1. Example : Toric Code
   a) Hamiltonian, ground state,
   b) G.S. degeneracy
   c) QPs = GS on punctured surface : $1, e, m, \psi$
   d) Fusion : $e \times e = m \times m = 1$, $e \times m = \psi$
   e) ~~Braiding~~ Twists
   f) Braiding
   h) Effective field theory : $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$
   i) Physical realizations

2. Definition of a Top. Phase in 2D
   a) Local H, $\exists\, \Delta, \xi$ w/ $\Delta > 0$ and $\xi < \infty$ for $L \to \infty$
   such that $\exists$ finite-~~set~~ dim $V = \text{span}\{|1\rangle, |2\rangle, \dots |N_\Sigma\rangle\}$ ← energy eigenstates
   of states w/ energies less than Δ
   satisfying: N_Σ depends only on top. of Σ
   $$\langle a | X | b \rangle = C\, \delta_{ab} + O(e^{-L/\xi})$$
   for any local operator X.

Got this far

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   b) T.C. satisfies this
   c) Σ = punctured sphere = qps. . Identity particle, anti-particle $\bar{a} \times a = 1$
   d) 3-punctured sphere = fusion spaces V_{ab}^{c} $a, b, \bar{c}$
   e) N_{ab}^{c}
   f) quantum dim d_a
   g) MCG on n-punctured sphere : $\begin{cases} \theta_a & \text{twists} \\ R_{ab}^{c} & \text{braid group} \end{cases}$
   ≈ Braid group.

   h) Alternate def. <u>Exercise</u>: show that T.C. satisfies this

3. Second Example : Ising anyons
   a) QP types, fusion rules
   b) Quantum dims.
   c) F-matrix

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4. MTC : a) Particle types, including 1, $a \to \bar{a}$
   ~~b~~ b) Pentagon
   ~~c~~ c) Hexagon
   ~~d~~ d) Hexagon for Ising
   ~~d~~ e) MTC ⟷ TQFT
   MTC ⟷ real world ⟷ TQFT

1. Def'n of top phase: ground states on surfaces

   a) Annulus: particle types a, identity 1, anti-particle $\bar{a}$

   b) 3-punctured sphere: Hilbert space $V_{ab}^{c} \longrightarrow N_{ab}^{c}$ 

   c) n-punctured sphere: $MCG = \dfrac{\mathrm{diff}\,\Sigma}{(\mathrm{diff}\,\Sigma)_0}$ for $\Sigma = S \setminus P_1 \cup P_2 \cdots P_{n+1}$

   $\longrightarrow$ Braid group $R_{\psi}^{em} = -1$, $R_{\psi}^{me} = 1$

   Dehn twists: $\theta_1 = \theta_e = \theta_m = 1$

   $\theta_\psi = -1$

   d) Chern-Simons theory, $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

   e) Physical Realizations

2. Second example: Ising anyons

   a) Particle types, fusion rules: $1, \sigma, \psi$

   $\sigma \times \sigma = 1 + \psi$

   b) MZM interpretation $i\gamma_1\gamma_2 = \pm 1 \Longleftrightarrow \sigma \cdot \sigma = 1 + \psi$

   $\left.\begin{matrix} i\gamma_1\gamma_2 = \sigma_z \\ i\gamma_1\gamma_4 = \sigma_x \end{matrix}\right\} \Longleftrightarrow F = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

   c) $N_{ab}^{c} \longrightarrow$ Quantum dimension $d = \sqrt{2}$

   d) F: associator

   e) Pentagon: $F \cdot \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot F$

   f) R-matrix & hexagon $R \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$