Lecture 3

Mesoscopic effects in hopping conductivity

Topics

1. ***Anomalous transmission of a barrier***

2. ***Hopping conductivity through an amorphous film***

3. ***Distribution function of hopping conductivity of a finite-size sample***

4. ***Hopping conductivity in 1D***

Theory of the passage of particles and waves through randomly inhomogeneous media

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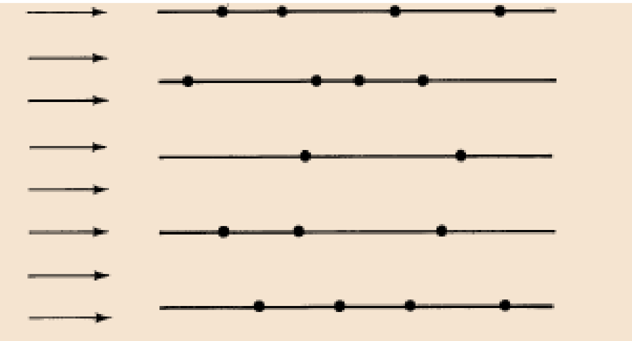
(Submitted 28 July 1982)

Zh. Eksp. Teor. Fiz. **83**, 2362–2376 (December 1982)



I.M. Lifshitz

system of filaments with random traps

D *-probability to pass through a trap*

Poisson's distribution

$$p(N) = \bar{N}^N e^{-\bar{N}}/N!$$

$$\langle \sigma \rangle = \sum_{N=0}^{\infty} D^N p(N) = \sum_{N=0}^{\infty} \frac{(D\bar{N})^N e^{-\bar{N}}}{N!}$$

$$\langle \sigma \rangle = \exp[-\bar{N}(1-D)]$$

$$\ln \langle \sigma \rangle = -\bar{N}(1-D).$$

$$\langle \ln \sigma \rangle = -\bar{N} \ln(1/D).$$

$$\ln(1/D) > 1 - D$$

$$\ln \langle \sigma \rangle > \langle \ln \sigma \rangle$$

average conductance exceeds ***exponentially*** *the conductance of a typical filament*

the product $D^N p(N)$ *is maximal for* $N = D\bar{N} = N_{\text{opt}} < \bar{N}$

sparse filaments with N *close to* N_{opt}

determine the average conductance

Tunnel transparency of disordered systems

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(Submitted 9 February 1979)
Zh. Eksp. Teor. Fiz. 77, 989–1016 (March 1979)

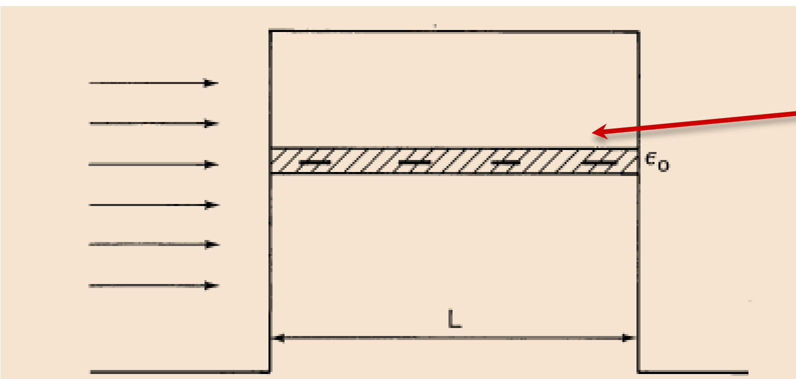
electrons with broadly distributed energies are incident on a barrier with randomly positioned impurities

$\Gamma_0 \propto \exp(-L/a)$ *energy interval where transmission is high*

$$\sigma(\varepsilon) = \frac{4\Gamma_0^2}{(\varepsilon-\varepsilon_0)^2 + 4\Gamma_0^2 \operatorname{ch}^2 2z/a},$$

impurity level position

distance from the center



resonant tunneling through a chain of four localized states

for a chain of N impurities

$\Delta_N \propto \exp(-L/Na),$ *increases with N*

miniband width

$$\sigma_N \propto \Delta_N w_N \propto \exp\left[-\frac{L}{Na} - N\ln\left(\frac{1}{nv_0}\right)\right].$$

$(nv_0)^N$ *probability of formation of a chain*

$$N = N_{\rm opt} = \left(\frac{L}{a\ln(1/nv_0)}\right)^{1/2}.$$

$$\ln\langle\sigma\rangle = -2\left[\frac{L}{a}\ln\left(\frac{1}{nv_0}\right)\right]^{1/2}.$$

rigorous calculation yields $v_0 = a^2 L$

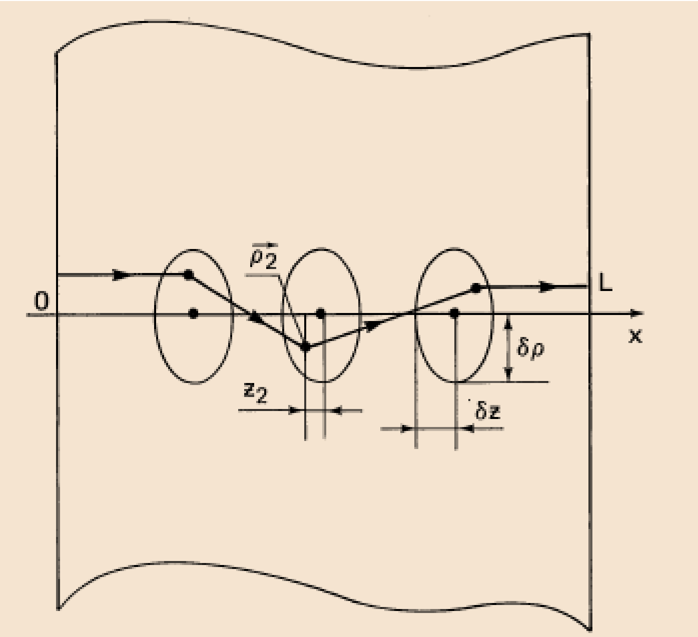
unlike tunnel conductivity $\propto \exp\left(-\frac{2L}{a}\right)$

Hopping conductivity of an amorphous film

$$R_{12} = \frac{1}{\sigma_0} \exp\left(\frac{2|\mathbf{r}_1 - \mathbf{r}_2|}{a} + \frac{\varepsilon_{12}}{T} \right)$$

$$\varepsilon_{12} = \tfrac{1}{2}(|\varepsilon_1| + |\varepsilon_2| + |\varepsilon_1 - \varepsilon_2|).$$

resistance of one hop



resistance of one chain

$$R_N = \frac{1}{\sigma_0} \exp\left(\frac{2L}{Na} + \frac{\delta z}{a} + \frac{(\delta\rho)^2 N}{La} + \frac{\delta\varepsilon}{T} \right)$$

probability of formation

$$w_N = [g\delta\varepsilon(\delta\rho)^2\,\delta z]^N.$$

the product $w_N R_N^{-1}$ *is maximal for*

$\delta\varepsilon = NT,$ $\delta z = Na,$ $\delta\rho = (La)^{1/2}.$

$$w_N R_N^{-1} = \sigma_0 \exp\left(-\frac{2L}{Na} - N \ln \frac{1}{gTa^2LN^2} \right)$$

$$N_{\text{opt}} = (2L/a\lambda)^{1/2}$$

$$\ln\langle\sigma\rangle = -2(2L\lambda/a)^{1/2}$$

solution of equation

$$\lambda = \ln(\lambda/gTaL^2).$$

Applicability: chains dominate when

$$|\ln\langle\sigma\rangle| < \left(\frac{T_0}{T}\right)^{1/4}$$

$$L < \frac{1}{(gTa)^{1/2}}$$

$$\lambda >> 1$$

We assumed that the chains are nearly straight:

$\delta z \approx \left(\frac{2La}{\lambda}\right)^{1/2}$ *and* $\delta\rho \approx (La)^{1/2}$ *must be smaller than* $\frac{L}{N} \approx \left(\frac{La\lambda}{2}\right)^{1/2}$ *these conditions are also met when* $\lambda >> 1$

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PHYSICAL REVIEW B **84**, 125447 (2011)

Hopping transport in systems of finite thickness or length

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(Received 18 July 2011; published 29 September 2011)

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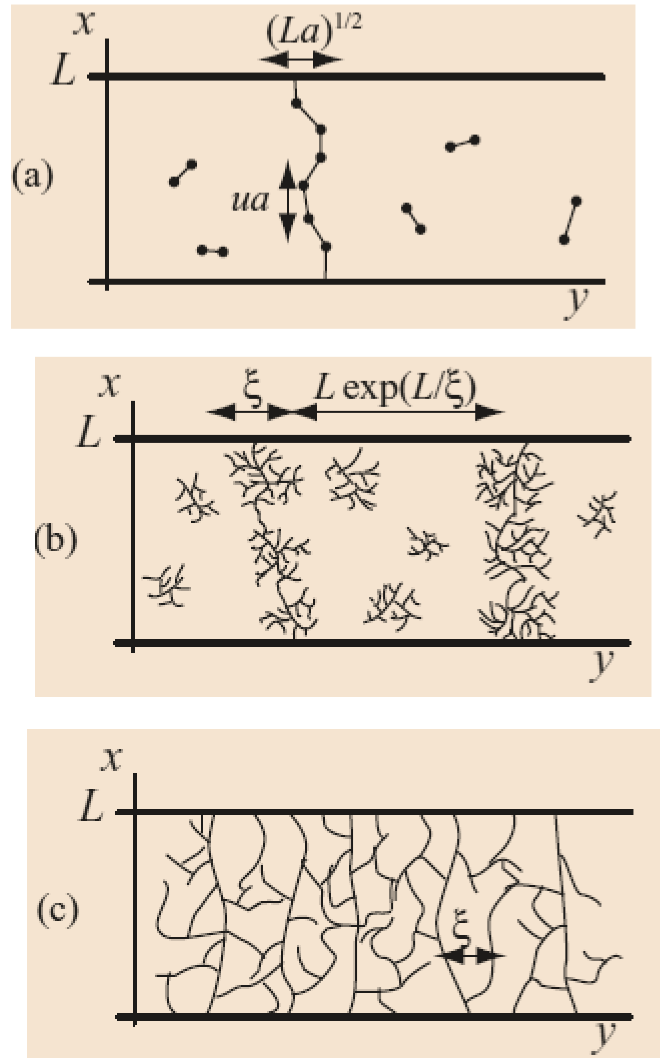
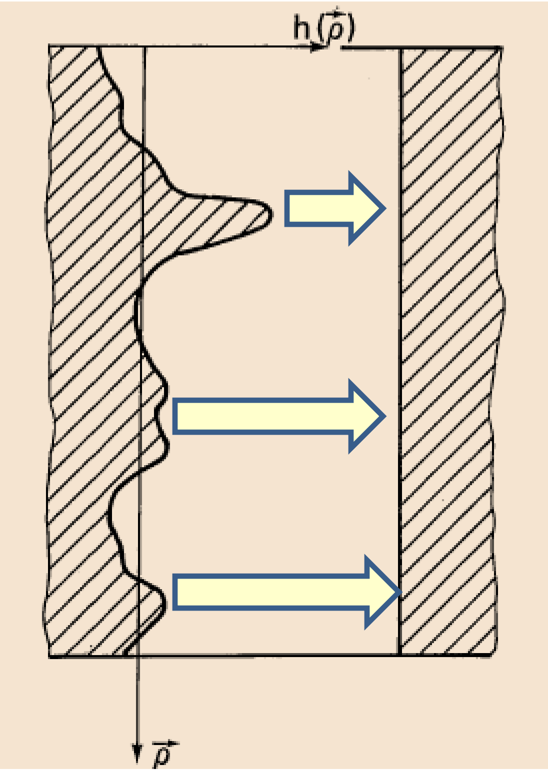


Evolution of a 2D network with increasing T
The network progresses from independent conducting strands to an interconnected grid

Sample of a finite area



*Since the distances between punctures are exponentially large, the situation may occur when **no optimal puncture** is present in a sample*

*Then the conductivity of a typical sample will be determined by a few punctures of highest transmittance **present in the sample***

conductivity will strongly fluctuate from sample to sample

conductivity will strongly depend on the sample area

*Ensemble of samples should be characterized by the **distribution function***

Peak position of the distribution function

Cross section of transmission of a puncture

$$A = A_0 \exp(-u)$$

Areal density of punctures

$$\rho(u) = \lim_{S\to\infty} \frac{1}{S} \sum_i \delta(u - u_i),$$

area of the sample

For punctures with anomalously high transmission

$$\rho(u) = \frac{1}{S_0} \exp[-\Omega(u)] \Leftarrow \Omega(u)$$ ***decreases with increasing*** u

characteristic puncture area

Sample-averaged conductivity

$$\langle\sigma\rangle = A_0 \int_0^\infty \mathrm{d}u\, e^{-u} \rho(u) = \frac{A_0}{S_0} \int_0^\infty \mathrm{d}u \exp[-u - \Omega(u)]$$

The integrand has a sharp maximum at $u = u_{\mathrm{opt}}$ ***such that*** $\Omega'(u_{\mathrm{opt}}) + 1 = 0.$

$$\ln\langle\sigma\rangle \cong \ln\left(\frac{S_0}{A_0}\langle\sigma\rangle\right) = -u_{\text{opt}} - \Omega(u_{\text{opt}})$$

Applies when the number of optimal punctures within the sample area is large $S\rho(u_{\text{opt}}) \gg 1$

Since $|\ln(S_0\rho(u_{\text{opt}}))| = \Omega(u_{\text{opt}}) \gg 1$ *the condition of applicability reduces to* $\nu > 1$

$$\nu = \frac{\ln(S/S_0)}{\Omega(u_{\text{opt}})}$$

For $\nu < 1$ *the conductivity of a typical sample will be dominated by a few punctures with highest transmission present in the sample*

this condition defines $u = u_{\text{f}} > u_{\text{opt}}$ *such that* $S\rho(u_{\text{f}}) \sim 1$ $\Rightarrow$ $\Omega(u_{\text{f}}) = \nu\Omega(u_{\text{opt}})$

u_f *defines the lower limit in* $\int_{u_f}^{\infty} du\,\rho(u)\exp(-u)$

$$\ln\left(\frac{S_0}{A_0}\sigma(\nu)\right) = -\nu\Omega(u_{\text{opt}}) - u_{\text{f}}(\nu)$$ *maximum of the distribution function*

Width of the distribution function

variance

$$\langle(\delta\sigma)^2\rangle = \langle\sigma^2\rangle - \langle\sigma\rangle^2 = \frac{A_0^2}{S}\int_0^\infty du\, \rho(u)\, e^{-2u}$$

from Poisson's statistics

$$\langle n_i^2\rangle - \langle n_i\rangle^2 = \langle n_i\rangle$$

The integrand has a sharp maximum at $u = u_d$ *such that* $\Omega'(u_d) + 2 = 0.$

$$\langle(\delta\sigma)^2\rangle \sim \frac{A_0^2}{SS_0}\exp[-2u_d - \Omega(u_d)]$$

$$u_d < u_{opt}$$

punctures responsible for variance are exponentially more sparse than punctures responsible for average

determines the width of the the distribution function if $S\rho(u_d) \gg 1$

$v > v_d$ *where* v_d *is the solution of* $\varphi(v_d) = 2,$

definition of the function φ

$$\Omega(u_f) = v\Omega(u_{opt})$$

$$\varphi(v) = -\Omega'(u_f(v)).$$

$$v = \frac{\ln(S/S_0)}{\Omega(u_{opt})}$$

The width of the distribution function of $\ln\sigma$ *for* $\nu > \nu_{\rm d}$

$$\Delta_0 \sim (\langle(\delta\sigma)^2\rangle)^{1/2}/\langle\sigma\rangle \sim \exp[u_{\rm opt} + (1-\tfrac{1}{2}\nu)\Omega(u_{\rm opt}) - u_{\rm d} - \tfrac{1}{2}\Omega(u_{\rm d})]$$

the width is inversely proportional to $S^{1/2}$

for $\nu < \nu_{\rm d}$

$$\Delta_0 \sim \frac{1}{\sigma(\nu)}\left(\frac{A_0^2}{SS_0}\int_{u_{\rm f}}^{\infty} {\rm d}u\, \exp[-2u - \Omega(u)]\right)^{1/2} = \frac{A_0}{S_0\sigma(\nu)}\exp[-u_{\rm f}(\nu) - \nu\Omega(u_{\rm opt})]$$

integration only over punctures present in a typical sample

$1 < \nu < \nu_{\rm d}$ ⇨ $\sigma(\nu) = \langle\sigma\rangle$ ⇨ $\Delta_0 \sim \exp[-u_{\rm f}(\nu) + u_{\rm opt} + (1-\nu)\Omega(u_{\rm opt})]$

$\nu < 1$ ⇨ $\ln\left(\frac{S_0}{A_0}\sigma(\nu)\right) = -\nu\Omega(u_{\rm opt}) - u_{\rm f}(\nu)$ ⇨ $\Delta_0 \sim 1$

for $\nu \ll 1$ *uncertainty in* $u_{\rm f}$ *determines the width of the distribution function*

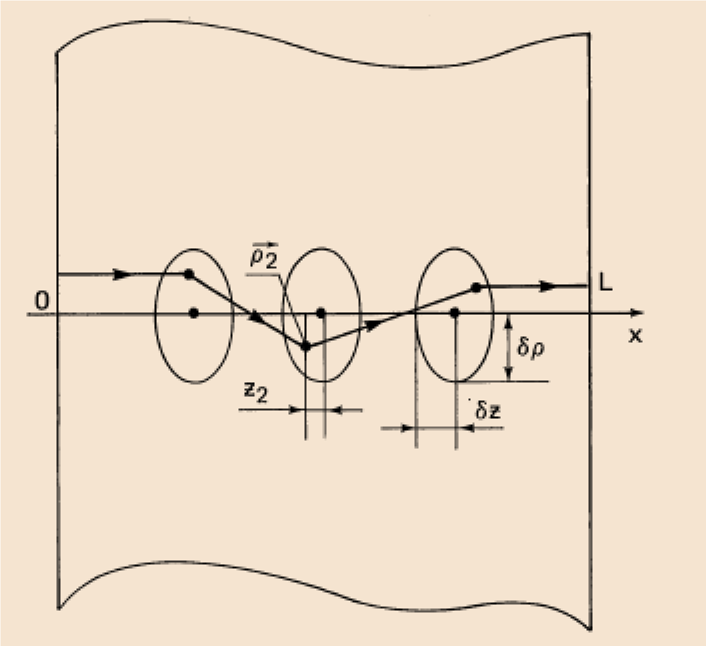
$S\rho(u_{\rm f}) \sim 1\text{–}2$ ⇨ $\delta u_{\rm f} \sim [S\rho'(u_{\rm f})]^{-1}$ ⇨ $\Delta_0 \sim \delta u_{\rm f} \sim 1/\varphi(\nu) \gg 1$

Example:

$$\sigma_N = \exp\left[-\frac{2L}{Na}\right]$$

$$w_N = C^{-N}$$

$$\rho(u) = \frac{1}{S_0}\exp[-\Omega(u)]$$



$$\Omega(u) = Q_0^2/4u \longrightarrow u_{\rm opt} = Q_0/2$$

$$\Omega(u_{\rm f}) = \nu\Omega(u_{\rm opt}) \longrightarrow u_{\rm f}(\nu) = Q_0/2\nu$$

$$\varphi(\nu) = -\Omega'(u_{\rm f}(\nu)) \longrightarrow \varphi(\nu) = \nu^2$$

$$\varphi(\nu_{\rm d}) = 2 \longrightarrow \nu_{\rm d} = 2^{1/2}$$

$\nu < 1$ ***peak position*** $\ln\sigma(\nu) = -\dfrac{Q_0}{2}\left(\nu + \dfrac{1}{\nu}\right)$

evolution of the width of the distribution function with area

$$\Delta_0 \sim \exp[-\tfrac{1}{2}Q_0(\nu + \nu^{-1} - 2)] \qquad 1 < \nu < 2^{1/2}$$

$$\Delta_0 \sim \exp[-\tfrac{1}{4}Q_0(\nu - 4 + 2^{3/2})] \qquad \nu > 2^{1/2}$$

Analytical expression for the distribution function

$$\sigma = \frac{A_0}{S}\sum_i n_i \exp(-u_i),$$

sum over the types of punctures

$$p(n_i) = \frac{\exp(-\bar{n}_i)}{n_i!}\,\bar{n}_i^{n_i}$$

each type of punctures is Poisson-distributed

$$Q = -\ln(S_0\sigma/A_0)$$

$$f(Q) = \mathrm{e}^{-Q}\sum_{n_i=0}^{\infty}\delta\left(\mathrm{e}^{-Q} - \frac{S_0}{S}\sum_i n_i \exp(-u_i)\right)\prod_k p(n_k)$$

Fourier transform of the δ *function*

$$f(Q) = \frac{\mathrm{e}^{-Q}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t\,\exp(\mathrm{i}t\,\mathrm{e}^{-Q}) \times \sum_{n_i=0}^{\infty}\prod_i \frac{\exp(-\bar{n}_i)}{n_i!}\left[\bar{n}_i \exp\left(-\frac{\mathrm{i}tS_0}{S}\exp(-u_i)\right)\right]^{n_i}$$

summation over n_i

$$f(Q) = \frac{\mathrm{e}^{-Q}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t\,\exp(\mathrm{i}t\,\mathrm{e}^{-Q})\prod_i \exp\left\{\bar{n}_i\left[\exp\left(-\frac{\mathrm{i}tS_0}{S}\mathrm{e}^{-u_i}\right) - 1\right]\right\}$$

in the continuous limit

$$f(Q) = \frac{\mathrm{e}^{-Q}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t\,\exp\left\{\mathrm{i}t\,\mathrm{e}^{-Q} + S\int_0^{\infty}\mathrm{d}u\,\rho(u)\left[\exp\left(-\frac{\mathrm{i}tS_0}{S}\mathrm{e}^{-u}\right) - 1\right]\right\}$$

For $\nu > \nu_d$ *the distribution function is gaussian*

$\Delta = \ln \sigma - \ln \langle \sigma \rangle$

Δ_0 *distribution function of* $\ln \sigma$

$$f(\ln \sigma) = \frac{\exp(-\Delta^2/2\Delta_0^2)}{(2\pi)^{1/2}\Delta_0}$$

$1 < \nu < \nu_d$ *non- gaussian but the peak position is still determined by optimal punctures*

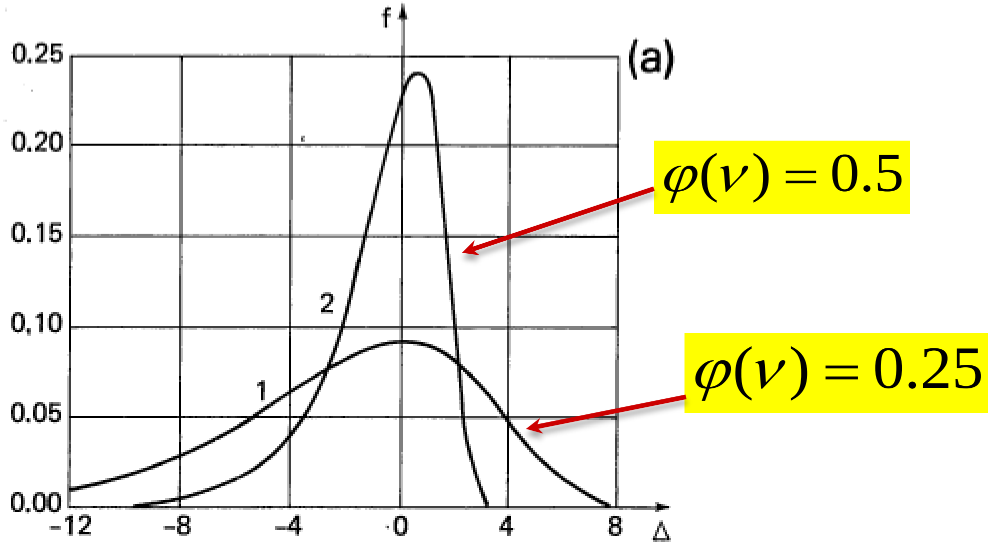
$\Delta = \ln \sigma - \ln \langle \sigma \rangle$

$$f(\ln \sigma) = \frac{1}{\pi \Delta_0} \int_0^\infty \mathrm{d}t \, \exp\left(t^\varphi \cos \frac{\pi\varphi}{2} \right) \cos\left(t \frac{\Delta}{\Delta_0} + t^\varphi \sin \frac{\pi\varphi}{2} \right)$$

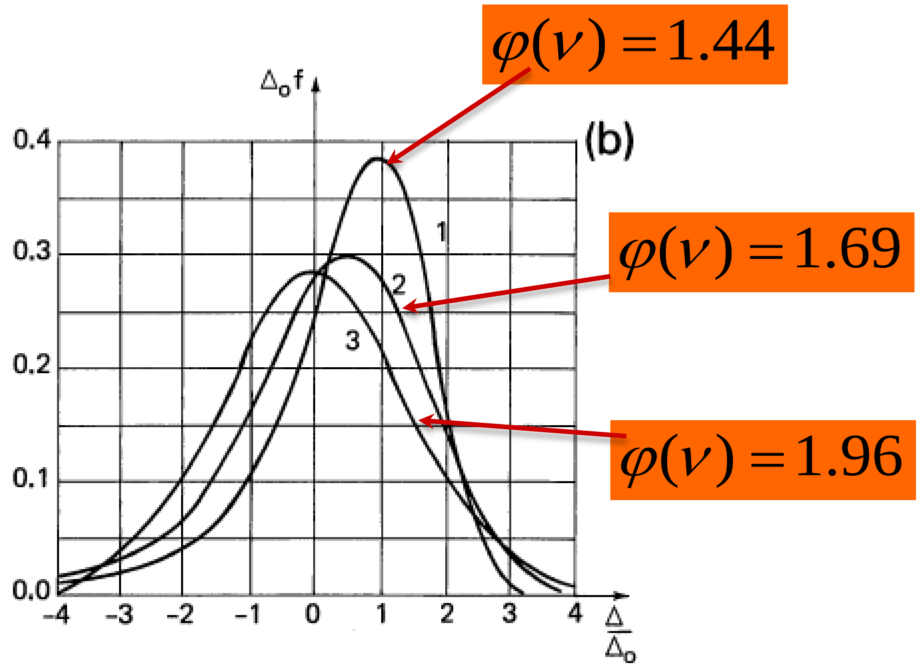
$\nu < 1$ *the peak position is determined by punctures with* $u = u_f > u_{opt}$

$\Delta = \ln \sigma - \ln \sigma(\nu)$

$$f(\ln \sigma) = \frac{e^\Delta}{\pi} \int_0^\infty \mathrm{d}t \, \exp\left(-t^\varphi \cos \frac{\pi\varphi}{2} \right) \cos\left(t\, e^\Delta - t^\varphi \sin \frac{\pi\varphi}{2} \right)$$



longer tail towards large conductivities



For $\nu < 1$ *the variance*

$$D = \langle (\ln \sigma)^2 \rangle - \langle \ln \sigma \rangle^2$$

is given by

$$D(\nu) = \frac{\pi^2}{6}\left(\frac{1}{\varphi^2(\nu)} - 1\right)$$

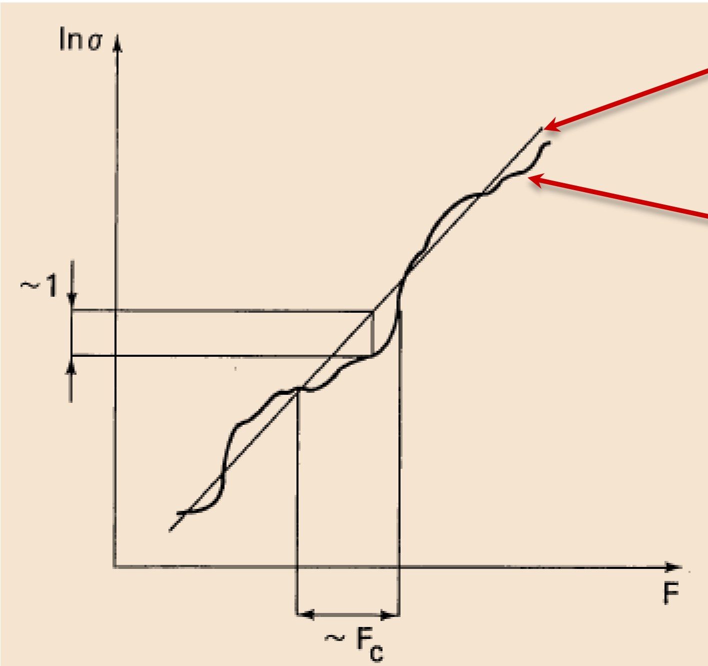
Asymptotic form for small area, $\nu \ll 1$

$$f(\ln \sigma) = \varphi \, \exp(-\varphi\Delta - e^{-\varphi\Delta})$$

$$F(\sigma) = \frac{\varphi}{\sigma}\left(\frac{\sigma_\nu}{\sigma}\right)^{\varphi} \exp\left[-\left(\frac{\sigma_\nu}{\sigma}\right)^{\varphi}\right]$$

Is it possible to measure the distribution function experimentally?

A single sample of a small area $\nu < 1$ *in external field* F



regular component $\delta u = -u_f \delta F / F_0$

with random component $\delta u' \sim u_f \dfrac{\delta F}{F_0[\beta\Omega(u_f)]^{1/2}}$

transmissions of two most transparent punctures within a given sample have different rate of change with F

the difference in log-transmissions exceeds the width of the distribution function at

$$\delta F \sim F_0[\beta\Omega(u_f)]^{1/2}/u_f\varphi(v) = F_c$$

⇒ *"individuality" of the sample is forgotten*

distribution function of mesoscopic fluctuations of conductivity of a given sample yields the distribution functions over samples

PHYSICAL REVIEW B VOLUME 8, NUMBER 2 15 JULY 1973

Hopping Conductivity in One Dimension

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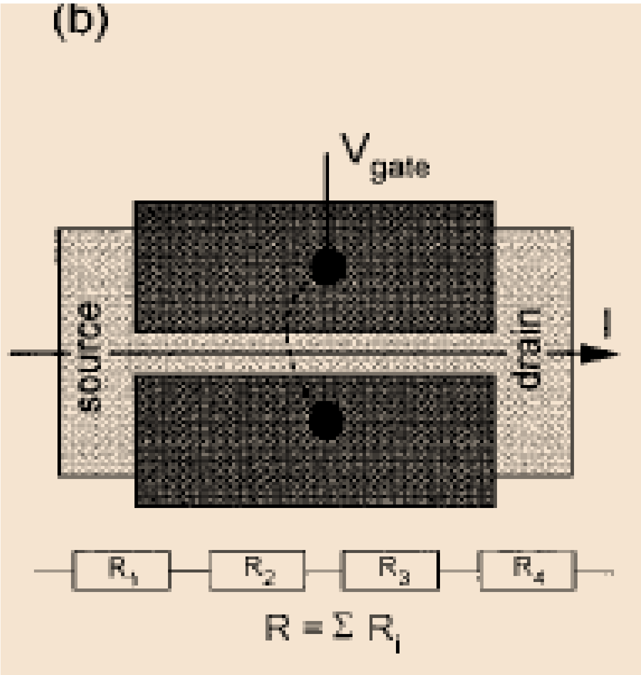
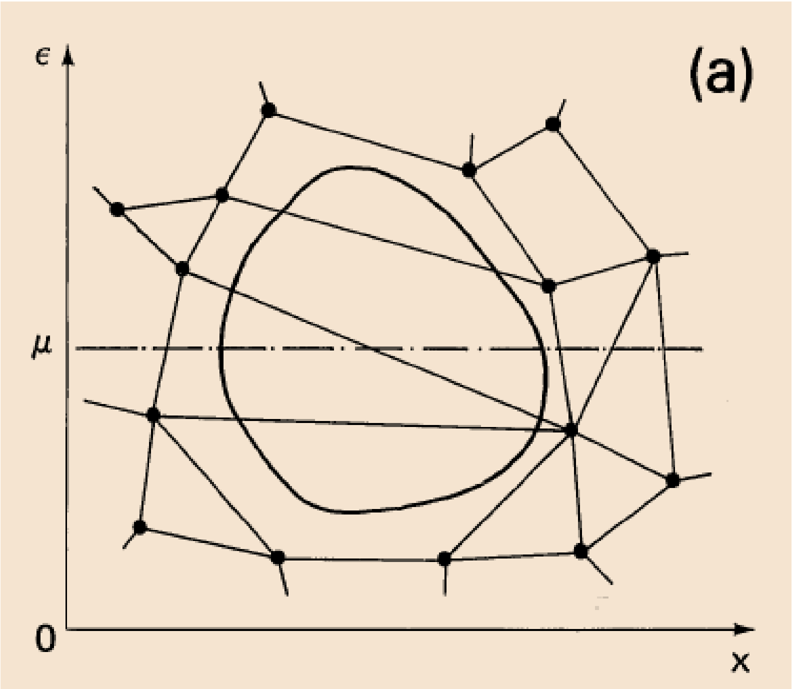
(Received 31 August 1972)

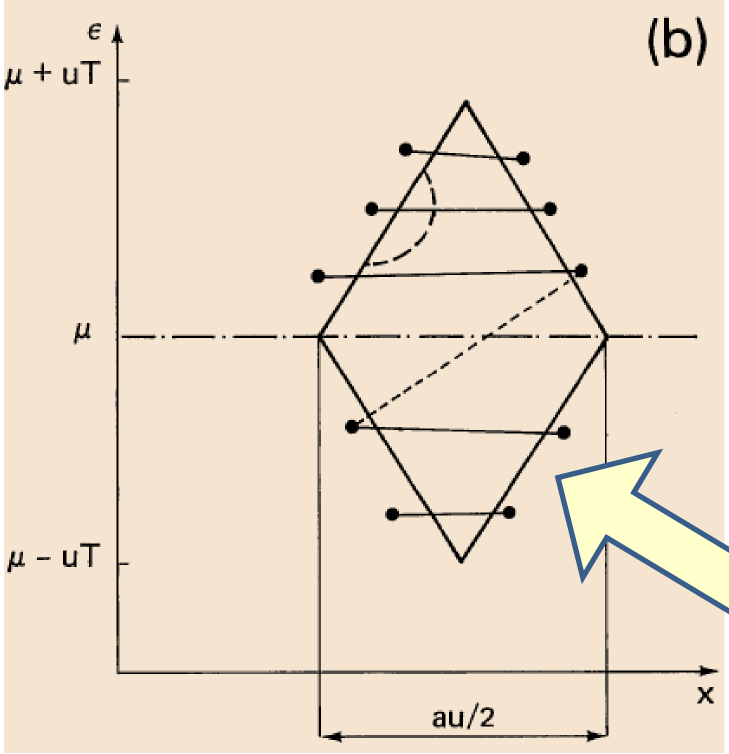
Mott's law in 1D $\ln\sigma = -(T_0/T)^{1/2}$, $T_0 = 1/ga$.



resistance of 1D sample



a break in a one-dimensional chain



For a given resistance of a break $\sigma_0^{-1}\, e^u$

its area is the smallest if it is rhombus-shaped with the diagonals $\dfrac{au}{2}$ *and* $2uT$

the resistances between two sites arranged symmetrically on the opposite sides of the rhombus are the same

Any decrease of the rhombus area results in the decrease of its resistance

the are of the rhombus $A = \frac{1}{2} a T u^2$

Contribution of breaks to the resistance

$$R(u) \propto \exp(u)\, \exp(-gaTu^2/2)$$

the product is maximal for

$$u = (gTa)^{-1} = T_0/T$$

$$\ln \langle R \rangle = T_0/2T$$

much bigger than $\langle \ln R \rangle = \left(\dfrac{T_0}{T} \right)^{1/2}$

Conductance in Restricted-Dimensionality Accumulation Layers

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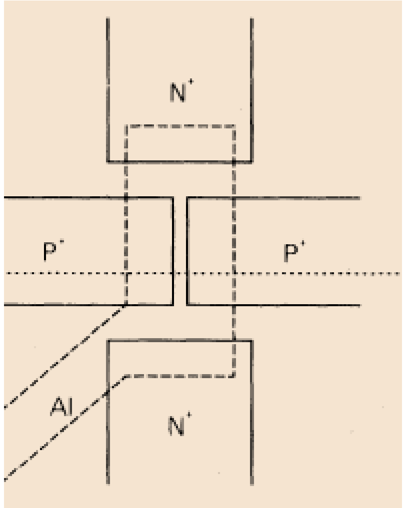
(Received 20 August 1981)



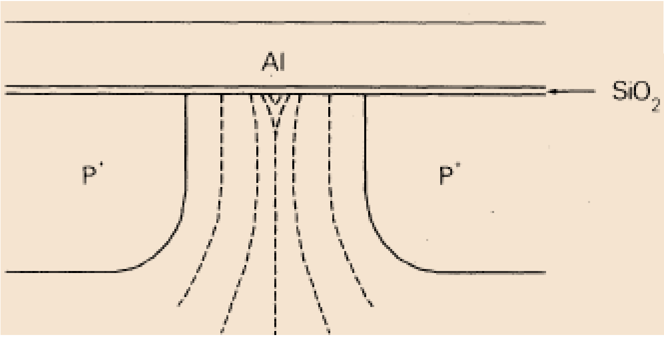


FIG. 1. The upper part shows an idealized plan of a sample. The two n^+ regions are the source and drain. The p^+ regions are the control electrodes. In this case the n-type substrate was 10-Ω-cm Si. The width between the controls was 1–2 μm. The length of the controls is 14 μm. The lower part shows a section through the device along the dotted lines. The diffusions were about 1 μm deep and the oxide was 300 Å thick. Potential lines are sketched for a positive gate voltage.

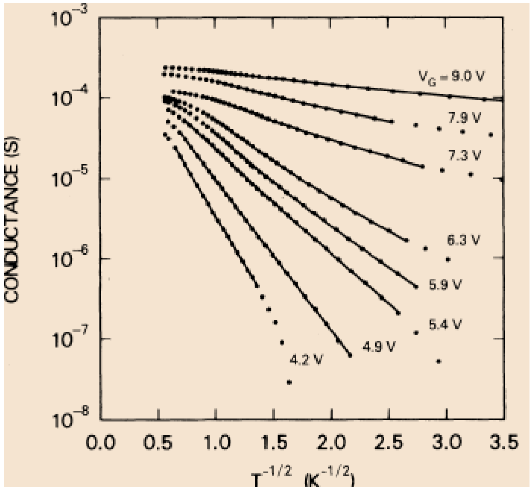


FIG. 3. The log of conductance plotted as a function of $T^{-1/2}$ for various gate voltages (V_g) with the substrate and controls grounded to the source. The solid lines are the best least-squares fits to the data.

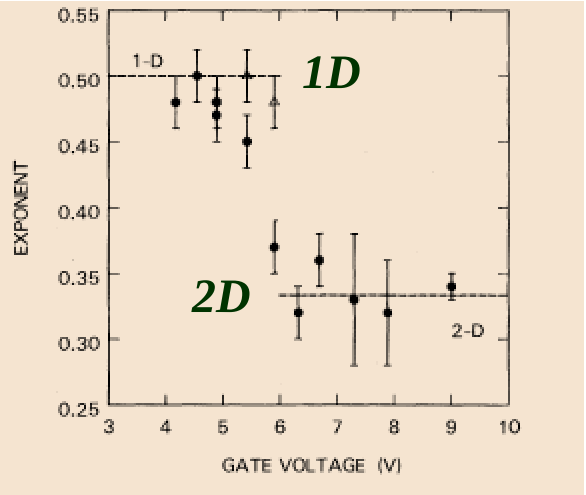


FIG. 4. The power laws that best fit the data in Fig. 3 as a function of gate voltage. The error bars are estimated by deleting points from the ends of the fits. The error from the least-squares fit parameters is better. The open triangles are fits to a restricted low-temperature range only.

R.A. Webb

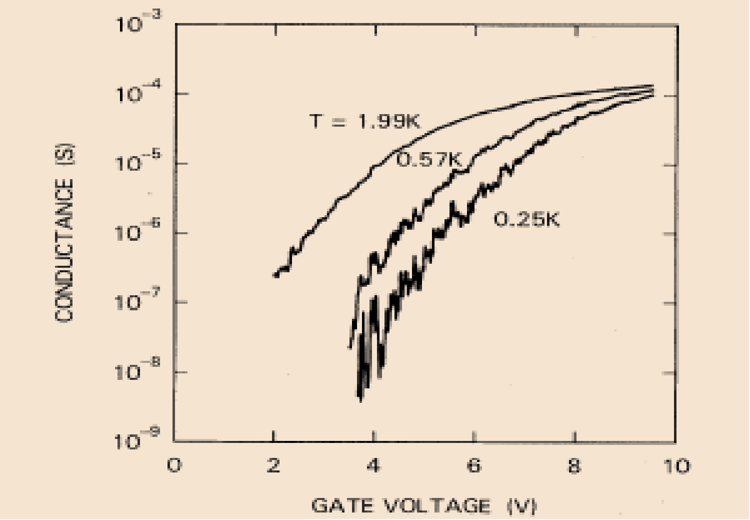


FIG. 2. Conductance as a function of gate voltage for three temperatures.

Upon decreasing temperature:

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Origin of the Peaked Structure in the Conductance of One-Dimensional Silicon Accumulation Layers

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(Received 6 February 1985)

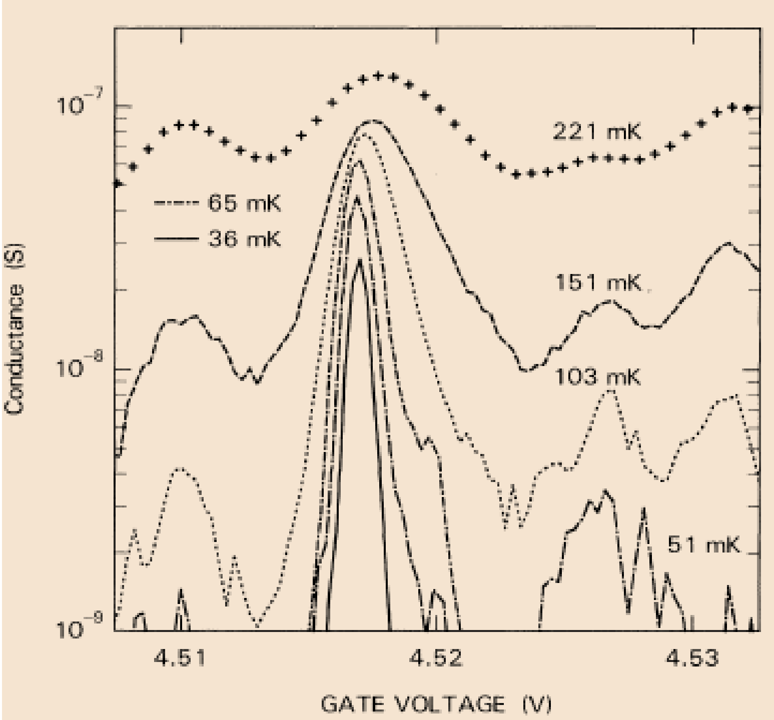

FIG. 1. Conductance as a function of gate voltage on an expanded gate voltage scale for selected temperatures. Only the large peak is displayed for the 65- and 36-mK curves.

No flat-topped peaks characteristic of one-hop–controlled processes were observed.

Nonmonotonic Variations of the Conductance with Electron Density in ~70-nm-Wide Inversion Layers

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(Received 18 August 1983)

The conductance of metal-oxide-silicon field-effect transistors with ~70-nm-wide inversion layers exhibits nonmonotonic variations with electron density below 15 K. The variations are largest at low electron concentrations and are the result of variations of the activation energy E_A. When E_A is largest the current is found to be limited by spatial barriers which contain tunneling channels at discrete energies, as in the model of Azbel.

mean width is ~ 70 nm.

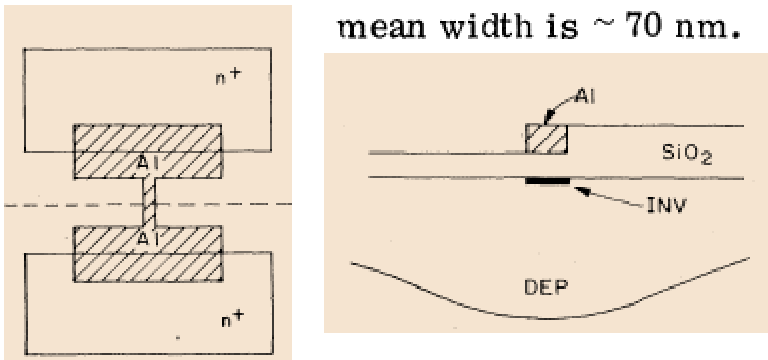

FIG. 1. Left: Schematic top view of the narrow-gate MOSFET. The resistivity of the p-type Si substrate is 3 Ω-cm. Right: Cross section through the device along the dotted line in the left figure. Shown are the narrow inversion layer situated under the narrow gate and the boundary of the depletion region.

The narrow gate is created by first reactive-ion etching a 50-nm step down into the 100-nm-thick gate oxide using photoresist as the mask, and then evaporating Al into the step at a glancing angle to the surface.

As sketched in Fig. 1, wide gates overlap the n^+ regions so that electrical contact to the narrow inversion layer is made through ~1-mm-wide inversion layers.

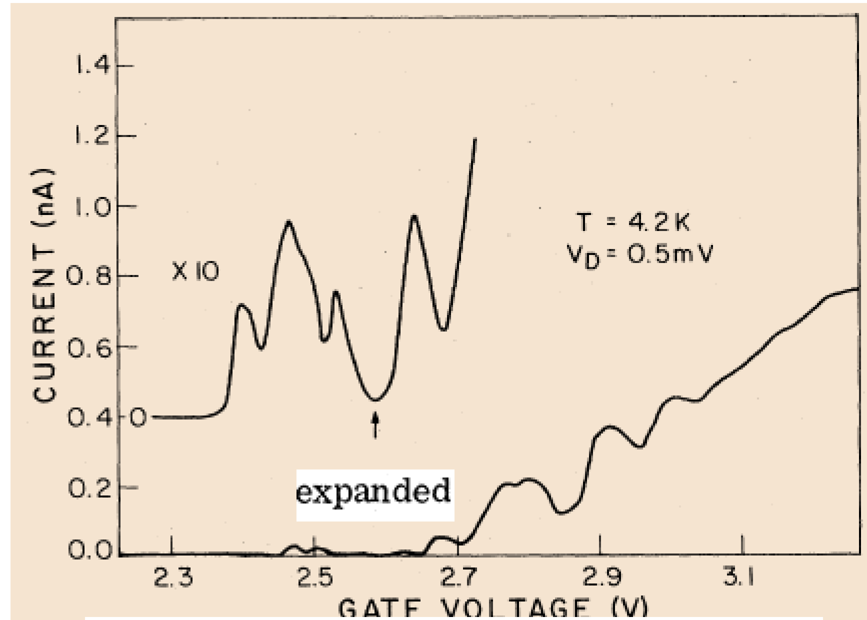

FIG. 2. Current vs gate voltage. The arrow indicates the gate voltage (2.585 V) of the deep minimum explored in this Letter.

In Fig. 2 we show an expanded version of the structure near threshold. When V_G is increased beyond the first few maxima the conductance decreases by as much as three orders of magnitude at 2 K with a gate-voltage change of 0.05 V. Note that the inversion layer contains only 10^3–10^4 electrons because it is so narrow, and the large decrease in the conductance is achieved with an increase of V_G corresponding to the addition of only ~200 electrons to the inversion layer.

VOLUME 53, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 1984

Variable-Range Hopping in Finite One-Dimensional Wires

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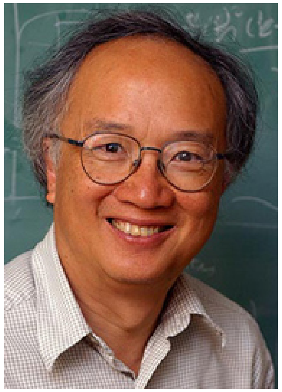


$\alpha = 0.02$

$-0.5 < E_i < 0.5$

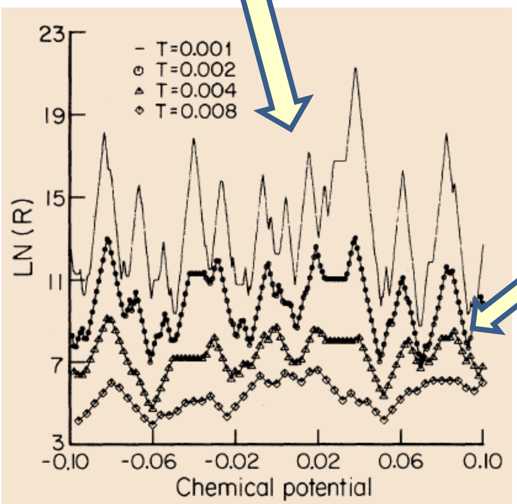
$$\ln(R_{ij}/\gamma) = 2\alpha|x_i - x_j| + (1/2kT)(|E_i - \mu| + |E_j - \mu| + |E_i - E_j|)$$

dependence of the resistance between two sites on the gate voltage

percolation simulation in 1D

at low temperatures plateaus are absent

current flows when resistors with $R_{ij} < R_c$ are switched on



"period" grows with T

FIG. 1. $\ln(R/\gamma)$ vs chemical potential μ for several temperatures. The chain length $N = 1000$ corresponds to 20 localization lengths.

$$N\alpha = 20$$

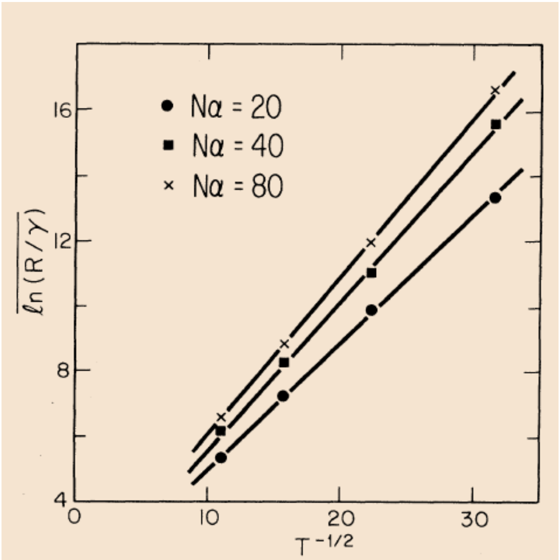


FIG. 2. $\langle \ln(R/\gamma) \rangle$ vs $T^{-1/2}$ for different chain lengths.

over Fermi level positions

in experiment

the sample length $\sim 10\,\mu m$

exceeds $\sim 20-70$ times

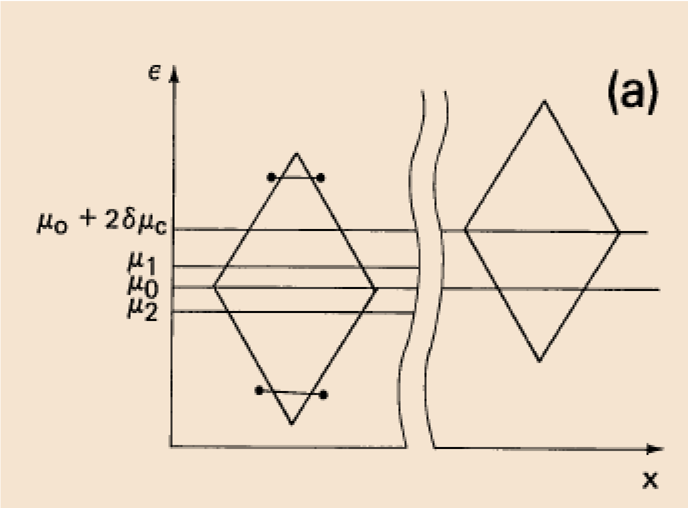
the localization length

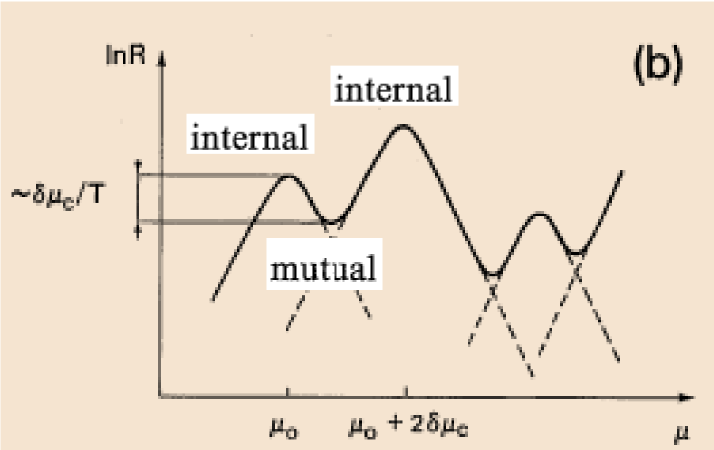
Average log-resistance obeys a 1D

Mott's law $\langle \ln R \rangle = \left(\dfrac{T_0}{T}\right)^{1/2}$

T_0 ***increases with wire length***

Qualitative picture of mesoscopic fluctuations: switching of most resistive breaks





rhombus resistance will fall off as

$\exp(-|\mu-\mu_0|/T)$

Fig. 15. (a) Schematic of internal and mutual break switching in a one-dimensional chain with variation of the Fermi level; (b) Fluctuations of the chain $\ln \sigma$ with variation of the Fermi level. The dashed lines specify the variation of the resistances of individual breaks.

absence of plateaus: sites on opposite sides of the Fermi level never determine the resistance

Quantitative description

$$\rho(u) = gTu\, \mathrm{e}^{-\Omega(u)}$$

$$\Omega(u) = \tfrac{1}{2} gTau^2 = Tu^2/2T_0$$

$$v = \frac{\ln(Lv^{1/2}/a)}{\Omega(u_{\mathrm{opt}})} = \frac{2T}{T_0} \ln\left(\frac{Lv^{1/2}}{a}\right)$$

$$v < 1$$

$$L\rho(u_{\mathrm{f}}) \sim 1$$

Mott's law

$$\ln\left(\frac{R(v)}{\mathscr{R}_0}\right) = u_{\mathrm{f}}(v) = \frac{v^{1/2}\, T_0}{T} = \left\{2\,\frac{T_0}{T} \ln\left[\frac{L}{a}\left(\frac{T}{T_0}\ln\frac{L}{a}\right)^{1/2}\right]\right\}^{1/2}$$

typical log-resistance

function $\varphi(\nu)$ *for 1D hopping*

Width of the distribution function:

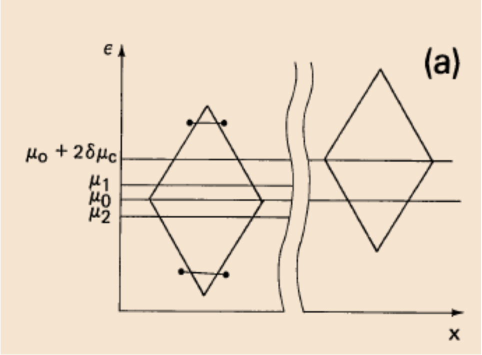
$$\varphi(\nu) = \nu^{1/2} = \Omega'(u_f(\nu))$$

$$\Delta \sim \frac{1}{\nu^{1/2}} = \left(\frac{T_0}{2T}\right)^{1/2} \ln^{-1/2}\left[\frac{L}{a}\left(\frac{T}{T_0}\ln\frac{L}{a}\right)^{1/2}\right]$$

$$\frac{\Delta}{\langle \ln R\rangle} = \frac{T}{T_0\nu} = \frac{1}{2\ln\left[\frac{L}{a}\left(\frac{T}{T_0}\right)^{1/2}\right]}$$

strong fluctuations

"Period "of mesoscopic fluctuations



For a general μ *the resistance is dominated by two breaks shifted in in energy by* $2\delta\mu_c$ *their log-resistances differ by* $\sim \delta\mu_c/T$

On the other hand, this difference should be $\sim \Delta$

$$\delta\mu_c \sim (T_0 T)^{1/2} \Big/ \ln^{1/2}\left[\frac{L}{a}\left(\frac{T}{T_0}\right)^{1/2} \ln^{1/2}\left(\frac{L}{a}\right)\right] \sim \frac{T}{\nu^{1/2}}$$

PHYSICAL REVIEW B VOLUME 33, NUMBER 12 15 JUNE 1986

New aspects of variable-range hopping in finite one-dimensional wires

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$$\Delta^2 = \langle (\ln\rho - \langle \ln\rho \rangle)^2 \rangle$$

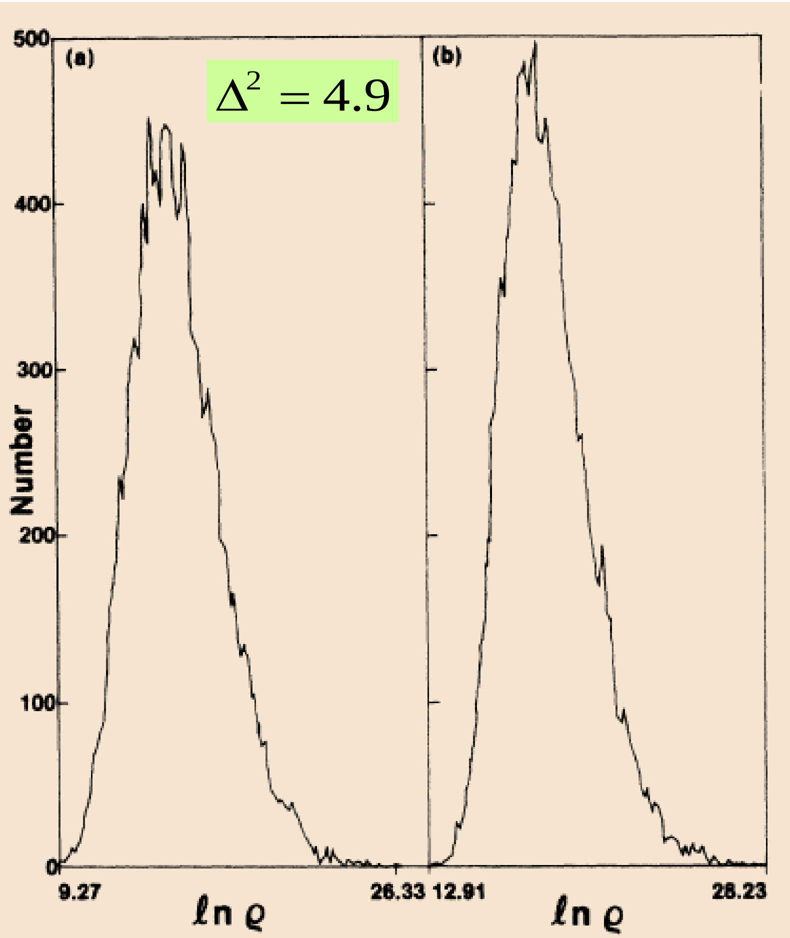
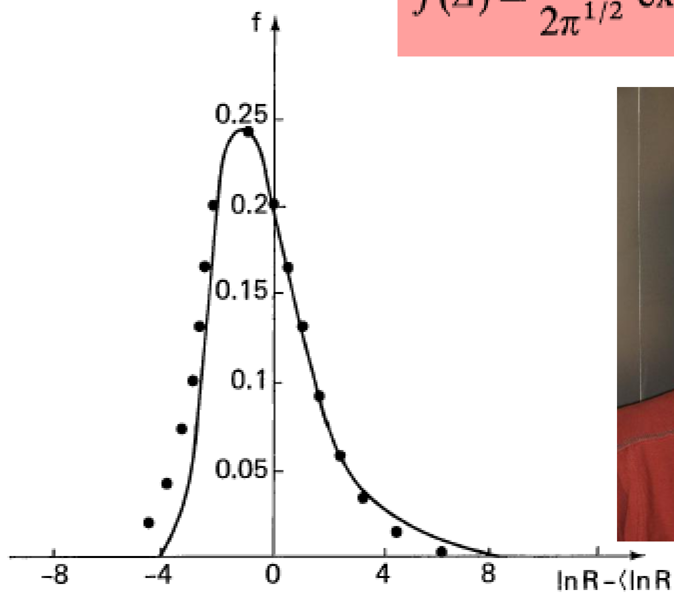


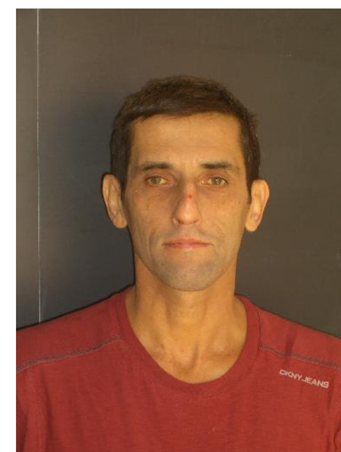
FIG. 1. Distributions of $\ln\rho$ for chains of $N = 2000$ sites (3: ensembles) and $N = 9000$ sites (36 ensembles), respectively. Note the high-resistance tails, and that the distribution of the longer chain is narrower.

It is evident that the distributions are asymmetric with longer high-resistance tails, which is a characteristic of all the systems. These distributions also become narrower with the increase in length of the systems.

$$D(\nu) = \frac{\pi^2}{6}\left(\frac{1}{\varphi^2(\nu)} - 1\right) \Rightarrow \varphi = 0.5$$

$$f(\Delta) = \frac{1}{2\pi^{1/2}} \exp\left(-\frac{\Delta}{2} - \frac{1}{4} e^{-\Delta}\right)$$





Igor Ruzin

Wide samples

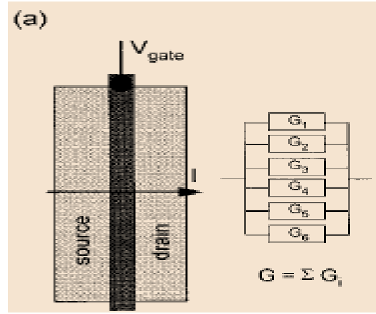
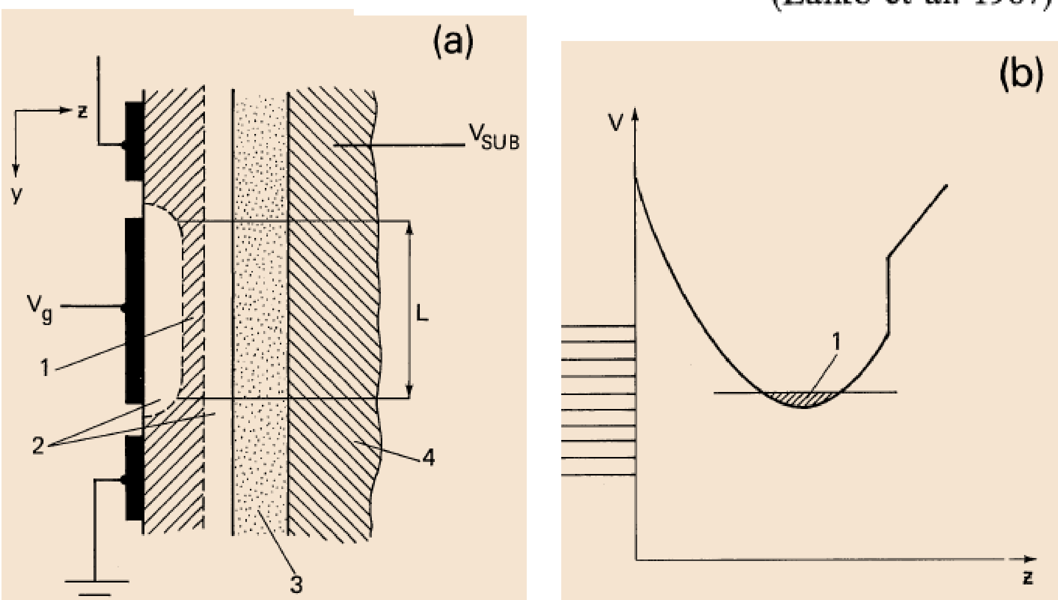


Fig. 21. Mesoscopic fluctuations in the conductance of MOSFET with variation of gate voltage (Laiko et al. 1987) for different temperatures.



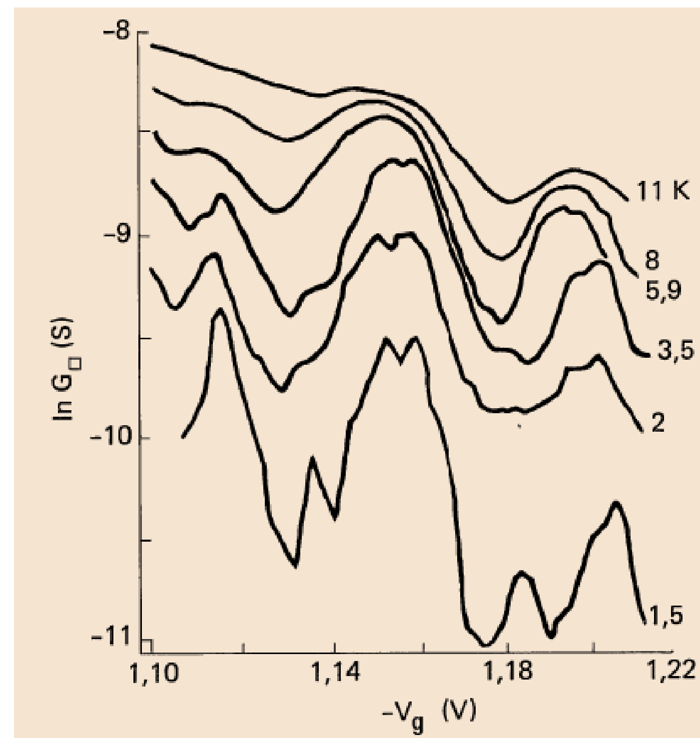
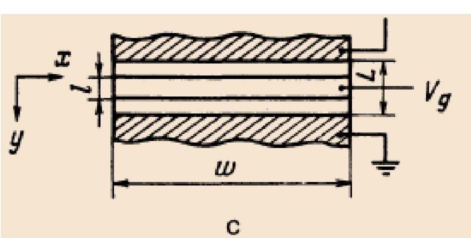


Fig. 20. (a) Cross-section of MOSFET (Orlov et al. 1986, Laiko et al. 1987): 1, conducting channel in GaAs; 2, depletion region; 3, semi-insulating GaAlAs layer; 4, conducting substrate; (b) Band diagram of the structure.



$w = 200\ \mu m$ $\quad l \approx 1.5\text{–}2\ \mu m$

Statistical properties of mesoscopic conductivity fluctuations in a short-channel GaAs field-effect transistor

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(Submitted 15 June 1989)

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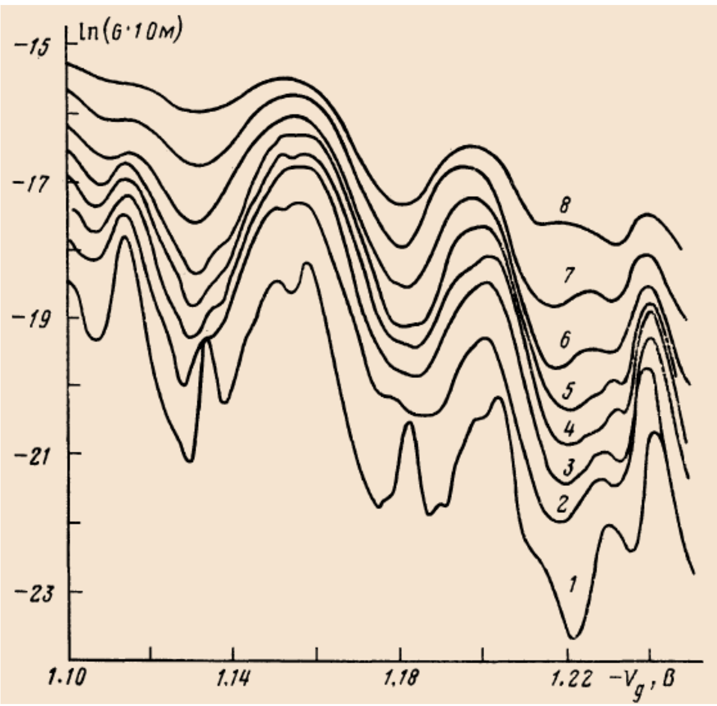

FIG. 1. Conductivity fluctuations in the channel of a GaAs FET at various temperatures T:1.5; 2.0; 2.5; 3.0; 3.5; 4.2; 6.0; 8.0 K (curves 1–8).

Alexander Savchenko

1951 - 2010

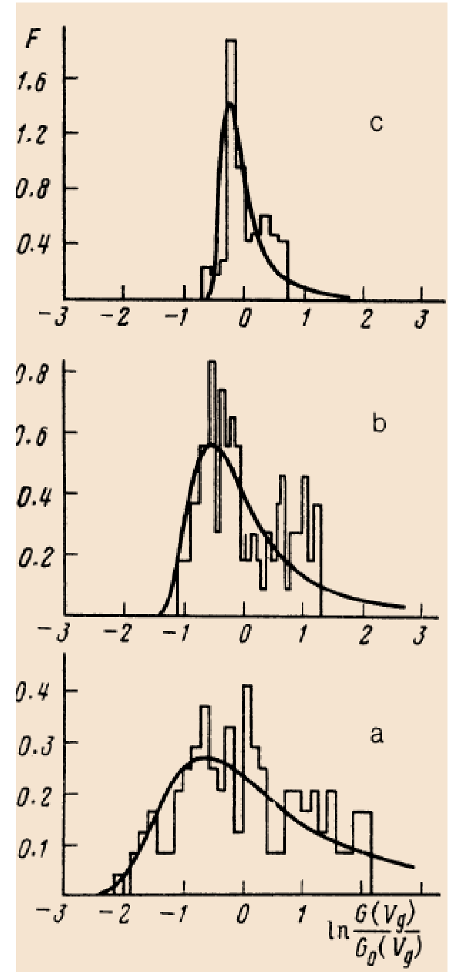

FIG. 3. Histogram of the distribution of the log conductivity of the experimental dependences for the three temperatures 1.5 (a), 4.2 (b) and 8 K (c) (curves 1,6 and 8 of Fig. 1) after subtracting the monotonic part from them. The smooth curves show the corresponding approximating functions.

for $|1-\varphi| \ll (\Omega''(u_{\text{opt}}))^{1/2}$

$$f(Q)=\frac{1}{\pi w}\int_0^\infty \exp\left(-\frac{\pi x}{2}\right)\cos\left(x\frac{\Delta_1}{w}-x\ln x\right)dx$$

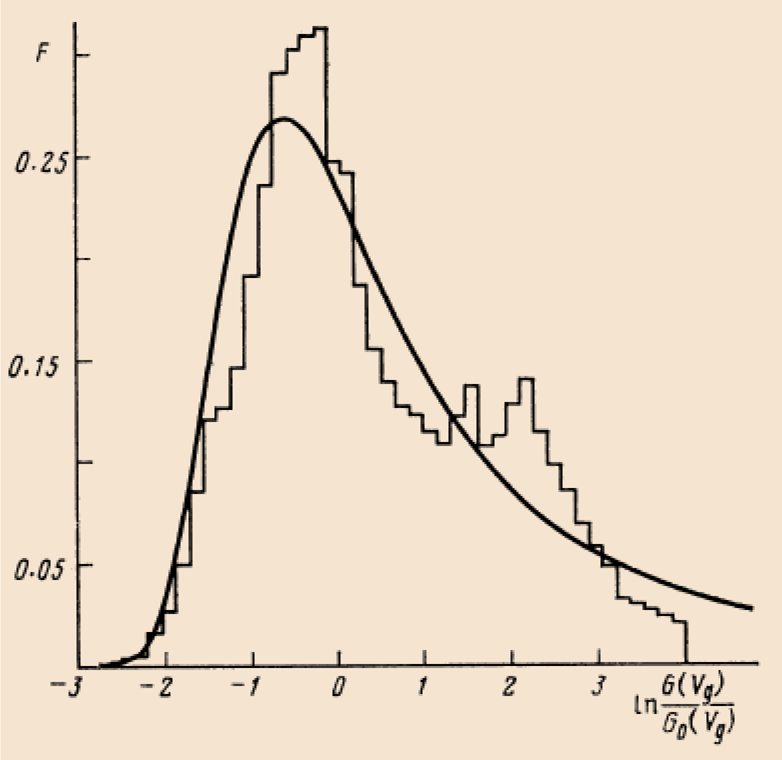

FIG. 4. Histogram resulting from averaging over the data reduced to $T = 1.5$ K for eight temperatures, and the corresponding theoretical distribution function of the log conductivity.

the experimental histograms
for all eight temperatures are approximated by the same function (11), in which the parameter $w(T)$ is determined each time by minimizing the mean-square deviation of the theoretical curve from the experimental histogram.

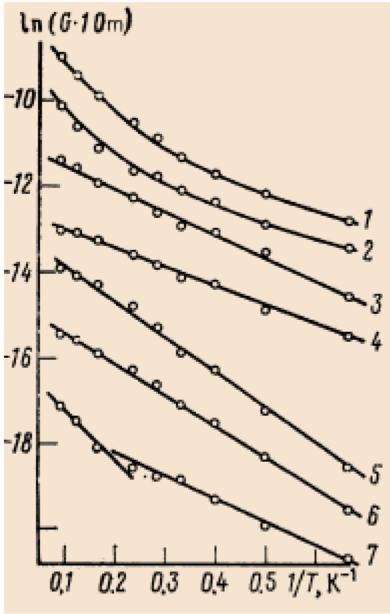

FIG. 5. Temperature dependence of the channel conductivity for various values of $|V_g|$: 1—1.105; 2—1.117; 3—1.148; 4—1.157; 5—1.195; 6—1.200; and 7—1.241 V (curves 1–6 are arbitrarily shifted along the ordinate axis).

as the temperature is increased a transition should occur from the regime of chainlike conduction to a regime where the sample conductivity is determined by a rather small number of regions having the form of branched clusters.

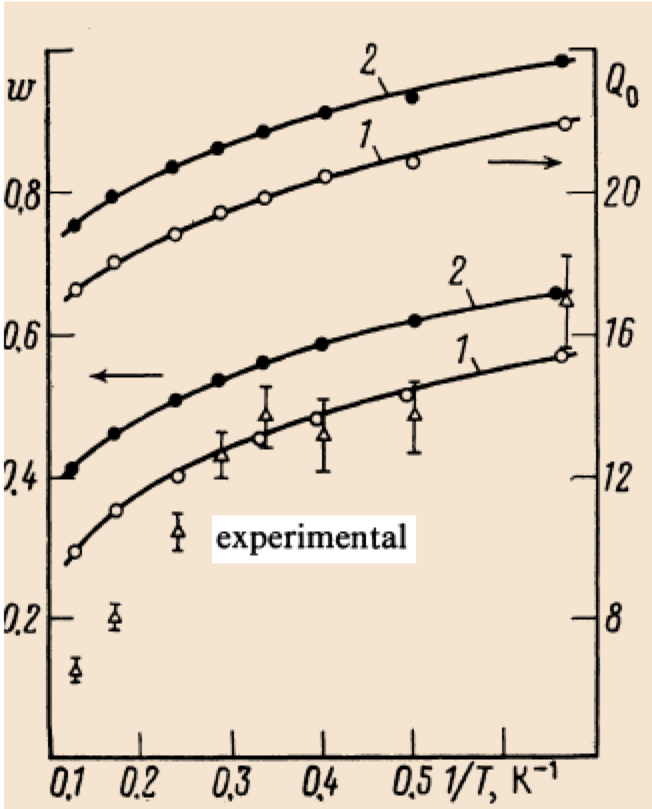

FIG. 6. Temperature dependence of the experimental value of the width w of the distribution function and the calculated parameters w and Q_0 for two values of the frequency τ_{ph}^{-1}: 10^{11} sec^{-1} (curve 1) and 10^{12} sec^{-1} (curve 2).

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Distribution-function analysis of mesoscopic hopping conductance fluctuations

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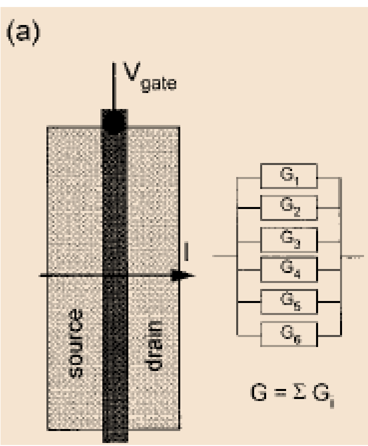
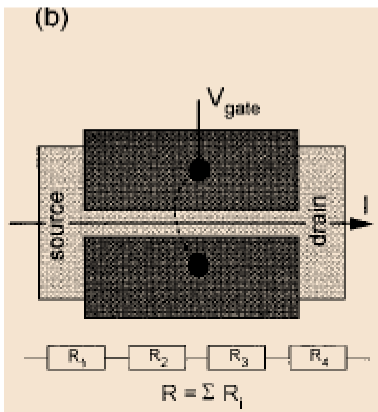
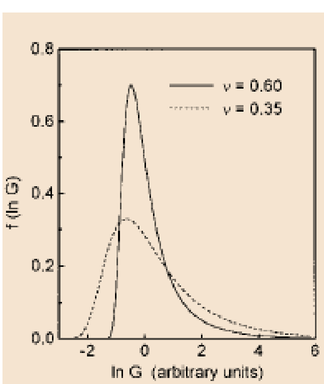
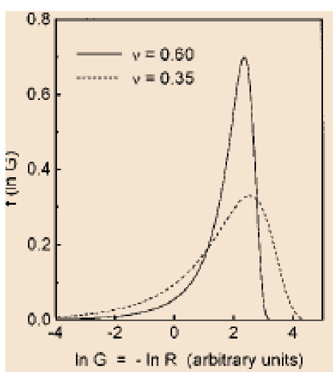




$$\langle Q^2\rangle - \langle Q\rangle^2 = \frac{\pi^2}{6}\left(\frac{1}{\nu} - 1\right)$$

$$s \equiv \langle (\ln R - \langle \ln R\rangle)^2\rangle^{1/2} \sim \left(\frac{T_0}{T}\right)^{1/2}\left[\ln\left(\frac{2L}{\xi}\right)\right]^{-1/2}$$





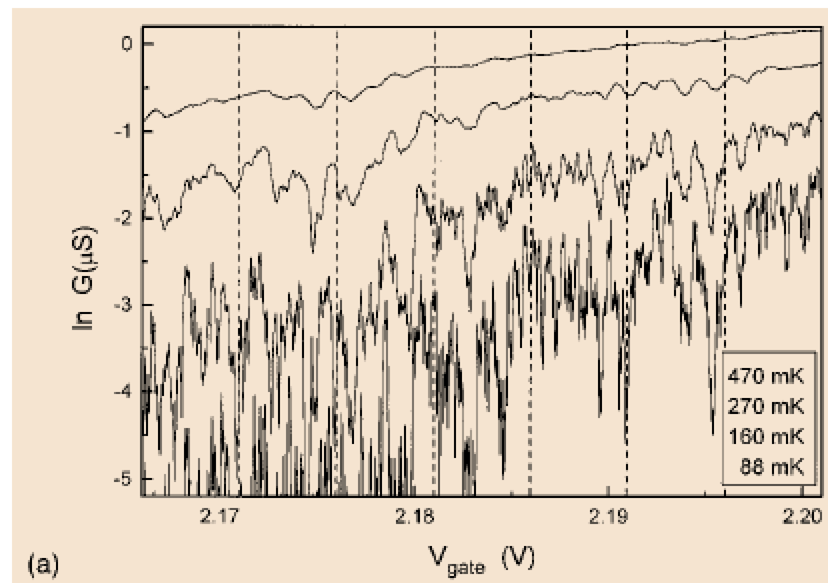


FIG. 2. (a) Part of the experimental characteristic on a logarithmic conductance scale from a 19.4×0.6 μm 1D Si MOSFET. Dotted lines demarcate gate-voltage intervals used for averaging.

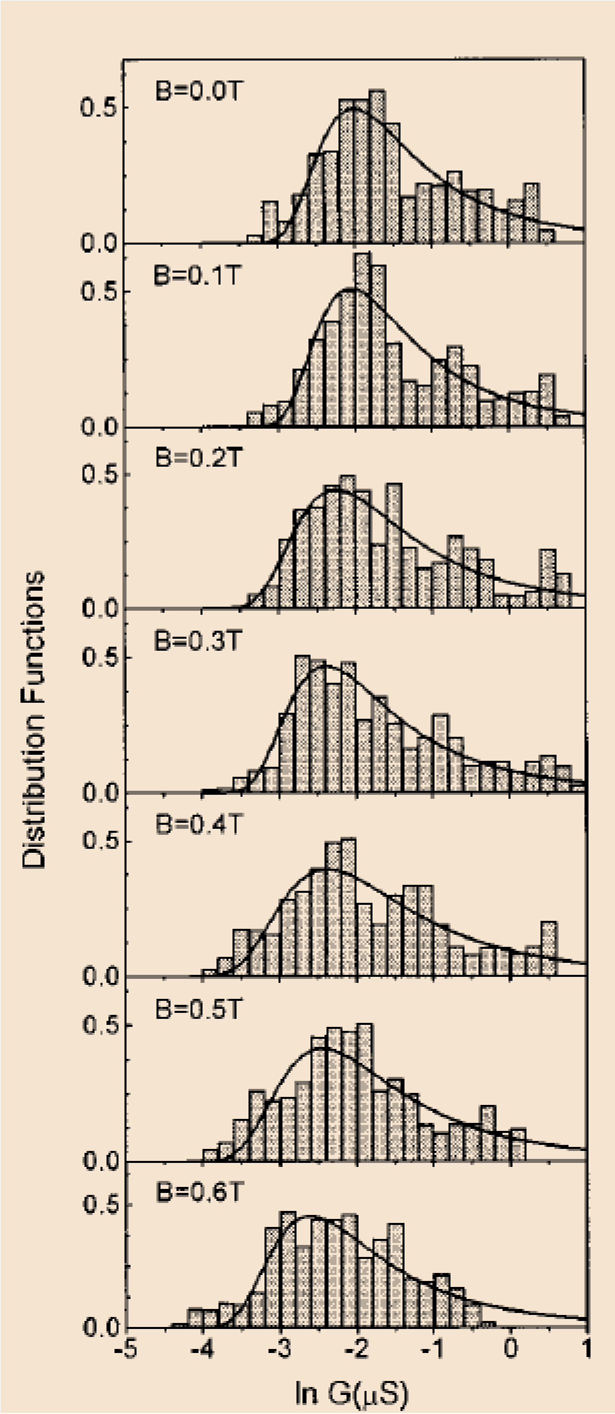


FIG. 8. (a) DF's obtained from a 2×100 μm Si MOSFET as a function of magnetic field and their fits to the 2D theory. (b) The average magnetoconductance obtained by the direct and fitting methods is negative. (c) The standard deviation of the distribution obtained by the two methods increases slightly.

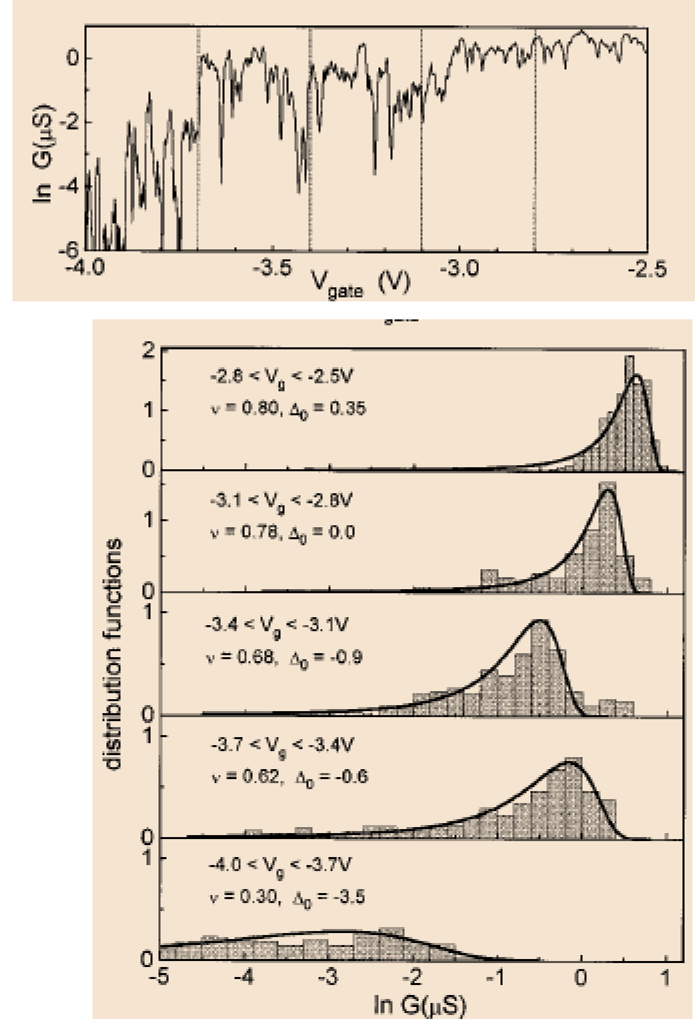


FIG. 3. Conductance fluctuations from a 1.8×0.2 μm 1D GaAs device and experimental DF's obtained from five adjacent gate-voltage intervals spanning the characteristic. At low gate voltages, the distribution is no longer exponentially wide while at high gate voltages the conductance becomes too small to measure. In between is a region where good fits to the theoretical 1D DF (solid curves) can be obtained.

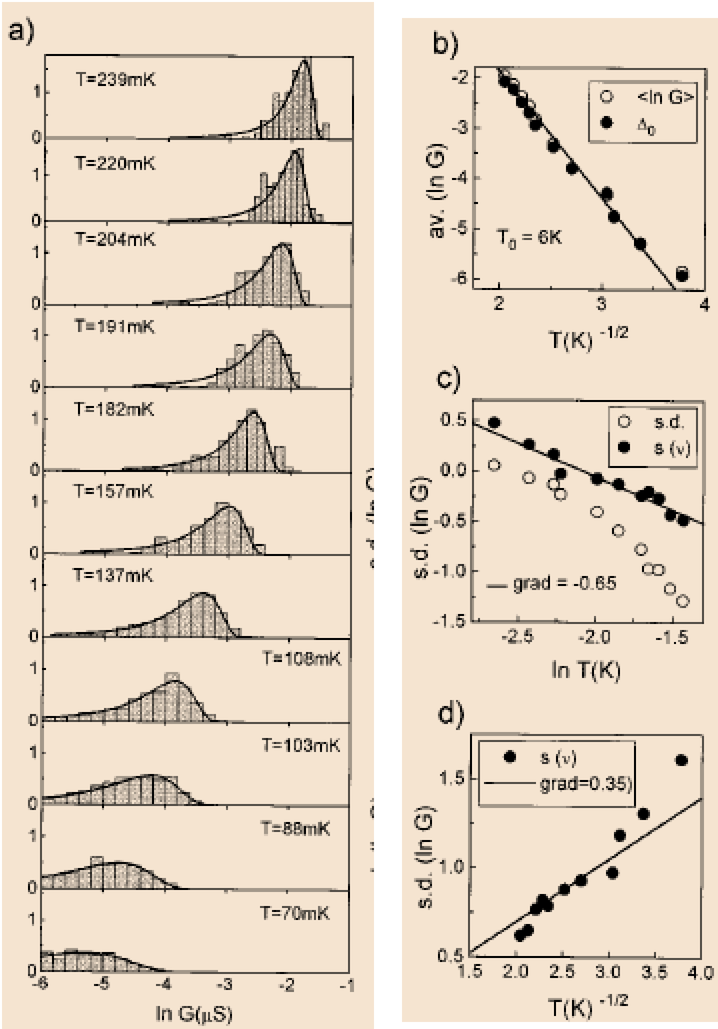
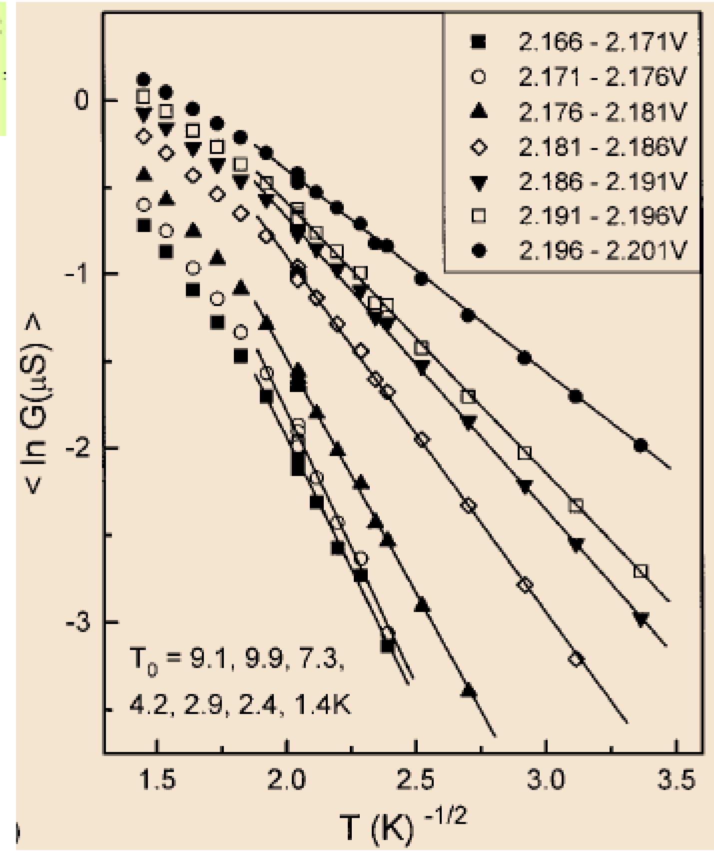


FIG. 5. Temperature dependence of the fluctuations from a 19.4×0.6 μm Si MOSFET. (a) Experimental DF's fit by the 1D theoretical form. (b) Fit to Eq. (1) of the average of $\ln G$ obtained both directly (hollow circles) and from the position Δ_0 of the fitted DF's (filled circles). (c) Temperature dependence of the fluctuation amplitude s both measured by the standard deviation of the data points (hollow circles) and calculated from the fits to the 1D DF's (filled circles). The gradient of the latter yields a $T^{0.65}$ power law. (d) Fitting the fluctuation amplitude to a $T^{1/2}$ power law yields the prefactor 0.35.

$$\langle \ln R\rangle \sim \left(\frac{T_0}{T}\right)^{1/2}\left[\ln\left(\frac{2L}{\xi}\right)\right]^{1/2}$$



Fits of these data to Eq. (1) averaged over the marked gate-voltage intervals together with the values of T_0 extracted below 0.3 K.

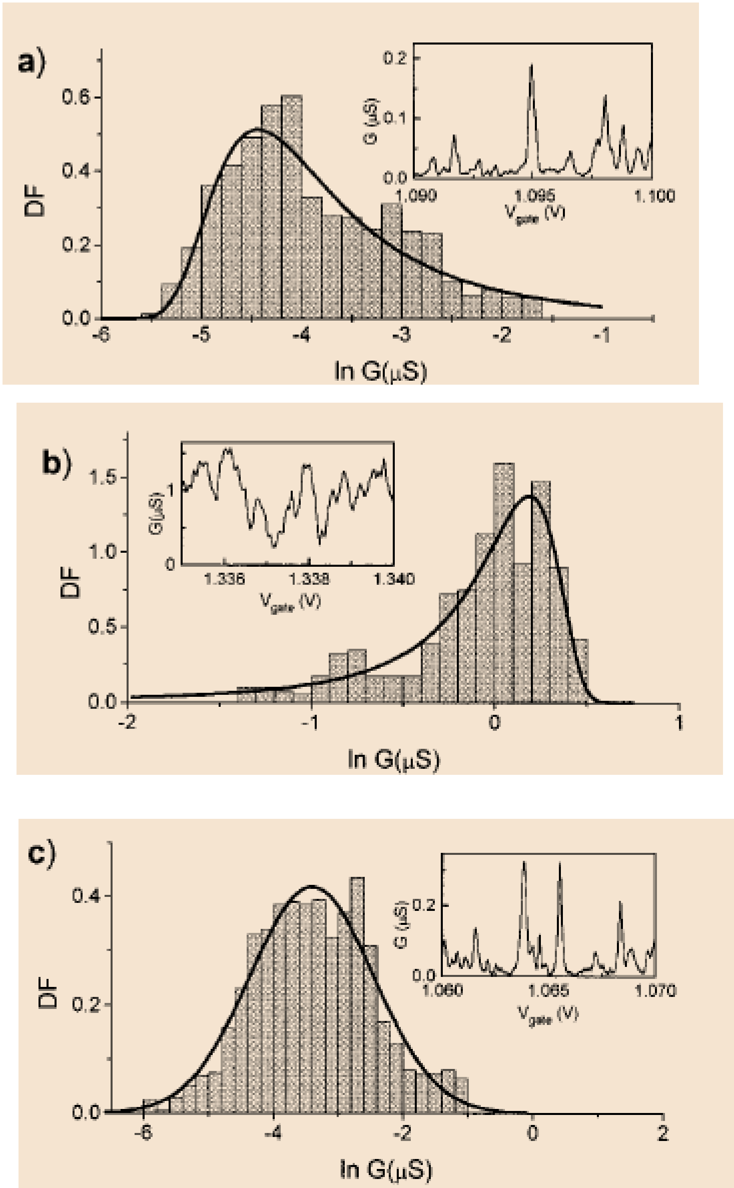


FIG. 4. Distribution functions obtained from three Si MOS-FET's fabricated on the same chip showing the characteristic 1D and 2D asymmetries. Lithographic channel dimensions are (length $\times$width): (a) 2×100 μm fit by 2D DF, (b) 5×2 μm fit by 1D DF, and (c) 1.5×100 μm fit by a Gaussian. Inset are the regions of the characteristic from which the histograms were obtained.

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Conductance fluctuations in large metal-oxide-semiconductor structures in the variable-range hopping regime

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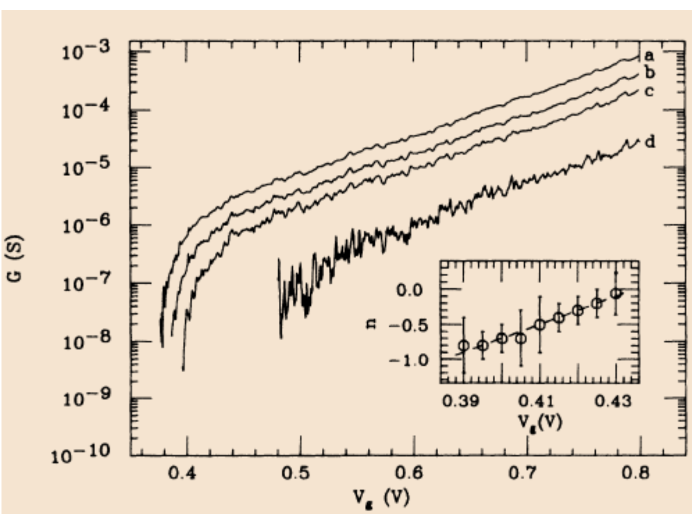
Conductance fluctuations due to variable-range hopping have been studied in 8-mm-wide silicon inversion layers of large area (3.2 mm^2). The temperature dependence of the average logarithm of conductance $\langle \ln G \rangle$ varies with the carrier density N_s from nearly activated to very weak. Fluctuations of $\ln G$ with the chemical potential μ occur on two different scales. The distribution function of the fluctuations in $\ln G$ is also analyzed, and the results are consistent with the model of conduction via exponentially rare, highly conducting, quasi-one-dimensional chains of hops.



on much larger samples with length $L = 0.4$ mm and width $W = 8$ mm. Several features of our data are consistent with the model of conduction via quasi-1D chains (punctures).

FIG. 1. Conductance vs gate voltage at (a) $T = 0.555$ K, (b) $T = 0.420$ K, (c) $T = 0.330$ K, (d) $T = 0.090$ K. Inset: The "best" exponent n for different gate voltages. The dashed line is a guide to the eye.

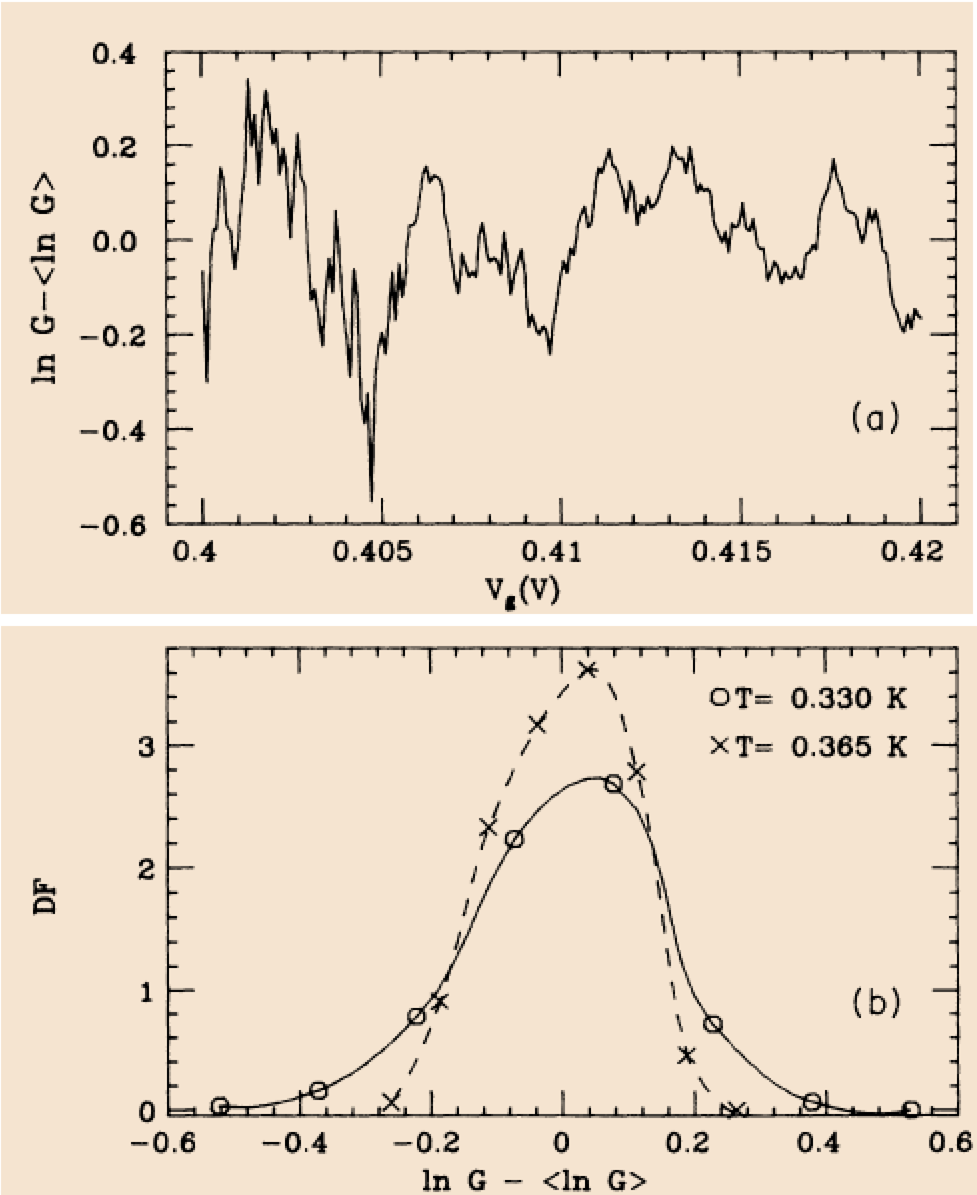


FIG. 2. (a) Fluctuations in the conductance logarithm with the gate voltage at $T = 330$ mK. (b) Histograms of the distribution of the conductance logarithm for $0.40\ \mathrm{V} \leq V_g \leq 0.42\ \mathrm{V}$ at two temperatures.