Shot Noise and the Non-Equilibrium FDT

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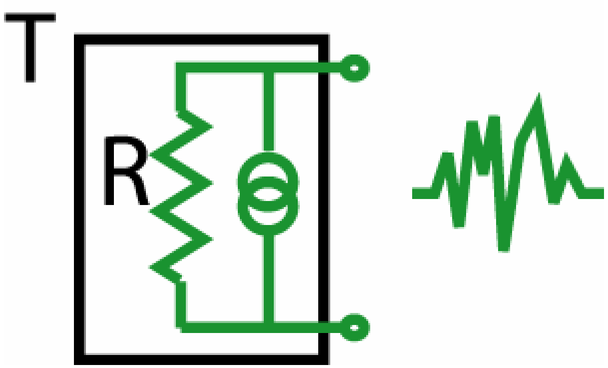
Outline

- Shot noise is quantum noise
- Shot noise of a tunnel junction
- Measurements of shot noise – testing the non-eq. FDT
- “Quantum shot noise”
  – measuring the frequency dependence of shot noise
- Experiments on the zero point noise in circuits
- Shot noise and the nonequilibrium FDT (time permitting)

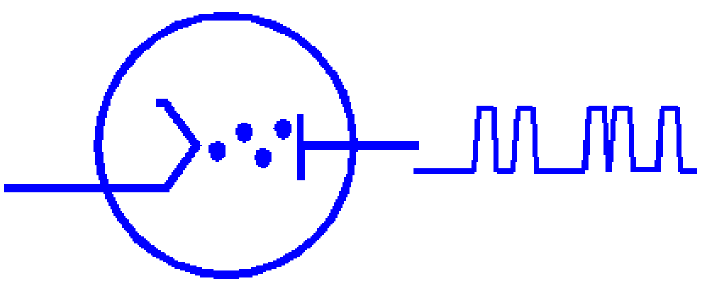
Fundamental Noise Sources



Johnson-Nyquist Noise

$$S_I(f) = \frac{4k_BT}{R}\left[\frac{A^2}{Hz}\right]$$

- Frequency-independent
- Temperature-dependent
- Used for thermometry

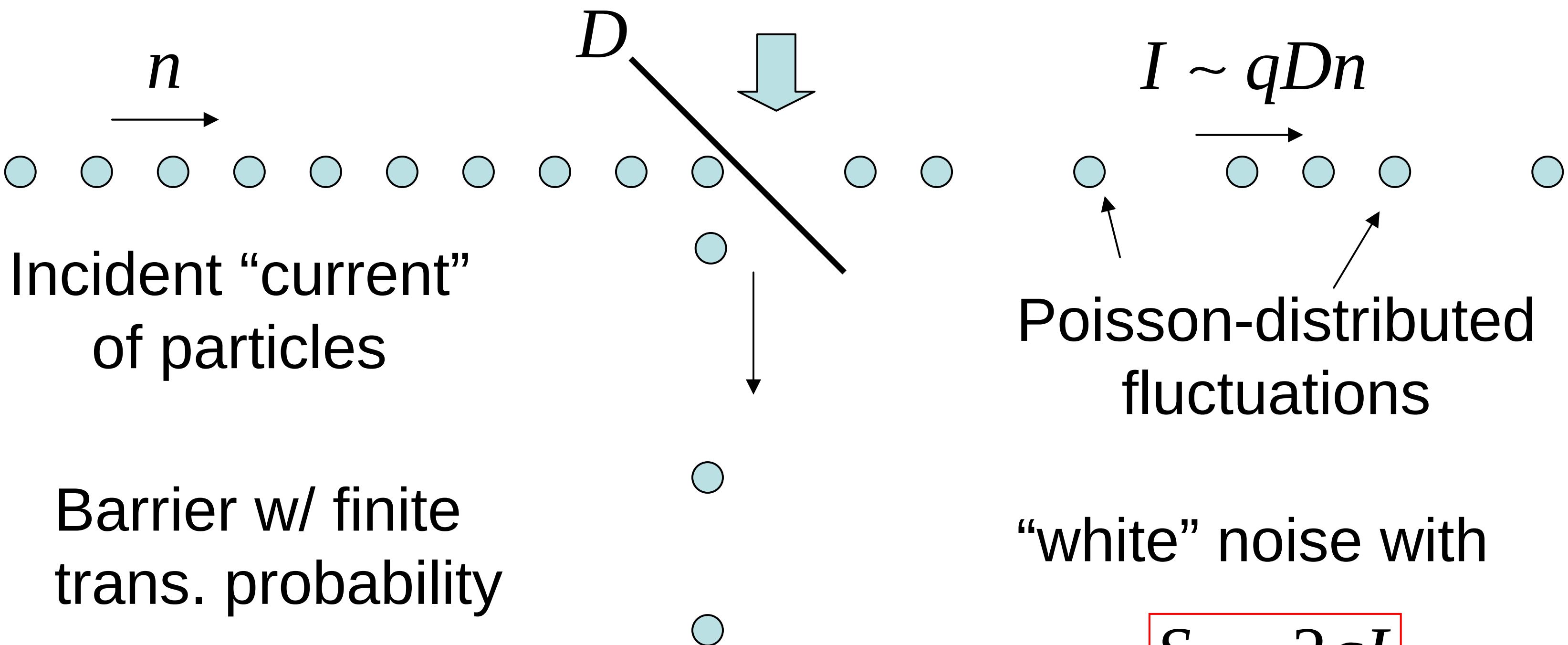


Shot Noise

$$S_I(f) = 2eI \quad \left[\frac{A^2}{Hz}\right]$$

- Frequency-independent
- Temperature independent

Shot Noise – “Classically”



“white” noise with

$$\boxed{S_I = 2qI}$$

Shot Noise is Quantum Noise

Einstein, 1909: Energy fluctuations of thermal radiation

“Zur gegenwartigen Stand des Strahlungsproblems,” Phys. Zs. **10** 185 (1909)

$$\left\langle (\Delta E)^2 \right\rangle = \left[\hbar\omega\rho(\omega) + \frac{\pi^2 c^3}{\omega^2}\rho^2(\omega) \right] V d\omega$$

particle term = shot noise! wave term

first appearance of wave-particle complementarity?

Can show that “particle term” is a consequence of $\left[a, a^\dagger\right] = 1$

(see Milloni, “The Quantum Vacuum,” Academic Press, 1994)

$$\left\langle \Delta n^2 \right\rangle = \left\langle a^\dagger a a^\dagger a \right\rangle - \left\langle a^\dagger a \right\rangle^2$$

$$= \left\langle a^\dagger (a^\dagger a + 1) a \right\rangle - \left\langle a^\dagger a \right\rangle^2$$

$$= \left\langle a^\dagger a^\dagger a a \right\rangle + \left\langle a^\dagger a \right\rangle - \left\langle a^\dagger a \right\rangle^2$$

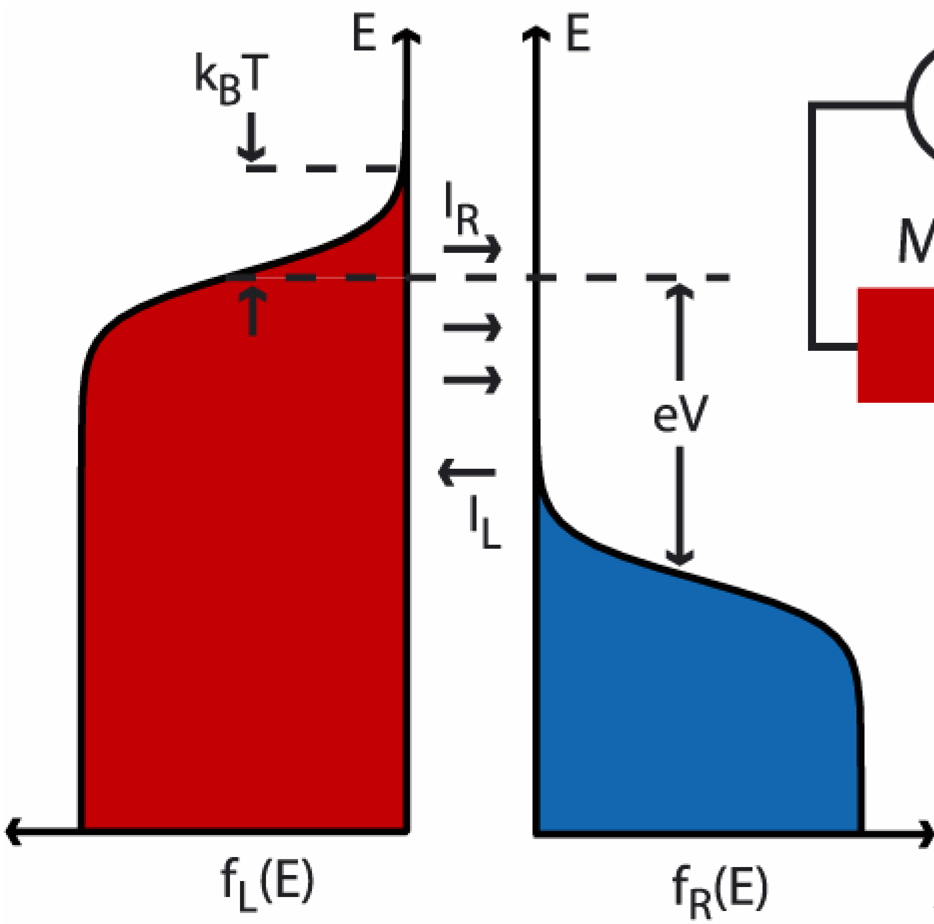
$$\boxed{\left\langle \Delta n^2 \right\rangle = \bar{n}^2 + \bar{n}}$$

$$\bar{n} = \left\langle n \right\rangle = \left(e^{\hbar\omega/kT} - 1 \right)^{-1}$$

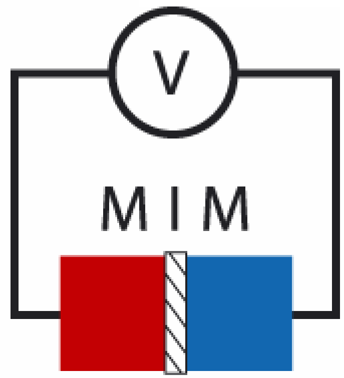
$$P_n = \bar{n}^n / \left(\bar{n} + 1 \right)^{n+1}$$

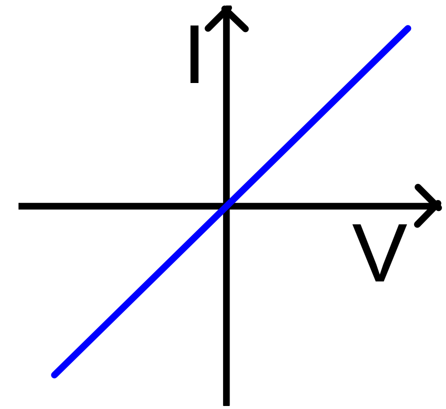
$$\left\langle a^\dagger a^\dagger a a \right\rangle = \sum n(n-1) P_n = 2\bar{n}^2$$

Conduction in Tunnel Junctions



Fermi functions





$$I_{L\to R} = \frac{G}{e}\int f_L(1 - f_R)\,dE$$

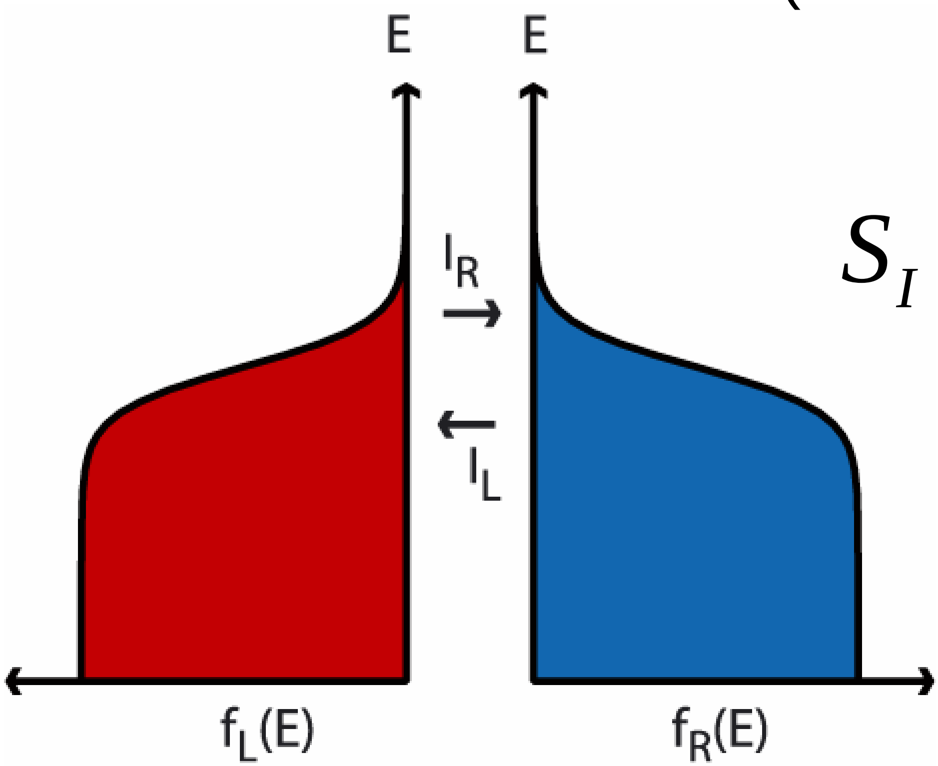
$$I_{R\to L} = \frac{G}{e}\int f_R(1 - f_L)\,dE$$

Difference gives current:

$$I = I_{L\to R} - I_{R\to L} = GV$$

Assume: Tunneling amplitudes and
D.O.S. independent of energy → Conductance (G) is constant
Fermi distribution of electrons

Non-Equilibrium Noise of a Tunnel Junction



(Zero-frequency limit)

Sum gives noise:

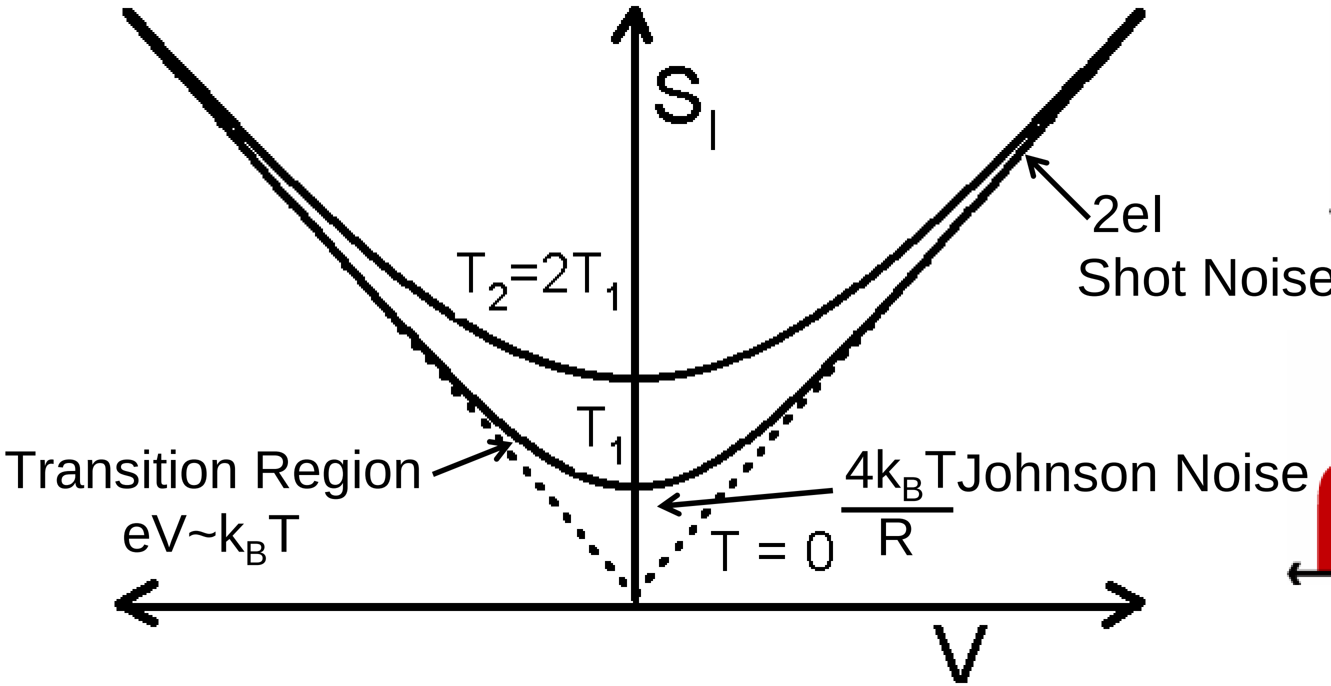
$$S_I(f) = 2e(I_{L\to R} + I_{R\to L})$$

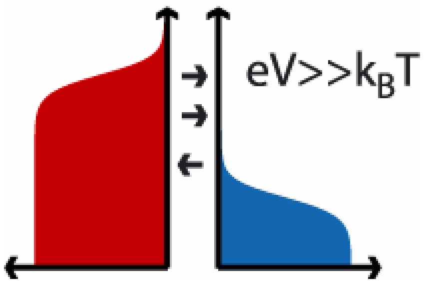
$$\boxed{S_I(f) = 2eI\coth\left(\frac{eV}{2k_BT}\right)}$$

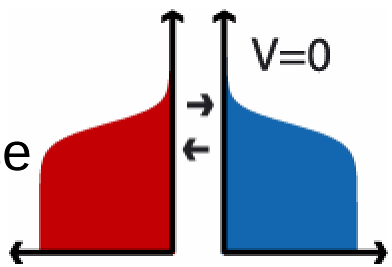
$$\boxed{I = V/R}$$

*D. Rogovin and D.J. Scalpino, Ann Phys. **86**,1 (1974)

Non-Equilibrium Fluctuation Dissipation Theorem

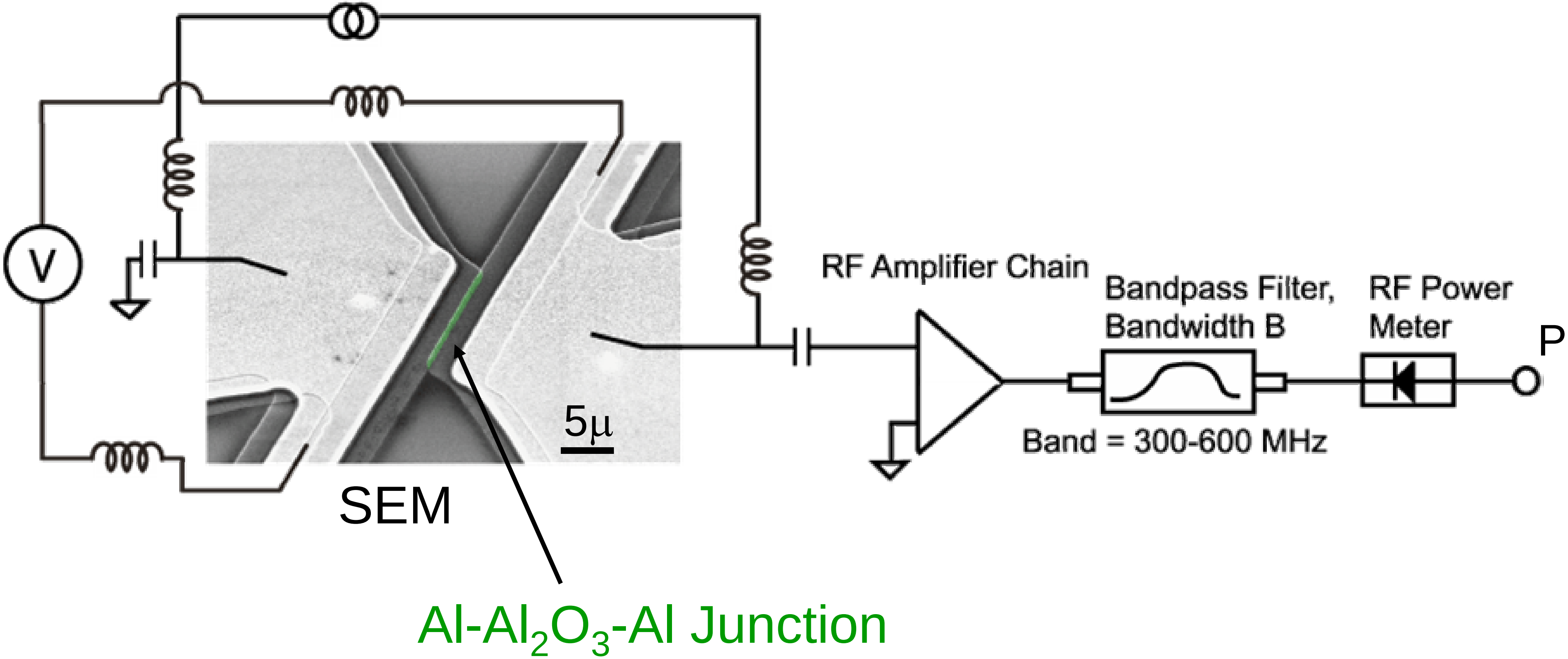






$$\boxed{S_I(f) = 2eV/R\coth\left(\frac{eV}{2k_BT}\right)}$$

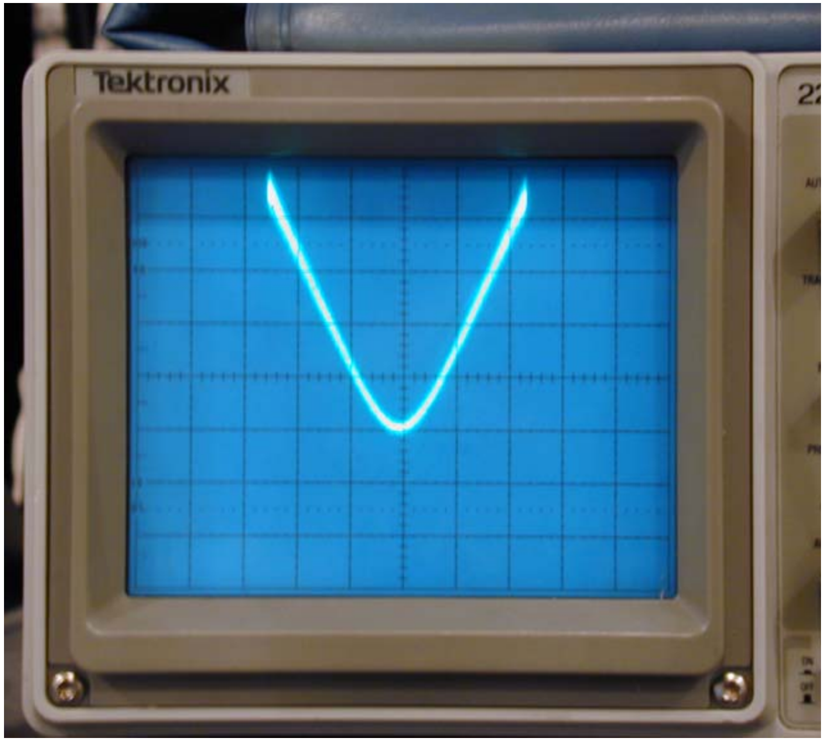
Noise Measurement of a Tunnel Junction



Measure symmetrized noise spectrum at $\hbar\omega < kT$

Seeing is Believing

$$\frac{\delta P}{P} = \frac{1}{\sqrt{B\tau}}$$

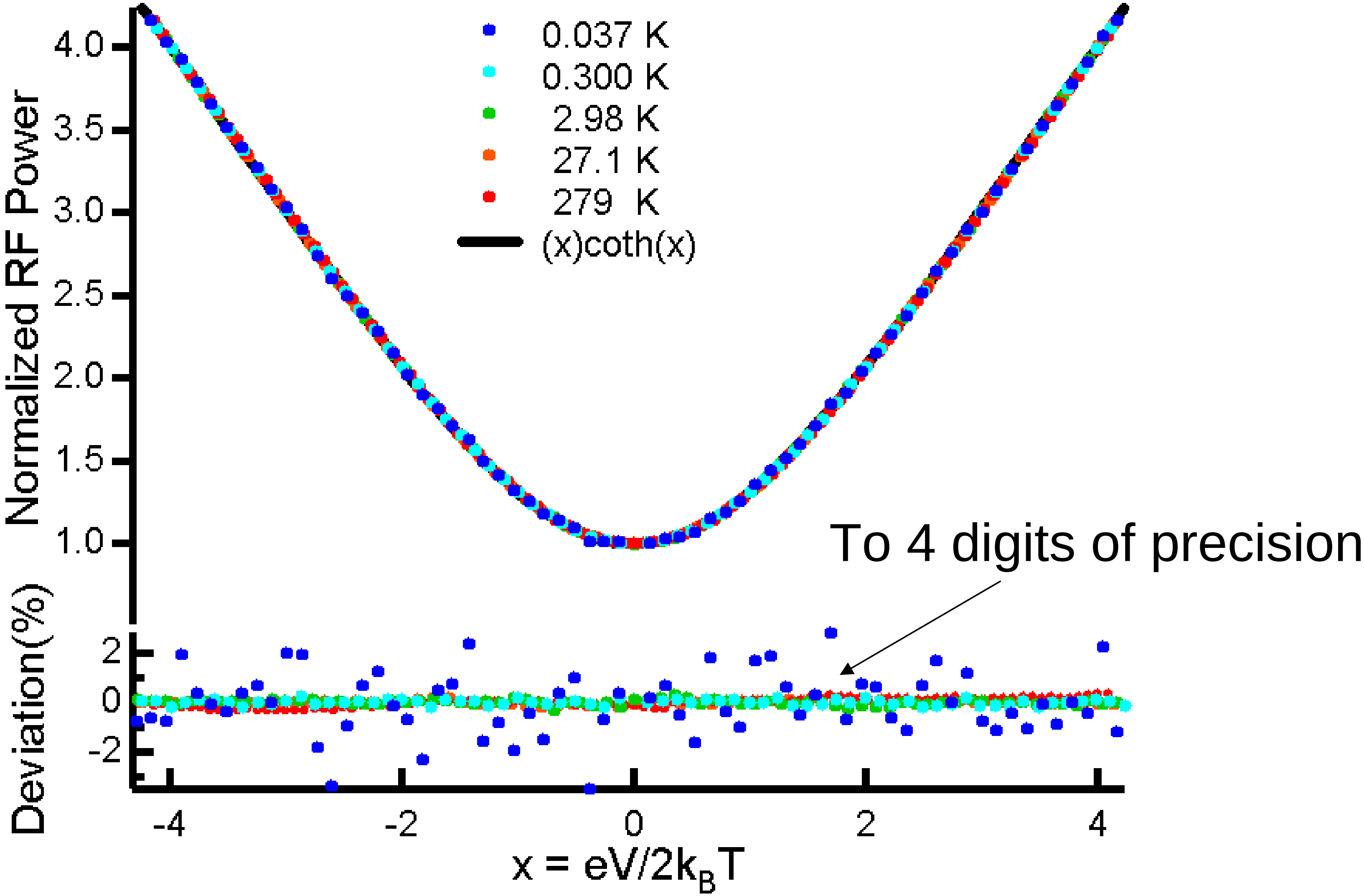


High bandwidth measurements of noise

$$B \sim 10^8\, Hz, \tau = 1 \text{ second} \longrightarrow \frac{\delta P}{P} = 10^{-4}$$

Test of Nonequilibrium FDT

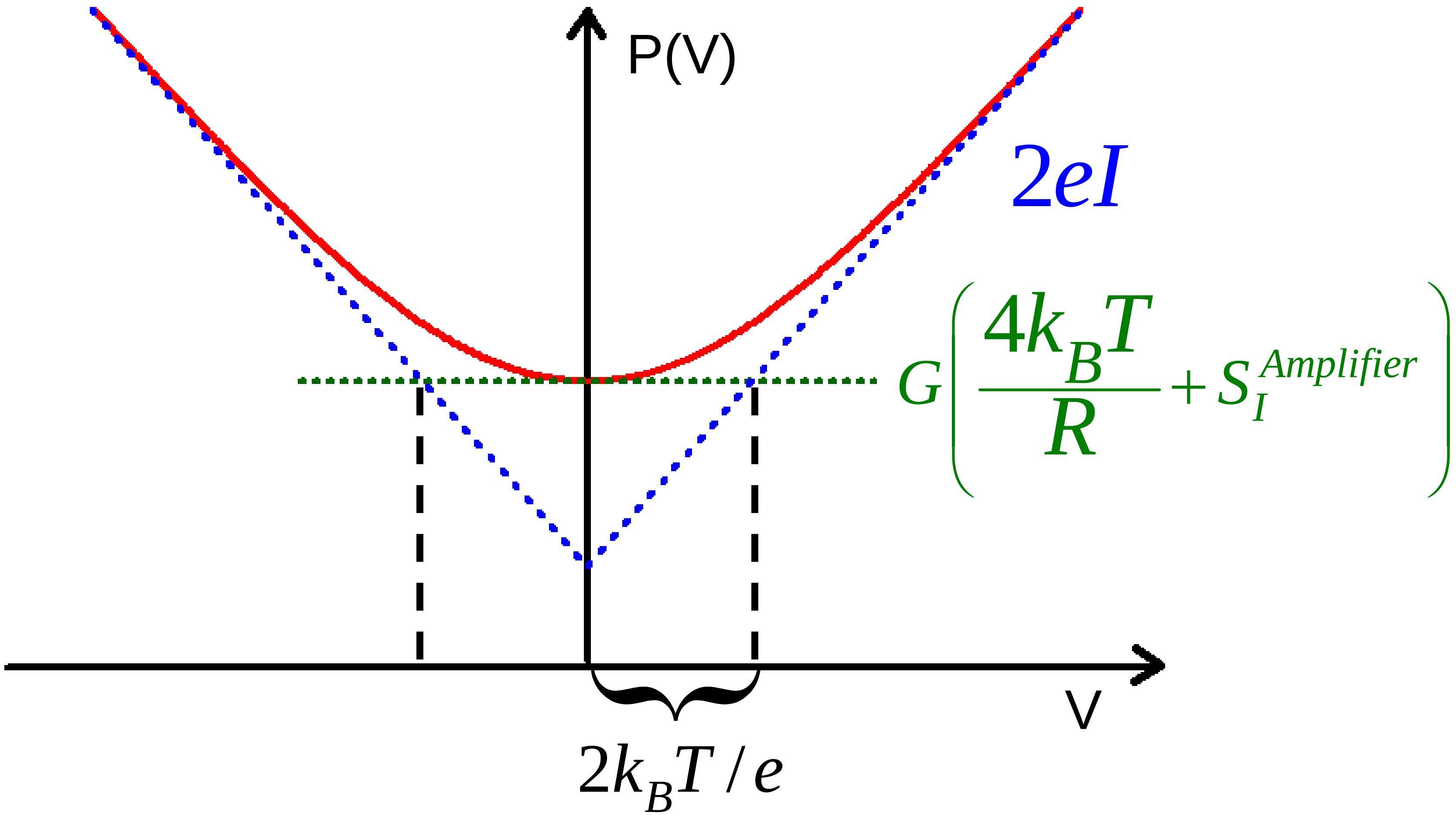
Agreement over four decades in temperature



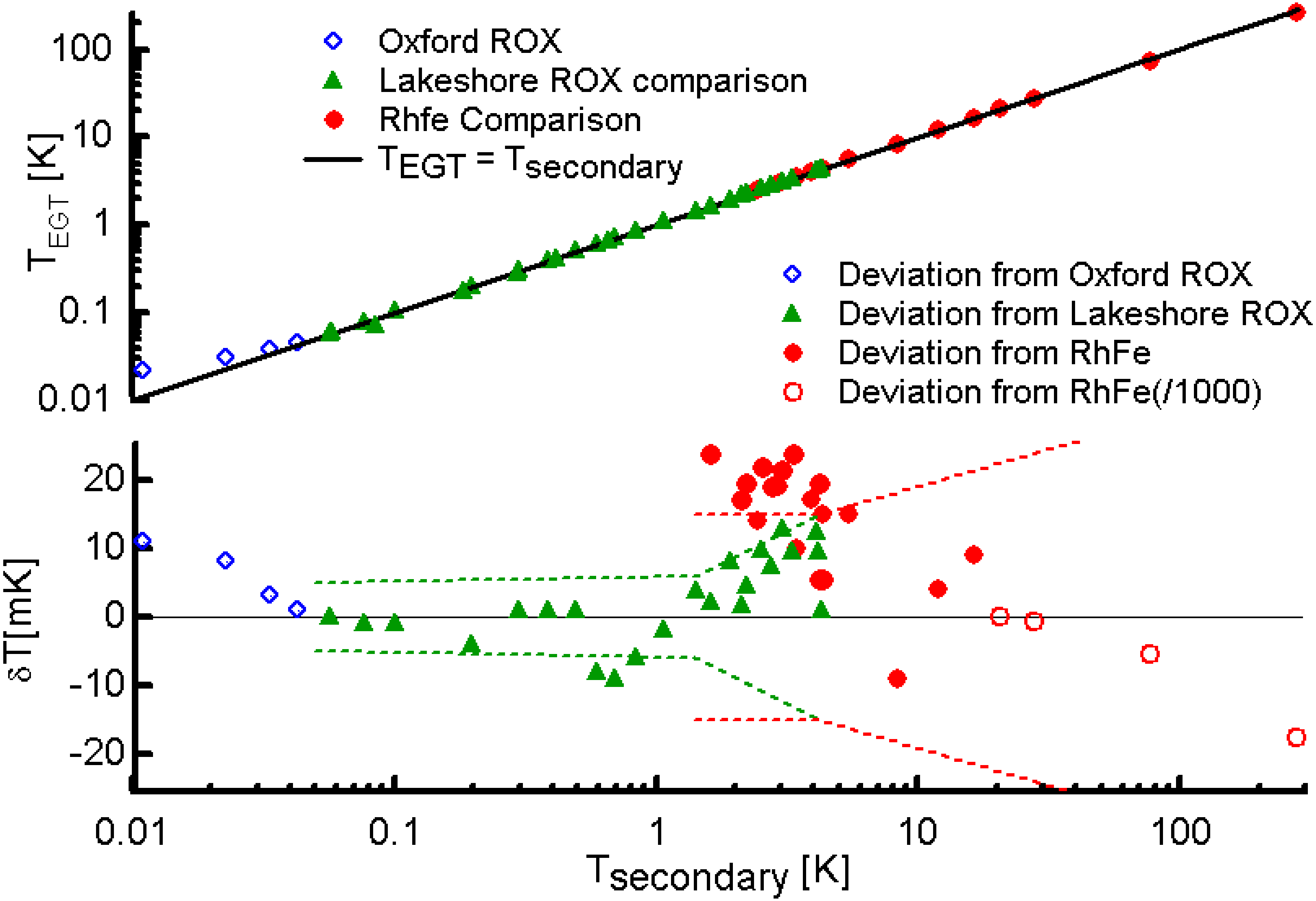
L. Spietz et al., Science **300**, 1929 (2003)

Self-Calibration Technique

$$P(V) = \mathrm{Gain}(\,S_I^{Amp} + S_I(V,T)\,)$$



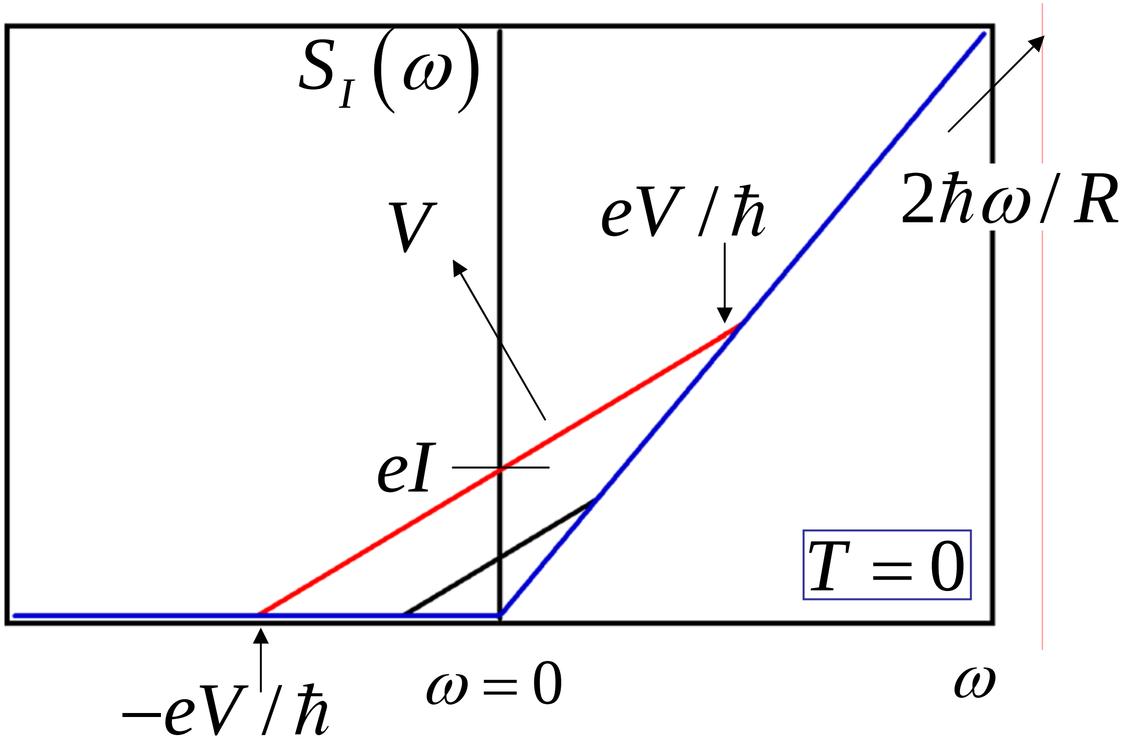
Comparison to Secondary Thermometers



Two-sided Shot Noise Spectrum

(Quantum, non-equilibrium FDT)

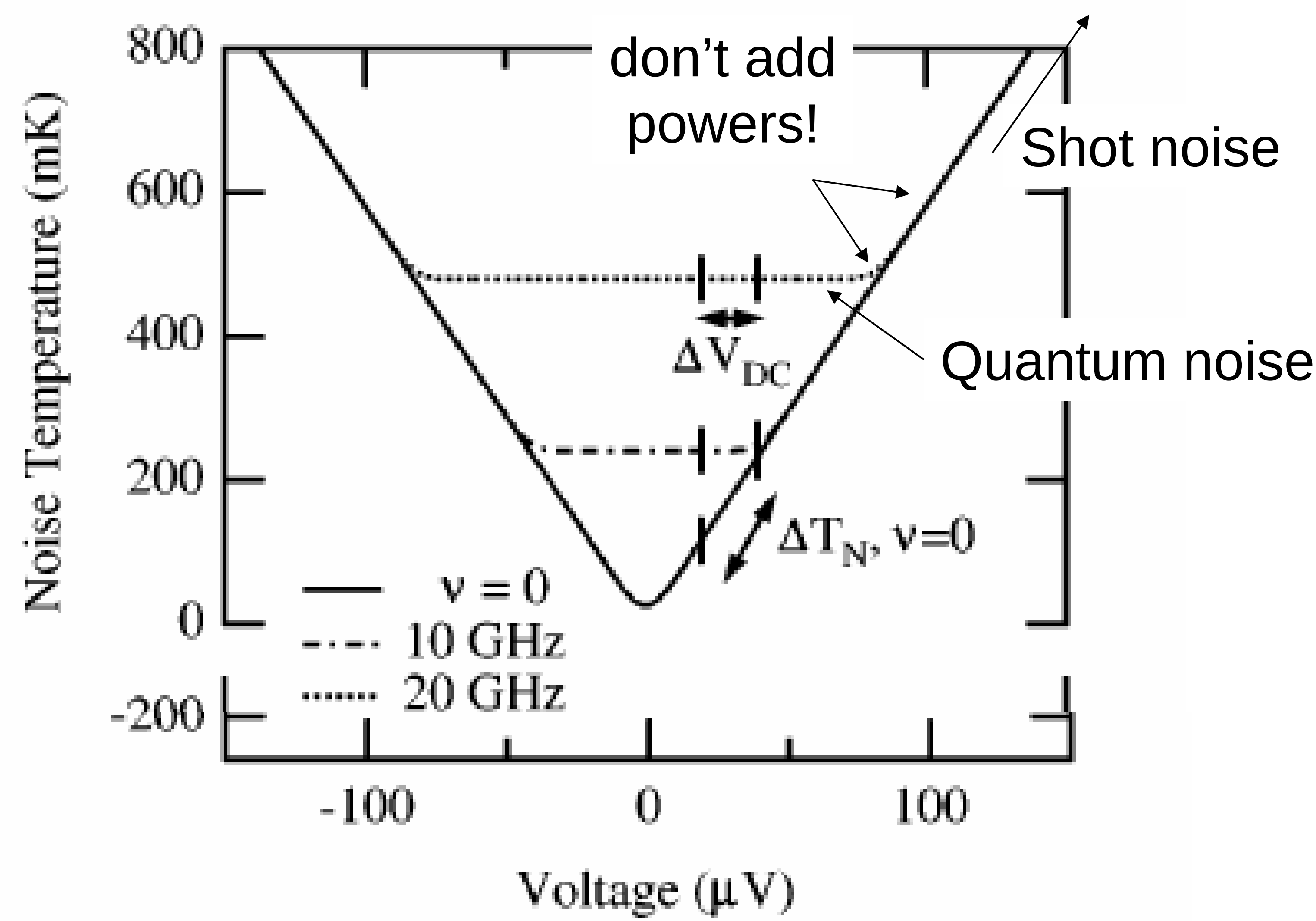
$$S_I(\omega) = \frac{(\hbar\omega + eV)/\mathrm{R}}{1 - e^{-(\hbar\omega + eV)/kT}} + \frac{(\hbar\omega - eV)/\mathrm{R}}{1 - e^{-(\hbar\omega - eV)/kT}}$$



Aguado & Kouwenhoven, PRL **84**, 1986 (2000).

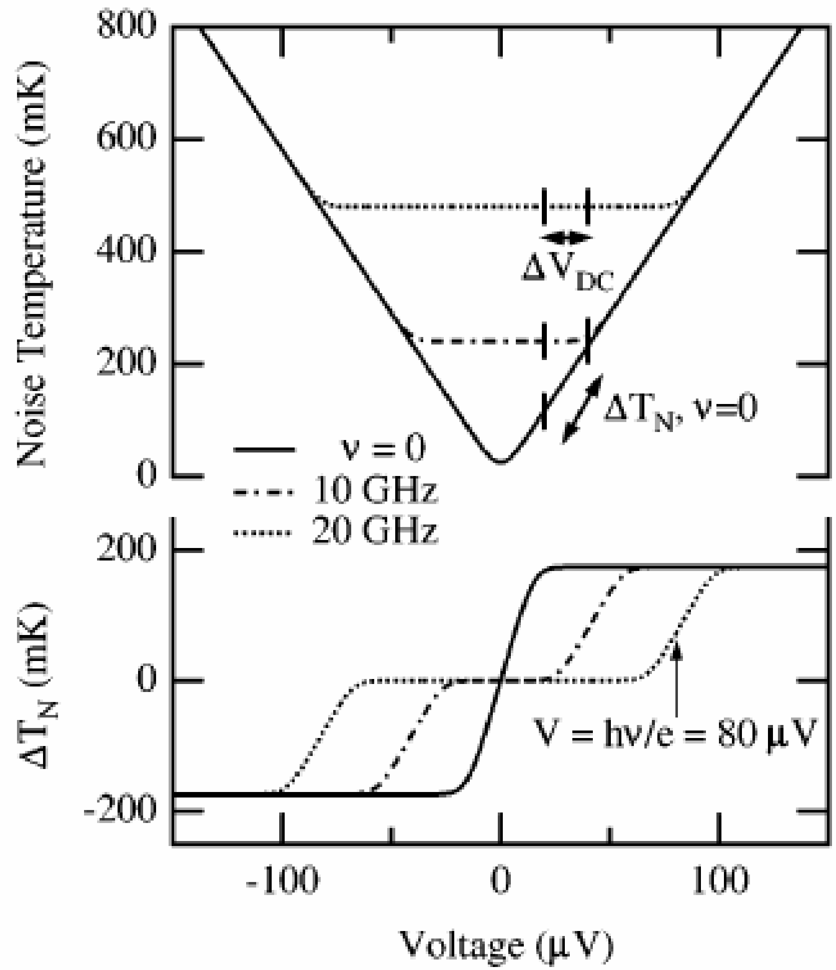
Finite Frequency Shot Noise

Symmetrized Noise: $S_{sym} = S(+\omega) + S(-\omega)$

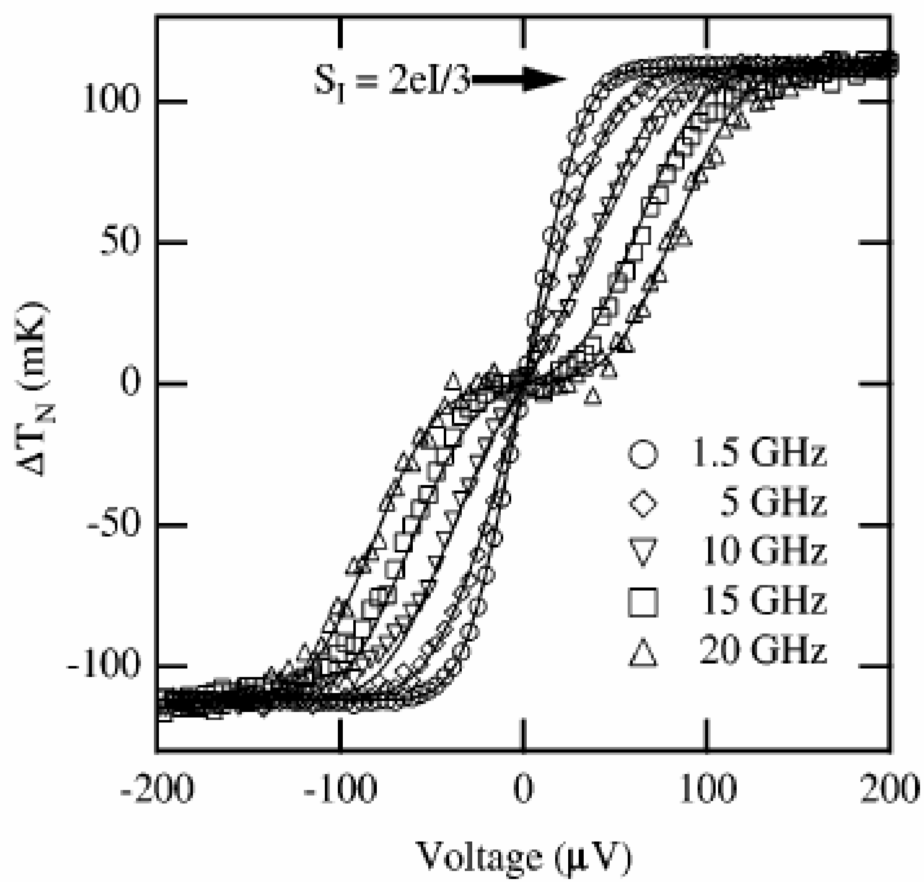


Measurement of Shot Noise Spectrum

Theory

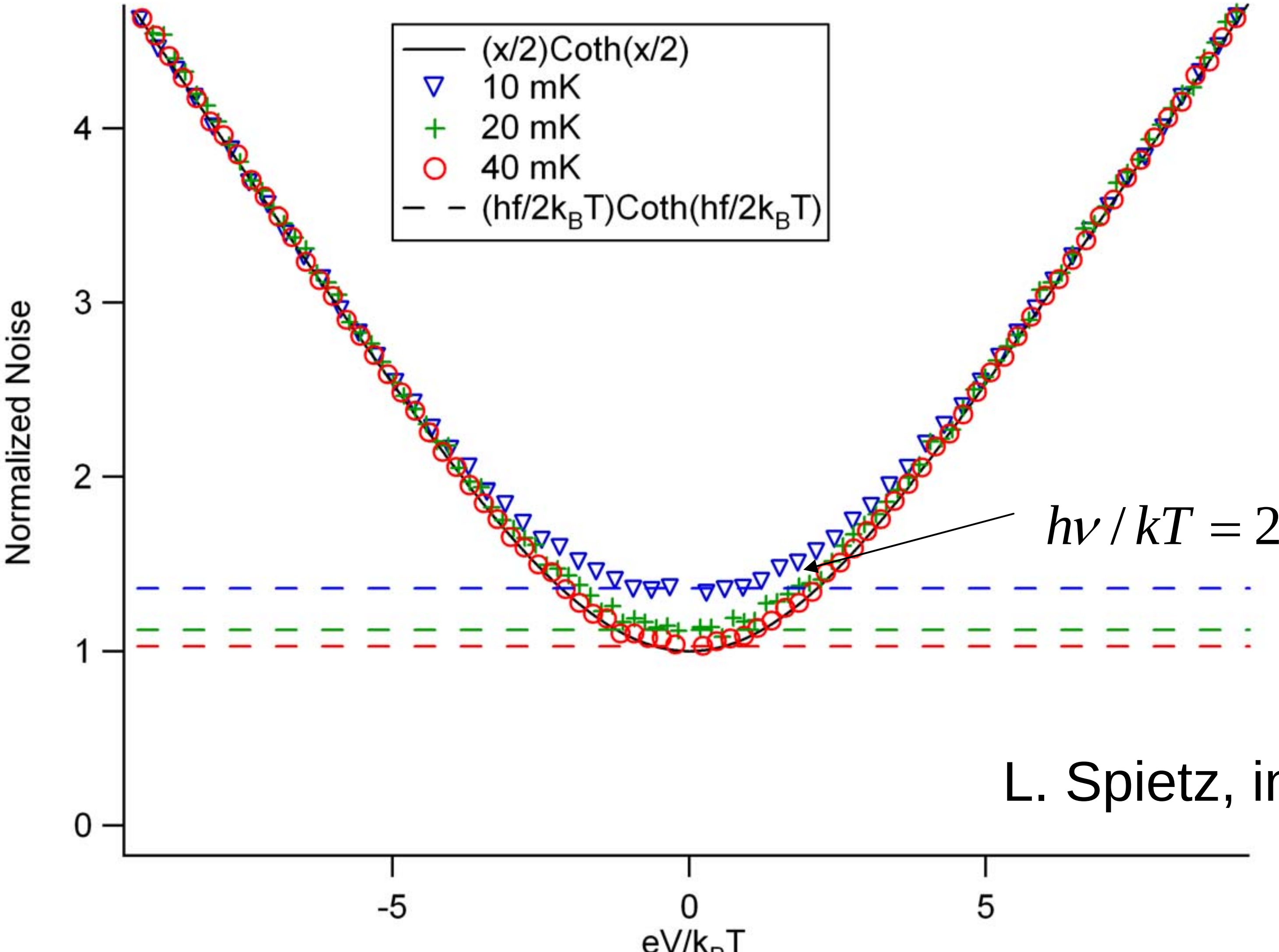


Expt.



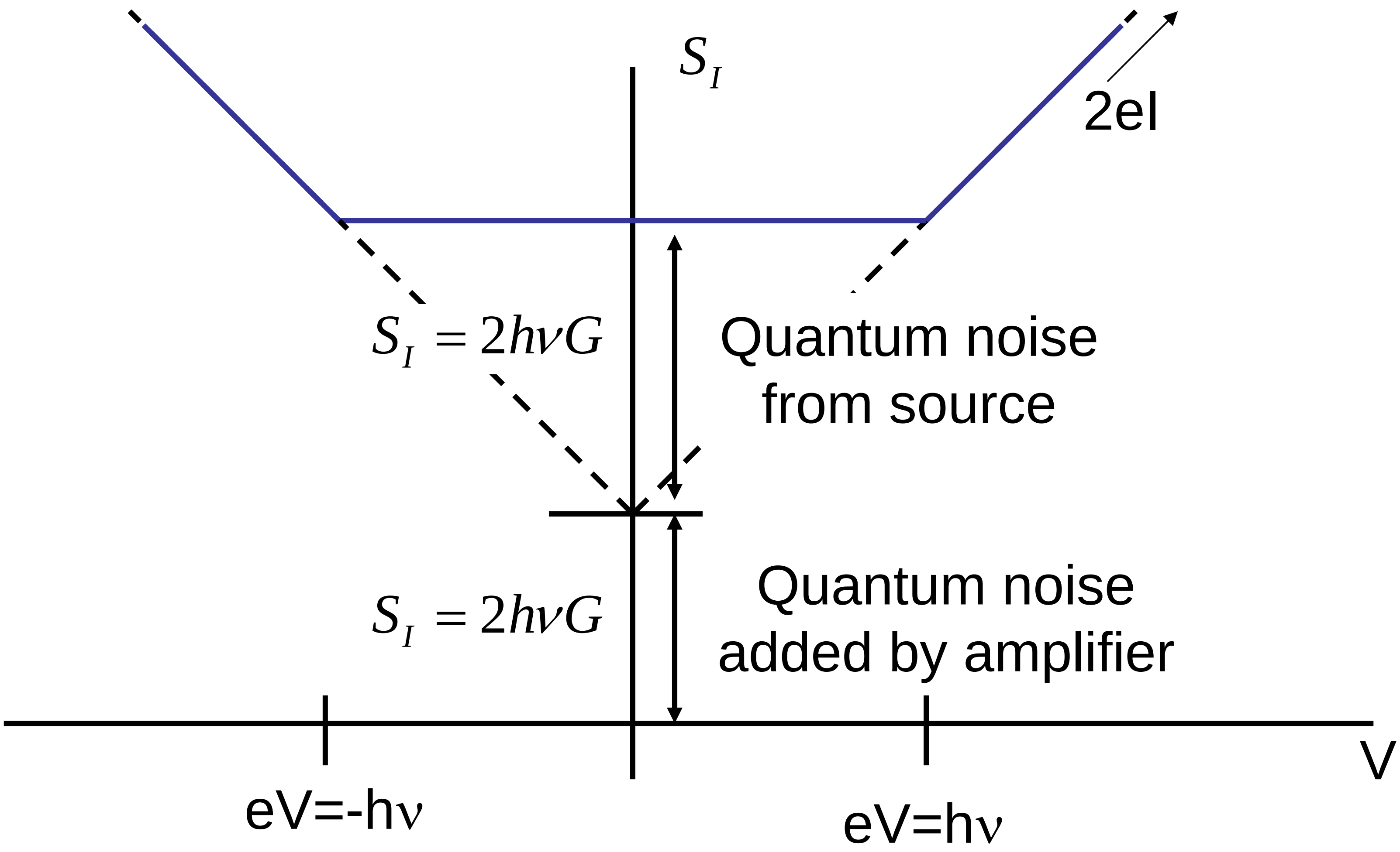
Schoelkopf et al., PRL **78**, 3370 (1998)

Shot Noise at 10 mK and 450 MHz



L. Spietz, in prep.

With An Ideal Amplifier and T=0



Summary – Lecture 1

- Quantizing an oscillator leads to quantum fluctuations present even at zero temperature.
- This noise has built in correlations that make it very different from any type of classical fluctuations, and these cannot be represented by a traditional spectral density- requires a “two-sided” spectral density.
- Quantum systems coupled to a non-classical noise source can distinguish classical and quantum noise, and allow us to measure the full density – next lecture!

Additional material on Johnson noise follows

Nyquist's Derivation of Johnson's Noise

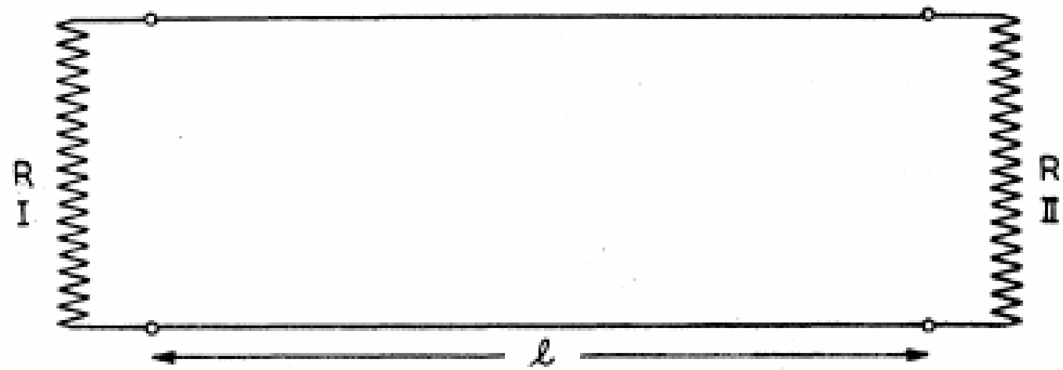
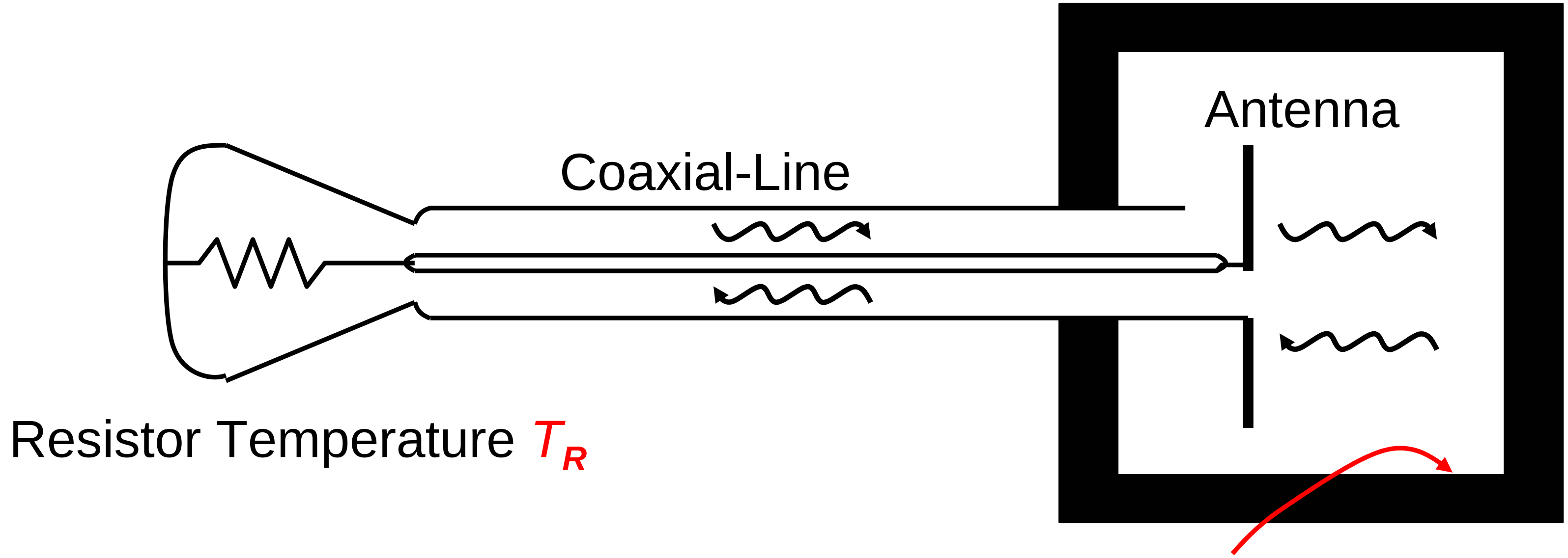


Fig. 3.

Connection Between Johnson Noise and Blackbody Radiation*

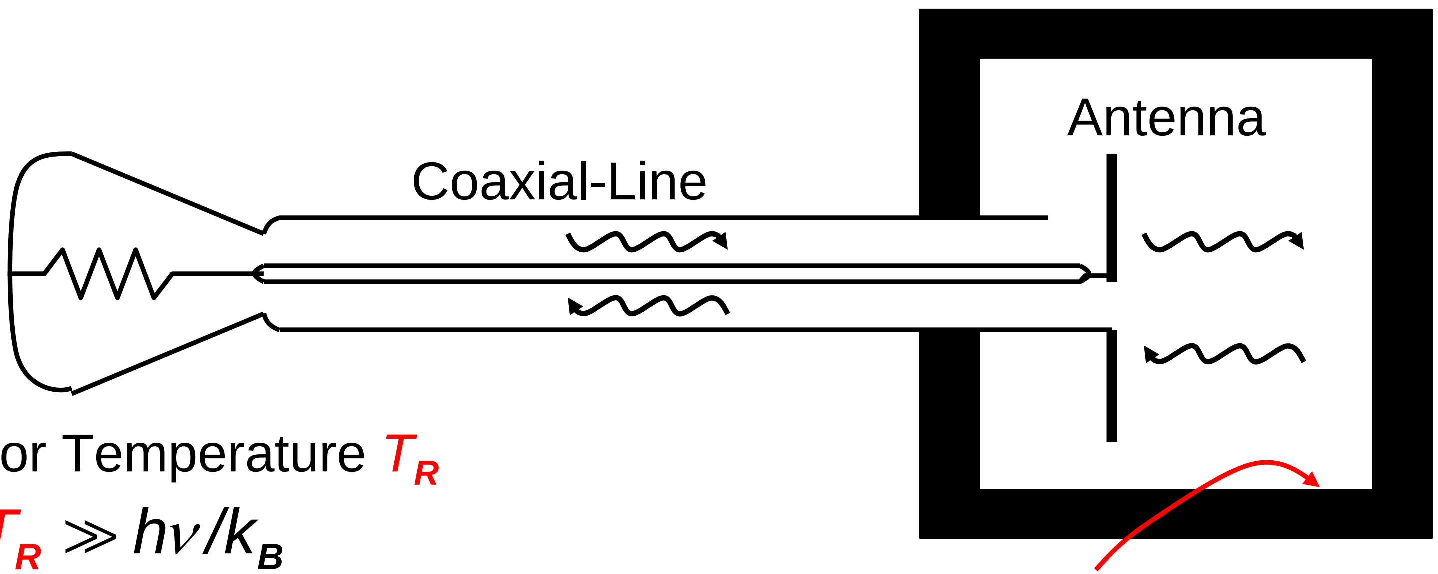
*R. H. Dicke, Rev. of Sci. Instrum. **17**, 268 (1946)



$$T_R = T_{bb} \longrightarrow$$

Johnson Noise Power ≡ Blackbody Radiation Power

Connection Between Johnson Noise and Blackbody Radiation in Rayleigh-Jeans Limit



Resistor Temperature T_R

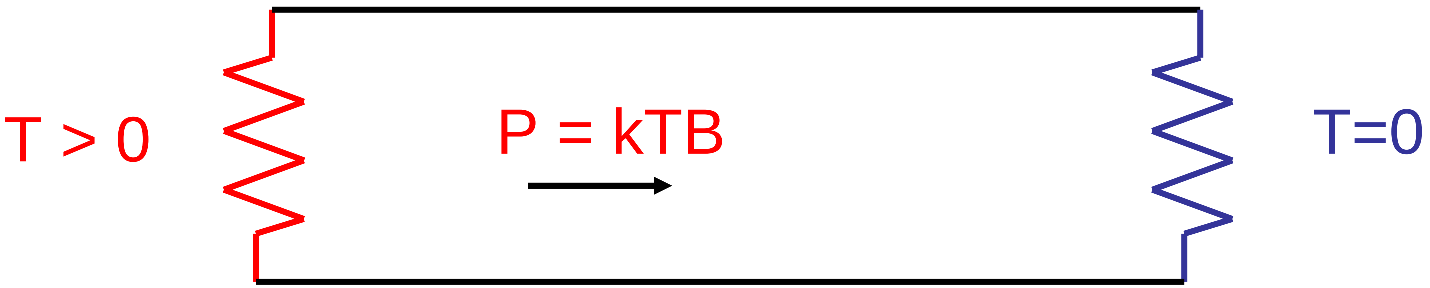
$T_R \gg h\nu / k_B$

$$\boxed{S_I = 4 k_B T / R}$$

Enclosure Walls Black Temperature T_{bb}
Rayleigh-Jeans limit

$$P_{Johnson} \sim k_B T_R B \longleftrightarrow P_{bb\text{-}Rayleigh\text{-}Jeans} \sim k_B T_{bb} B$$

Radiative Cooling of a Resistor?



Total radiated power: $kTB_{\max} = kT \times \frac{kT}{h}$

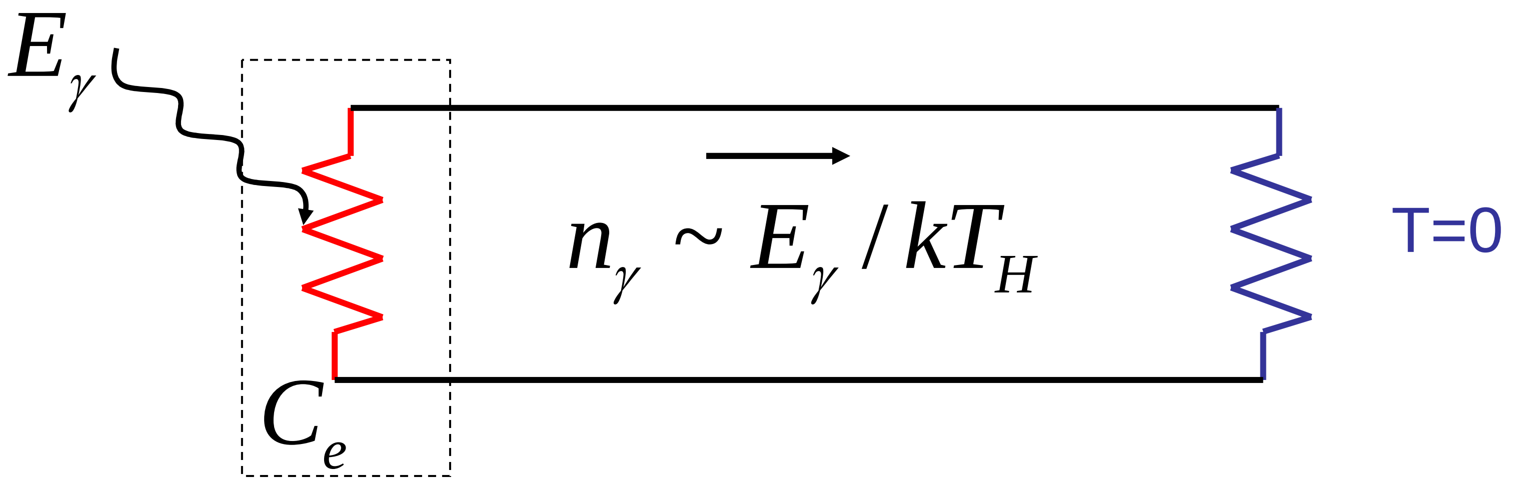
Total conductance: $G_{photon} = dP/dT = \frac{k^2}{h} T$

More correct: $$P = \int_0^\infty h\nu n(\nu) d\nu = \int_0^\infty \frac{h\nu}{e^{h\nu/kT} - 1} d\nu$$

One quantum of thermal conductance per electromagnetic mode

Schmidt, Cleland, and Schoelkopf, PRL 93, 045901 (2004)

Resistor as Ideal Square Law Detector



Single photon heats one resistor to $T_H = E / C_e$

If no other thermal conductances, cools entirely by radiation!

Photon number gain is large!: $n_\gamma \sim E_\gamma / kT_H$

Where's the nonlinearity?