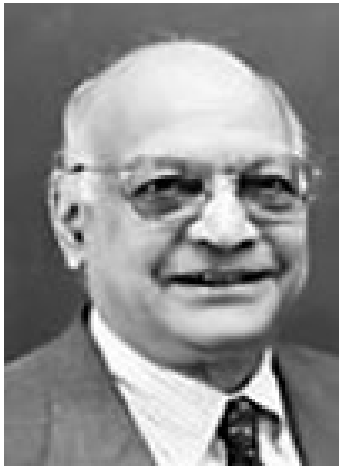
*N.F. Mott*

*V. Ambegaokar*

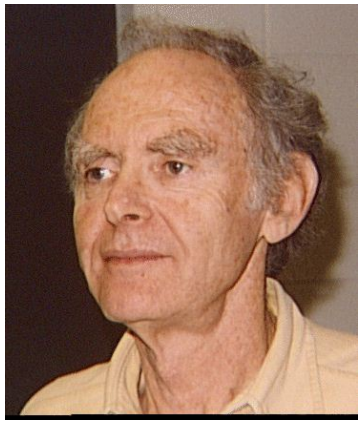


*B.I. Halperin*

*J. Langer*

*M. Pollak*

*E. Abrahams*



*B. Spivak*

B.I. Shklovskii A.L. Efros

# Electronic Properties of Doped Semiconductors

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

# Lecture 1

## Topics:

1. *Impurity states in semiconductors*
2. *Basic experimental facts*
3. *Impurity band structure*
4. *Electron transition between two localized states*
5. *Mott's law*
6. *Miller-Abrahams random resistor network*
7. *Percolation treatment*
8. *Spin relaxation in the hopping regime*

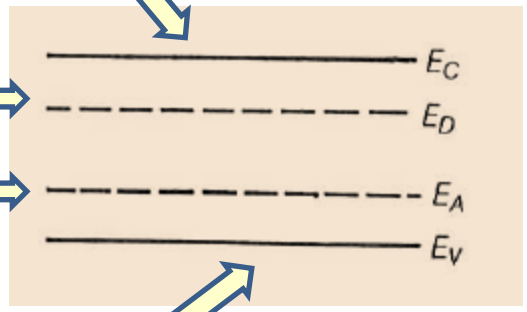
# Impurity levels inside a forbidden gap

bottom of conduction band

shallow donor

shallow acceptor

top of the valence band




energy spectrum of conduction band

$$\left[E_j(-i\nabla) - \frac{e^2}{\kappa r}\right] F_j(\mathbf{r}) = E\,F_j(\mathbf{r})$$

binding energy

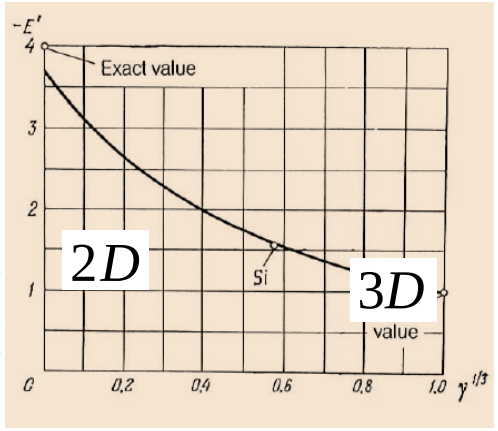
envelope of the donor wave function

binding energy in the units of $E_t = \hbar^2/2m_t a_t^2$

$a_t = \hbar^2\kappa/e^2 m_t$

for anisotropic spectrum:

$$E(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m_t} + \frac{p_z^2}{2m_\ell}$$




asymptotic behavior of the envelope at large distances

$$F(\vec{r}) \propto \int d\vec{p}\, \frac{\exp\left[\frac{i}{\hbar}\vec{p}\cdot\vec{r}\right]}{|E| + E(\vec{p})}$$

$\gamma = m_t/m_\ell$

$$F(\vec{r}) \propto \exp\left[-\left(\frac{2|E|}{\hbar}\left(m_l z^2 + m_t x^2 + m_t y^2\right)\right)^{1/2}\right]$$

| Material | $E_{1s}$, [meV] | | $E_{2p_0}$, [meV] |
|---|---|---|---|
| Si (theor) | 31.27 | | 11.51 |
| Si (P) | 45.5 | 33.9 | 32.6 | 11.45 |
| Si (As) | 53.7 | 32.6 | 31.2 | 11.49 |
| Si (Sb) | 42.7 | 32.9 | 30.6 | 11.52 |
| Ge (theor) | 9.81 | | 4.74 |
| Ge (P) | 12.9 | 9.9 | 4.75 |
| Ge (As) | 14.17 | 10.0 | 4.75 |
| Ge (Sb) | 10.32 | 10.0 | 4.74 |

**Table 1.2.** Donor ionization energies in Ge and Si. Theoretical values were calculated in the effective-mass approximation. Experimental values correspond to the impurities indicated in parentheses. Energies of all *s*-state levels, split due to the chemical shift, are listed separately

PHYSICAL REVIEW VOLUME 99, NUMBER 2 JULY 15, 1955

# Electrical Properties of Germanium Semiconductors at Low Temperatures*

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(Received April 11, 1955)

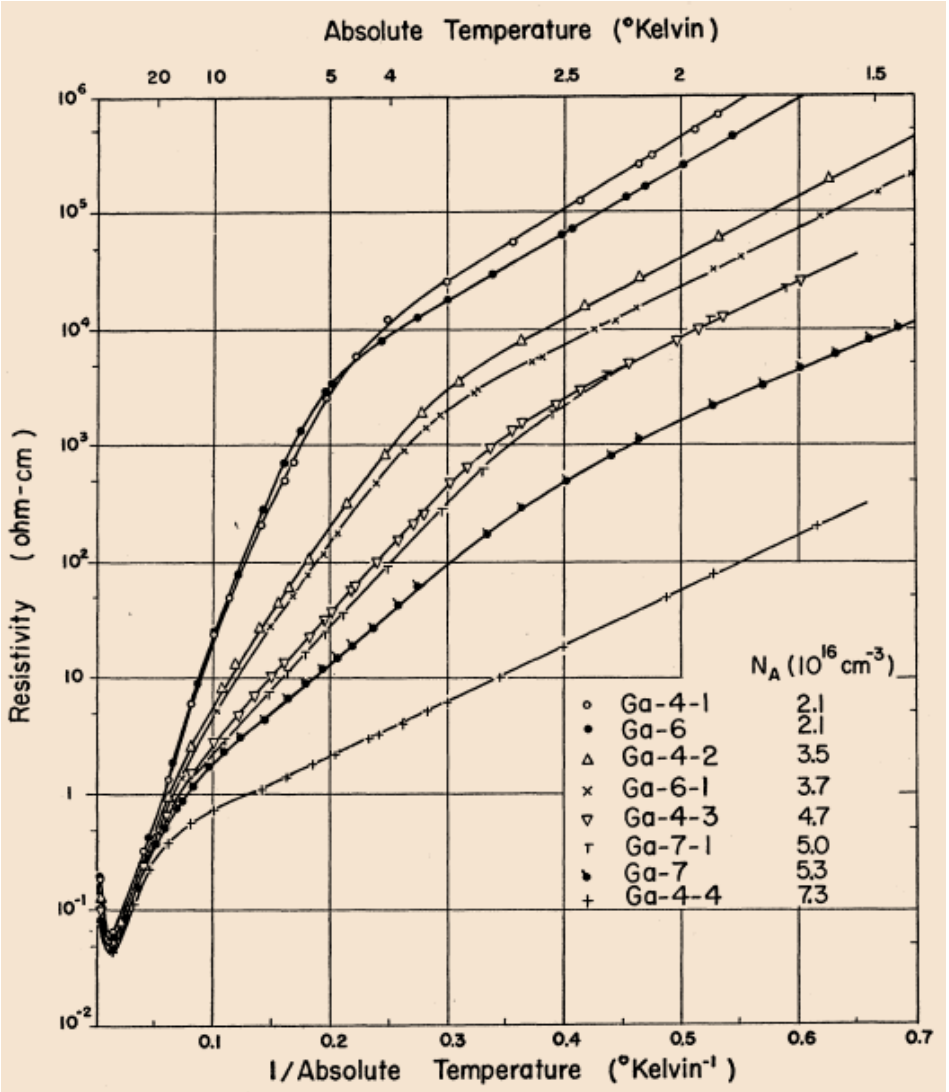


FIG. 8. Resistivity measurements of $p$-type germanium with large concentrations of gallium.

The resistivity can be described approximately by the equation[10]

$$1/\rho = C_1 e^{-\epsilon_1/kT} + C_3 e^{-\epsilon_3/kT},$$

with $\epsilon_1 \approx 1\times10^{-2}$ ev and $\epsilon_3 \approx 2\times10^{-3}$ ev.

$\frac{\varepsilon_1}{T} = 10$ ⇐ $T = 10K$ ⇒ $\frac{\varepsilon_3}{T} = 2$

$\exp\left[-\frac{\varepsilon_1}{T}\right]$ ***dominates***

$\frac{\varepsilon_1}{T} = 50$ ⇐ $T = 2K$ ⇒ $\frac{\varepsilon_3}{T} = 10$

$\exp\left[-\frac{\varepsilon_3}{T}\right]$ ***dominates***

The explanation of these observations due to Hung, who assumed that the localized impurity states interact and form a conduction band, seems plausible for the following reasons. The width of the impurity band should drastically decrease with increasing separation between the impurity atoms. Therefore one would expect the mobility of the charge carriers in the impurity band to decrease rapidly with decreasing concentration of impurities. This effect is demonstrated by the

# Anisotropic Electrical Properties of Amorphous Germanium

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(Received 21 June 1972)

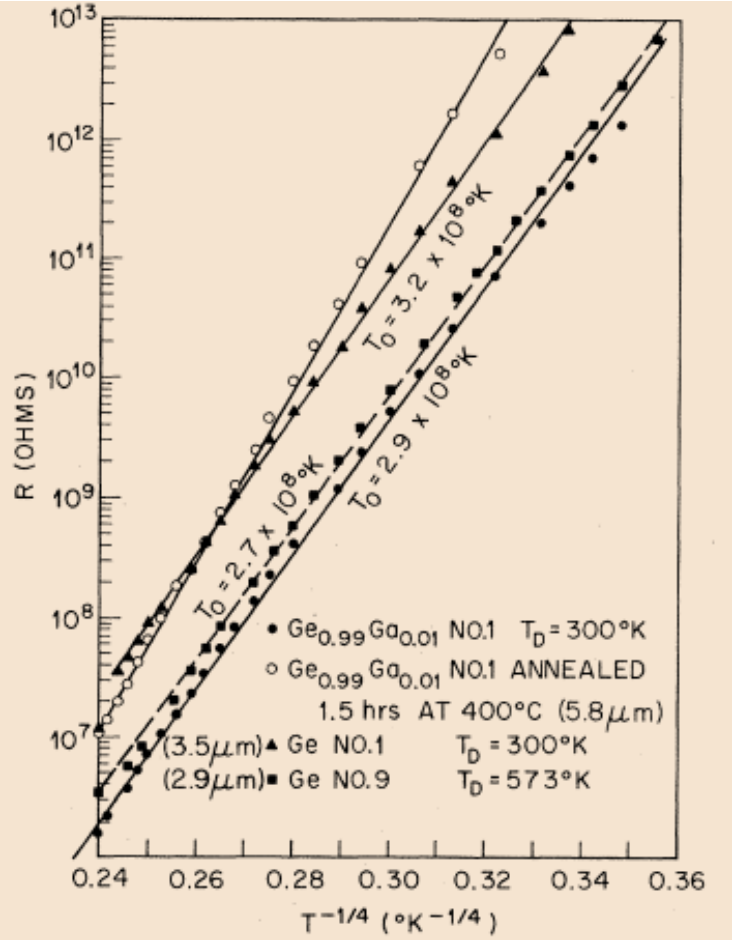
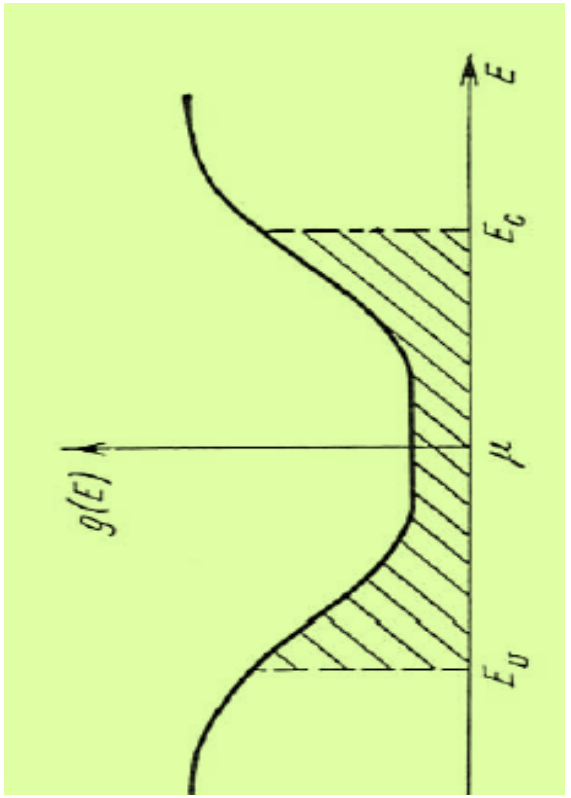



FIG. 1. Temperature dependence of the planar resistance for films deposited at and above 300°K. $Ge_{0.99}Ga_{0.01}$ sample No. 1, $d = 5.8\ \mu$m, $\rho_{RT} = 600\ \Omega$ cm; Ge sample No. 1, $d = 3.5\ \mu$m, $\rho_{RT} = 1500\ \Omega$ cm; Ge sample No. 9, $d = 2.86\ \mu$m, $\rho_{RT} = 600\ \Omega$ cm.

$$\rho = \rho_0 \exp[(T_0/T)^{1/4}]$$

$$\approx 40$$



VOLUME 32, NUMBER 21 PHYSICAL REVIEW LETTERS 27 MAY 1974

# Mott-Anderson Localization in the Two-Dimensional Band Tail of Si Inversion Layers

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(Received 31 January 1974)

Experimental evidence for a transition from localized to extended states in a two-dimensional band tail is obtained by measuring the conductivity in $n$- and $p$-type inversion layers in Si as a function of electron density and temperature (4.2 to 0.4°K). A transition from metallic to thermally activated conductivity is observed as a function of electron density, while the temperature dependence at electron densities below the transition shows both thermally activated nearest-neighbor hopping and variable-range hopping as proposed by Mott.

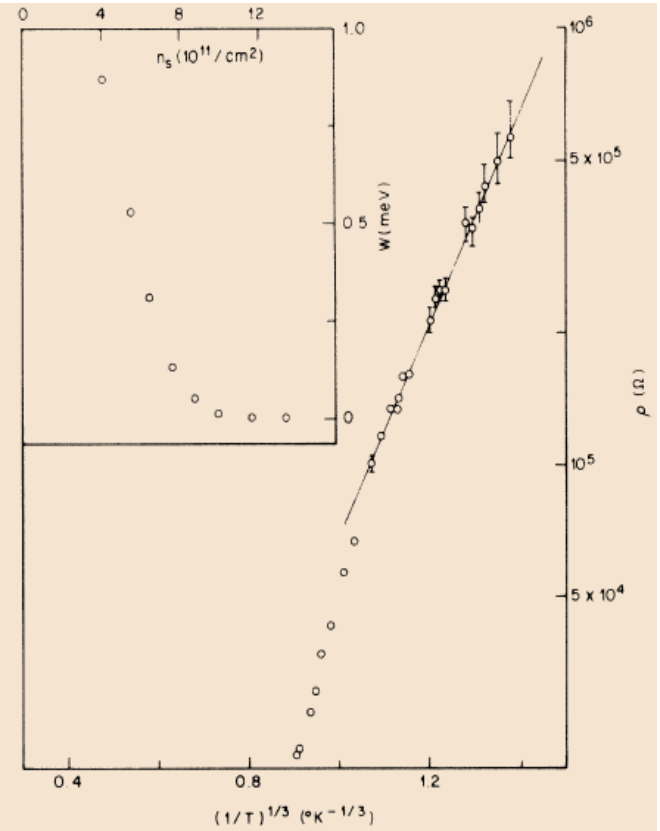


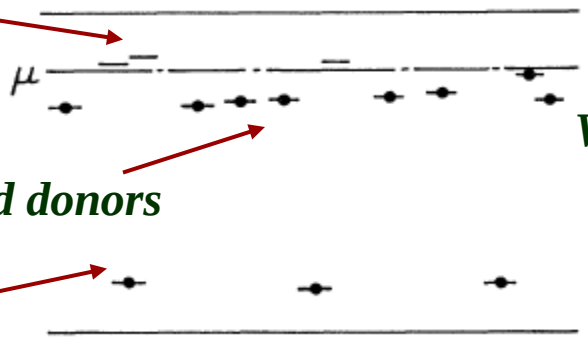

FIG. 2. $\ln\rho$ versus $(1/T)^{1/3}$ at $n_s = 6.4 \times 10^{11}/\text{cm}^2$. The slope of the solid line gives the parameter $T_0 = 177$°K. The inset shows the activation energy $W$ as a function of $n_s$.

$$T = 1K \Rightarrow \left(\frac{T_0}{T}\right)^{1/3} = 8$$

In Fig. 2, the data for $n_s = 6.4 \times 10^{11}/\text{cm}^2$ are replotted on a $\ln\rho$-versus-$(1/T)^{1/3}$ scale. It is clear that $\rho$ follows the $\exp[(T_0/T)^{1/3}]$ dependence on $T$ for $T \lesssim 1$°K. However, the temperature range is much too limited to determine unambiguously the correct power of $1/T$.

The parameter $T_0$ deduced from our data is 500°K for $n_s = 5.2 \times 10^{11}/\text{cm}^2$ and 180°K for $n_s = 6.4 \times 10^{11}/\text{cm}^2$. According to Mott's argument, $T_0 \approx 27\alpha^2/\pi k\eta$ in the case of a 2D system. Here $k$ is the Boltzmann constant and $\alpha^{-1}$ is the range of the localized state. Our data yield $\alpha^{-1} \approx 110$ Å for $n_s = 5.2 \times 10^{11}/\text{cm}^2$ and $\approx 180$ Å for $n_s = 6.4 \times 10^{11}/\text{cm}^2$. We can also estimate $\alpha^{-1}$ from[4] $\alpha(E_F) \approx [2m^*(E_c - E_F)/\hbar^2]^{1/2}$. Using $m^* = 0.2m_0$ and $E_F = n_s/\eta$, we obtain $\alpha^{-1} \approx 80$ Å for $n_s = 5.2 \times 10^{11}/\text{cm}^2$ and $\alpha^{-1} \approx 100$ Å for $n_s = 6.4 \times 10^{11}/\text{cm}^2$.

## *Lightly doped*




$N_D a^3 << 1$

*Weak overlap of the impurity wave functions*

*Low degree of compensation*

$$K = \frac{N_A}{N_D} << 1$$

$\mu$ *is determined from the condition that* $K$ *percent of donors*

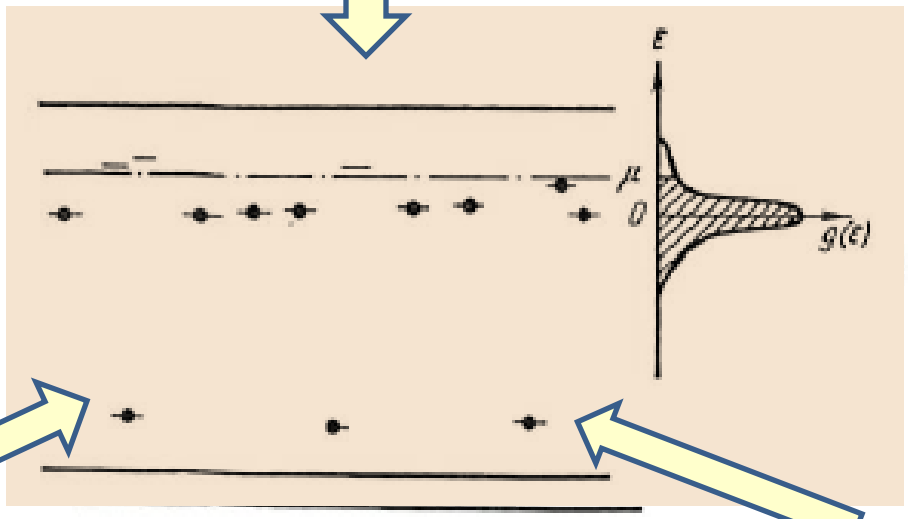
*with energies* $\varepsilon_i > \mu$ *are empty*

*Acceptors shift the level up*

*Empty donors shift the level down*

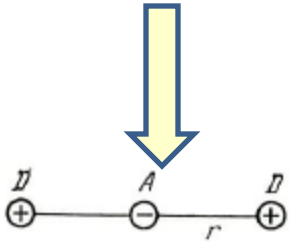
$$\epsilon_i = \frac{e^2}{\kappa}\left[\sum_{\ell}^{\text{acc}} \frac{1}{|r_i - r_\ell|} - \sum_{k \neq i}^{\text{don}} \frac{1 - n_k}{|r_i - r_k|}\right]$$

*Typically, an empty donor is the nearest neighbor of an acceptor*




*Some acceptors empty two donors in their neighborhood- 2 complex*

*Some acceptors do not have a donor in the neighborhood- 0 complex*




*Positive energy shift by acceptor overweighs negative energy shift by second donor*

*Neutrality condition:*

$$N_0(\mu) = N_2(\mu)$$

*increases*

*decreases*

*For 0-comlex to exist there must be no donors in a sphere*

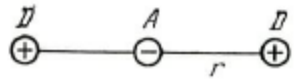
*with radius* $r_\mu = e^2/\kappa\mu$ *around a fixed acceptor*

$$N_0(\mu) = N_A \exp\left(-\frac{4\pi}{3}\frac{e^6 N_D}{\kappa^3\mu^3}\right)$$

*Shift by acceptor* *Shift by donor*

*2-comlexes*

$$\epsilon_1 = \frac{e^2}{\kappa|\mathbf{r}_1|} - \frac{e^2}{\kappa|\mathbf{r}_1-\mathbf{r}_2|}$$




$$\epsilon_2 = \frac{e_2}{\kappa|\mathbf{r}_2|} - \frac{e^2}{\kappa|\mathbf{r}_1-\mathbf{r}_2|}$$

$$N_2^{>}(\mu) = N_A N_D^2 \int d\mathbf{r}_1 \int_{r_2>r_1} d\mathbf{r}_2\, \Theta[\epsilon_1(\mathbf{r}_1,\mathbf{r}_2)-\mu]\,\Theta[\epsilon_2(\mathbf{r}_1,\mathbf{r}_2)-\mu]$$

*probability that two donors are the nearest neighbors of the acceptor*

$$N_2^{<} = N_A N_D^2 \int d\mathbf{r}_1 \int_{r_1<r_2} d\mathbf{r}_2 \exp\left(-\frac{4\pi}{3}N_D r_2^3\right)\Theta[\epsilon_1(\mathbf{r}_1,\mathbf{r}_2)-\mu]\,\Theta[\epsilon_2(\mathbf{r}_1,\mathbf{r}_2)-\mu]$$

$$N_0(\mu) = N_2(\mu) \Rightarrow \mu = 0.61\epsilon_D = 0.99\, e^2 N_D^{1/3}/\kappa$$

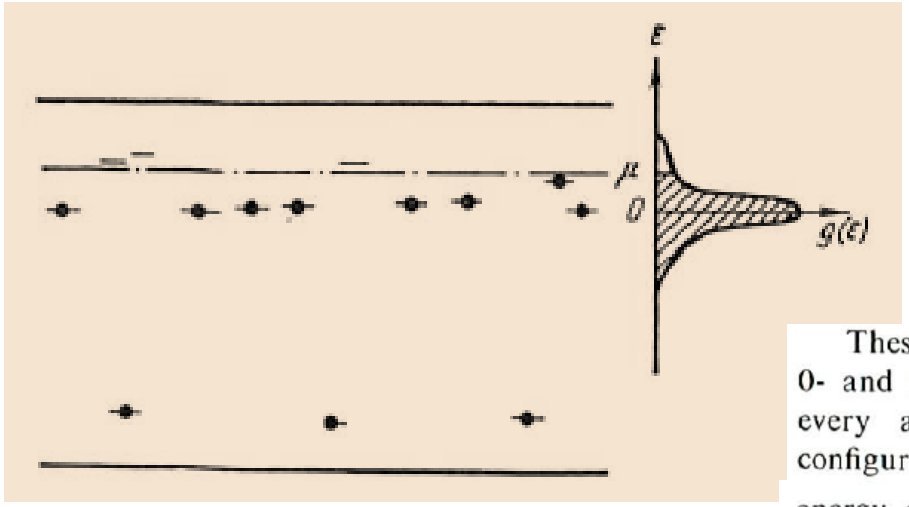
*0-comlexes constitute 1.3% of the acceptor concentration*

# *Temperature dependence of the nearest-donor hopping conductivity*

$$\exp\left(-\frac{\mu}{T}\right) = \exp\left(-\frac{0.99e^2 N_D^{1/3}}{\kappa T}\right)$$

*probability that at* $T \ll \mu$ *a donor is empty*

*condition* $N_D a^3 \ll 1$ *ensures that* $\mu \ll \frac{\hbar^2}{ma^2}$ ⇨ *impurity band is narrow*




***Width of the density-of-states peak is determined by long-range fluctuations***

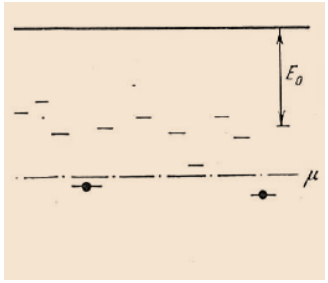
$$\gamma = 2\sqrt{2\pi}\,\frac{e^2}{\kappa r_0}\,(N_0 r_0^3)^{1/2} = 0.26\,\epsilon_D\,(N_A/N_D)^{1/4}$$

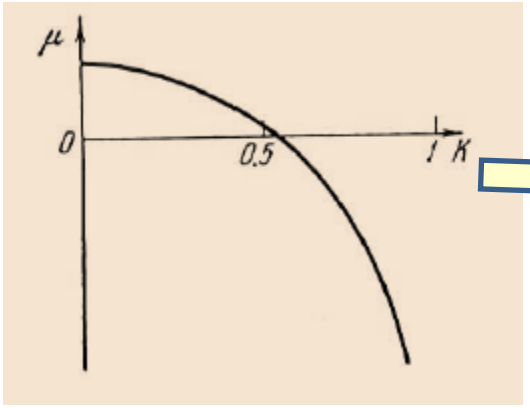
$$r_0 = 0.58\,N_A^{-1/2} N_D^{1/6}$$

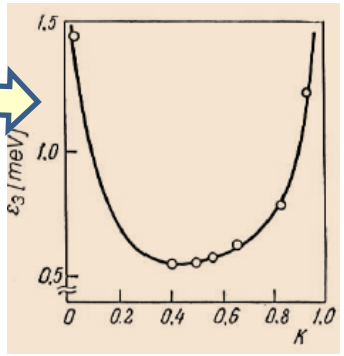
These figures may tempt one to think that in the first approximation both 0- and 2-complexes can be ignored. However, this would be incorrect. If every acceptor produced a 1-complex, such complexes would include configurations where the nearest donor is far from its acceptor. The site energy $\epsilon$ for such a donor is almost zero. But this donor must be ionized, since we are dealing with a 1-complex, and hence we would have $\mu=0$ rather than $0.61\epsilon_D$. Thus it is not permissible to neglect the 0- and 2-complexes in calculating the Fermi level.

## I. Close pairs of donors:

*With increasing* $K$




$$dN = 4\pi N_D R^2 dR \qquad \varepsilon = \frac{e^2}{\kappa R}$$

$$g(\varepsilon) = \frac{dN}{d\varepsilon}$$

$$g(\epsilon) = \frac{3}{2}\frac{\epsilon_D^3}{\epsilon^4} N_D \qquad \int_{-\infty}^{\mu} g(\epsilon)\, d\epsilon = n$$

$$\mu = -\left(\frac{2\pi}{3}\right)^{1/3} \frac{e^2 N_D^{2/3}}{\kappa n^{1/3}} = -\frac{\epsilon_D}{2^{1/3}(1-K)^{1/3}}$$

## II. Long-range fluctuations:

*Fluctuation of number of donors in a cube with side* $r$

$$\mu \sim \frac{e^2 \left(N_D r^3\right)^{1/2}}{r}$$

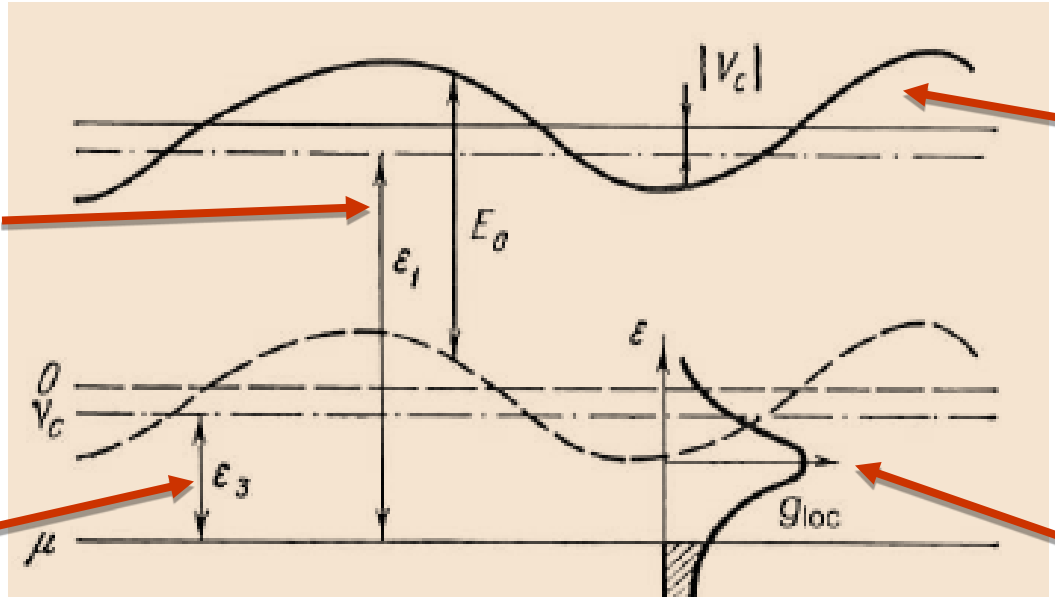
$r$ *is determined by the condition:*

*excess concentration of charges*

$$\frac{\left(N_D r^3\right)^{1/2}}{r^3} \sim n = N_D(1-K)$$

*yields the same result for* $\mu$

At $T=0$ electrons tend to occupy states of lowest energy. Therefore they will move into regions with a positive impurity charge. If the excess impurity density $\Delta N$ is lower than the average electron density $n$, then the fluctuations will be completely neutralized by a small variation in the electron density. If, on the other hand, $\Delta N \gg n$, the fluctuations cannot be neutralized by electrons at all. Indeed, even if all electrons moved from negative regions, their density in the positive regions would only double, still remaining small compared to the impurity charge.




*activation into conduction band*

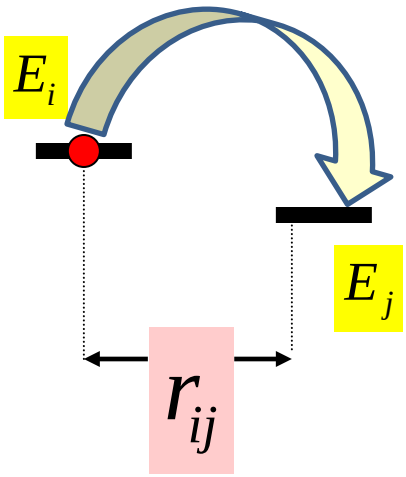
*Long-range potential*

*activation energy for hopping*

*Local density of states due to donor pairs*

**Fig. 3.5.** Energy diagram of a highly compensated semiconductor, taking the long-range potential relief into account. Meandering lines (solid and dashed) represent the bottom of the conduction band and the impurity ground-state energy, both following the behavior of the potential energy $V(\mathbf{r}) = -e\phi$. The straight solid line on the top and dashed line correspond to these unperturbed energies. The Fermi level is shown by a solid line at the bottom. Also shown is the local density of states at some point in space. The region of occupied states is shaded. The dash-dotted lines correspond to the percolation levels (Sect. 5.2); $\varepsilon_1$ and $\varepsilon_2$ are the activation energies (Sect. 8.2)

## *Phonon-assisted transition between two sites:*




*Golden rule* *sound velocity* *phonon wave vector*

$$\gamma_{ij} = \frac{2\pi}{\hbar} \frac{V_0}{(2\pi)^3} \int |M_{\mathbf{q}}|^2 \, \delta(\hbar s q - \Delta^j{}_i) \, d\mathbf{q}$$

$E_i - E_j$

*overlap integral*

$$\Psi_i' = \Psi_i + \frac{I_{ij}}{\Delta^j{}_i} \Psi_j \qquad \Psi_j' = \Psi_j - \frac{I_{ij}}{\Delta^j{}_i} \Psi_i$$

$$\gamma_{ij} = \gamma_{ij}^0 \exp(-2r_{ij}/a) \, N(\Delta^j{}_i)$$

*Matrix element* *deformation potential*

$$|M_{\mathbf{q}}|^2 = \left[\frac{\hbar q E_1^2}{2dV_0 s}\right] \left[\frac{I_{ij}}{\Delta^j{}_i}\right]^2 N_q (1 - \cos \mathbf{q}\cdot\mathbf{r}_{ij}) \left[1 + \left[\frac{qa}{2}\right]^2\right]^{-4}$$

*density* *Planck distribution* *Fourier transform of the donor wave function*

PHYSICAL REVIEW B VOLUME 4, NUMBER 8 15 OCTOBER 1971

# Hopping Conductivity in Disordered Systems

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(Received 14 May 1971)

## Resistance corresponding to a single hop

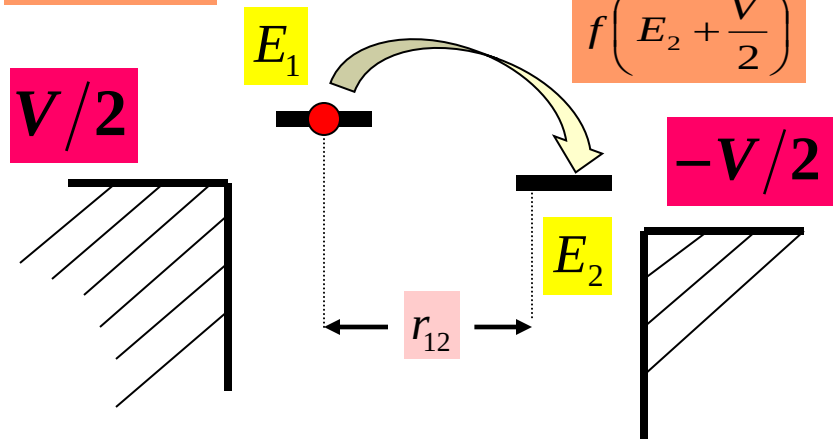
$$I_{12} = \gamma_{12} f\left(E_1 - \frac{V}{2}\right)\left[1 - f\left(E_2 + \frac{V}{2}\right)\right]$$

$$\gamma_{12} = \gamma_{21} \exp\left[\frac{E_1 - E_2}{T}\right]$$

$$I_{21} = \gamma_{21} f\left(E_2 + \frac{V}{2}\right)\left[1 - f\left(E_1 - \frac{V}{2}\right)\right]$$

*average occupation* $f\left(E_1 - \frac{V}{2}\right)$

*average occupation* $f\left(E_2 + \frac{V}{2}\right)$




*in equilibrium* $V = 0 \Rightarrow I_{12} = I_{21}$

*at finite* $V \ll T$ *and* $|E_1|, |E_2| \gg T$

$$I = I_{12} - I_{21} = \frac{V}{R_{12}}$$

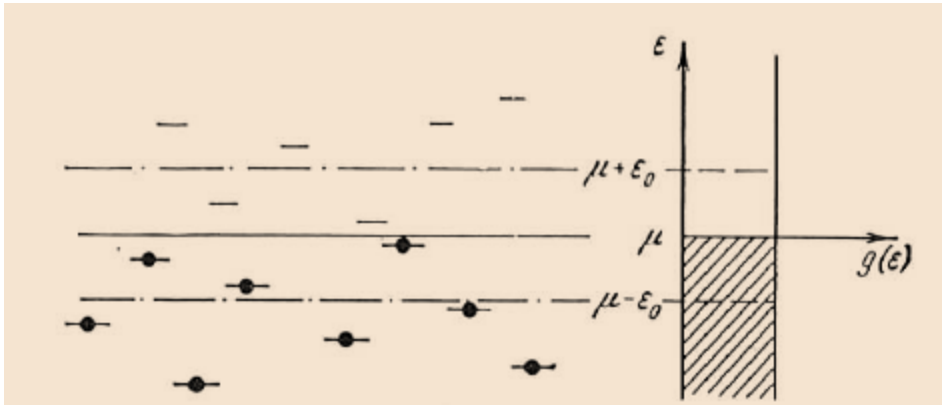
*where*

$$R_{12} = R_0 \exp\left[\frac{2r_{12}}{a} + \frac{|E_1| + |E_2| + |E_1 - E_2|}{2T}\right]$$

$$E_a = |E_1 - E_2| \text{ for } E_1 \cdot E_2 < 0$$

$$E_a = \max\{E_1, E_2\} \text{ for } E_1 \cdot E_2 > 0$$

# Mott's law




*constant density of states within the strip*

$$|\varepsilon_i - \mu| \le \varepsilon_0$$

*concentration of sites within the strip*

$$N(\varepsilon_0) = 2g(\mu)\varepsilon_0$$

*Transport resulting from energy levels within the strip*

$$\frac{\rho}{\rho_0} = \exp\left[\frac{1}{N(\varepsilon_0)^{1/3}a} + \frac{\varepsilon_0}{T}\right] = \exp\left[\frac{1}{(g\varepsilon_0)^{1/3}a} + \frac{\varepsilon_0}{T}\right]$$

*has a sharp minimum at*

$$\varepsilon_0 = \varepsilon_0(T) = \frac{T^{3/4}}{(ga^3)^{1/4}}$$

$$\ln\left[\frac{\rho(\varepsilon_0(T))}{\rho_0}\right] = \left(\frac{T_0}{T}\right)^{1/4}$$

$$r = [g\varepsilon_0(T)]^{-1/3} = a\left(\frac{T_0}{T}\right)^{1/4}$$

$$T_0 = \frac{1}{g(\mu)a^3}$$

*typical hopping distance increases with decreasing* $T$

***variable-range hopping***

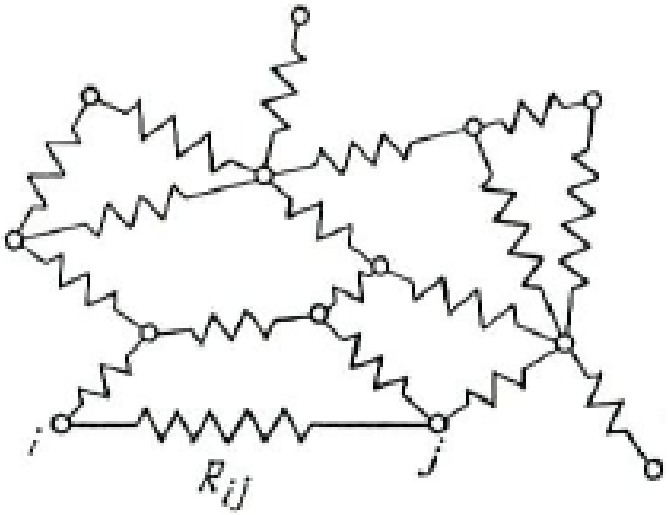
# *Numerical factor in log-resistance from percolation theory*

PHYSICAL REVIEW VOLUME 120, NUMBER 3 NOVEMBER 1, 1960

## Impurity Conduction at Low Concentrations*†

ALLEN MILLER‡ AND ELIHU ABRAHAMS
*Physics Department, Rutgers University, New Brunswick, New Jersey*
(Received June 23, 1960)

***Nearest-neighbor hopping: tunneling dominates over activation***




$$\frac{r}{a} \sim \frac{1}{N_D^{1/3} a} > \frac{\varepsilon}{T} \sim \frac{e^2 N_D^{1/3}}{\kappa T}$$

$$\frac{e^2}{\kappa a} > T > \frac{e^2}{\kappa a}(N_D a^3)^{2/3}$$

$$\ln R_{ij} = \frac{2 r_{ij}}{a}$$

***The sites with*** $r_{ij} < r_c$ ***are connected***

***Infinite cluster appears when***

***average number of neighboring donors inside a sphere with*** $r = r_c$

$$B_c = \frac{4\pi}{3} N_D r_c^3 = 2.7 \pm 0.1$$

$$\ln R = \frac{1.73}{N_D^{1/3} a}$$

## *Numerical factor in log-resistance from percolation theory: Variable-range hopping*

### *Bonding criterion*

$$\frac{2r_{ij}}{a}+\frac{\varepsilon_{ij}}{T}<\xi$$

$$r_{\max}=\frac{a\xi}{2} \qquad \varepsilon_{\max}=T\xi$$

*dimensionless distance*

$$s_i=\frac{r}{r_{\max}}$$

$$s_{ij}=|s_i-s_j|=\frac{r_{ij}}{r_{\max}}$$

*dimensionless energy*

$$\Delta_i=\frac{\varepsilon_{ij}}{\varepsilon_{\max}}$$

$$\Delta_{ij}=\frac{1}{2}\left[|\Delta_i|+|\Delta_j|+|\Delta_i-\Delta_j|\right]$$

$$s_{ij}+\Delta_{ij}<1$$

*Dimensionless concentration of sites within energy strip* $|\Delta_i|<1$

$$n(\xi)=2g\varepsilon_{\max}r_{\max}^3=\frac{gTa^3}{4}\xi^4$$

*Infinite cluster appears at* $n(\xi)=n_c=5.3$

*In 2D*

$$\ln R=\left[\frac{14}{ga^2T}\right]^{1/3}$$

$$\ln R=\left[\frac{21}{ga^3T}\right]^{1/4}$$

*Mott's law with numerical factor*

$$n_{VRH}^{c} \int_{\Delta_{ij}<1} \left(1-\Delta_{ij}\right)^{3} \frac{d\Delta_i\, d\Delta_j}{2} = n_{NNH}^{c}$$

$$= \frac{3}{20}$$

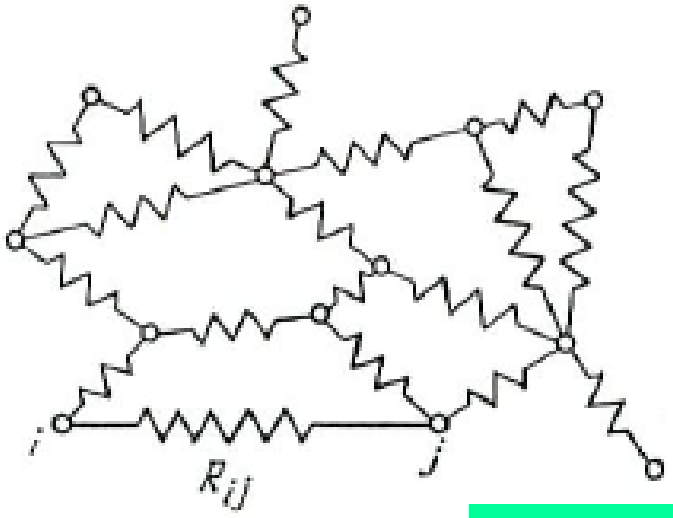
So far we have been dealing with an example of a very complicated anisotropy, namely that of a four-ellipsoid donor state. Another example of an anisotropic wave function whose shape is provided by donor states is germanium subjected to a large uniaxial stress. It is well known [6.17] that under uniaxial compression in the {111} direction, the energy of one of the conduction band minima goes down, while that of the other three minima goes up. At pressures of order $10^9$ dyne/cm$^2$ this splitting becomes so large that the electron ground state on a donor is no longer associated with all four ellipsoids as it is in the absence of pressure, but with one ellipsoid only. Further increases in pressure essentially produce no new change in the wave function. Therefore the value of order $10^9$ dyne/cm$^2$ can be called the limiting or "maximal" pressure. At the "maximal" pressure only one term corresponding to a selected ellipsoid remains in the expressions (6.2.1), (6.2.2), and (6.2.7). As a result we obtain an expression of the form (6.2.10) with $\xi_{ij}$ given by

$$\xi_{ij} = 2\left[\frac{x_{ij}^2 + y_{ij}^2}{a^2} + \frac{z_{ij}^2}{b^2}\right]^{1/2}$$

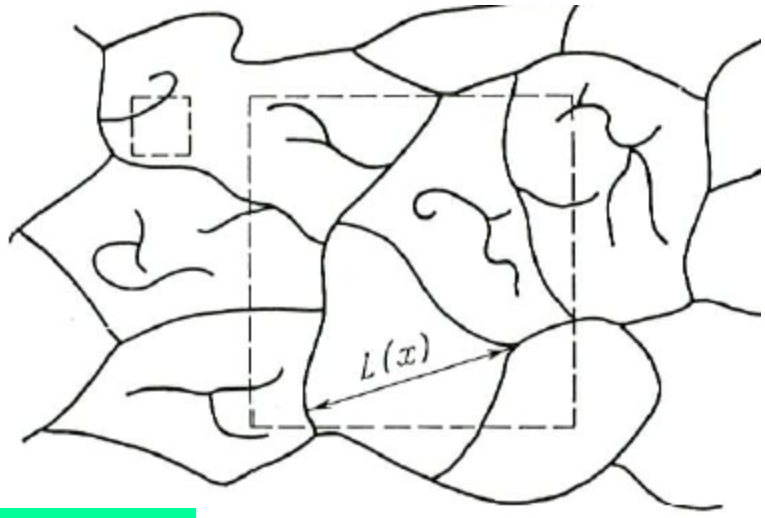
$$N_D \int d\vec{r}\, \theta\left(\xi - \xi_{ij}\right) = B_c$$

$$\ln R = \frac{1.73}{N_D^{1/3}\left(a^2 b\right)^{1/3}}$$

*Random resistor network*




*Near the percolation threshold*




$$L(\xi) \sim a\xi_c \left( \frac{\xi_c}{\xi - \xi_c} \right)^{\nu}$$

*hopping length*

*critical exponent*

$$\nu_2 = \frac{4}{3} \qquad \nu_3 \approx 0.9$$

*Resistance of the sample is dominated by infinite cluster with* $(\xi - \xi_c) \sim 1$

*"Unit cell" of the current-carrying network*

$$L_c \sim a\xi_c^{\nu+1}$$

## *Hopping transport in amorphous film of a thickness* **d**

$$d << a\xi_c = \bar{r}$$

*2D Mott's law with*

$$T_0 = \frac{\beta}{[g\,d]a^2}$$

$$d >> \bar{r}$$

*No thickness dependence if* **d** *exceeds*

$$L_c \sim a\xi_c^{\nu+1}$$

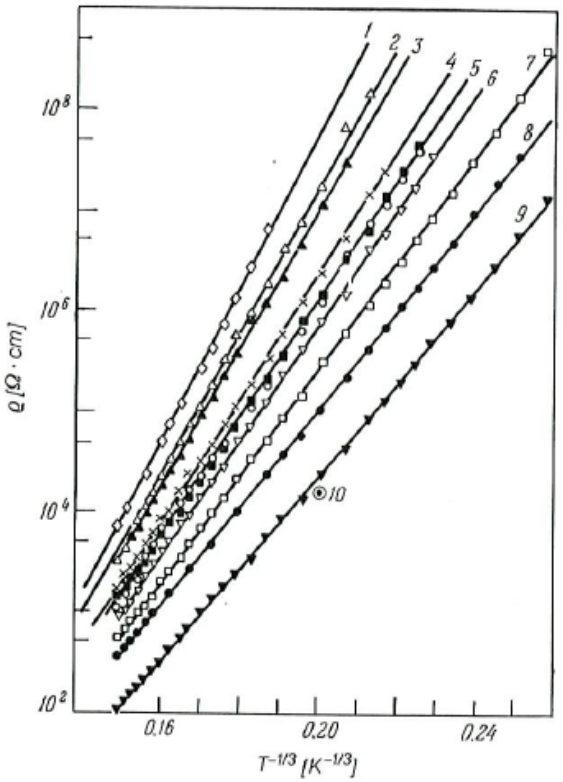



**Fig. 9.3.** Temperature dependences of the resistivities in amorphous carbon films of varying thickness $d$. The values of $d$ are (in Å): (*1*) 150, (*2*) 200, (*3*) 170, (*4*) 225, (*5*) 250, 300, (*6*) 275, (*7*) 400, (*8*) 575, (*9*) 1200. Point 10 corresponds to $d$ = 6890 Å [9.33]

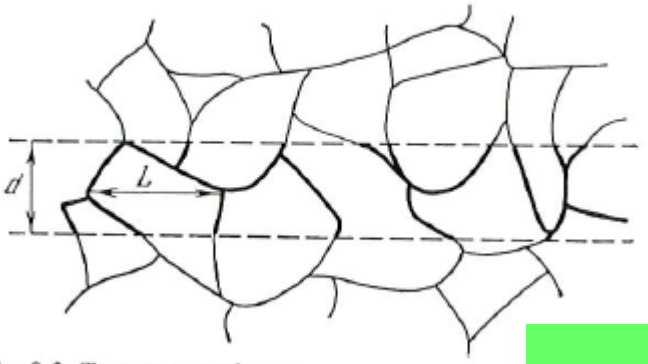


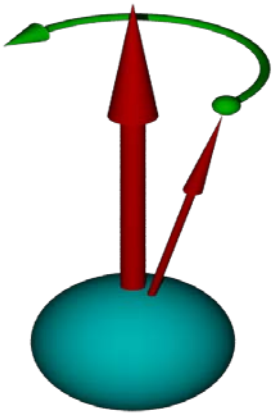

**Fig. 9.4.** The relation of bulk and film infinite clusters. Dashed lines show the film boundaries, solid lines the parts of the bulk infinite cluster lying within the film. $L$ is the correlation radius, $d$ the film thickness

$$L(\xi_{cd}) = d$$

$$\xi_{cd} = \xi_c\left[1 + D\left(\frac{\bar{r}}{d}\right)^{1/\nu_3}\right]$$

$$\ln\left(\frac{\rho(d)}{\rho(\infty)}\right) = D\left(\frac{T_0}{T}\right)^{1/4}\left(\frac{\bar{r}}{d}\right)^{1/\nu_3}$$

## *Dyakonov-Perel spin relaxation in diffusive regime*



*Over the mean free path* $l = v\tau$

*electron spin rotates by a random angle* $\delta\varphi << 1$

*Initial spin orientation is forgotten after* **N** *steps:* $N(\delta\varphi)^2 \sim 1$ *random walk*

*Spin relaxation time:*

$$\tau_s = N\tau = \frac{\tau}{(\delta\varphi)^2}$$

*Spin diffusion length:*

$$l_s = \sqrt{N}\tau = \frac{l}{\delta\varphi}$$

*Relaxation due to spin-orbit coupling:*

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{so} \times \vec{S} \Leftarrow \hat{H}_{SO} = \vec{\Omega}_{SO} \cdot \hat{\sigma}$$

$$\delta\varphi = \Omega_{so}\tau$$

$$\tau_s = \frac{1}{\Omega_{so}^2 \tau}$$

$$l_s = \frac{v}{\Omega_{so}}$$

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# Dyakonov-Perel spin relaxation near the metal-insulator transition and in hopping transport

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*for free electrons in conduction band*

$$A = \frac{m\Omega_{so}^2}{ne^2}$$

*relation* $\tau_s = \frac{1}{\Omega_{so}^2 \tau}$ *can be cast in the form* $\frac{1}{\tau_s} = A\sigma$

*Relation holds in the hopping regime*

$$\sigma = \frac{ne^2\tau}{m}$$

*Drude conductivity*

***Electron rotates its spin only during time intervals when it tunnels between the sites***

$$\tau_s = N\tau_{hop} = \frac{|\tau_{tun}|}{(\delta\varphi)^2}\left(\frac{\tau_{hop}}{|\tau_{tun}|}\right) = \frac{1}{\Omega_{so}^2|\tau_{tun}|}\left(\frac{\tau_{hop}}{|\tau_{tun}|}\right)$$

$\propto \sigma_{hop}^{-1}$

*waiting time*

*tunneling time*

*For localized electron the dominant mechanism of spin relaxation is interaction with random hyperfine field created by nuclei inside the donor orbit*

$\vec{\omega}_1^N$ $\vec{\omega}_2^N$

$r_{12} \sim a\xi_{12}$

*unlike SO-coupling, electron spin precesses while electron waits for the hop*

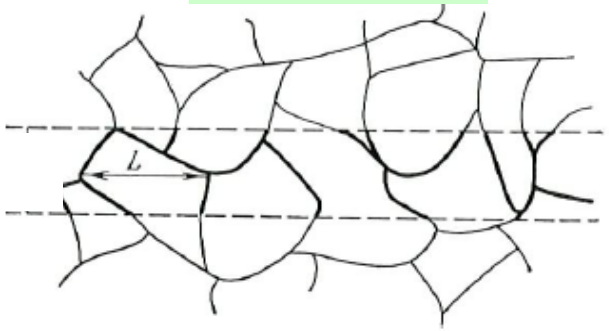
$$\delta\varphi = \omega^N \tau_{hop} = \omega^N \tau_0 \exp(\xi_{12}) >> 1$$

*spin rotation is dominated by most resistive hops*

*If* $$\delta\varphi_c = \left\langle \left( \omega^N \right)^2 \right\rangle^{1/2} \tau_0 \exp(\xi_c) >> 1$$

*the initial spin orientation is "forgotten" at distances less than* $L_c \sim a\xi_c^{\nu+1}$

*If* $\delta\varphi_c << 1$ *initial spin orientation will be "forgotten" at the diffusion stage*




$$l_s = \frac{L_c}{\delta\varphi_c} \sim \frac{L_c \exp(-\xi_c)}{\omega^N \tau_0}$$

*spin diffusion length contains correlation radius of infinite cluster*