BCS to BEC Crossover and the Unitarity Fermi Gas

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2014 Boulder School on
"Modern aspects of Superconductivity"

Review articles:

M. Randeria and E. Taylor,
Ann. Rev. Cond. Mat. Phys. **5**, 209 (2014)

Introductory review by
M. Randeria, W. Zwerger & M. Zwierlein,
Plus 13 Chapters by leading experts in:
"BCS-BEC Crossover and the Unitary Fermi Gas"
ed. by W. Zwerger (Springer, 2012)

S. Giorgini, L. P. Pitaevskii and S. Stringari,
Rev. Mod. Phys. 80, 1215 (2008).

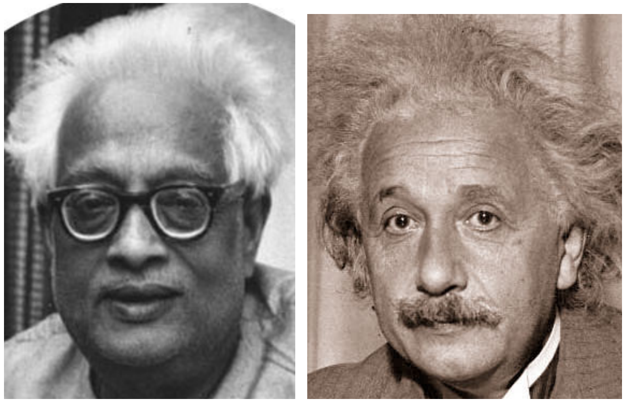
"Ultracold Fermi Gases", Proc. of the Varenna
'Enrico Fermi' Summer School 2006,
W. Ketterle, M. Inguscio and C. Salomon (editors).

Outline:

- (pre)History & Introduction
- Qualitative ideas of BCS-BEC crossover
- Theoretical progress
- Some key experiments
- Exact Results for strongly interacting regime
- Connections to other areas in physics
- Outlook

BCS (1956-57)

- Fermions
- Pairing
- Condensation of pairs
- superconductivity



BEC (1924-25)

- Bosons
- Macroscopic occupation of a quantum state
- superfluidity

Before 2004:

BCS-BEC crossover was a problem of purely theoretical interest
… with diverse motivations, but
no direct experimental relevance!

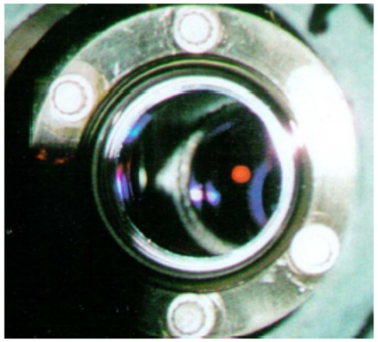
- D. M. Eagles (1969) [Doped semiconductor $SrTiO_3$]
- A. J. Leggett (1980) [Superfluid He^3]
- P. Nozieres & S. Schmitt-Rink (1985) [Excitons, …]
- M. Randeria & collaborators (1990's) [HTSC]

2004: BCS-BEC crossover in ultracold Fermi gases

Realized in the lab!

Experiments:

Jin (JILA)
Ketterle (MIT)
Salomon (ENS)
Grimm (Innsbruck)
Hulet(Rice)
Thomas (Duke)



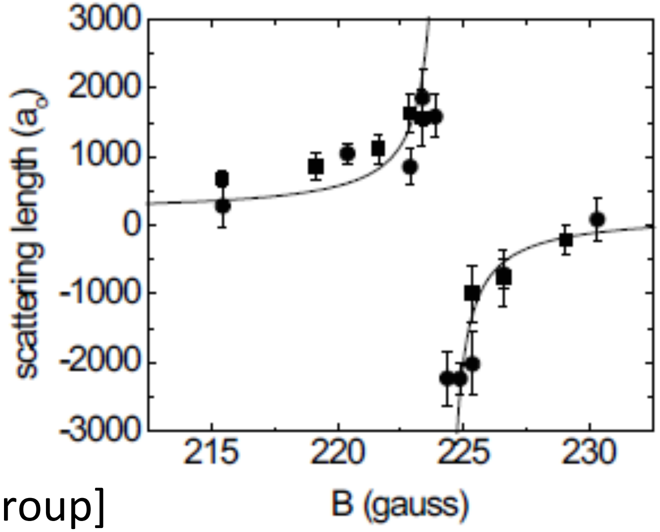
$^{6}Li, \ ^{40}K$

$10^5 - 10^7$ atoms

dilute : $k_F^{-1} \sim 0.3\mu m$

$E_F \sim 100\ nK - 1\ mK$

$T \gtrsim 10nK$



[Jin group]

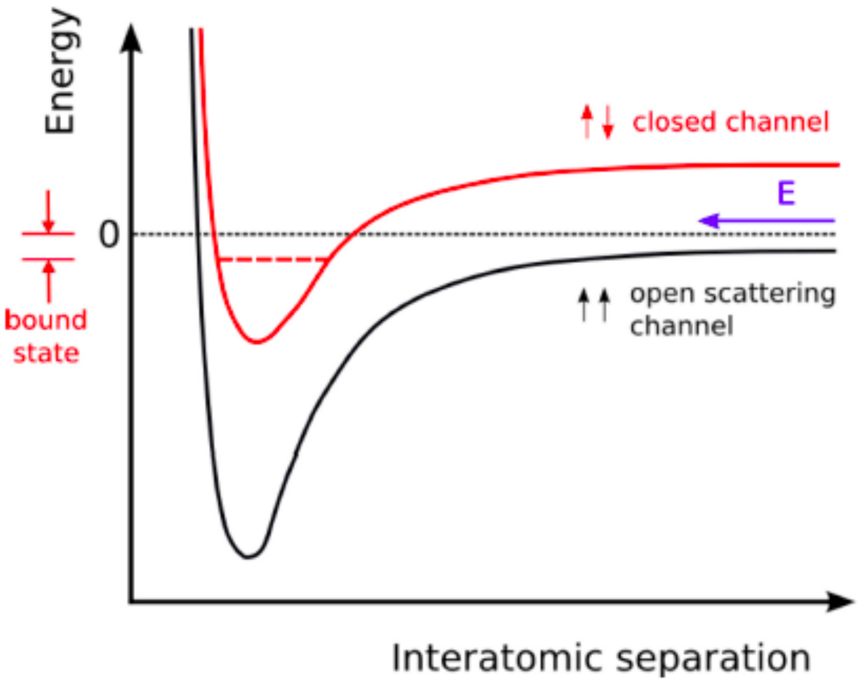
"spin up" & "down"
two hyperfine states

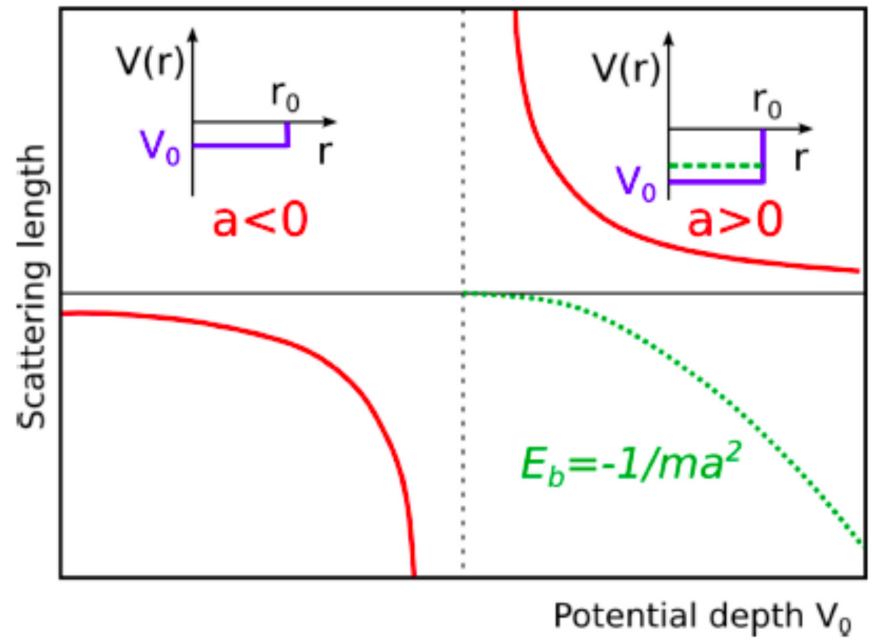
Tunable two-body interaction:
Feshbach Resonance
$|a| \to \infty$

Feshbach Resonance:

Two-channel description ←→ Single channel model*

*broad resonance





Tuning parameter = B-field

$(k_F r_0 \ll 1)$

Cheng, Grimm, Julienne & Tiesinga. RMP (2010)
See: Leo Radizihovsky's lectures

Feshbach Resonance (simplified)

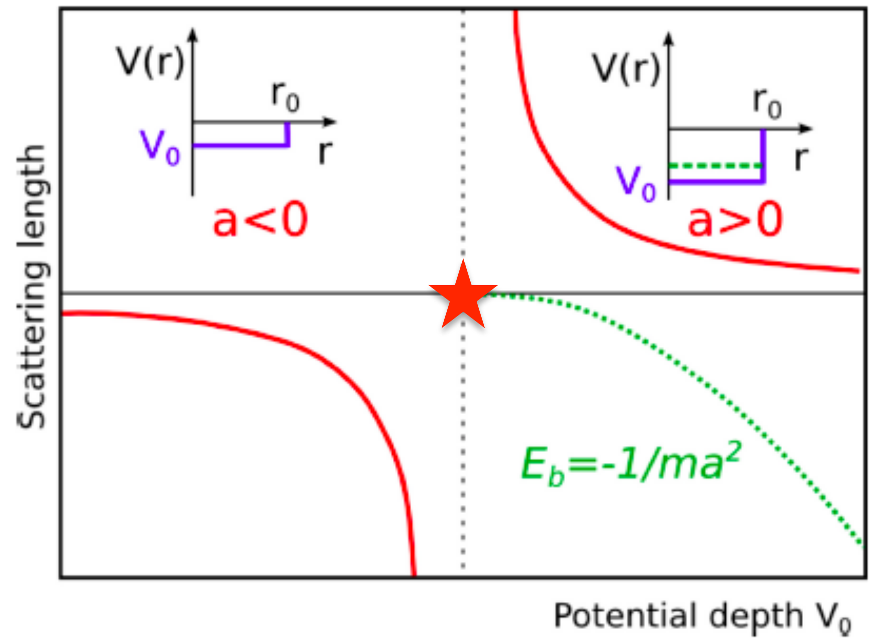
Two-body problem in 3D:

Low-energy $kr_0 \ll 1$
effective interaction:
s-wave scattering length a

$$f(k) = \frac{1}{k\cot\delta_0(k) - ik}$$

$$\approx \frac{-1}{1/a + ik}$$

$$\frac{d\sigma}{d\Omega} = |f|^2$$



$$\boxed{\text{Unitarity} \rightarrow \; |a| = \infty}$$

threshold for
→ two-body
bound state
(in vacuum)

Attractive Fermi Gas:

$$\mathcal{H} = \overline{\psi}_\sigma(x)\left[-\frac{\nabla^2}{2m} - \mu\right]\psi_\sigma(x)$$
$$-g(\Lambda)\overline{\psi}_\uparrow(x)\overline{\psi}_\downarrow(x)\psi_\downarrow(x)\psi_\uparrow(x)$$

Dilute Gas: range $r_0 \ll k_F^{-1}$ interparticle distance

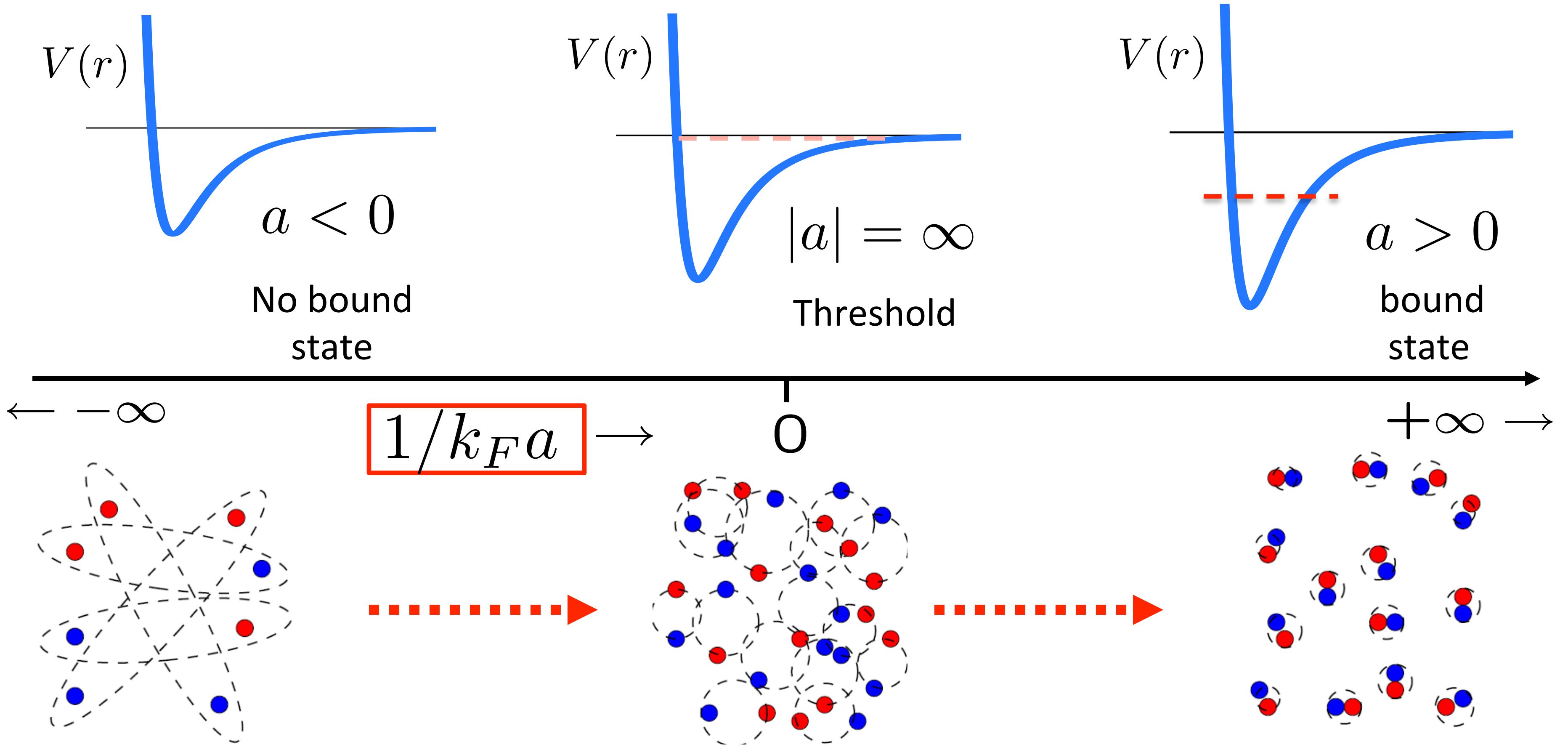
- $\mu \rightarrow n \sim k_F^3$
- $g(\Lambda) \rightarrow a$

$\left.\right\}$ Dimensionless Coupling constant $1/(k_F a_s)$

$$\frac{-1}{g(\Lambda)} = \frac{m}{4\pi a} - \sum_{k<\Lambda} \frac{m}{k^2} \qquad \Lambda \simeq \frac{1}{r_0} \rightarrow \infty$$

For an equivalent real-space approach with range $r_0 \rightarrow 0$
See: Y. Castin & F. Werner in Zwerger Book

BCS-BEC crossover [Leggett (1980); Nozieres & Schmitt-Rink (1985)]



BCS limit

- cooperative Cooper pairing
- pair size $\gg k_F^{-1}$

Unitarity

$|a| = \infty$

pair size $\simeq k_F^{-1}$

strongly interacting gas

BEC limit

- tightly bound molecules
- pair size $\ll k_F^{-1}$

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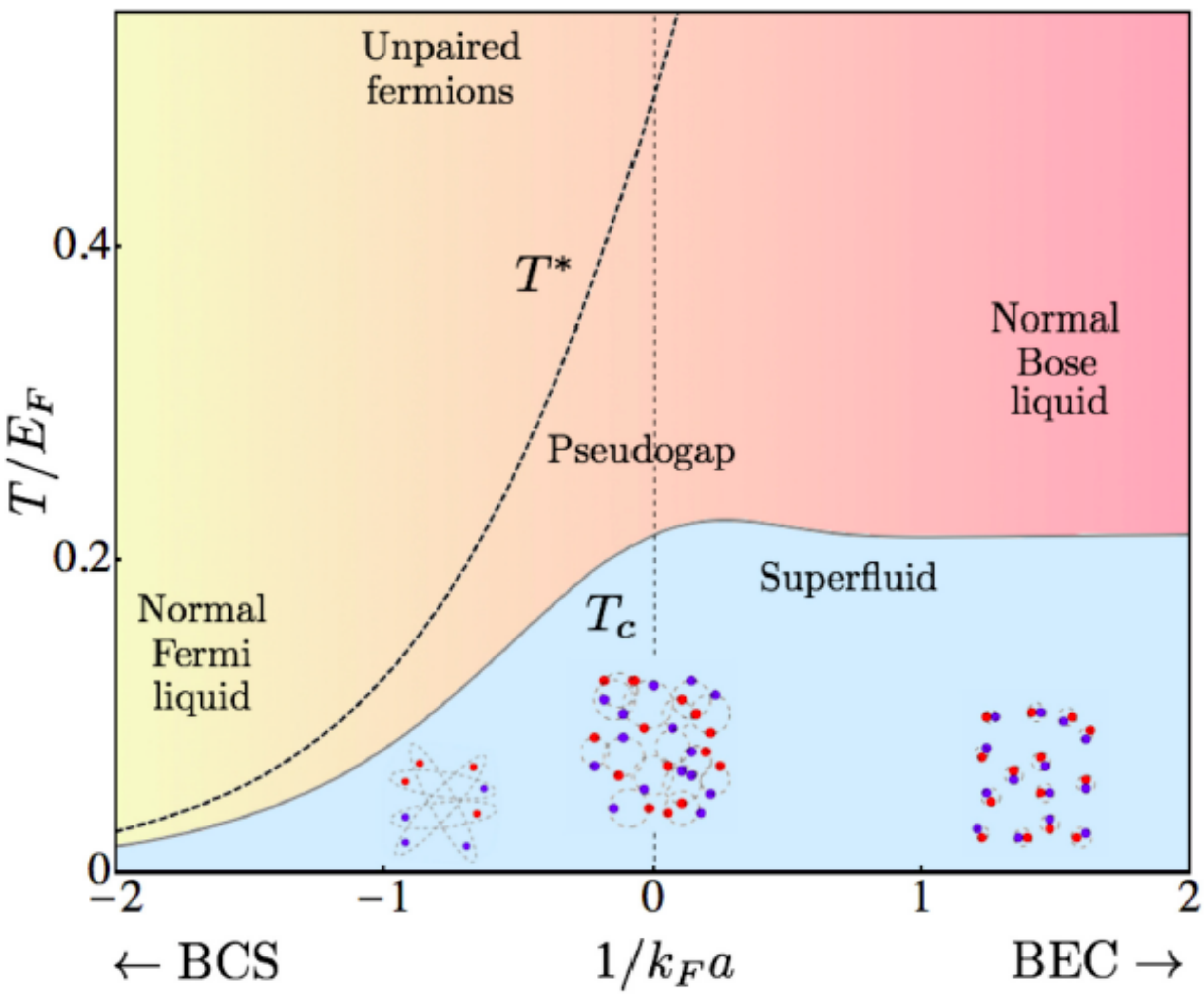
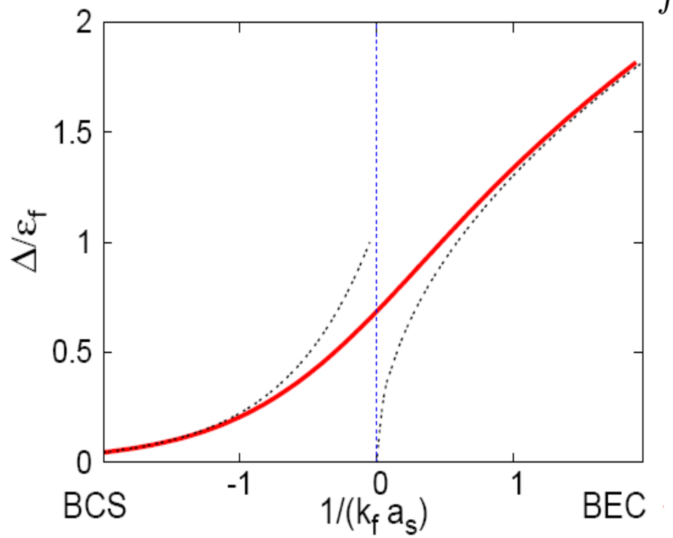
Qualitative description of BCS-BEC crossover



Based on: Sa deMelo, MR & Engelbrecht, PRL (1993)

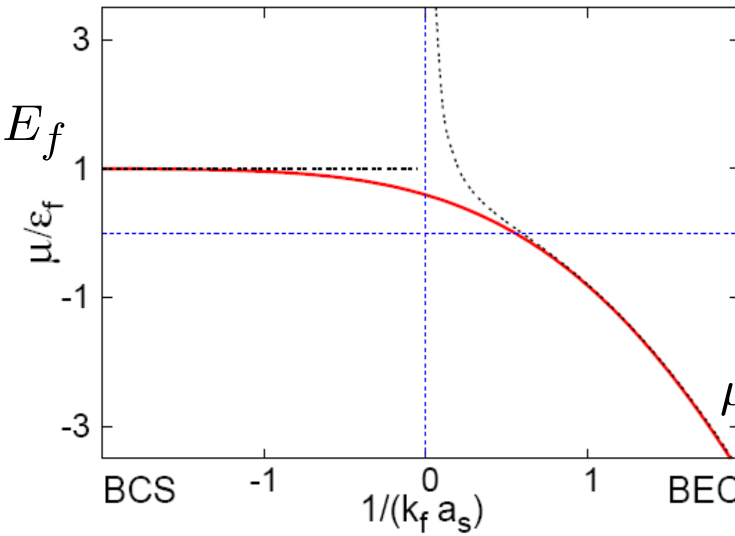
T=0 BCS-Leggett Mean Field Theory:

$\Delta \sim E_f/\sqrt{k_F a_s}$



$\Delta \simeq E_F e^{c/k_F a_s}$

$\mu = E_f$



$\mu \simeq -\frac{E_b}{2}$

$E_b = \frac{\hbar^2}{m a_s^2}$

SadeMelo, MR, Engelbrecht, PRL (93), PRB(97)

- MFT Qualitatively correct at T=0:
  all the way from Cooper pairs to composite bosons!
- will address Quantitative limitations later

  - Note: crossover region $-1 \leq \frac{1}{k_F a_s} \leq +1$
  - No small parameter near unitarity!

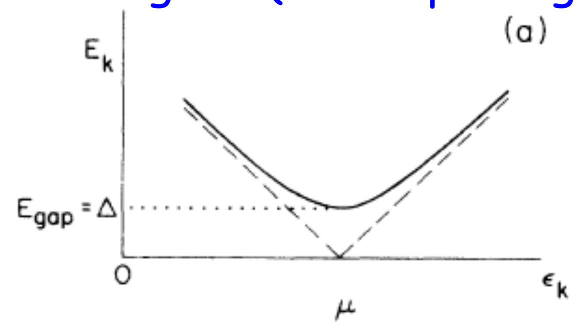
Energy Gap for Fermionic Excitations:

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$$

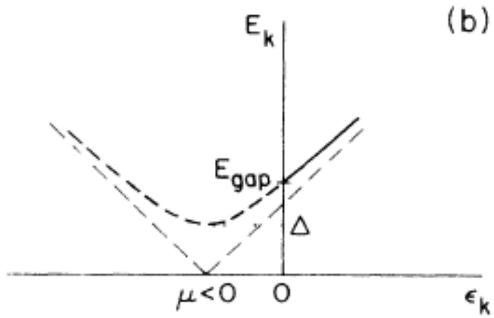
$$E_{\text{gap}} = \min_{k \geq 0} E_k$$

$$= \begin{cases} \Delta & (\mu > 0) \\ \sqrt{|\mu|^2 + \Delta^2} & (\mu < 0) \end{cases}$$

BCS regime ("weak pairing")



BEC regime ("strong pairing")



→ Phase transition (<u>not</u> a crossover)
for non-s-wave pairing, e.g. p+ip [Read & Green]

Gapless Goldstone excitations: phonon $\omega = cq \quad q \to 0$

BCS

$$\boxed{c^2 = v_F^2/3}$$

$\longrightarrow$

BEC

$$\boxed{c^2 = n_b U_b / m_b}$$

$$n_b = n/2$$
$$m_b = 2m$$
$$U_b = 4\pi\hbar^2 a_b / m_b$$

Petrov, Salomon, Shlyapnikov $\boxed{a_b = 0.6a}$

BCS-BEC @ Finite Temperatures

T*: Pairing
saddle-point/MFT

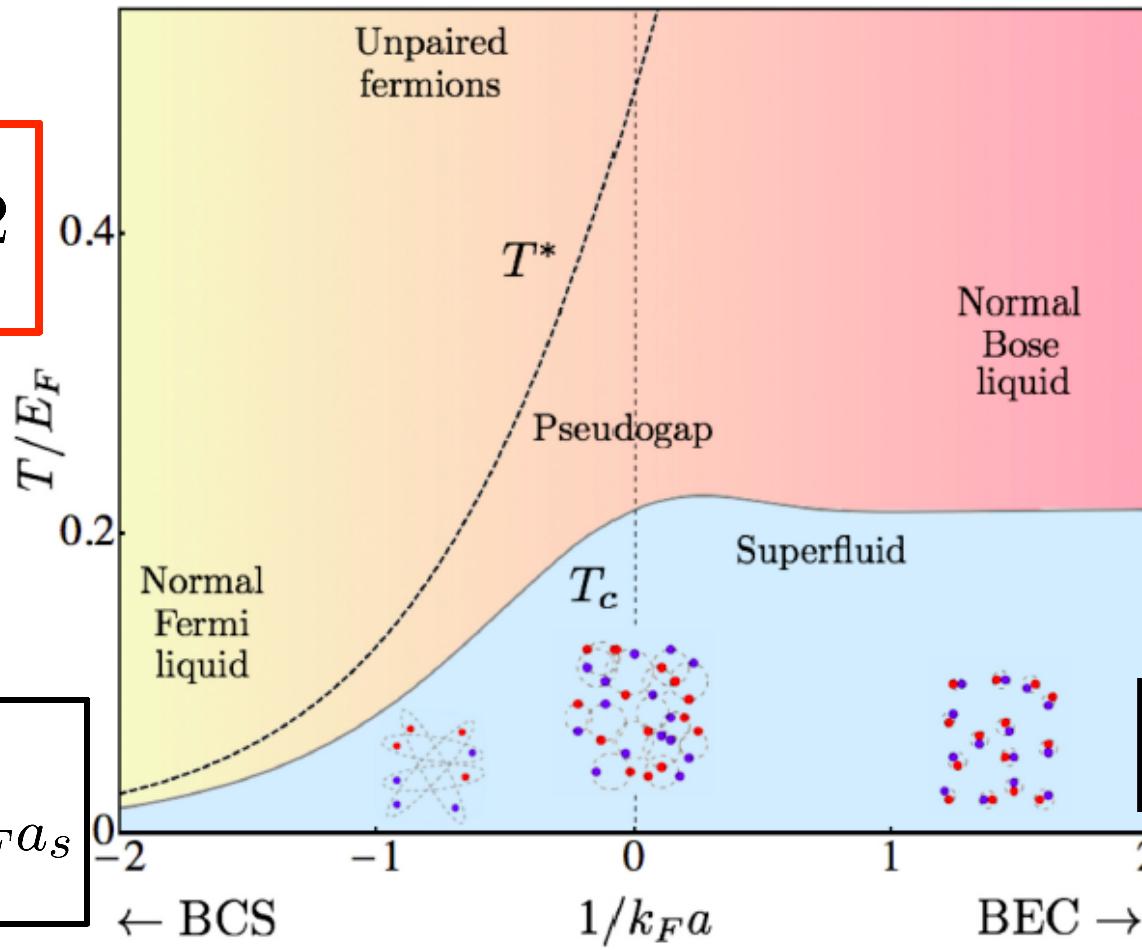
Saha ionization

$$T^* \sim \frac{E_b}{\log(E_b/E_F)}$$

$$E_b = \frac{\hbar^2}{m a_s^2}$$

maximum

$$\boxed{\frac{T_c}{E_f} \simeq 0.2}$$



Tc: Phase Coherence
saddle-point
+ Gaussian
fluctuations
= NSR

$$T_c \sim \frac{\hbar^2 n^{2/3}}{m}$$

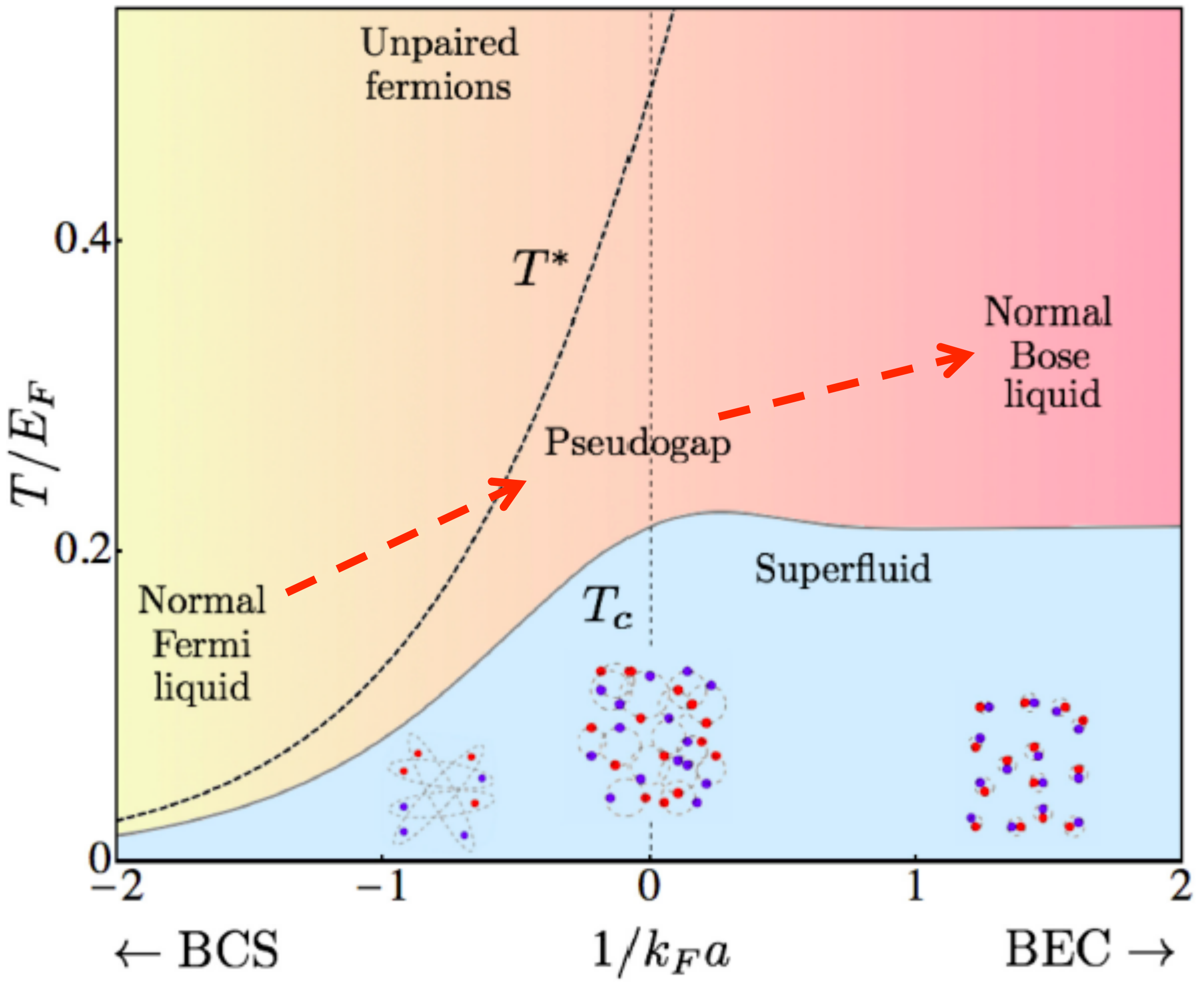
BCS limit

$$T_c \simeq T^* \sim E_F e^{c/k_F a_s}$$

G-MB correction

SadeMelo, Randeria & Engelbrecht, PRL (1993)

Evolution from
Normal Fermi → Normal Bose Gas?
Is the system quantum degenerate at these high T?



*Pairing Pseudogap
Randeria, Trivedi, Scalettar & Moreo PRL (1992)
Trivedi & Randeria, PRL (1995)

→ possible Breakdown of Fermi-liquid description in pseudogap regime

Recent QMC:
Tc = 0.15 Ef
Burovski et al, PRL (2008)
T* = 0.2 Ef
Magierski et al, PRL (2009)

Experiments?
See later!

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- Theoretical progress

  No small parameter at unitarity!

  * Field theory

  * Quantum Monte Carlo
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Unitary Fermi Gas

$\lvert a \rvert = \infty$	$\xi_s = E_0/(\frac{3}{5}NE_F)$	Δ/E_F	T_c/E_F
MFT ($T=0$)/NSR (T_c)	0.59 [27]	0.68 [27]	0.2 [26]
ϵ-expansion [51]	0.377(14)	0.60	0.180(12)
QMC	$<$ 0.383(1) [57]	0.5 [58, 60]	0.152(7) [62, 63]
Experiment	0.376(5) [65] *	0.44 [61]	0.167(13) [65]

MFT & NSR: [26] SadeMelo, Randeria & Engelbrecht. PRL 71, 3202 (1993)
[27] Engelbrecht, Randeria, & SadeMelo. PRB 55, 15153 (1997)

ε-expansion: [51] Nishida & Son, PRA (2007) and in "The BCS-BEC Crossover and the Unitary Fermi Gas", ed. W. Zwerger, (Springer, 2011)

QMC: [57] Astrakharchik et al., PRL 93, 200404 (2004)
[58] Bulgac, Drut & Magierski. PRA 78, 023625 (2008)
[60] Carlson & Reddy. PRL 95, 060401 (2005)
[62] Burovski et al, PRL 96,160402, (2006)
[63] Burovski et al, PRL 101, 090402 (2008)

Experiments: [61] Schirotzek et al, PRL 101, 140403 (2008)
[65] Ku et al, Science 335, 563 (2012)
*revised [G. Zurn, PRL 110, 135301 (2013)]

Table from:
Randeria & Taylor
Ann. Rev. CMP
(2014)

Universality:

Results, independent of microscopic details (e.g. Li, K, …) across the entire BCS-BEC crossover provided $k_F r_0 \ll 1$

All (dimensionless) results can be expressed as

$$\mathcal{F}(T/E_f, 1/k_F a)$$

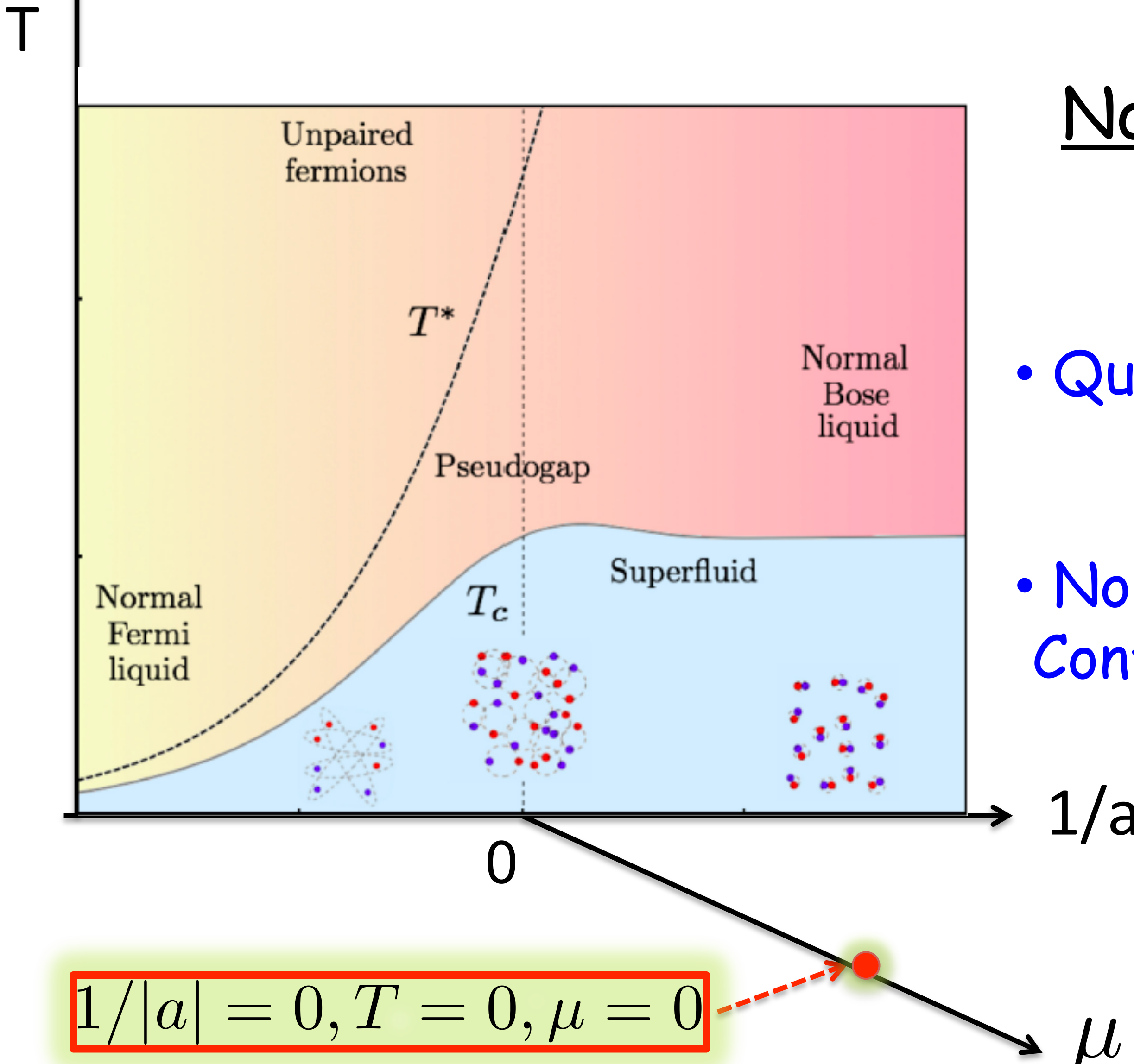
No interaction scale at Unitarity $|a| = \infty$

Universal results for All observables $\mathcal{F}(T/E_F)$ Bertsch (2003) Ho (2004)

e.g. ground state Energy per particle $E(T=0) = \xi_s \left(\frac{3E_F}{5} \right)$

ξ_s is a Universal number

Scale-invariance at Unitarity: $|a| = \infty$



<u>No</u> length scale at

$$T = 0, \mu = 0$$

- Quantum critical point

Sachdev & Nikolic (2007)

- Non-relativistic Conformal Field Theory

Son (2008)

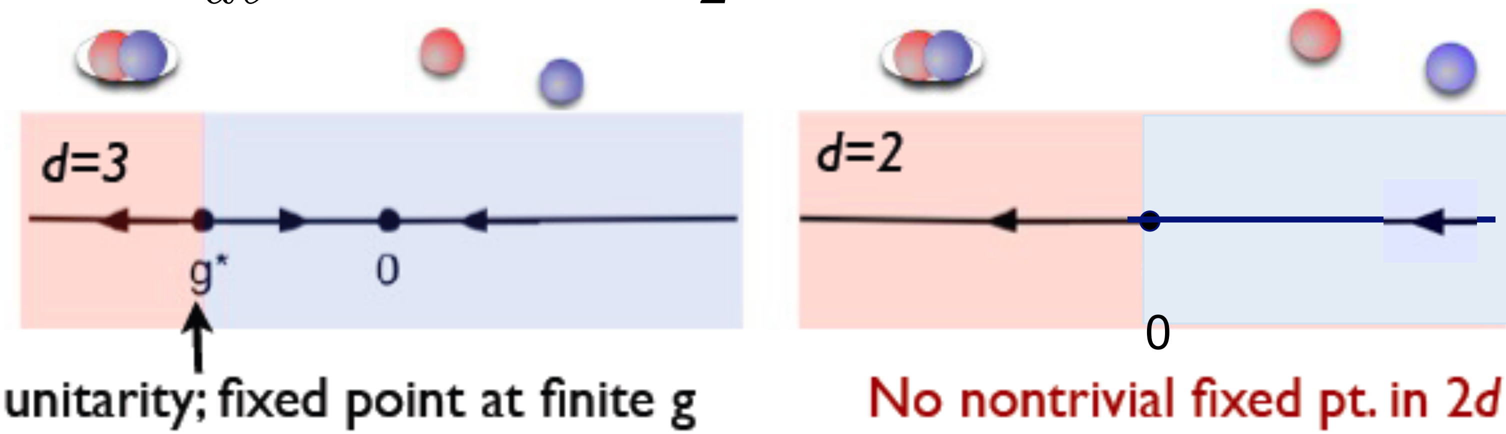
$\updownarrow$

AdS/CFT duality?

Renormalization Group:

$$\frac{dg}{d\ell} = (2-d)g - \frac{g^2}{2}$$

Nikolic & Sachdev (2007)



2D = lower critical dimension
Randeria, Duan, Shieh (1989)

Dimensionality expansions:

$$d = 2 + \varepsilon$$ Expand about free fermions

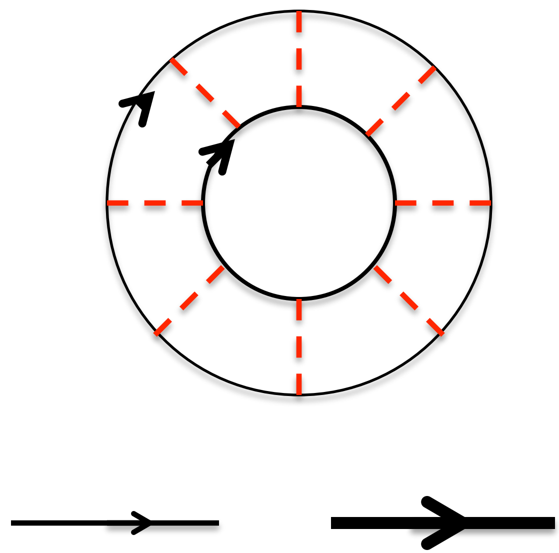
$$d = 4 - \varepsilon$$ Expand around free bosons in Two-channel formulation

Nishida & Son (2006)

Large N expansion:

Veillette, Sheehy & Radzihovsky
Nikolic & Sachdev

Analytical Approximations: Mean-Field + Pair Fluctuations



$$G_0 \quad vs. \quad G$$

Diagrammatic Approx:
Levin et al.
Strinati, Perali et al.
Hu, Liu & Drummond

Gaussian Approx:
Diener, Sensarma & Randeria

Large N approx
Veillette, Sheehy & Radzihovsky
Nikolic & Sachdev

Luttinger-Ward/Conserving Approx:
Zwerger, Hausman et al.

Why bother?
(when there is no small parameter!)
Analytical theories can give insights. E.g.: Why is
$\xi_s = E_0/(\frac{3}{5}NE_F)$ reduced by 40% from its MF value?

Quantum Monte Carlo (QMC) Simulations:

- Best available tool for non-perturbative problems

- Fermion sign problem (sometimes absent!
  e.g., lattice problem
  with zero range attraction)

- Analytic continuation problem: $(\tau \text{ or } i\omega_n \rightarrow \omega + i0^+)$

Many types of QMC:

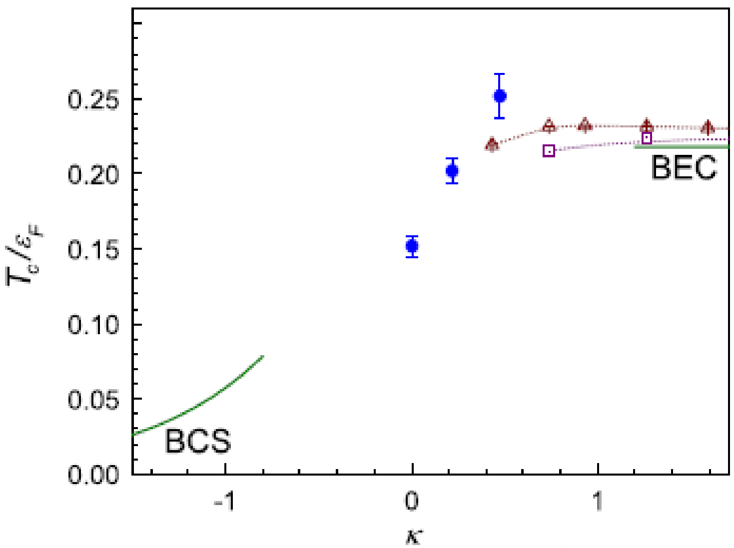
* T=0 Diffusion QMC -- wave function [Trento; Urbana; LANL, …]
* Finite temperature QMC
  -- imaginary-time functional integrals [Amherst; Seattle; ETH; …]
  -- diagrammatic MC [Amherst]

Quantum Monte Carlo

Transition Temperature

$$T_c^{\text{unitarity}} \simeq 0.15 E_F$$

$$T_c^{\max} \simeq 0.2 E_F$$

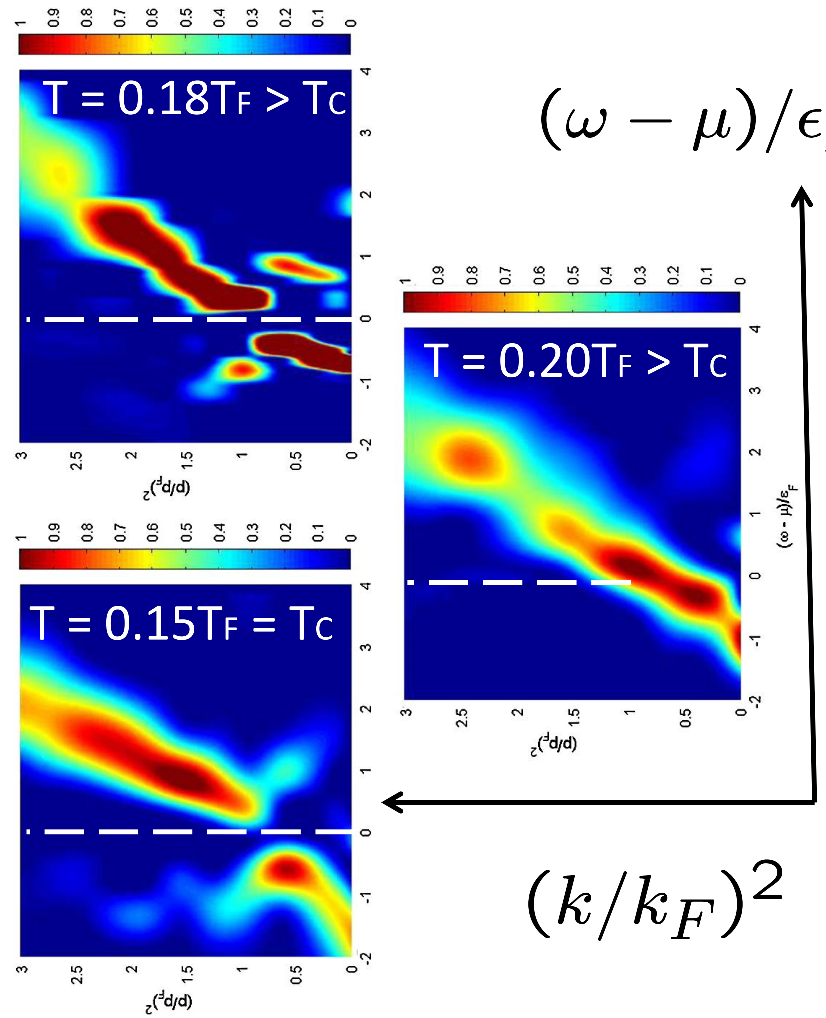


$1/k_F a$

Burovski et al, PRL (2008)

Pseudogap $T_c < T < T^*$

$A(\mathbf{k}, \omega)$ unitarity



$(\omega - \mu)/\epsilon_F$

$(k/k_F)^2$

P. Magierski et al, PRL (2009, 2011)

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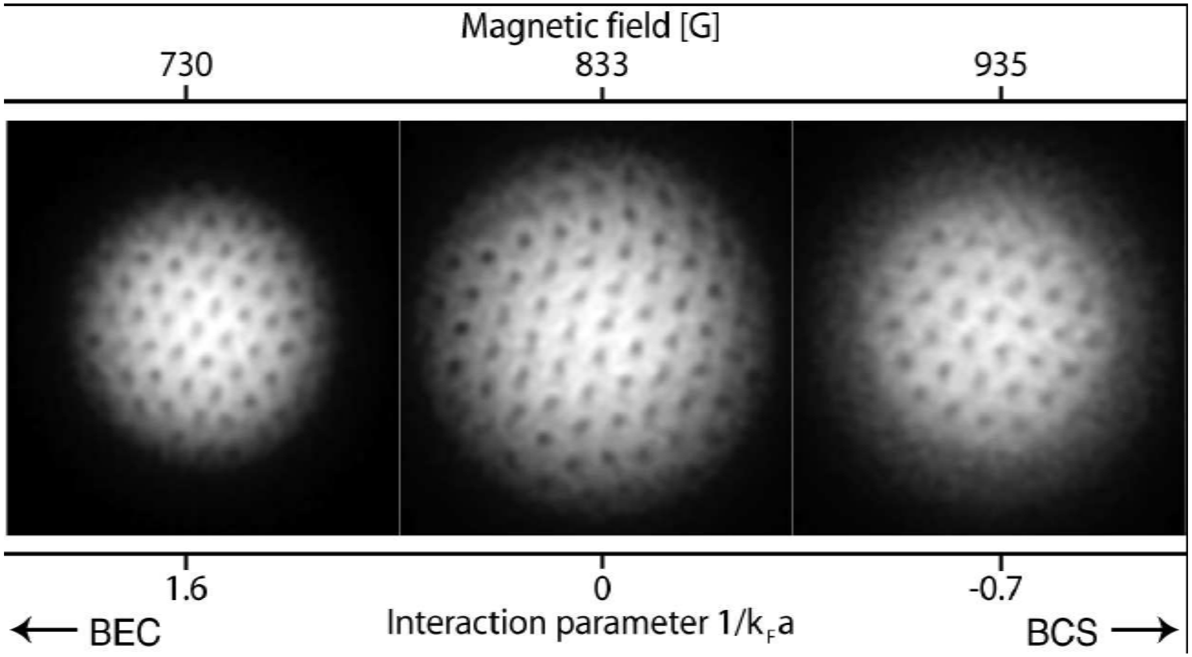
Some Key Experiments:

* vortices
* thermodynamics
* spectroscopy
* transport

Quantized Vortices in Rotating Superfluid Fermi Gases

$$\oint \mathbf{v_s} \cdot d\mathbf{l} = \frac{h}{2m}$$

^{6}Li Fermi gas through a Feshbach Resonance



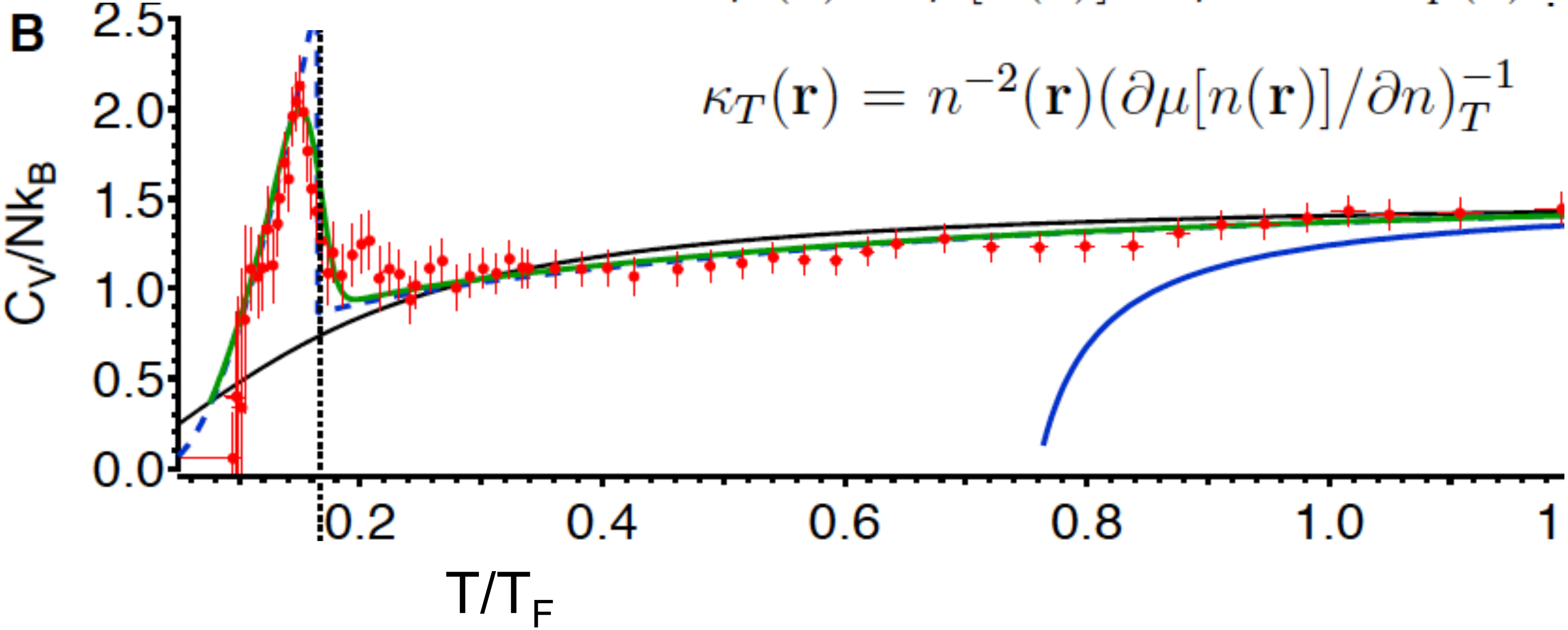
M.W. Zwierlein *et al.*, Nature, **435**, 1047, (2005)

Thermodynamics of the Unitary Fermi Gas

[MIT; ENS; Tokyo groups]

$$\mu(\mathbf{r}) \equiv \mu[n(\mathbf{r})] = \mu_0 - V_{\text{trap}}(\mathbf{r})$$

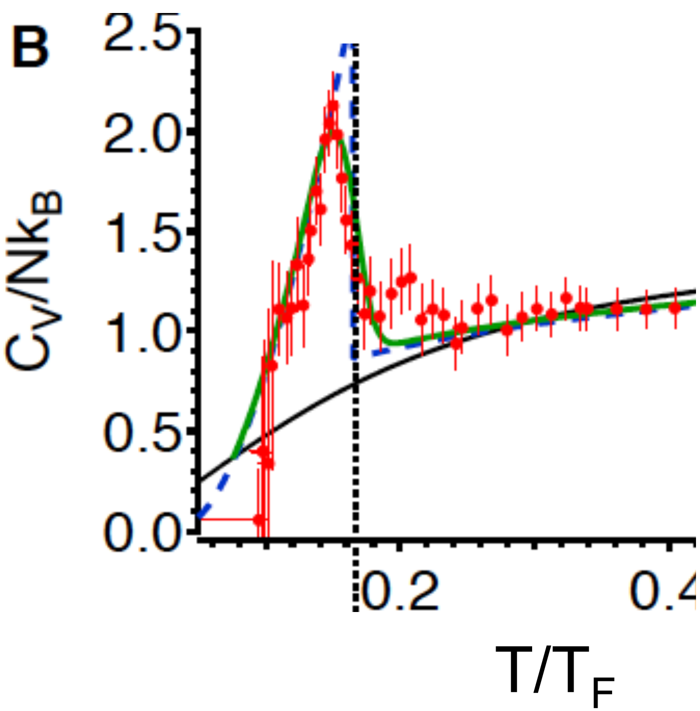
$$\kappa_T(\mathbf{r}) = n^{-2}(\mathbf{r})(\partial\mu[n(\mathbf{r})]/\partial n)_T^{-1}$$

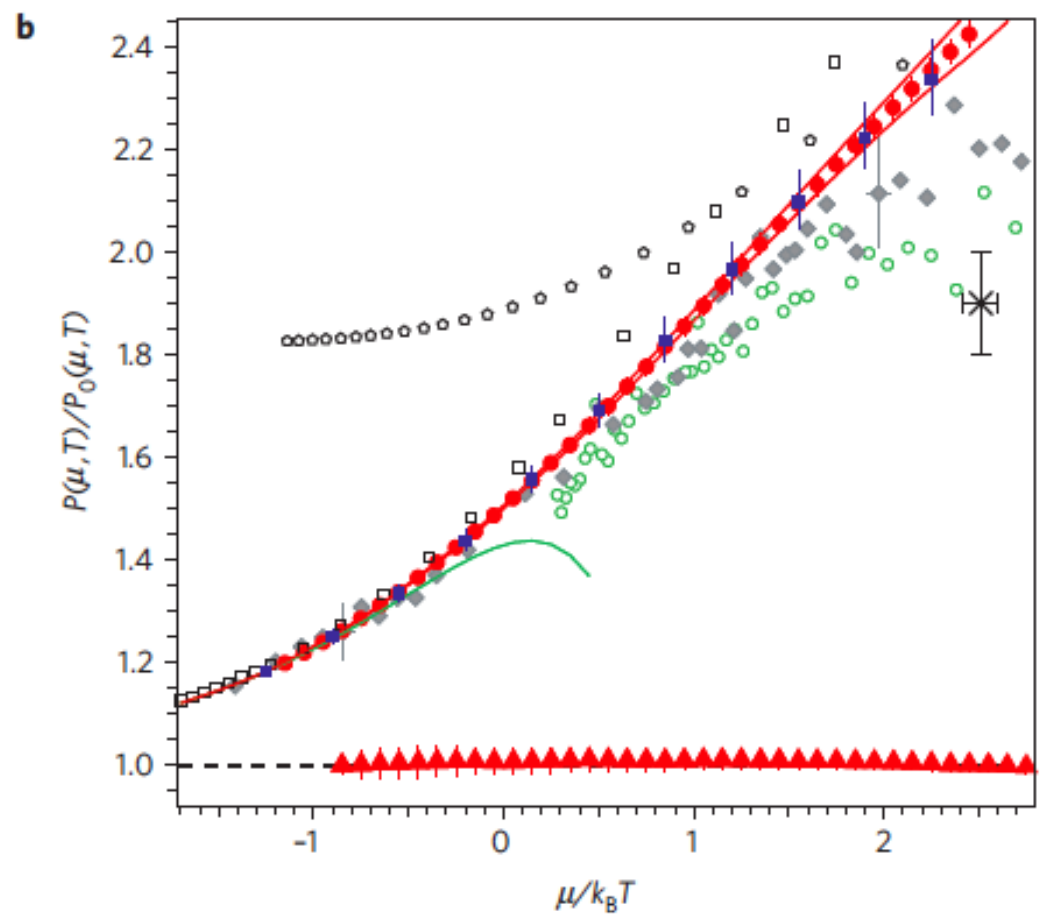


M. J. H. Ku et al., Science (2012) [Zwierlein group]

- λ−Transition
- Determination of $\xi_s = E_0/(\frac{3}{5}NE_F)$

Thermodynamics of the Unitary Fermi Gas





M. J. H. Ku et al., Science (2012) [Zwierlein group]

K. Van Houcke et al., Nature Phys. (2012)
Expt: Zwierlein group
BD QMC: UMass group

RF spectroscopy:

$$A_\sigma(\mathbf{k},\omega) = -\mathrm{Im}G_\sigma(\mathbf{k},\omega + i0^+)/\pi$$

Fermion spectral function = probability to make an excitation at momentum **k** and energy ω

k-resolved RF ~ ARPES [Jin]

$$I(\mathbf{k},\omega) \propto n_F(\epsilon_\mathbf{k} - \mu_\sigma - \omega) A_\sigma(\mathbf{k}, \epsilon_\mathbf{k} - \mu_\sigma - \omega)$$

k-integrated RF [Grimm; Ketterle]

$$I(\omega) \equiv \sum_\mathbf{k} I(\mathbf{k},\omega)$$

RF threshold not at Δ but (in MF) at:

$$E_{\mathrm{th}} = \sqrt{\Delta^2 + \mu^2} - \mu$$

QMC: Carlson & Reddy (2008)   Expt: Schirotzek et al., (2008)

$$E(\mathbf{k}) = [(\hbar^2 k^2/2m^* - \tilde{\mu})^2 + \Delta^2]^{1/2} \qquad \tilde{\mu} = \mu - U$$

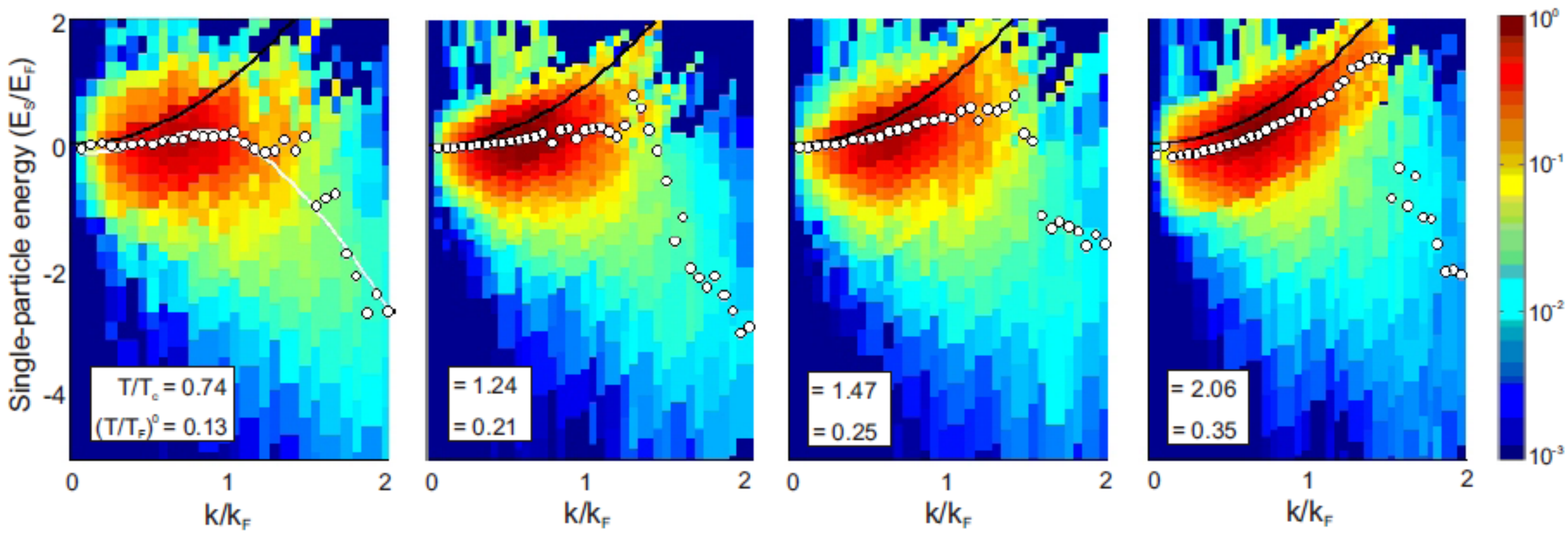
Observation of a pairing pseudogap:

"energy gap" in the normal state near unitarity $|a| = \infty$

k-resolved RF spectroscopy ←→ Angle Resolved Photoemission (ARPES)

Stewart, Gaebler & Jin, Nature *(2008)*

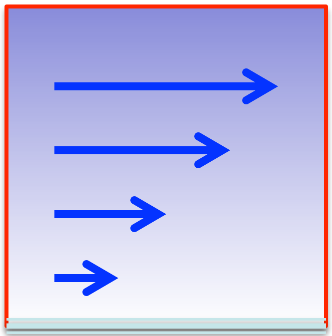
$$f(\omega)A(\mathbf{k},\omega)$$



Gaebler et al, Nature Phys. (2010)

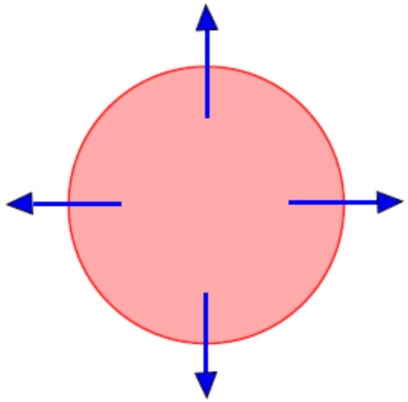
- unusual dispersion ☑
- well-defined quasiparticles? anomalous line-shape?

Transport coefficients: shear & bulk viscosity



$$\frac{dE}{dt} = -\eta \int d^3\mathbf{r} \left(\frac{\partial u_x}{\partial y} \right)^2$$

$\boxed{\eta}$ → Shear viscosity
Dissipation in presence of a flow gradient



$$\frac{dE}{dt} = -\zeta \int d^3\mathbf{r} \left(\frac{\partial u(r)}{\partial r} \right)^2$$

$\boxed{\zeta}$ → Bulk viscosity
Dissipation with isotropic u
$\nabla \cdot \mathbf{u} \neq 0$

Transport in strongly interacting fluids

Shear viscosity η

Boltzmann equation: $\eta \sim np\ell$ $\qquad \ell =$ Mean free path

Sharp "quasiparticles" $p\ell \gg \hbar \quad \Rightarrow \eta/n \gg \hbar$

$$\eta/s \gg \hbar/k_B$$

Minimum viscosity conjecture

based on AdS/CFT [Kovtun, Starinets, Son (2005)]

(shear viscosity)/(entropy density) ratio of <u>all</u> fluids obeys:

$$\boxed{\eta/s \geq \hbar/(4\pi k_B)}$$

status of bound <u>not</u> clear,
even in string theory

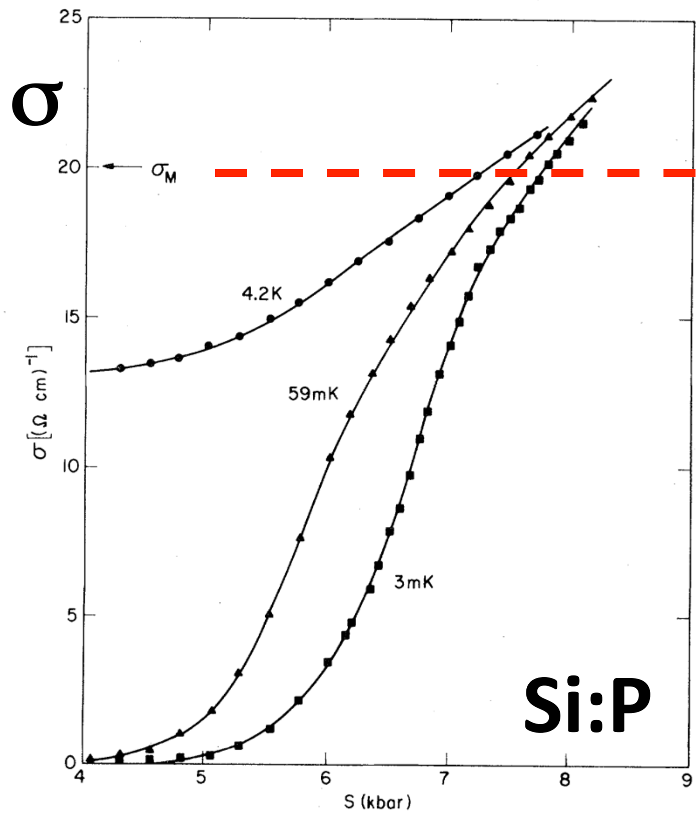
→ No known violations in the laboratory!

A digression: Mott & Ioffe-Regel $\ell \geq \ell_{\min} \simeq k_F^{-1}$

→ minimum conductivity conjecture

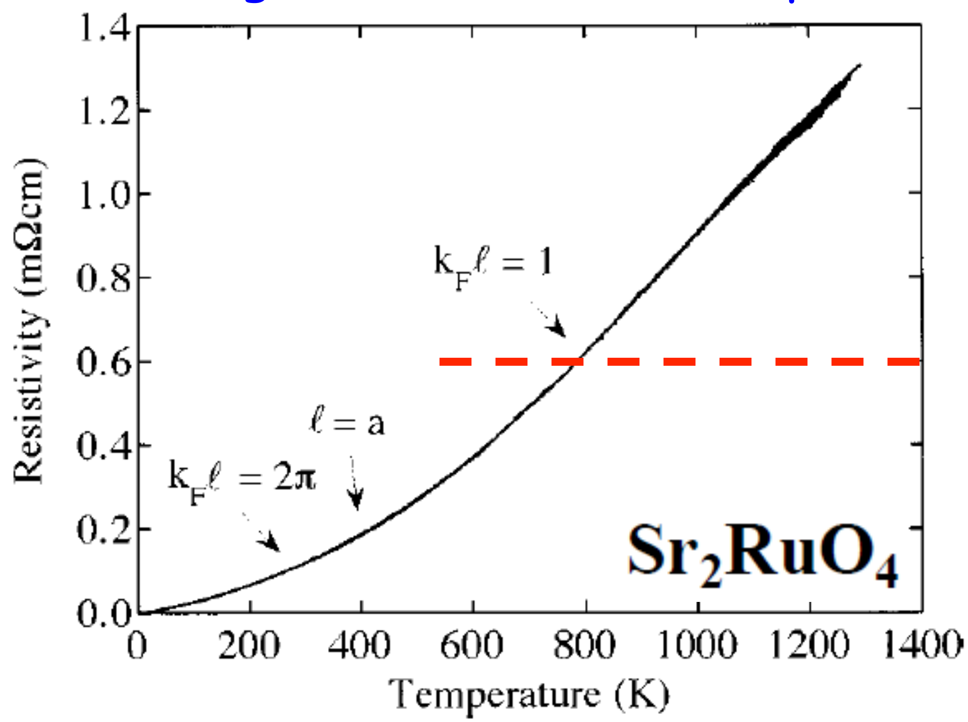
Experiments → conjecture is false for charge transport!

T=0 Metal-Insulator transition



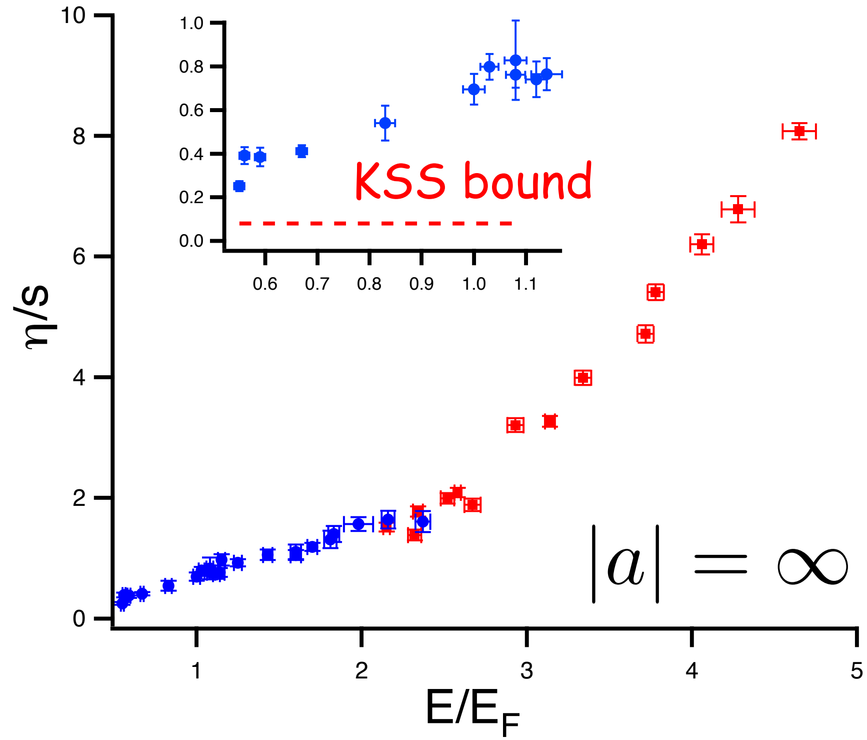
Rosenbaum et al, PRB **27**, 7509 (1983)

High T incoherent transport



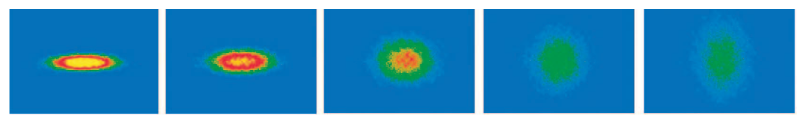
Tyler et al, PRB **58** R10 107 (1998)

η/s Experiments: Unitary Fermi gas



Hydrodynamic modeling of:

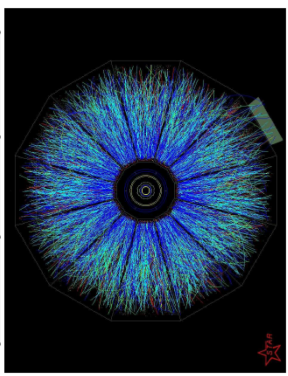
← 🔴 "Elliptic Flow"



← 🔵 Damping of radial Breathing mode

C. Cao, et al. Science **331**,58 (2011)

Fluid	$T\ [K]$	$\eta\ [Pa \cdot s]$	$\eta/n\ [\hbar]$	$\eta/s\ [\hbar/k_B]$
^{6}Li ($\|a_s\| \simeq \infty$)	23×10^{-6}	$\leq 1.7 \times 10^{-15}$	≤ 1	~ 0.4
QGP	2×10^{12}	$\leq 5 \times 10^{11}$	-	≤ 0.4



QGP (RHIC)

Data from: T. Schafer & D. Teaney, Rept.Prog.Phys. (2009)

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Exact Results valid for all $1/k_F a$ and all T/E_F

* Tan Relations

S. Tan (2005/2008)
OPE: Braaten & Platter (2008)
Castin & Werner (2009)
Zhang & Leggett (2009)

* Sum Rules

RF spectroscopy: Baym et al, PRL (2007)
Punk & Zwerger, PRL (2007)
Zhang & Leggett, PRA (2008)

Viscosity
Spectral Functions: Taylor & Randeria PRA (2010), PRL (2012)
Enss, Haussmann, Zwerger, Ann. Phys.(2011)

* Review: E. Braaten's chapter in Zwerger book

Tan's Contact "C"

Two-body problem: $\phi(r) \approx \left(\frac{1}{r} - \frac{1}{a}\right)$ at <u>short</u> distances

range $r_0 \to 0$

In the many-body problem:

- Density correlations

$$\langle n_\uparrow(r) n_\downarrow(0)\rangle \approx C\left(\frac{1}{r} - \frac{1}{a}\right)^2 \qquad r \ll k_F^{-1}$$

- Momentum distribution:

$$n(\mathbf{k}) \approx C/k^4 \qquad k \gg k_F$$

Contact

$$\boxed{C = k_F^4 \mathcal{F}(T/E_f, 1/k_F a)}$$

Exact results: Tan Relations

$$n(\mathbf{k}) \approx C/k^4$$

$$k \gg k_F$$

Energy relation

- $$\varepsilon = \underbrace{\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{k^2}{2m} \left[n(\mathbf{k}) - C/k^4\right]}_{\text{Kinetic Energy: Divergent in zero range limit}} + \frac{C}{4\pi m a}$$

Pressure relation

- $$P = 2\varepsilon/3 + C/(12\pi m a)$$

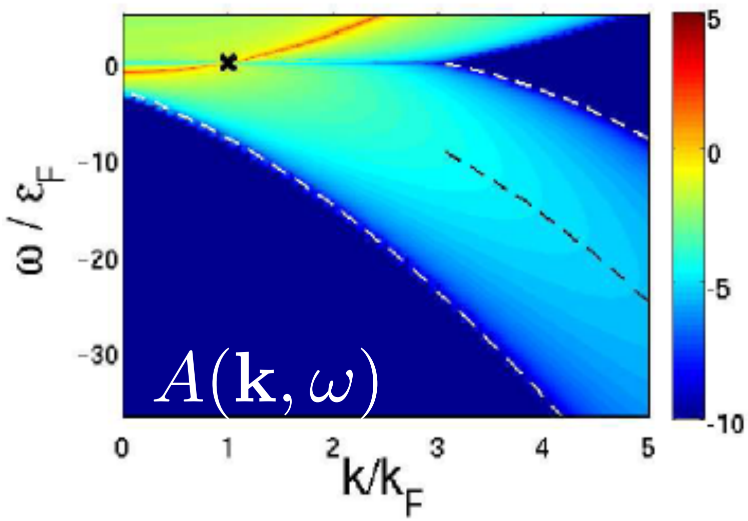
"The tail that wags the dog" -- E. Braaten

Adiabatic relation

- $$\left(\partial\varepsilon/\partial a^{-1}\right)_s = -C/(4\pi m)$$

Contact & Tails of Dynamical Correlations:

Momentum-resolved RF



Schneider & MR (2010)

RF Intensity

$$\lim_{\omega\to\infty} I_\sigma(\omega) \approx \frac{1}{4\pi^2\sqrt{m}}\frac{C}{\omega^{3/2}}$$

Pieri, Perali & Strinati (2009)
Schneider & MR (2010)

Dynamic Structure factor

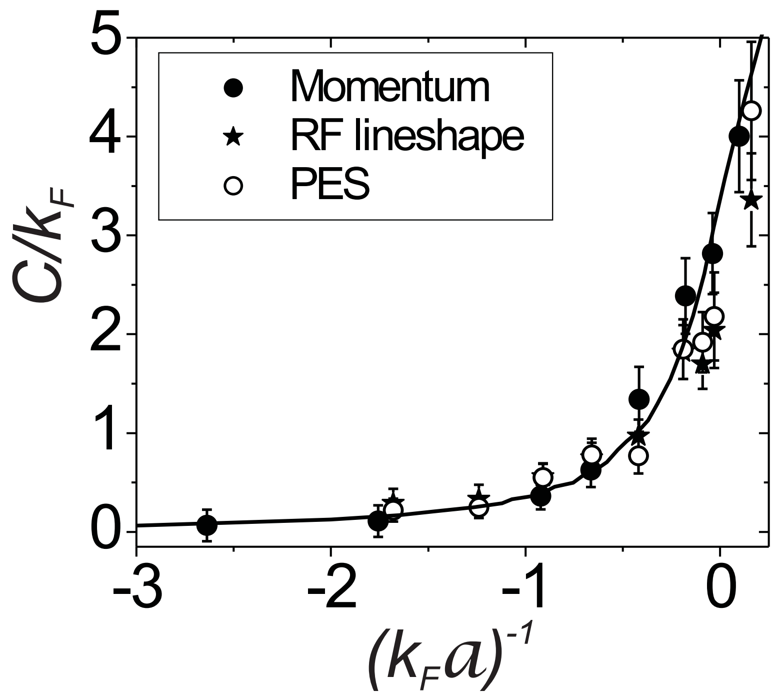
$$\lim_{\omega\to\infty}\lim_{q\to 0} S(\mathbf{q},\omega) \approx \frac{2Cq^4}{45\pi^2 m^{1/2}\omega^{7/2}}$$

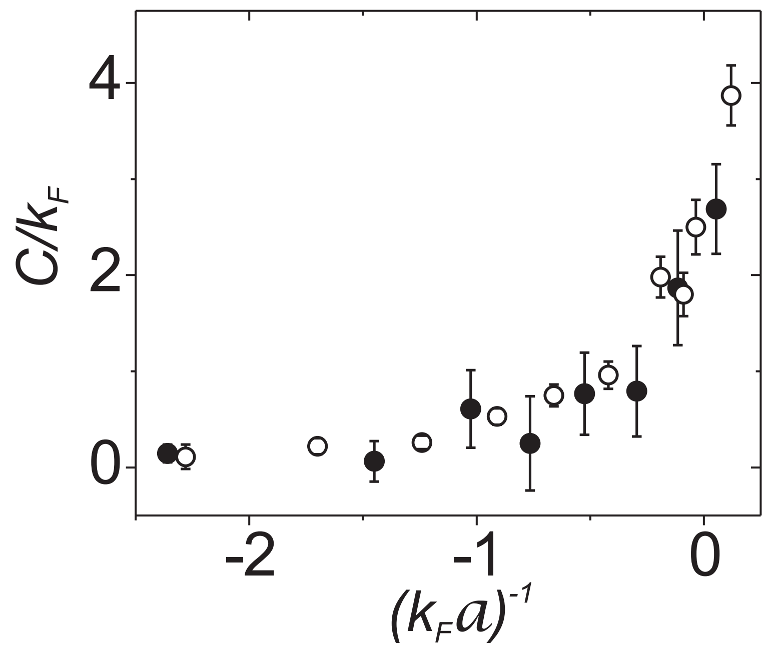
Son & Thomson (2010)
Taylor & MR (2010)

Experimental studies of "Contact"

JILA: Stewart et al, PRL 104, 235301 (2010)

[See also: Vale group
Hoinka, PRL 110, 055305 (2013)]





k and ω Tails:

$$n(\mathbf{k});\;\; I(\omega);\;\; \int f(\omega) A(\mathbf{k},\omega)$$

RF k-resolved RF

Thermodynamics:

$$\left(\frac{\mathrm{d}E}{\mathrm{d}a^{-1}}\right)_S = -\frac{\hbar^2}{4\pi m}\, C$$

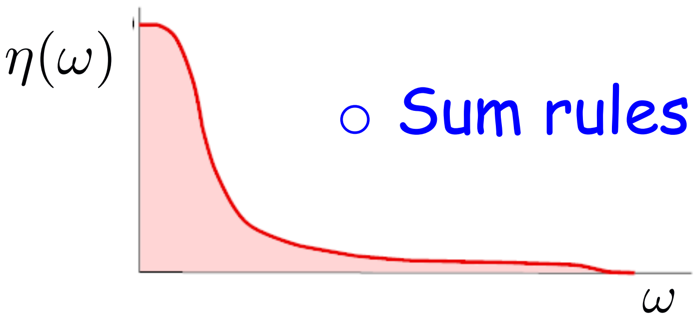
Viscosity sum rules

PRA **81**, 053610 (2010)
PRL **109**, 135301 (2012)

Ed Taylor
(McMaster)

- Linear Response (Kubo) $\eta(\omega), \zeta(\omega)$
- Kramers-Kronig: causality → analyticity
- Lehmann spectral representation

Valid for arbitrary interactions, all temperatures, all phases



- Sum rules

$$\left.\begin{aligned} &\int d\omega\ \eta(\omega) \\ &\int d\omega\ \zeta(\omega) \end{aligned}\right\} = \text{Thermodynamic Quantities}$$

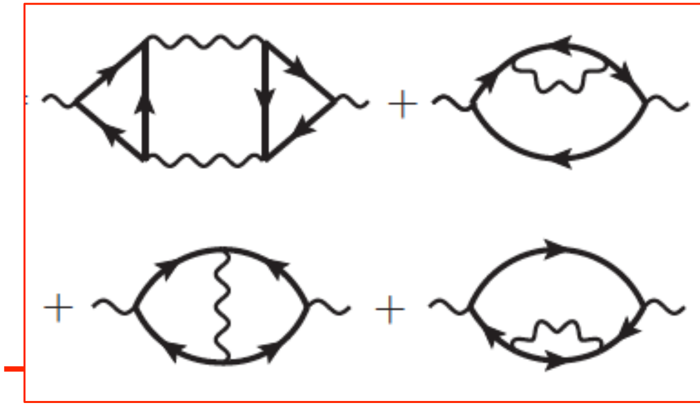
Sum rules useful for
- analyzing experiments
- constraining approximate calculations
- proving rigorous results

Sum-Rule, as calculated, has an
ultraviolet divergence in zero-range limit $\Lambda = 1/r_0 \to \infty$

$$I \equiv \int \frac{d\omega}{\pi}\eta(\omega) = \frac{\varepsilon}{3} + \alpha\frac{C}{a} + \beta C\Lambda$$

[Linear in 3D; log in 2D]

"Two-body problem" has
exactly the same divergence
solvable limit: density n → 0, T→ 0



$$I_0 \equiv \int \frac{d\omega}{\pi}\eta_0(\omega) = \frac{\varepsilon_0}{3} + \alpha\frac{C_0}{a} + \beta C_0\Lambda$$

The difference $I - \left(\frac{C}{C_0}\right) I_0$ is finite!

$C =$ Contact

$$\boxed{\frac{1}{\pi}\int_0^\infty d\omega\left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}}\right] = \frac{\varepsilon}{3} - \frac{C}{12\pi m a}}$$

(3D)

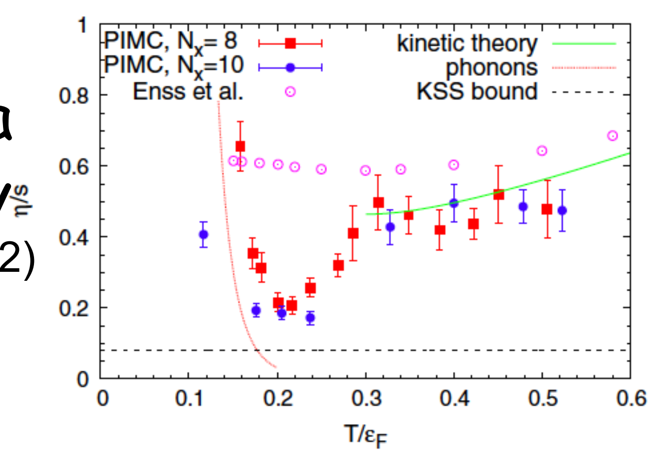
Shear viscosity Sum Rule in 3D

$$\boxed{\frac{1}{\pi}\int_0^\infty d\omega\left[\eta(\omega)-\frac{C}{15\pi\sqrt{m\omega}}\right]=\frac{\varepsilon}{3}-\frac{C}{12\pi m a}}$$

Valid for all T and all scattering lengths a

$$P=\text{pressure}\quad \varepsilon=\text{energy/vol.}\quad \rho=\text{mass density}$$
$$c_s\equiv(\partial P/\partial\rho)^{1/2}\ \text{fixed}\ s=S/N$$

- Constraints for approximate calculations
  Enss, Haussmann, Zwerger, Ann. Phys. (2011)
- Constraints for numerical procedures for analytic continuation of QMC data from imaginary time → real frequency
  Wlazłowski, Magierski, Drut, PRL (2012)
- No progress on bounds yet



Bulk Viscosity Sum Rule in 3D

$$\boxed{\frac{1}{\pi}\int_0^\infty d\omega\, \zeta(\omega) = \; P - \varepsilon/9 - \rho c_s^2/2}$$

Valid for <u>all</u> T and <u>all</u> scattering lengths a

At unitarity $\quad P - \varepsilon/9 - \rho c_s^2 = \dfrac{1}{72ma^2}\left(\partial C/\partial a^{-1}\right)_s$

$$= 0 \longleftarrow \mathcal{F}(T/E_f, 1/\cancel{k_F} a)$$

$$\frac{1}{\pi}\int_0^\infty d\omega\, \zeta(\omega) = 0$$

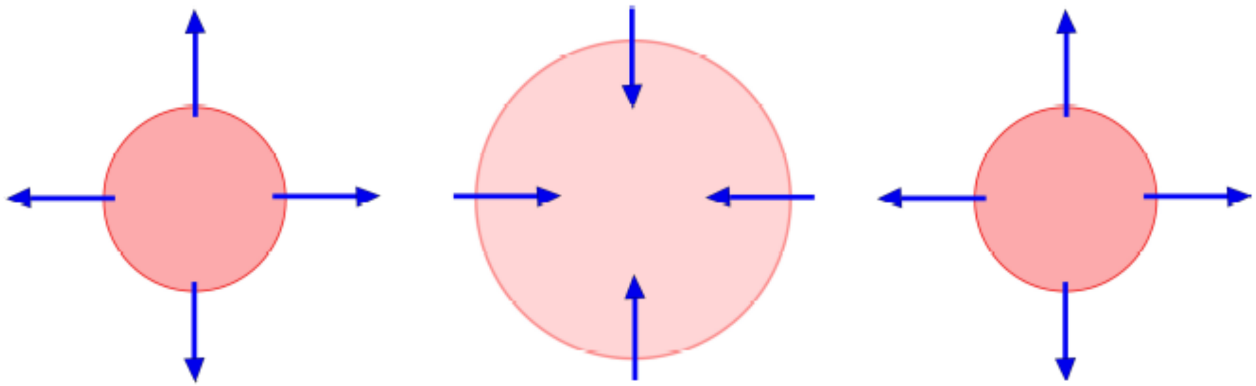
But $\quad \zeta(\omega) \geq 0$ $\qquad$ 2nd law of thermodynamics

$$\Rightarrow \boxed{\zeta(\omega) = 0}$$

for <u>all</u> ω and T at $|a| = \infty$

Consequence of Scale invariance

Vanishing bulk viscosity at unitarity



- Bulk viscosity → relax to equilibrium
  after uniform dilation

- Scale invariance at unitarity
  → w.f. after scale change remains eigenstate of H
  → gas never leaves equilibrium under dilation
  $\zeta(0) = 0$ [Werner & Castin (2006); CFT: Son (2007)]

- Our result generalizes this to <u>all</u> frequencies

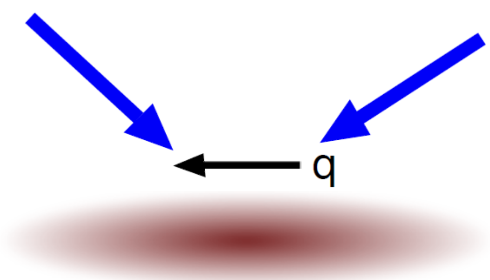
Measuring the shear viscosity spectral function in 3D

$\zeta(\omega) = 0$ at unitarity $\Rightarrow$

$$\frac{4\eta(\omega)}{3} + \cancel{\zeta(\omega)} = \lim_{q\to 0} \frac{\omega}{q^2} \mathrm{Im}\chi_L(\mathbf{q},\omega)$$

Continuity equation $\Rightarrow$ $\mathrm{Im}\chi_{\rho\rho} = (q^2/\omega^2)\mathrm{Im}\chi_L$

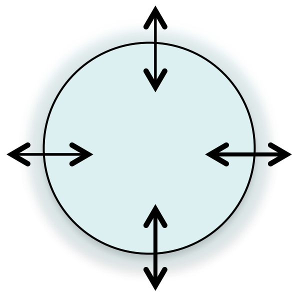
$$\boxed{\eta(\omega) = \lim_{q\to 0} \frac{3\omega^3}{4q^4} \mathrm{Im}\chi_{\rho\rho}(\mathbf{q},\omega)}$$

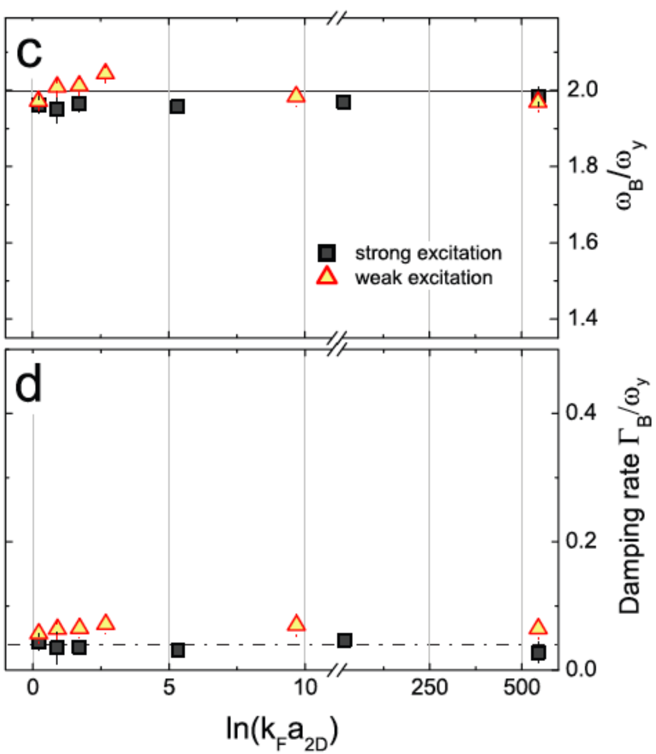


Prediction for two-photon Bragg spectroscopy

Apparent scale invariance in 2D Fermi gases

<u>Expt</u>: E. Vogt et al.,
PRL 108, 070404 (2012)





Monopole breathing mode in 2D

* Frequency $\boxed{\omega_m = 2\omega_0}$
* No damping $\boxed{\zeta = 0}$

Characteristic of
scale-invariant behavior!
-- without any fine tuning
-- in a system with a scale:
dimer binding energy $|\varepsilon_b| = 1/ma_2^2$

Why?

E. Taylor & MR, PRL 109, 135301 (2012)

- Variational bound on ω_b
- 2D Sum rule constraint on ζ

Both implicate

$$\gamma_d \equiv (1 + 2/d)P - \rho(\partial P/\partial\rho)_s$$

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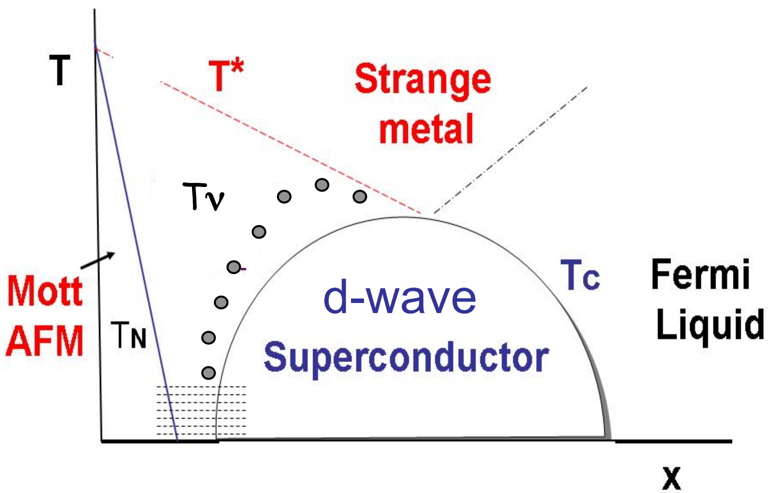
High Tc Superconductors

- Highest known Tc (in K)
  - * cuprates
- Charged electrons
- Repulsive interactions
- d-wave SC
- doped Mott insulator
- competing orders: AFM, CDW, …
- single band
- repulsion U >> bandwidth
- $\xi \sim 10$ A
- $T_c \sim \rho_s \ll \Delta$ (underdoped)
- anomalous normal states
  - strange metal
  - pseudogap

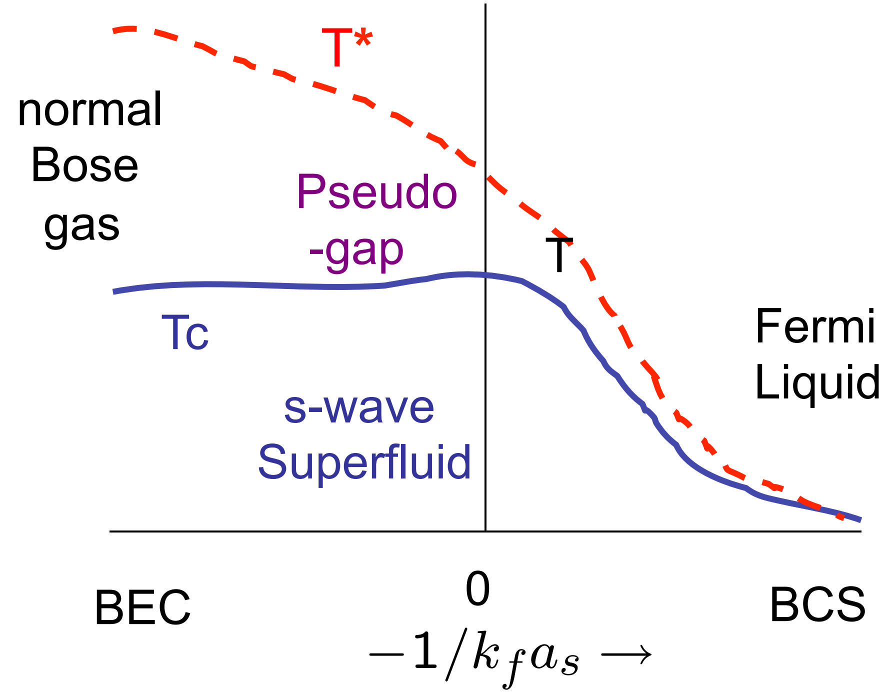
BCS-BEC crossover

- Highest known $T_c/E_f \sim 0.2$
  - * ultracold atomic gases
- Neutral Fermi atoms
- Attractive interactions
- s-wave SF
- only pairing instability
- single band
- attraction >> E_f
- $\xi \sim 1/k_f$
- $T_c \sim \rho_s \ll \Delta$ (for $a_s > 0$)
- pairing pseudogap
- Mean-field theory fails for Tc

High Tc Cuprates



BCS-BEC crossover

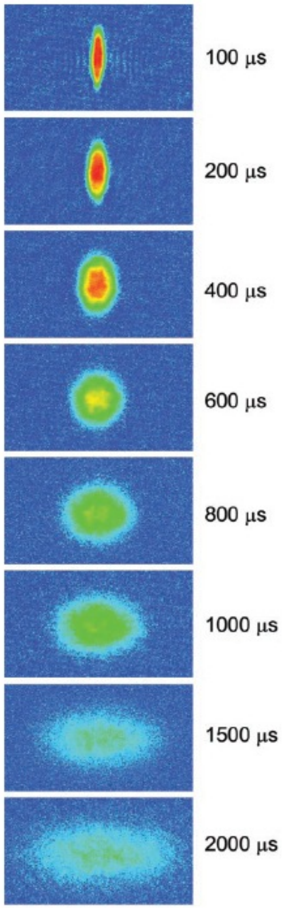


Superconductivity is strongest
- near crossover from
  pair-breaking (Δ)
  to phase-fluctuation (ρ_s)
  dominated regimes

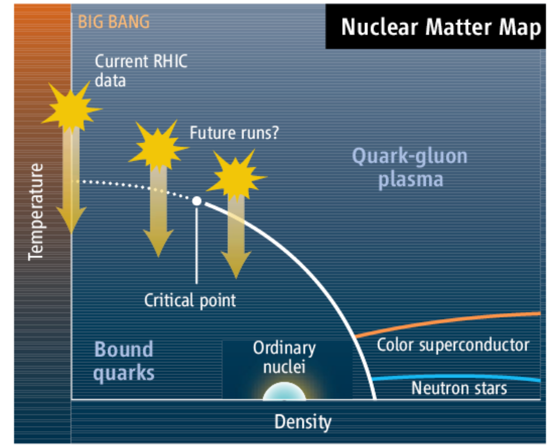
Pseudogap when $\Delta > \rho_s$

Cuprate pseudogap is much more complex: Mott physics & competing orders

Connections with Nuclear & High Energy Physics



* hydrodynamic expansion of unitary Fermi gas [Thomas]
  ←→"elliptic flow" in quark-gluon plasma at RHIC
  ←→ Viscosity/entropy bounds Ads/CFT [Son et al]

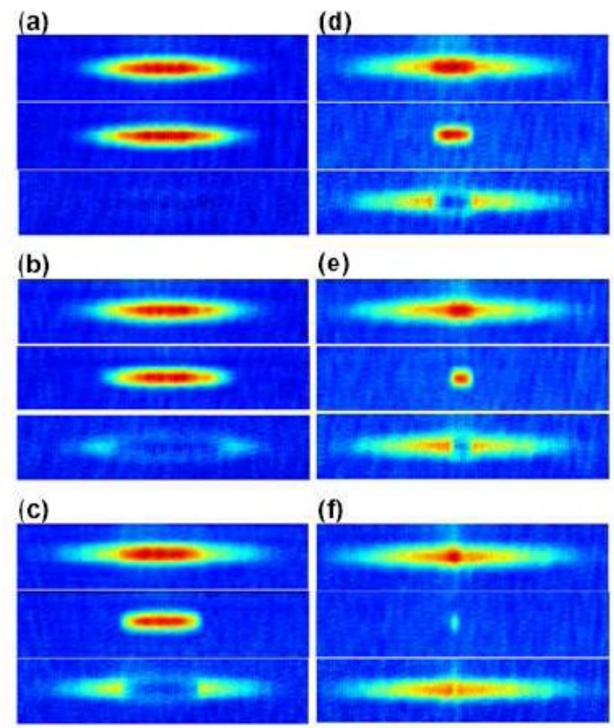


* Color SC in Quark matter
  ←→ Phases of Fermi gases with

$$n_\uparrow \neq n_\downarrow$$ [Ketterle; Hulet]

- Phase separation
- no evidence for FFLO at unitarity

$$m_\uparrow \neq m_\downarrow$$ [Grimm]



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BCS-BEC Crossover: Experiments & Theory

- A remarkable new chapter in many-body physics
- Unitary Fermi gas: a new paradigm for strongly interacting systems
- Open questions
  -- Is there a general upper bound on Tc/Ef?
  -- 2D systems
  -- vortices, solitons, …
  -- transport
  -- non-equilibrium dynamics
  -- non-s-wave, SOC & topological states …

The End!