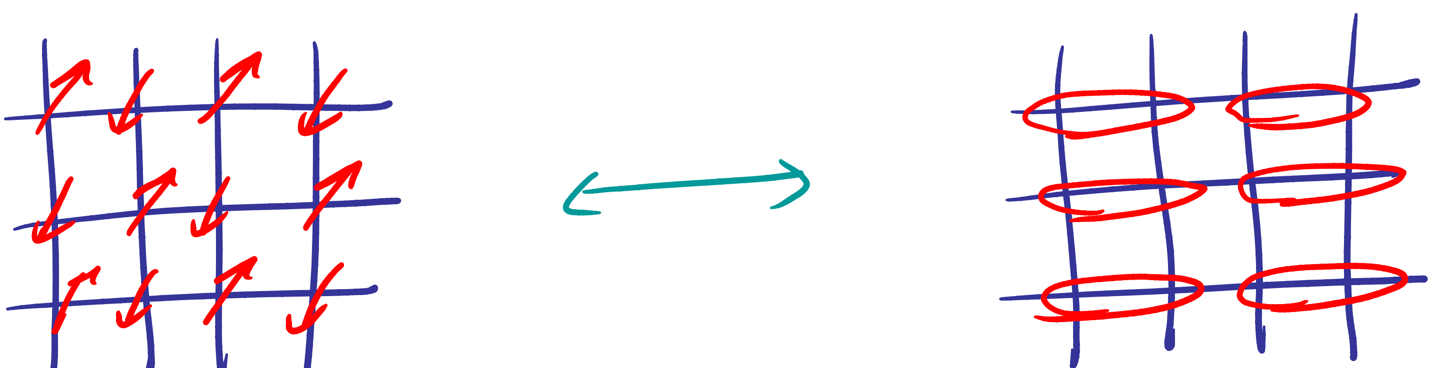
- Many lessons for competing order physics in correlated electron systems.

Phase transitions in quantum magnetism

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \ldots$$

- Spin-1/2 quantum antiferromagnets on a square lattice.
- ``……'' represent frustrating interactions that can be tuned to drive phase transitions.

Study transitions between Neel & VBS states

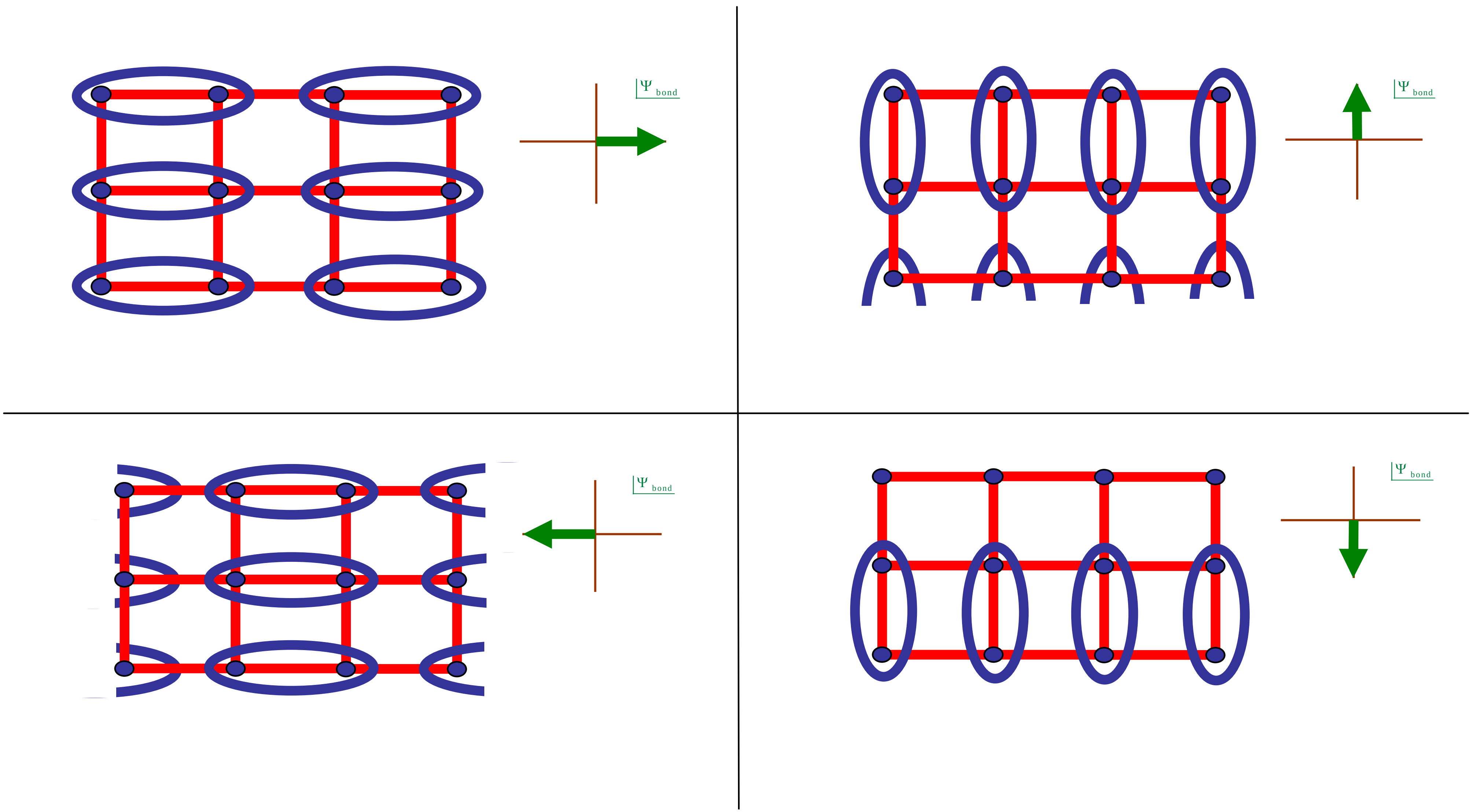


Neel

VBS (Z_4 order parameter)

VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



Neel-valence bond solid(VBS) transition

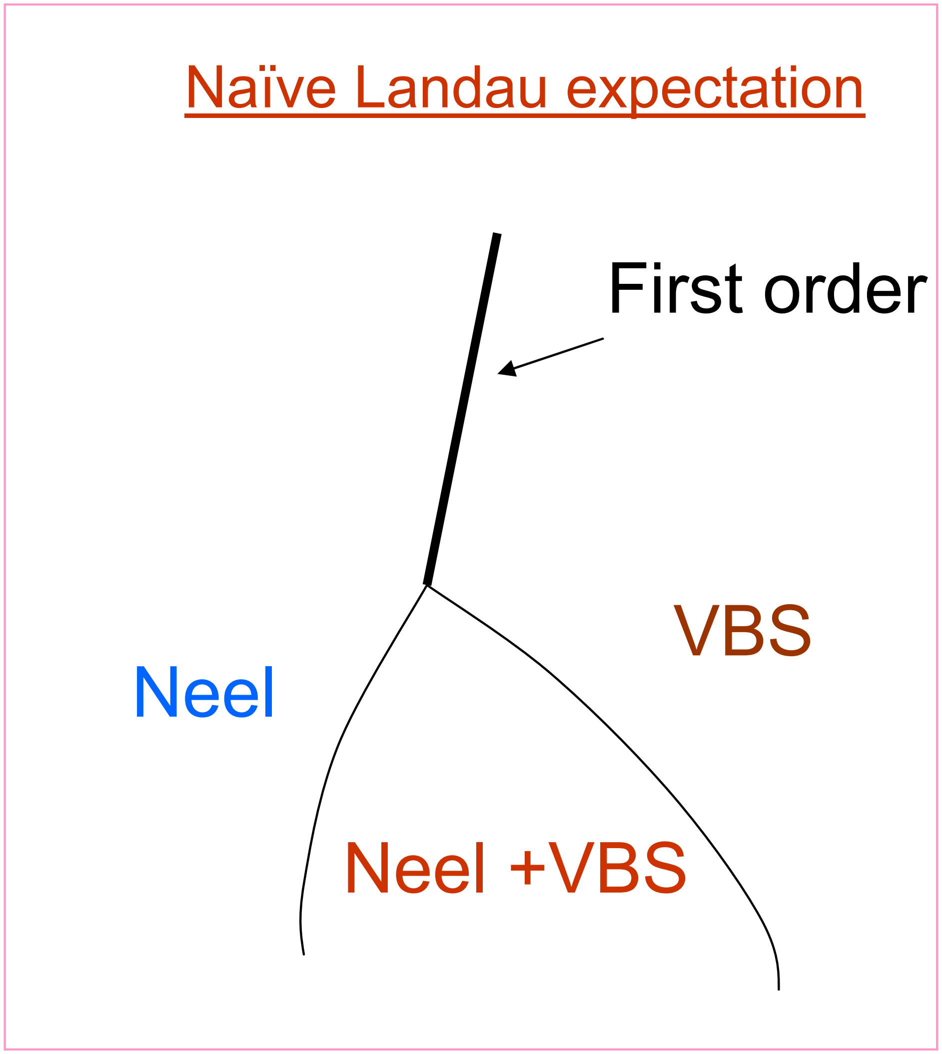
- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.

- Landau – Two independent order parameters.

- no generic direct second order transition.

- either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.



Neel-Valence Bond Solid transition

- Naïve approaches fail

Attack from Neel ≠Usual O(3) transition in D = 3

Attack from VBS ≠ Usual Z_4 transition in D = 3

(= XY universality class).

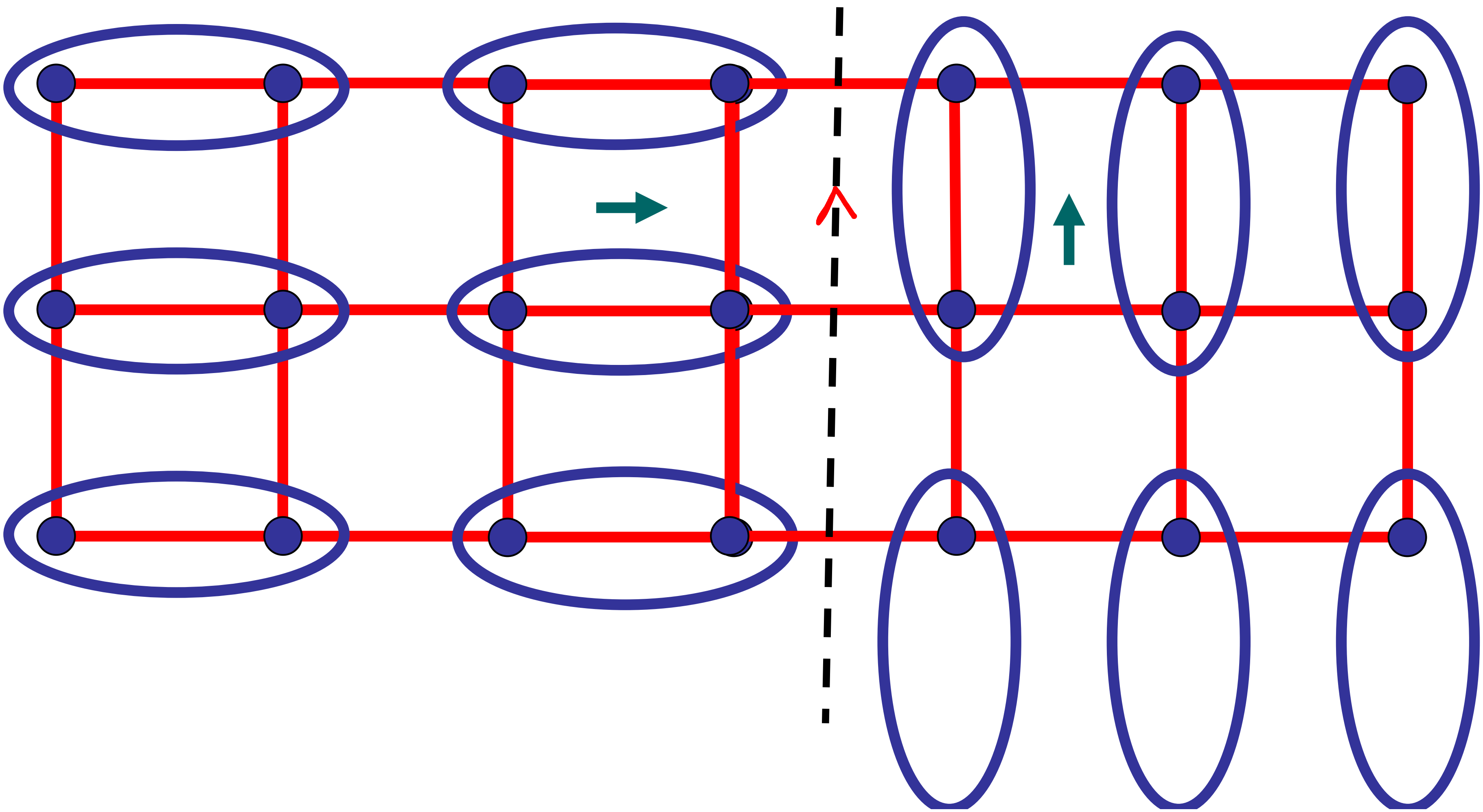
Why do these fail?

Topological defects carry non-trivial quantum numbers!

Attack from VBS (Levin, TS, '04)

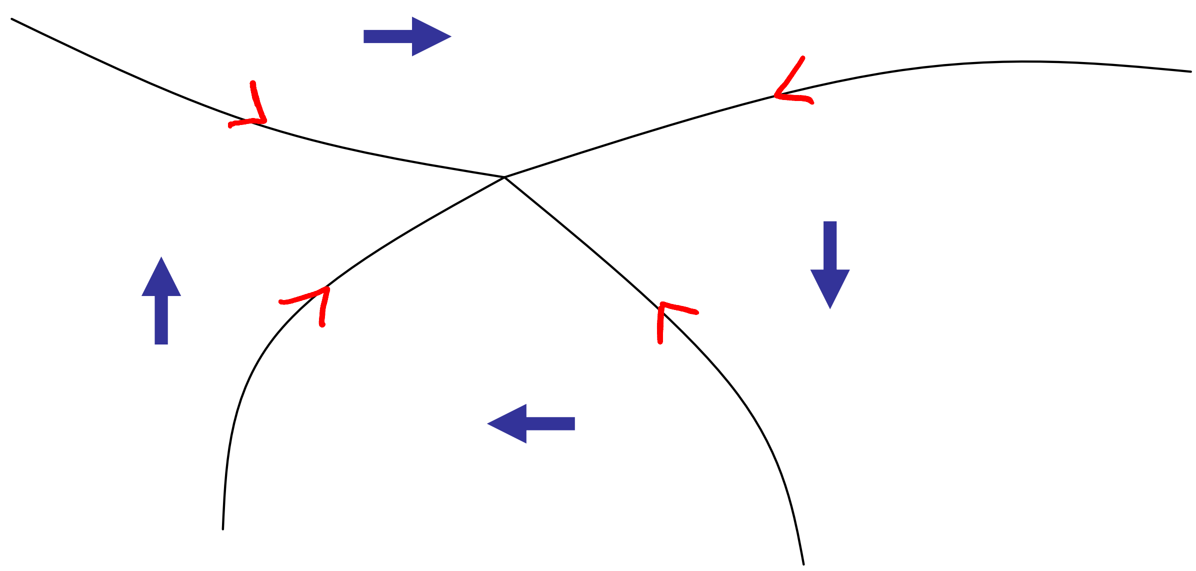
Topological defects in Z_4 order parameter

- Domain walls – elementary wall has π/2 shift of clock angle



Z_4 domain walls and vortices

- Walls can be oriented; four such walls can end at point.
- End-points are Z_4 vortices.

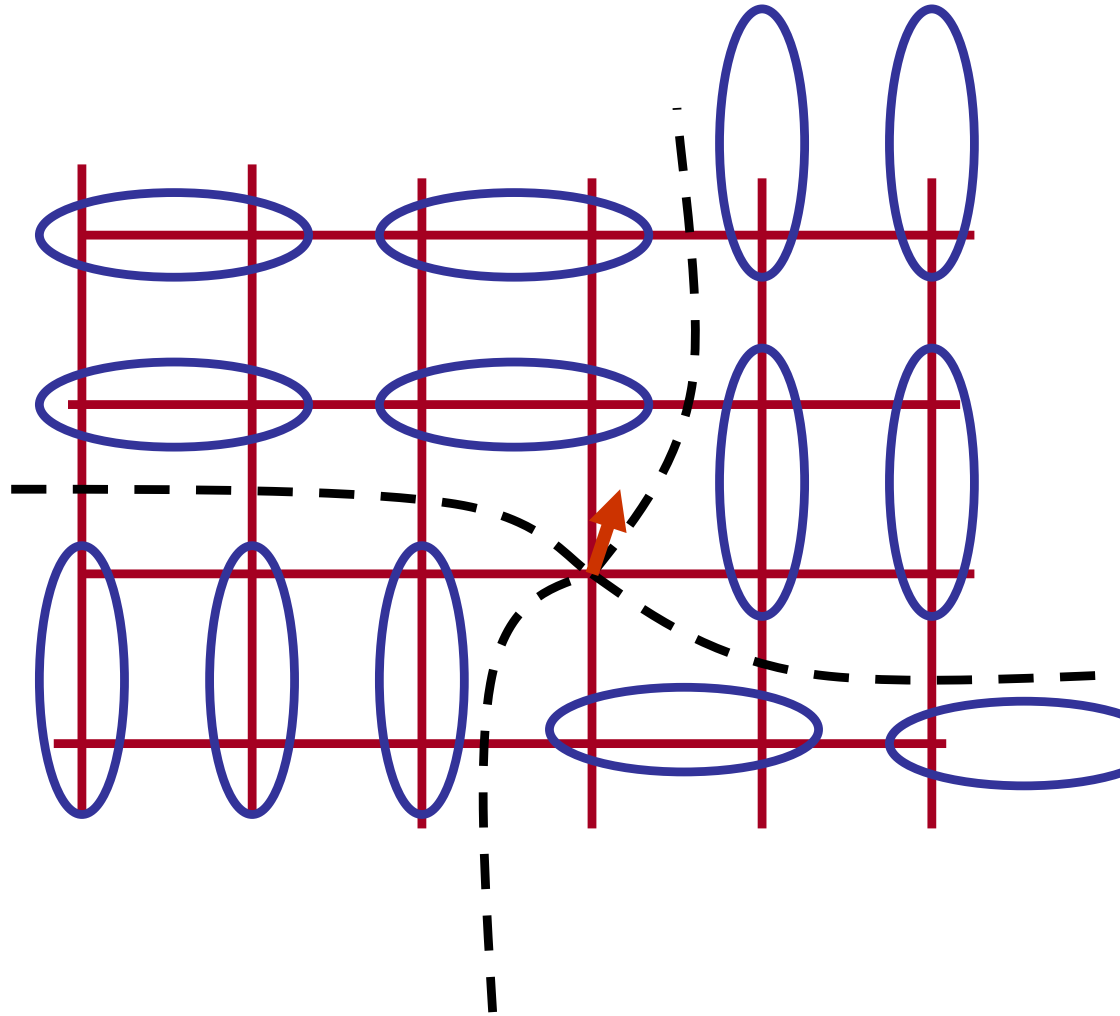


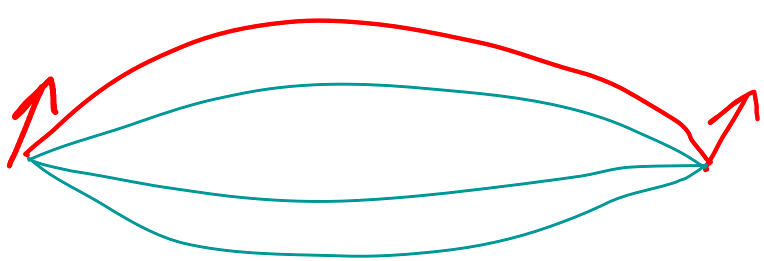
Z_4 vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

Z_4 vortices are spin-1/2 ``spinons''.

Domain wall energy
$\Rightarrow$ linear confinement
in VBS phase.





$E \sim L$

Z_4 disordering transition to Neel state

- As for usual (quantum) Z_4 transition, expect clock anisotropy is irrelevant.

(confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).

Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)

Phase mode - ``photon''

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons

=> Critical spinons minimally coupled to fluctuating U(1) gauge field*.

*non-compact

Critical theory
``Non-compact CP_1 model''

$$S = \int d^2 x d\tau \, |(\partial_\mu - i a_\mu) z|^2 + r |z|^2 + u |z|^4$$
$$+ (\varepsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2$$

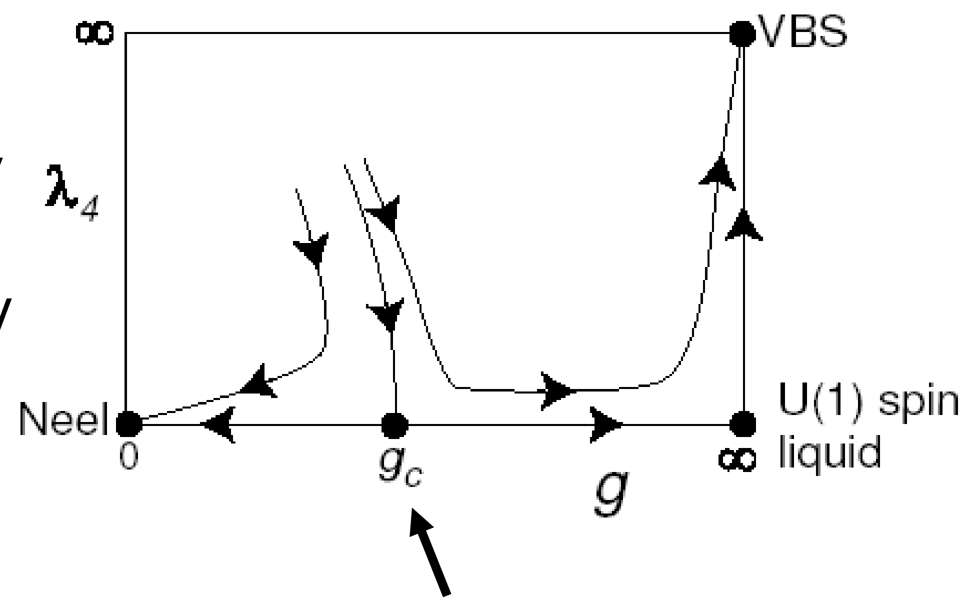
z = two-component spin-1/2 spinon field
a_μ = non-compact U(1) gauge field.
Distinct from usual O(3) or Z_4 critical theories*.

Theory not in terms of usual order parameter fields but involve fractional spin objects and gauge fields.

*Distinction with usual O(3) fixed point due to non-compact gauge field (Motrunich,Vishwanath, '03)

Renormalization group flows

Clock anisotropy
= quadrupled
Instanton fugacity



Deconfined critical fixed point

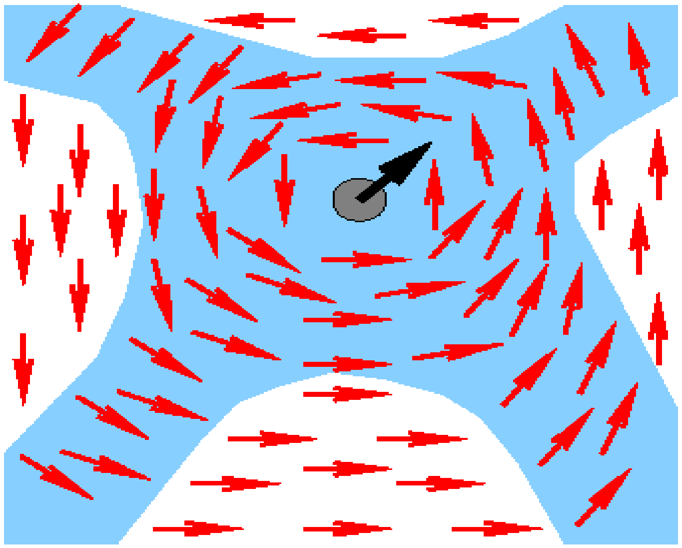
Clock anisotropy is ``dangerously irrelevant''.

Precise meaning of deconfinement

- Z_4 symmetry gets enlarged to XY

$\Rightarrow$ Domain walls get very thick and very cheap near the transition.

=> Domain wall energy not effective in confining Z_4 vortices (= spinons)



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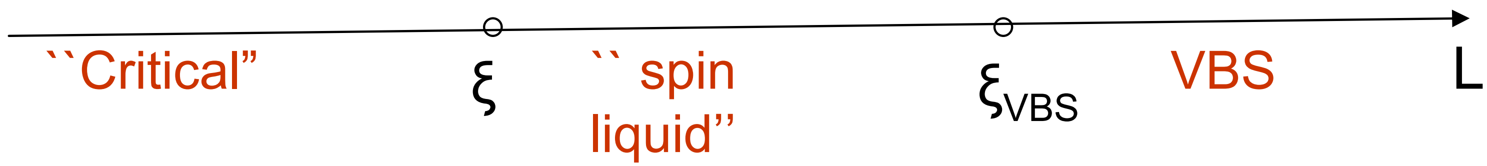
Formal: Extra global U(1) symmetry not present in microscopic model :

Two diverging length scales in paramagnet

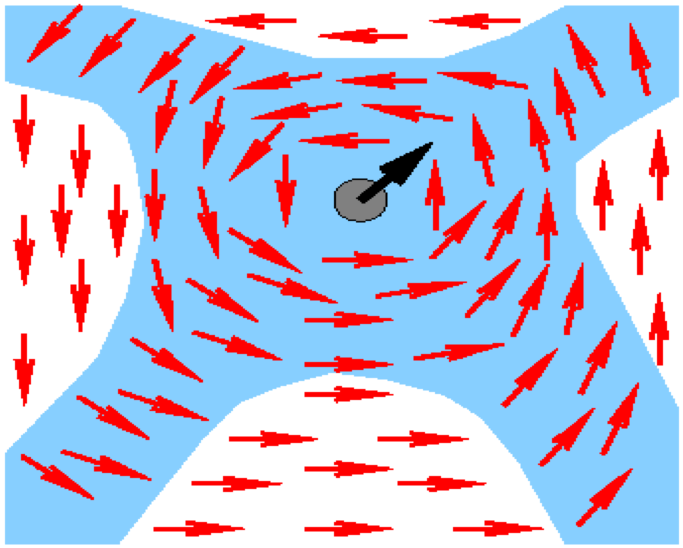


ξ: spin correlation length

ξ_{VBS} : Domain wall thickness.

$\xi_{VBS} \sim \xi^{\kappa}$ diverges faster than ξ

Spinons confined in either phase but `confinement scale’ diverges at transition – hence `deconfined criticality’.



Other examples of deconfined critical points

1. VBS- spin liquid (Senthil, Balents, Sachdev, Vishwanath, Fisher, ’04)
2. Neel –spin liquid (Ghaemi, Senthil, ‘06)
3. Certain VBS-VBS
(Fradkin, Huse, Moessner, Oganesyan, Sondhi, ’04; Vishwanath, Balents,Senthil, ‘04)

4. Superfluid- Mott transitions of bosons at fractional filling on various lattices (Senthil et al, ’04, Balents et al, ’05,…….)
5. Spin quadrupole order –VBS on rectangular lattice
(Numerics: Harada et al, ’07;Theory: Grover, Senthil, 07)

……..and many more!

Apparently fairly common

Numerical/experimental sightings of Landau-forbidden quantum phase transitions

Weak first order/second order quantum transitions between two phases with very different broken symmetry surprisingly common….

Numerics

Antiferromagnet – superconductor (Assaad et al 1996)
Superfluid – density wave insulator on various lattices
(Sandvik et al, 2002, Isakov et al, 2006, Damle et al, 2006))
Neel -VBS on square lattice (Sandvik, Singh, Sushkov,…………….)

Spin quadrupole order –dimer order on rectangular lattice (Harada et al, 2006)

Experiments:

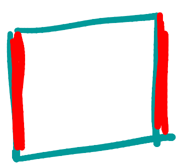
$UPt_{3-x}Pd_x$ SC – AF with increasing x. (Graf et al 2001)

Best numerical evidence: Neel-VBS on square lattice

(Sandvik '07
Melko & Kaul '07)

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - Q \sum_{\langle\langle (ij)(kl) \rangle\rangle} \left(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} \right) \left(\vec{S}_k \cdot \vec{S}_l - \frac{1}{4} \right)$$

$(ij), (kl)$: parallel neighbor bonds



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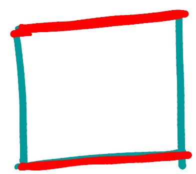
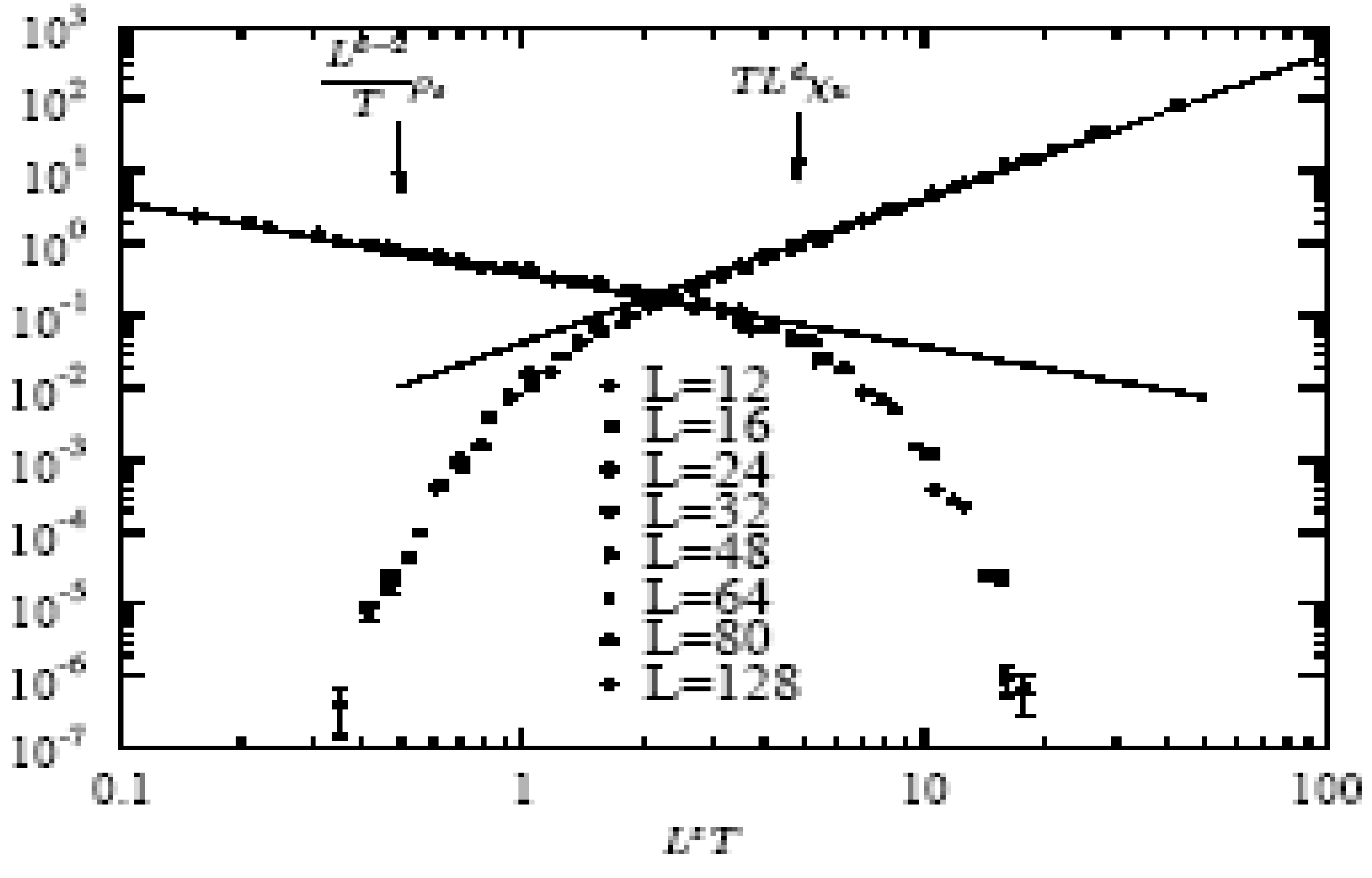


Q/J small : Neel order

Q/J large : VBS order

A sample scaling plot

Melko, Kaul '07

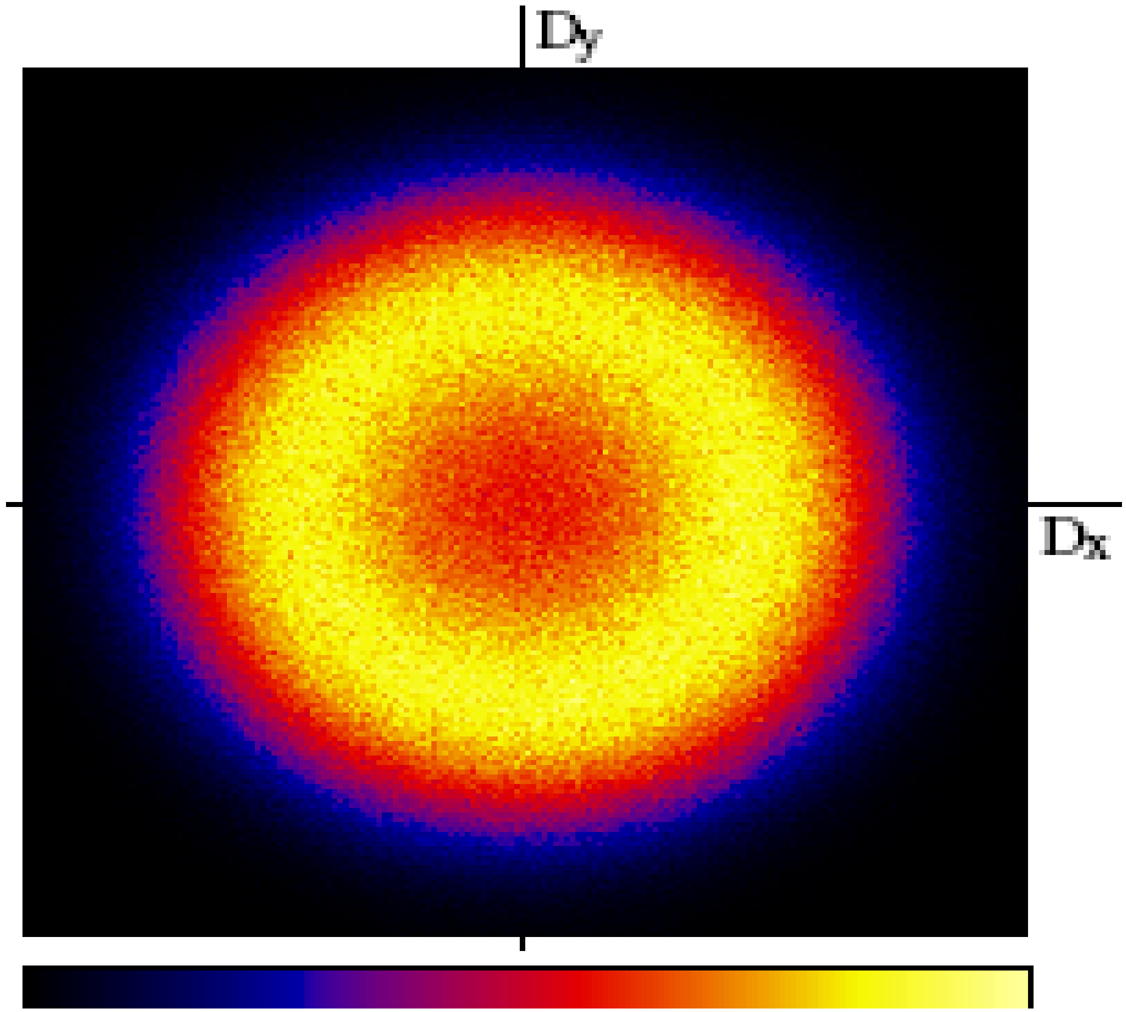


(Sandvik 2007)

Emergent XY symmetry for dimer order

Histogram of dimer order parameter shows full XY symmetry

⇒ irrelevance of Z_4 anisotropy!

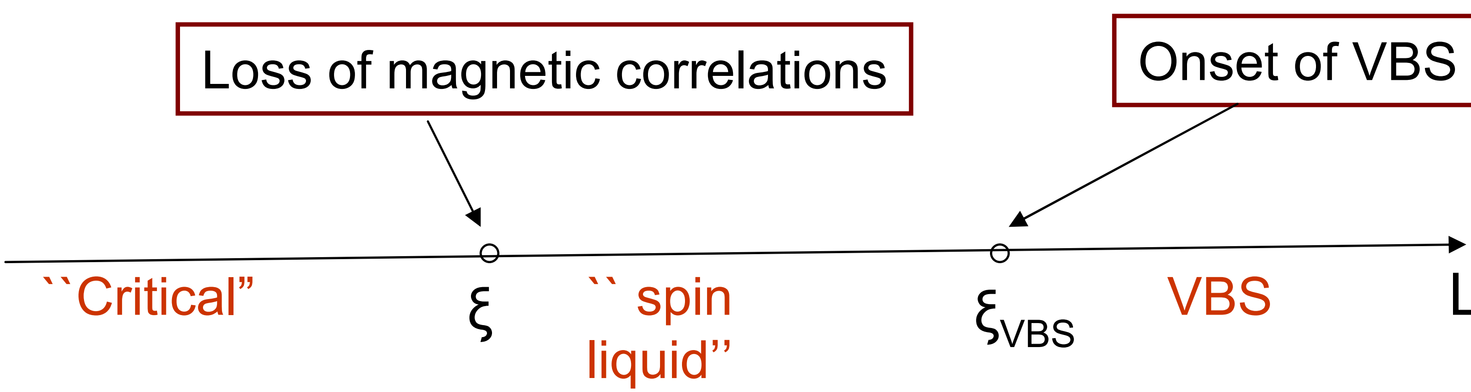


Some lessons-I

- Direct 2nd order quantum transition between two phases with different competing orders possible (eg: between different broken symmetries)

Separation between the two competing orders

not as a function of tuning parameter but as a function of (length or time) scale



Some lessons-II

- Striking ``non-fermi liquid'' (morally) physics at critical point between two competing orders.

Eg: At Neel-VBS, spin spectrum is anamolously broad - roughly due to decay into spinons- as compared to usual critical points.

Most important lesson:

Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics even if natural order parameters exist.

Strong impetus to radical approaches to non fermi liquid physics at

magnetic critical points in rare earth metals (and to optimally doped cuprates).

Outlook

- Theoretically important answer to 0^{th} order question posed by experiments:

Can Landau paradigms be violated at phases and phase transitions of strongly interacting electrons?

- But there still is far to go to seriously confront non-Fermi liquid metals in existing materials……….!

Can we go beyond the 0^{th} order answer?