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Fundamental Interactions I

overview:
- Ubiquitous forces
- Electrostatics
- Hydrogen bonds, hydrophobicity

Soft Matter

"squishy"

→ elastic modulus $G \sim \frac{T_c}{L^3}$ T_c ~ characteristic temperature

room temp. $(k_B = 1)$

Ex: $T_c \sim 0.1\,eV \sim 10\ T$

Hard $L \sim 10^{-8}$ cm

Soft $L \sim 10$ Å

$G_{soft} \sim 10^{-3}\, G_{hard}$

E_1 ~ atomic ~ eV

E_2 ~ interaction ~ 0.1 eV

E_3 ~ entropy ~ 0.01 eV

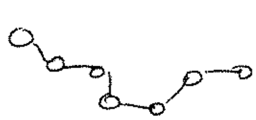
} Soft matter

Colloids - solid particles in solvent. 100 Å - 1 μm

Emulsions - liquid droplets

foams - gas in solvent

polymers - connected "beads"



Dilute Solution Thermodynamics

concentration $\quad c \equiv \frac{N}{V} \quad \frac{\#}{\text{vol.}}$

dilute : c is small

assume short-range forces

free energy

free energy density $\rightarrow$

$$\frac{F|_{\text{volume}}}{T} = \frac{F_0}{T} + B_1 c + \frac{1}{2} B_2 c^2 + \frac{1}{6} B_3 c^3 + \dots + \underbrace{c(\ln c - 1)}_{\text{entropy of mixing}}$$

F_0: pure solvent

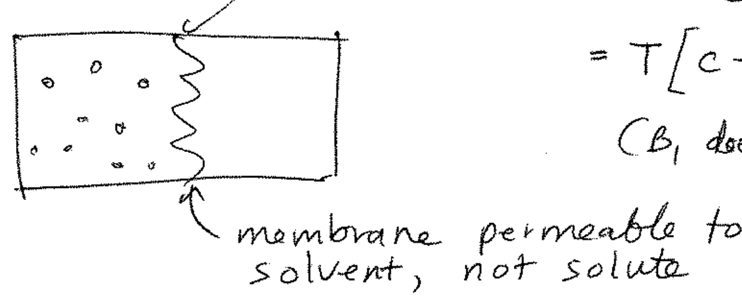
Virial expansion

$(T \equiv k_B T)$ room temp.

chemical potential (of solute)

$$\mu = \frac{\partial F}{\partial N}\Big|_{V,T} = T(B_1 + B_2 + \ln c + \dots)$$

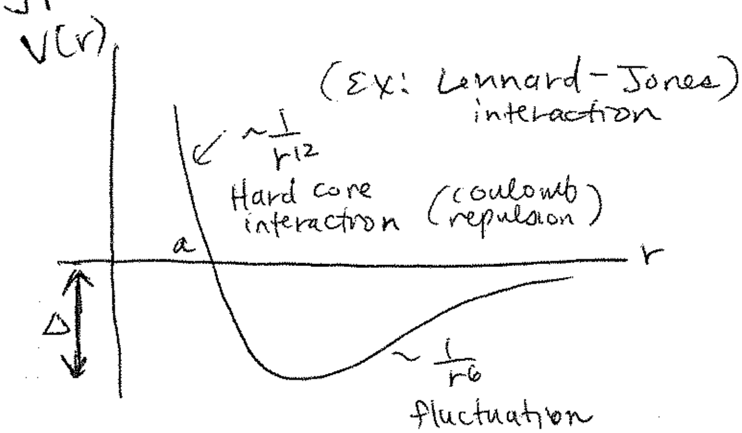
Osmotic pressure : $\Pi = c \frac{\partial F}{\partial c} - F$



$$= T\left[c + \frac{B_2}{2} c^2 + \frac{1}{3} B_3 c^3 + \dots\right]$$

(B_1 does not affect Π)

typical interaction between molecules (in solvent)



2nd virial coefficient:

$$B_2 = \int \left[1 - e^{-V(r)/T}\right] d^3r$$

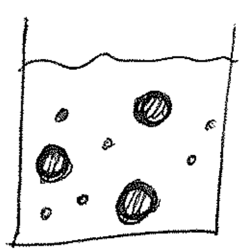
if $T \gg \Delta$:

$B_2 \sim a^3 > 0$ (note: dimension of volume)

if $T < \Delta$:

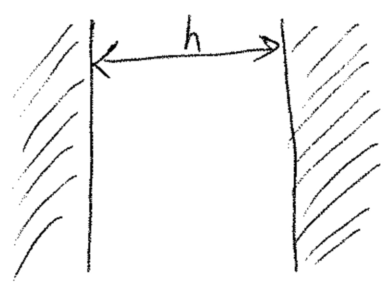
$B_2 \simeq -a^3 e^{\Delta/T}$

If $B_2 > 0 \Rightarrow$ repulsive interaction of solutes. (good solvent)



Depletion Interactions (Oosawa)

Zoom in on 2 big particles (approx. flat)



radius of small particles $r = b$

If $h < b$:

- small particles can't fit → depletion zone between big particles
- pressure on big particles from the outside: $\Pi \sim cT$

$\Rightarrow$ attraction between big spheres

$\Pi = -cT$

range $\sim b$

Hydrogen atom

(cgs)

$$E = \frac{p^2}{2m} - \frac{e^2}{r}$$

Bohr quantization

$$\oint \vec{p}\cdot d\vec{r} = n \cdot \hbar$$

n = 1, 2, 3; $\hbar$ = Planck's const.

$$2\pi r p = n\hbar$$

(remember $\frac{1}{40}$ eV ≈ k_BT)

$$E_1 = 13.6\,\text{eV} \leftarrow E_n = -\frac{1}{2}\frac{m e^4}{n^2 \hbar^2}$$

, m = electron mass

$$r_1 = 0.5\,\text{Å} \leftarrow r_n = \frac{n^2\hbar^2}{m e^2}$$

1s state: $\psi \sim e^{-r/r_1}$ spherically symmetric

dipole $\langle \mu \rangle = 0$

Atom in electric field:

$$\vec{\mu} = \alpha \vec{E}$$

, $\alpha \equiv$ polarizability

$$\sim \frac{e\,l}{e}\, l^2 \sim l^3$$

dimensions $[\alpha]$: volume

$$\sim r_1^3$$

(the only length scale is r_1)

2 hydrogen atoms

$$1 \bullet \xleftrightarrow{\;\;r\;\;} \bullet\, 2$$

$\vec{\mu}_1$

$$E_{21} \sim \frac{\mu_1}{r^3} \Rightarrow \mu_2 = E_{21}\,\alpha \Rightarrow U \sim \frac{-\vec{\mu}_1 \cdot \vec{\mu}_2}{r^3} = -\frac{\alpha \mu_1^2}{r^6} \approx -\frac{e^2 a^5}{r^6}$$

E-field at 2 due to 1.   induced dipole at 2 due to 1   dipole-dipole interaction

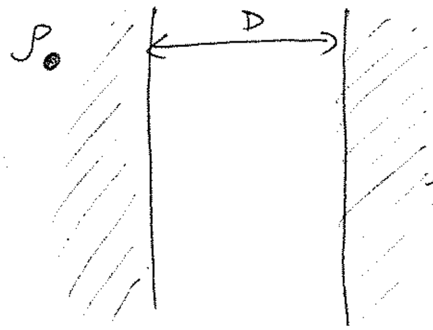
$\alpha \approx r_1$

$$\langle \mu_1^2 \rangle = (ea)^2$$

short range interaction, weak.
Arises from quantum fluctuations
(not pairwise additive)

this was derived neglecting retardation.

references: Parsegian, Ninham-Mohantay, Abrikosov et al



interaction energy (per area)

$$U_{area} = \frac{-\rho^2 \alpha^2 \epsilon}{D^2}$$

ϵ ← atomic energy $\frac{e^2}{a}$

dimensional analysis

$$= -\frac{A}{12\pi} \frac{1}{D^2}$$

← approx. breaks down for 2 infinite conductors (w/ ∞ polarizability ~size)

$A \equiv$ Hamaker constant

(~eV in vacuum)

Lifshitz – immerse in blackbody radiation

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

$$H = \sum_q \hbar \omega_q (n_q + 1/2)$$

↑ Bose-Einstein distribution

$$n_q = \frac{1}{e^{\hbar \omega_q / T} - 1}$$

$$F = -T \ln Z$$

$$= T \sum_q \ln \left[2 \sinh \frac{\hbar \omega_q}{2T} \right]$$

$$\Pi = \frac{-\partial F}{\partial D} \rightarrow -\frac{\hbar c}{D^4}$$

(need to subtract out modes of ~~[illegible]~~ wavelengths not integral multiples of D/2)