

1. Example : Toric Code
 - a) Hamiltonian, ground state,
 - b) G.S. degeneracy
 - c) QPs = GS on punctured surface : l, e, m, ψ
 - d) Fusion : $e \times e = m \times m = 1, e \times m = \psi$
 - e) ~~Excitations~~ Twists
 - f) Braiding
 - h) Effective field Theory : $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$
 - i) Physical realizations

2. Definition of a Top. Phase in 2D

- a) Local $H, \exists \Delta, \xi$ w/ $\Delta > 0$ and $\xi < \infty$ for $L \rightarrow \infty$ such that \exists finite-~~set~~ $\dim V = \text{span} \{ |1\rangle, |2\rangle, \dots, |N_\xi\rangle \}$ ← energy eigenstates of states w/ energies less than Δ satisfying : N_ξ depends only on top. of Σ

$$\langle a | X | b \rangle = C \delta_{ab} + O(e^{-L/\xi})$$
 for any local operator X .

Got this far

- b) T.C. satisfies this
- c) $\Sigma =$ punctured sphere = gps. . Identity particle, anti-particle $\bar{a} \times a = 1$
- d) 3-punctured sphere = fusion spaces V_{ab}^c a, b, \bar{c}
- e) N_{ab}^c
- f) quantum dim d_a
- g) MCG on n -punctured sphere : $\begin{cases} \theta_a \text{ twists} \\ R_{ab}^c \text{ braid group} \end{cases}$
- h) Alternate def. Exercise: show that T.C. satisfies this

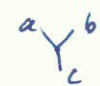
3. Second Example : Ising anyons

- a) QP types, fusion rules
- b) Quantum dims.
- c) F-matrix

4. MTC : $\begin{matrix} a) \text{ Particle types, including } 1, a \rightarrow \bar{a} \\ b) \text{ Pentagon} \\ c) \text{ Hexagon} \end{matrix} \quad \begin{matrix} d) \text{ Hexagon for Ising} \\ e) \text{ MTC} \leftrightarrow \text{TQFT} \\ \quad \swarrow \text{real world} \end{matrix}$

1. Def'n of top phase: ground states on surfaces

a) Annulus: particle types a , identity 1 , anti-particle \bar{a}

b) 3-punctured sphere: Hilbert space $V_{ab}^c \rightarrow N_{ab}^c$ 

c) n -punctured sphere: $MCG = \frac{diff \Sigma}{(diff \Sigma)_0}$ for $\Sigma = S^2 \setminus \{P_1, \dots, P_{n+1}\}$



→ Braid group $R_{\frac{cm}{4}} = -1, R_{\frac{me}{4}} = 1$
 Dehn twists: $\theta_1 = \theta_c = \theta_m = 1$
 $\theta_{\frac{c}{4}} = -1$

d) Chern-Simons theory, $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

e) Physical Realizations

2. Second example: Ising anyons

a) Particle types, fusion rules: $1, \sigma, \psi$
 $\sigma \times \sigma = 1 + \psi$

b) MZM interpretation $i\sigma, \tau_z = \pm 1 \leftrightarrow \sigma \cdot \sigma = 1 + \psi$
 $\left. \begin{matrix} i\tau_x, \tau_z = \sigma_z \\ i\sigma, \tau_y = \sigma_x \end{matrix} \right\} \leftrightarrow F = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

c) $N_{ab}^c \rightarrow$ Quantum dimension $d = \sqrt{2}$

d) F : associator

e) Pentagon: $F \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \cdot F$

f) R -matrix & hexagon $R \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

