Why study mesomopic physics? There are a lot of ways in which non- um samples defler from both atomic systems and larger devices I won't try to lay out a systematic list - will just give examples : · Single-electron affects - Colomb blockude can control electrons one by one · Near coossever between classical and quantum physics ~ 1 pen scale - phase coheverce, weak localization Aharanov - Bohm ~ 100 mm individual quarter states in semiconductors at low T ~ 10 nm individual quantum states in metals · Can now manipulate single stores and molecules and control their coupling to the outside world. Lots of important scales in the Inm-Irem range · elastic and inelastic mean free paths of electrons · spin-flip diffusion length (important for spin-polarized transport) · domain-wall widths in magnets · superparamagnetic crossover in magnets in superconductors, superconducting coherence and magnetic penetration lengths many others .. New discoveries are being made, both experimentally and theoretically. Knowledge being developed is beading to some practical applications. A time of productive ferment, if not always the most organized field.

Plan for Dan Ralph's Lectures (Tentative)

- Basic introduction to Coulomb blockade in quantum dots Simple spin physics, Zeeman spin splitting Spin-orbit effects on Zeeman splitting, including random-matrix-theory ideas
- 2. Interacting Electrons within Quantum Dots -- the Universal Hamiltonian
 - (a) Weak exchange interactions -- non-zero spin states prior to the Stoner instability
 - (b) Superconducting pairing
 - (c) Ferromagnetic quantum dots
- 3. Review of Kondo Experiments Single-Molecule Devices
- 4. Nano-magnetics (in metals)
 - (a) basics of "giant magnetoresistance" (GMR)
 - (b) spin-transfer torques and magnetic dynamics

Coulomb Blockade and Tunneling Effects in Nanostructures Vy 10 I majore the circuit contains an object that provides a pool for electrons to enter - e.g. metal grain, someconductor dot, redox state in a molecule, localized charge state If the grain is small, it has a small capacitance C to outside would = Gait ignore the classical charging energy Ec- 2c even for one electron. (100×100) MM2 C- 0.5 fF for aluminum oxide junctions 2 Ec ~ 0.2 met or (2k)kg for (5x5) Mm2 junctions, C- laf E= 80mel - (800 K) ks Conditions for Coulomb blockade KT, eV = Ec electrons in electrodes can have montficient 0 energy to hop onto the island and off the other side to avoid quantum fluctuations, need "energy uncertainty" << 22 3 7 44 4 20 Infetime of charge on capacitor P RC ~ 2c R>>> 2 4 ha= 25.8 "resistance quartum " When Rook and eV is comparable to Ec, at most one excere election at a time can occupy the island = single election tradeling,

With a timable gake voltage Vg, can make a single-election transistor. Up tunes the energy cost for adding an elector. E(n electrony) = (ne) - neve 5 Energy of ished = for V=0. = Ic [ne - VgCg] - (VgCg) Now what matters is E(n±1) - E(n) = energy required to change the number of electrons by one. This means we can ignore the last term because it is redependent of n. ⇒ E(n) = zc [Vg cg - ne] = tc [Qo-ne] where Qo = Vg Cg is called the "offset charge", but really it is due to the effects of an external potential. graphically: E(1, Q.) = [Q.-ne] 3e Q. = VgCg The different charge states have different energies. With a gate voltage, you can change how many electrons are in the ground state of the island For a fixed value of Qo, there is a minimum energy cost for the island to change change number (vertical arrows) At Qole = = + integer values, 2 different change states are degenerate ground states. At these volves of gate voltage, Changes can flow with zero Coulomb energy cost. The gate voltage has tunch away the Coulomb blockede. Conductance at Ec Qo=VgG te ie

Another picture for the seems physics : (assume that the Coulomb blockade is overcame by adding an electron) () Island (2) allowed states for electron addition (for now, assume a = DE(Va) right left continuum of states electrode electrode GL (I also assume that G+G MR Cg is small confired to Ci and Ci) C.+C The threshold on tenneling for this sign of voltage is when eV CL Citcz ML = Folly or = AE(1g) eV C2 = AE(Ug) For the opposite sign of bies Vg 5h. fts the energy levels on the islead up a down inlative to the Fermi level in the electrodes. When the levels are Shifted down to Esp, there is no charging Energy. One can draw a similar dragram for the case when the Colomb blockede is overcome by subtracting an electron



Coulomb-Staircase Curves



Effects of Gate Voltage on Coulomb Blockade

color scale: dl/dV



Coulomb-Blockade Effects in One Molecule



- High resistance (> megaOhms) single electron charging.
- Coulomb blockade > 150 meV (unstable beyond this).







Tunneling through Individual Quantum States





Why study these spectra of discuste states? - Experiments show that the spectra are not well described by simple models of non-interacting electrons. Instead, they reflect all the dotterent forces and interactions acting on the electrons, with deflerent types of interactions having distinguishable affects => These spectra are a good way to bear about the consequences of electron interactions, at a very findamental level. (Many analogies to atomic and nuclear physics.) There are different types of forces and interactions acting on electrons in semiconductors, normal metals, maquets, superconductors, carbon nanotubes, other molecules, etc. In this becture and the next, I will discuss the consequences of different kinds of forces 1) Spin - Orbit interactions (for Al, Cu, Ag, Au) 2) Weak exchange interactions and non-zero zpin states (compare Gats and Au) 3) Superconducting gaining (Al) 4) Strong exchange and magnetic anisotropy forces (ferromagnetic Co)

Basics of Spin in Quantum Dots (non magnetic) It is possible to tell it a nonmagnetic dot has an even or odd number of electrons, even it the number of electrons is in the 1000's. One procedure: Look at the lowest-energy traneling reservence. If it valengors Zerman spin splitting, then the transition corresponds to even > odd. If only one transition appears, moving to higher energy as a function of B, you have odd I even Single - electron picture: for electron addition odd + even, lower E even-sodd both states empty state is blocked by the odd electron. and amilable E for tenneling Only one state available for tenneling electron subtraction : for even-> odd both states odd-even . caly filled state is E available available. Alternation Many-body picture: (holds as long as the ground state is spin O for even electrons, spin to for odd) even > odd odd - even J=+ 1 in B field 5= ± + Kramer's Doublet singlet only lower state our pred. Only one transition & visitable ((Show deta for Al + Cur)

Magnetic-Field Dependence of Aluminum Levels







In general, copper is a little more complicated than aluminum.



Note for Al: g=2 for all states to within measurement incertainfies linear dependence of energy on B Simple crossings between up and down spin states For Cu: () g < 2 for all states @ g deffers from state to state 3 a voided crossings of up and down spin states What causes the difference? The shifts in spin energy vary depending on how close in energy are other orbital states - we should suspect spin - orbit coupling Cu (2=29) is a heavier element than Al (2=13) 50 scattering ~ 24 References for spin-orbit scattering in metals. R.J. Elliot Phys Rev. 96, 266 (1954) J Fubian and S. Day Sarma, J. Vac. Sci. Technol. B 17, 1708 (1999) Electrons scatter off of impurities, interfaces, quain boundaries, or phonons. This rotates the spin, Mixies spin-up and spin-down components in the wave function. But time-reversal symmetry still holds - Kramer's degeneracy is still valid.

with spin-orbit interactions, the two energy eigenstates are not pure spin up or spin down (the total electron spin does not commute with H). Instead they are linear superpositions of I and V: 1n, "7"> = ~ 19> + Bal+> 7 Related by time-revenal 1n "+"> = - p= 17> + a= 14> sy metry 1) g-factor for spin Zeeman splitting: gn = 2 < n,"7" | J= | n, "">= 2 (| wal2 - 1/Bul2) = 2 (1-21Bul2) = 2. (For weak spin-orbit interactions, real particles have negligible orbital contribution to the g-factor, because orbital anyther manatum is greached. unlike atoms. The orbital contribution will be considered below in the regime of strong JO.) Simple perturbation theory can explain the qualitative features I have described in the Cu dots. (J. Sone, J. Phys. Soc. Jpn. 42, 1457 (1977)) Auctuations in g-factors 2 $g_n = 2 \int 1 - 2 \frac{\left[\left< \frac{\psi_{m\psi}}{H_{so}} \right] \frac{\psi_{m\psi}}{H_{so}} \right]^2}{(E_n - E_m)^2} \int \frac{\left[\left< \frac{\psi_{m\psi}}{H_{so}} \right] \frac{\psi_{m\psi}}{H_{so}} \right]^2}{(E_n - E_m)^2}$ matrix elements will very between the unperturbed single-election states You + Yh the every denominators will also vary. > for different orbital states n, gn can be different. 3 avoided crossing? 2 1 Hool in crossing rayion, 2x2 Hamiltonian $\mathcal{P}_{i} = \begin{pmatrix} E_{i} + \frac{1}{2}g\mu_{0}B & H_{0} \\ H_{50}^{*} & E_{2} - \frac{1}{2}g\mu_{0}B \end{pmatrix}$

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Random Matrix Theory Predictions: Distributions of the principal g-factors for different strengths of spin-orbit interaction.





• With one fitting parameter per sample, both the average g-factor and the standard deviation are described well.

• Significant differences in spin-orbit strength even for particles made of the same metal.



Variations from Quantum State to Quantum State







Principal-axis directions are randomly oriented in space.

Check agreement between experiment and RMT predictions

Single parameter fit: λ (spin-orbit scattering strength) is determined by matching the experimental and theoretical values of $\langle g_1^2 \rangle + \langle g_2^2 \rangle + \langle g_3^2 \rangle$.



Theory and experiment are in excellent agreement.

One Mystery: The g factors in gold nanoparticles are smaller than expected



If we assume the orbital part does not contribute, the spin contribution to the average g factor gives an estimate for τ_{so} consistent with weak localization measurements.

Are the nanoparticles much more disordered than we expect so that they can quench the orbital contribution, or is there some shortcoming in the theory?

Summary · Coulomb blockade, single-electron tunneling are ubiquitous when thinking about the physics of small-scale objects weakly coupled to electrodes We developed pictures to understand the I-V characteristics level width & leT < level spacing & charging energy. when : This corresponds to relatively high veristance tranel junctions with Ron We (For lower-venistance junctions, cotunneling and Kondo physics come into play.) . The study of spin Zeeman splitting is a good tool to learn about spin-orbit coupling. · Mesoscopic affects bring some surprises - e.g. very large fluctuations in g-factors between different orbital levels in the same metal quantum dot. Random matrix theory can be applied in explaining many results, but some mysteries remain. Next time: Using measurements of electron-in-a-Lox enery levels to explore the consequences of different types of electron-electron interactions, and applications of a "universal Hamiltonian"

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