$$\frac{d\vec{p_i}}{dt} = -\nabla \sum_{i\neq j} \phi(|\vec{r}_i - \vec{r}_j|) = \sum_{\vec{k}} i\vec{k}\, \underset{\downarrow \text{F.T.}}{\phi_{\vec{k}}}\, \delta\rho_{-k}\, \delta\rho_k$$

"SLOW COMPONENTS OF FLUCTUATING" ↳ density modes propagating in opposite directions

$$e^{iQ\mathcal{L}t} \underset{\text{mouaif}}{\simeq} P_2 e^{i\mathcal{L}t} P_2$$ with P_2 projecting onto $\delta\rho\,\delta\rho$, $P_2 B = \sum_{k_1 \dots k_4} \delta\rho_{k_1}\delta\rho_{k_2} \langle \delta\rho_{k_3}\delta\rho_{-k_4} \cdot B\rangle \cdot \langle \delta\rho_{-k_1}\delta\rho_{-k_2}\delta\rho_{k_3}\delta\rho_{k_4}\rangle$

$$\Rightarrow \langle R_{-\vec{k}}\, e^{iQ\mathcal{L}t} R_{\vec{k}}\rangle \simeq \langle (P_2 R_{-\vec{k}}) e^{i\mathcal{L}t} (P_2 R_{\vec{k}})\rangle$$

$$P_2 R_{\vec{q}} = \sum_{\vec{k}_1 \vec{k}_2} \delta\rho_{\vec{k}_1}\delta\rho_{\vec{k}_2} \underbrace{\langle \delta\rho_{-k_3}\,\delta\rho_{k_4}\, R_{\vec{q}}\rangle \cdot \langle \delta\rho_{-k_1}\,\delta\rho_{-k_2}\,\delta\rho_{k_3}\,\delta\rho_{k_4}\rangle^{-1}}_{V_q(k_3,k_4)\ \text{vertex (static)}}$$

with $\vec{k} = \vec{k}_1 - \vec{k}_2$, $V_{\vec{q}}(\vec{k}_1,\vec{k}_2) = V_{\vec{k},\vec{q}-\vec{k}}$

$$= \frac{i\rho k_B T}{2mN}\left((\hat{q}\cdot\vec{k})\, C(k) + \hat{q}\cdot(\vec{q}-\vec{k})\, C(|\vec{k}-\vec{q}|)\right) \qquad C(k) \equiv \frac{1}{\rho}\left(1 - \frac{1}{S(k)}\right)$$

FACTORIZATION APPROXIMATION

so that $\langle R_{-k}(0)\, e^{iQ\mathcal{L}t} R_k(0)\rangle \simeq \sum |V|^2 \langle \rho_{k_1}\rho_{k_2}\rho_{k_3}(t)\rho_{k_4}(t)\rangle \simeq \sum |V|^2 \left(\delta_{-k_1 k_3} + \delta_{-k_2 k_4}\right) F(k_1,t) F(k_2,t)$

conserving momentum

Missing a few steps, but the final equation reads:

MCT
$$\frac{d^2F(q,t)}{dt^2} + \frac{q^2 k_B T}{m S(q)} F(q,t) + \int_0^t d\tau\, K(q,t-\tau)\frac{\partial F}{\partial\tau}(q,\tau) = 0$$
$$K_{MCT}(q,t) = \frac{\rho k_B T}{16\pi^3 m}\int d\vec{k}\, |V_{\vec{q}-\vec{k},\vec{k}}|^2 F(|\vec{q}-\vec{k}|,t) F(k,t)$$

$$\sum_{\vec{k}} \to \frac{V}{(2\pi)^3}\int d\vec{k}$$

→ Closed equations on $F(k,t)$
→ Theory for liquid state that from T, ρ and $S(k)$ derives the dynamic! ↳ static!
→ connections with exact dynamics of MF p-spins!

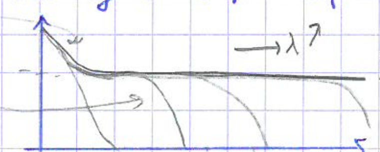
Looking at the consequences:

Approximating in a first step $S(q) \simeq A\delta(q-q^*)$ peak $\to \ddot{\phi} + \Omega^2\phi(t) + \lambda\int d\tau\, \phi^2(t-\tau)\,\dot{\phi}(\tau) = 0$ (with $\phi \to F(q^*,t)$)

$f(k)+t^{-a}$

$f(k)-t^{b}$

$$\frac{\Gamma(1-a)^2}{\Gamma(1-2a)} = \frac{\Gamma(1+b)^2}{\Gamma(1+2b)}$$



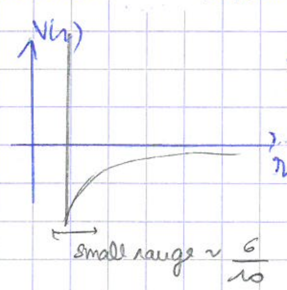
$\lambda_c = 4 \to$ freezing of the correlations + relaxation time divergence

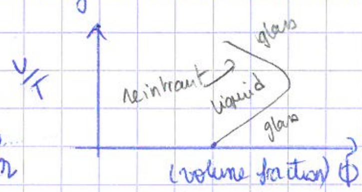
$\tau \sim (\lambda - \lambda_c)^{-\gamma}$

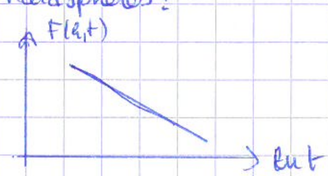
$\gamma = \frac{1}{2a} + \frac{1}{2b}$

Leutheusser 1984.

Using the theory to describe short range attractions between hard spheres:







↳ speed up relaxation by adding attraction
⇒ PREDICTED BY MCT BEFORE ANY EXPERIMENT.

19/07/2017

Failures

1) Violation of the Stokes-Einstein relations $D\tau \sim \tau^{\alpha}$ → only weakly predicted by MCT

2) Non gaussian parameters NGP $\sim \frac{3\langle \delta r^4(t)\rangle}{5\langle \delta r^2(t)\rangle^2}$

Configurational entropy
= complexity?

Model equally described:

$$H = g \sum_{\alpha<\beta<\gamma} J_{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \qquad J \sim \mathcal{N}(\bar{J}; 1/N^{2})^{(p-1)} \quad \phi_\alpha \in \mathbb{R}$$

$$\Rightarrow \frac{\partial \phi_\alpha}{\partial t} = -\mu(t)\phi_\alpha - 4g \sum_{\beta<\gamma} J_{\alpha\beta\gamma}\phi_\beta\phi_\gamma + \eta_\alpha$$ with η_α such that $\langle \eta_\alpha(t)\eta_\beta(t)\rangle = 2T\delta_{\alpha\beta}\delta(t)$

$\mu(t)$: lagrange multiplier $\frac{1}{N}\sum_\alpha \phi_\alpha^2 = 1$

SYK models: $H = \sum_{i<j<k<l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$ ~ seem to be related to small black holes models

↳ already incorporated in MCT?

→ DISCUSSIONS: see Reichmann's note.

MCT solved in $d\to\infty$ ($\vec{k}$ follows the dimension), exact mean field model?
↳ Apparently no, incon... encies arise from numerical simulations.

Maimbourg, Kurchan, Zamponi PRL (2016)

χ^4 Mode coupling theory liquid states:

Biroli/Bouchaud 2004 EPL

$$A(k,t) = \sum_{ij} \Theta(a - |r_i(0) - r_j(t)|)\, e^{i\vec{k}\cdot\vec{r}_i(0)} \sim \sum_q \rho_q \rho_{k-q} \,(H)_q$$

two space, two times order parameter

$$S_4(k,t) = \langle A(-k,t) A(k,t)\rangle - \langle A\rangle^2$$

Biroli, Bouchaud, Miyazaki, Reichman PRL (2006)
↳ external field spatially varying $U_0(x)$ → breaking space translational invariance

$$F(k_1,k_2,t) = \frac{1}{N}\langle \delta\rho_{-\vec{k}_1}(0)\, \delta\rho_{\vec{k}_2}(t)\rangle$$

⇒ Associated mode coupling equation:

$$\frac{\partial^2 F}{\partial t^2}(k_1,k_2,t) + \int d\vec{k}_1'\, \Omega(\vec{k}_1,\vec{k}_1')\, F(\vec{k}_1',k_2,t) + \int d\vec{k}_1' \int d\tau\, M(k_1,k_1',t-\tau)\frac{\partial F}{\partial \tau} F(\vec{k}_1',\vec{k}_2,t) = 0$$

↳ dynamic susceptibility: $\chi_q(k,t=0) = \left.\frac{\delta F(k,q+k,t)}{\delta U(q)}\right|_{U=0}$.

Kim, Saito, Miyazaki, Biroli, Reichman 2013 JPCB

⇒ equation $\frac{\partial^2 \chi_q}{\partial t^2} + \dots$

what we learn: 1) growing length scales $\xi \sim |T-T_c|^{-1/4}$
2) spacetime $\tau \sim \xi^z$, $z \in = 4\gamma$
3) growing length in β regime