Antiferromagnetism and Superconductivity

Boulder School for Condensed Matter and Materials Physics
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Outline

1. Antiferromagnetism and quantum criticality in insulators
2. Onset of antiferromagnetism in metals, and d-wave superconductivity
3. Competing density wave order, and the pseudogap of the cuprate superconductors
4. Non-Fermi liquids
1. Antiferromagnetism and quantum criticality in insulators

2. Onset of antiferromagnetism in metals, and d-wave superconductivity

3. Competing density wave order, and the pseudogap of the cuprate superconductors

4. Non-Fermi liquids
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in Néel state.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ \begin{pmatrix} \uparrow \downarrow \end{pmatrix} - \begin{pmatrix} \downarrow \uparrow \end{pmatrix} = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} \uparrow \downarrow \end{pmatrix} - \begin{pmatrix} \downarrow \uparrow \end{pmatrix} \right) \]
Quantum critical point with non-local entanglement in spin wavefunction

Excitation spectrum in the paramagnetic phase
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Sharp spin 1 particle excitation above an energy gap (spin gap)
Excitation spectrum in the Néel phase
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Derivation of field theory of critical point
Description using Landau-Ginzburg field theory

\[ \mathcal{S} = \int d^2r d\tau \left[ \left( \partial_\tau \varphi \right)^2 + c^2 (\nabla_r \varphi)^2 + (\lambda - \lambda_c) \varphi^2 + u (\varphi^2)^2 \right] \]
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c) \varphi^2 + u (\varphi^2)^2 \]

\[ \lambda > \lambda_c \]

Spin \( S = 1 \) “triplon”
Excitation spectrum in the paramagnetic phase

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Spin waves ("Goldstone" modes) and a longitudinal "Higgs" particle

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Excitation spectrum in the Néel phase

Spin waves ("Goldstone" modes) and a longitudinal "Higgs" particle

$$V(\bar{\phi}) = (\lambda - \lambda_c)\bar{\phi}^2 + u(\bar{\phi}^2)^2$$

$$\lambda < \lambda_c$$
$O(3)$ order parameter $\vec{\phi}$

$$S = \int d^2 r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\phi}^2 + u (\vec{\phi}^2)^2 \right]$$

$CFT3$
Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents $\nu$, $\beta/\nu$, and $\eta$. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of $\alpha_c$. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the $\chi^2$/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

<table>
<thead>
<tr>
<th>$\alpha_c$</th>
<th>$\nu^a$</th>
<th>$\beta/\nu^b$</th>
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<td>1.9096 - $\sigma$</td>
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$^a L > 12.$
$^b L > 16.$
$^c L > 20.$
$^d$ Previous best estimate of Ref. 19.
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Table IV: Fit results for the critical exponents \( \nu \), \( \beta / \nu \), and \( \eta \). We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of \( \alpha_c \). The bottom group are results for the plaquette model. Numbers in [...] brackets denote the \( \chi^2 \)/d.o.f. For comparison relevant reference values for the 3D \( O(3) \) universality class are given in the last line.

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Field-theoretic RG of CFT3
E. Vicari et al.
An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
Pressure in TICuCl$_3$
TlCuCl₃ at ambient pressure

FIG. 1. Measured neutron profiles in the a* c* plane of TlCuCl₃ for i=(1.35,0,0), ii=(0,0,3.15) [r.l.u]. The spectrum at T=1.5 K

Observation of $3 \rightarrow 2$ low energy modes, emergence of new longitudinal mode (the “Higgs boson”) in Néel phase, and vanishing of Néel temperature at quantum critical point.

**Prediction of quantum field theory**

Potential for $\bar{\varphi}$ fluctuations: $V(\bar{\varphi}) = (\lambda - \lambda_c)\bar{\varphi}^2 + u(\bar{\varphi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\bar{\varphi} = 0$:

$$V(\bar{\varphi}) \approx (\lambda - \lambda_c)\bar{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$
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Néel phase, $\lambda < \lambda_c$

Expand $\varphi = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \varphi_1$: $V(\varphi) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$
Prediction of quantum field theory

\[ \frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2} \]

\[ V(\vec{\phi}) = (\lambda - \lambda_c)\phi^2 + u(\phi^2)^2 \]

\( \Phi(0, 0, 1) = (0, 4, 0) \)

\( V(\vec{\phi}) = (\lambda - \lambda_c)\phi^2 + u(\phi^2)^2 \)

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Quantum criticality at non-zero temperature