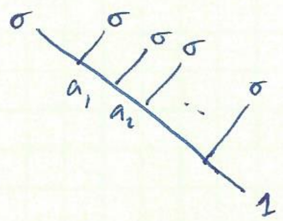
• Ising anyons $\sigma \cdot \sigma = 1 + \psi$



$$N_{\sigma\sigma}^{a_1} \, N_{a_1\sigma}^{a_2} \, N_{a_2\sigma}^{a_3} \cdots N_{a_{n-1}\sigma}^{1}$$

$$N_{a\sigma}^{b} \rightarrow (N_\sigma)_a^{\ b} = \begin{array}{c c} \begin{array}{ccc} 1 & \sigma & \psi \end{array} & \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ \sigma \\ \psi \end{array} \end{array}$$

eigenvalues $\lambda = 0, \pm\sqrt{2}$

$\therefore$ # of states $\sim (\sqrt{2})^n$

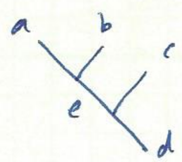
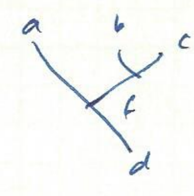
"quantum dimension" $d_\sigma = \sqrt{2}$

\# states per particle

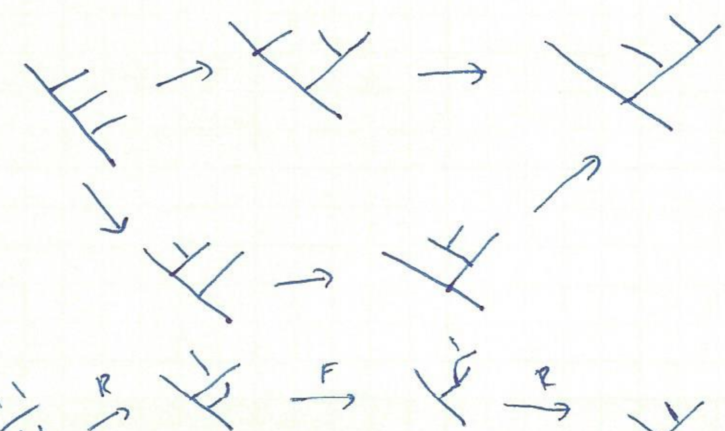
• Quantum dim : Total q. d. $D \equiv \sqrt{d_a^2}$

generalizes $|D|$ discr. gp. of a lattice.

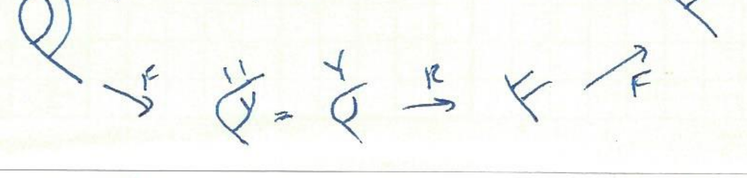
$$S = \frac{1}{\sqrt{D}} \sum_c d_c \left(R^{ab}_c \, R^{ba}_c \right) =$$ 

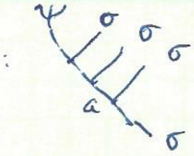
•  $= \sum_f \left[F^{abc}_d\right]_e^{\ f}$ 

• Pentagon:

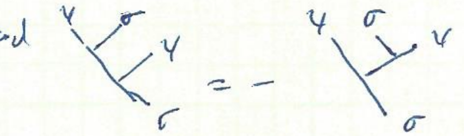


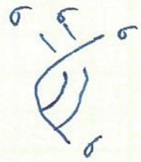
• Hexagon:



Ising anyons: Pentagon: 

$\rightarrow$ $F^{\sigma\sigma\sigma}_{\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

and 

Hexagon: 

$\rightarrow$ $\underbrace{R^{\sigma\sigma}_{1} = e^{-\pi i/8}, \quad R^{\sigma\sigma}_{\psi} = e^{3\pi i/8}}_{= e^{\pi i/8}\, e^{\pm\frac{\pi}{4}\gamma_1\gamma_2}}$

$R^{\sigma\psi}_{\sigma} = -i$

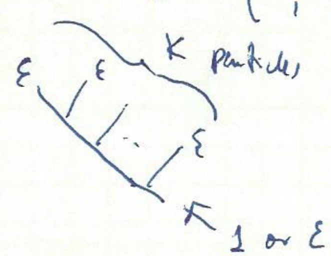
$\theta_\sigma = e^{+\pi i/8}$, $\theta_\psi = -1$, $\theta_1 = 1$ $\qquad \theta_a = \pm\left(R^{\bar{a}a}_{1}\right)^*$

• ~~[illegible]~~ $\quad \frac{1}{\sqrt{D}}\sum d_a^2\,\theta_a = e^{\pi i/8} \Rightarrow c = 1/2$ $\qquad |\,{}^1/\rho = \theta_a\,|$

• Fibonacci anyons $\quad 1, \varepsilon$

$\varepsilon \cdot \varepsilon = 1 + \varepsilon$

$N_\varepsilon = \begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix} \Rightarrow d_\varepsilon = \frac{1+\sqrt{5}}{2} = \phi$ "golden ratio"



$n^1_K = n^\varepsilon_{K-1}$

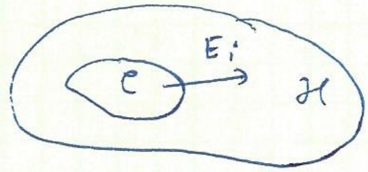
$n^\varepsilon_K = n^\varepsilon_{K-1} + n^1_{K-1} = n^\varepsilon_{K-1} + n^\varepsilon_{K-2}$

Fibonacci numbers

$F^{\varepsilon\varepsilon\varepsilon}_{\varepsilon} = \begin{pmatrix}\phi^{-1} & \phi^{-1/2}\\ \phi^{-1/2} & \phi^{-1}\end{pmatrix}$

$R^{\varepsilon\varepsilon}_{1} = e^{4\pi i/5} \qquad R^{\varepsilon\varepsilon}_{\varepsilon} = e^{-3\pi i/5}$

- Error-correcting Code



$\mathcal{C} \subset \mathcal{H}$

code subspace ↑ ↖ Hilbert space

$$\boxed{P_C E_j^{\dagger} E_i P_C = c_{ij} P_C}$$

e.g. Shor 9-qubit code :

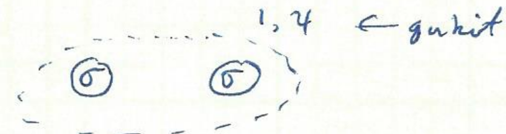
$$|0\rangle_L = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)\cdot\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|1\rangle_L = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)\cdot\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

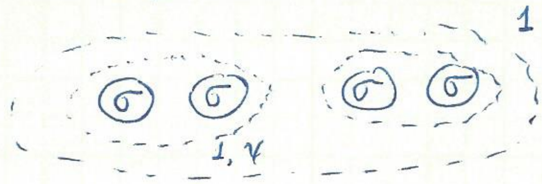
Compare to def. of a top. phase: $\boxed{\langle a|\,x\,|b\rangle = C\,\delta_{ab} + O(e^{-L/\xi})}$

↑ The different ground states of a top. phase (e.g. on an n-punctured plane) can serve as a code subspace

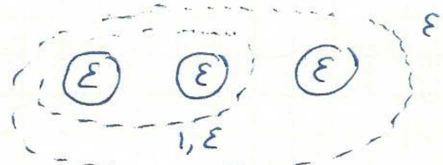
- Ising anyons : Dense encoding



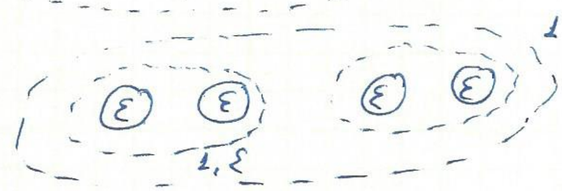
Sparse encoding



- Fibonacci anyons : Dense encoding

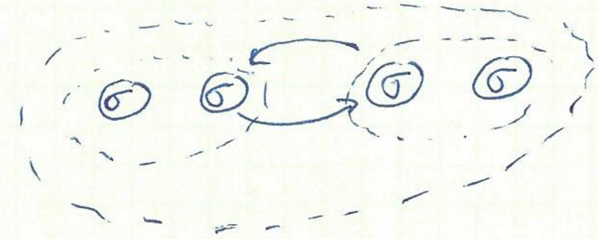


Sparse encoding



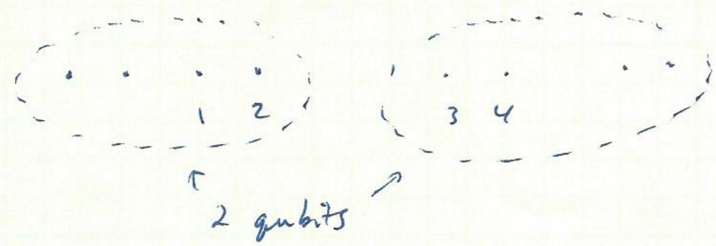
Gate Sets: H, $\Lambda(\sigma_z)$, T form a complete set of universal gates

Ising H:



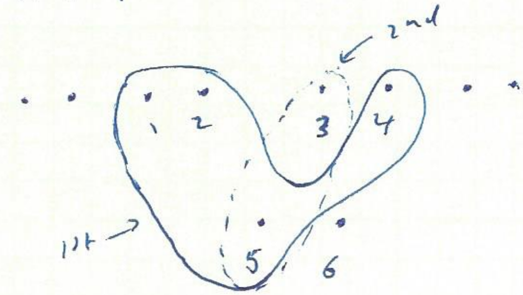
$$\Lambda(\sigma_z) = e^{i\frac{\pi}{4}}\, e^{-i\frac{\pi}{4}\gamma_1\gamma_2\gamma_3\gamma_4}\, e^{-\frac{\pi}{4}\gamma_1\gamma_2}\, e^{-\frac{\pi}{4}\gamma_3\gamma_4}$$

where



Need to apply $e^{-i\frac{\pi}{4}\gamma_1\gamma_2\gamma_3\gamma_4}$

Use an ancilla and perform measurements:



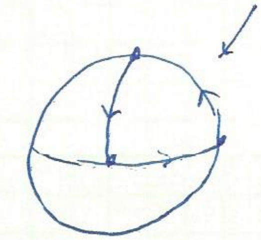
Bravyi '05

$$e^{i\frac{\pi}{4}\gamma_1\gamma_2\gamma_3\gamma_4} = 2\, e^{\frac{\pi}{4}\gamma_3\gamma_6}\, \Pi_+^{(2)}\, \Pi_-^{(4)}$$

$$= 2i\, e^{\frac{\pi}{2}\gamma_1\gamma_2}\, e^{\frac{\pi}{2}\gamma_3\gamma_4}\, e^{\frac{\pi}{4}\gamma_3\gamma_6}\, \Pi_-^{(2)}\, \Pi_-^{(4)}$$

$$\vdots$$

$$T = \begin{pmatrix} 1 & \\ & e^{\pi i/4} \end{pmatrix} \qquad S = T^2 = \begin{pmatrix} 1 & \\ & i \end{pmatrix} :$$



octant on Bloch sphere

Need to produce $\frac{1}{\sqrt{2}}\left(|0\rangle + e^{\frac{\pi i}{4}}|1\rangle\right)$

Then $T\left(a|0\rangle + b|1\rangle\right)$ can be obtained from

$$\left(a|0\rangle + b|1\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle + e^{\frac{\pi i}{4}}|1\rangle\right)$$

$\downarrow$ Joint parity

$$a|0\rangle|0\rangle + b\,e^{\frac{\pi i}{4}}|1\rangle|1\rangle$$

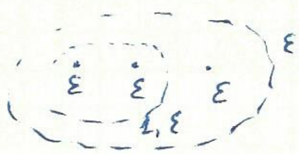
$\downarrow$ CNOT

$$\left(a|0\rangle + b\,e^{\pi i/4}|1\rangle\right)|1\rangle$$

15 copies of $\frac{1}{\sqrt{2}}\left(|0\rangle + e^{\frac{\pi i}{4}}|1\rangle\right)$ error ε $\xrightarrow{\text{Reed-Muller code}}$ $\underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle_L + e^{\frac{\pi i}{4}}|0\rangle_L\right)}_{\text{1 copy of } \frac{1}{\sqrt{2}}\left(|0\rangle + e^{\frac{\pi i}{4}}|1\rangle\right) \text{ w/ error } 35\varepsilon^3}$

<u>How to make T?</u>

• Fibonacci



universal gate set!

$$= \begin{pmatrix} e^{4\pi i/5} & \\ & e^{-3\pi i/5} \end{pmatrix}$$

$$= \begin{pmatrix} \phi^{-1} & \phi^{-1/2} \\ \phi^{-1/2} & \phi^{-1} \end{pmatrix}\begin{pmatrix} e^{4\pi i/5} & \\ & e^{-3\pi i/5} \end{pmatrix}\cdot$$

$$\begin{pmatrix} \phi^{-1} & \phi^{-1/2} \\ \phi^{-1/2} & \phi^{-1} \end{pmatrix}$$

<u>How to obtain Fibonacci anyons?</u>

<u>Algorithms tailored to Fibonacci!</u>

Need $|j\rangle \rightarrow \frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i\frac{jk}{2^n}}|k\rangle$

Quantum Fourier Transform

Have $e^{\pi i}, e^{\frac{2\pi i}{4}}, e^{2\pi i/5}, e^{\frac{2\pi i}{10}}, e^{2\pi i/20}$