Paired superfluidity in resonant atomic gases

Leo Radzihovsky

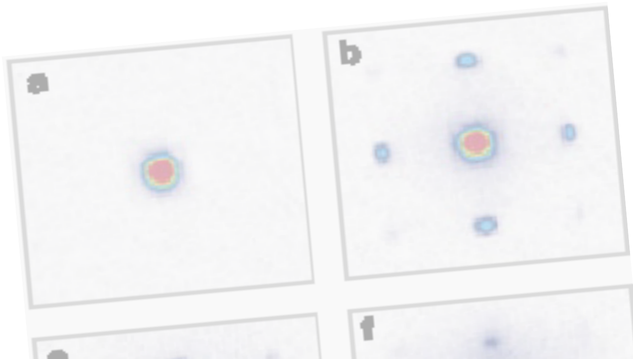
for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Gurarie, L.R., PRB 75, 212509 (2007)
Veillette, Sheehy, L.R., PRA 75, 043614 (2007)

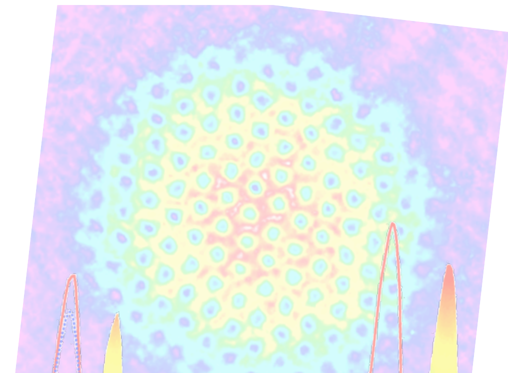
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)
Nicolic, Sachdev, PRA 75, 033608 (2007)
Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

$: NSF

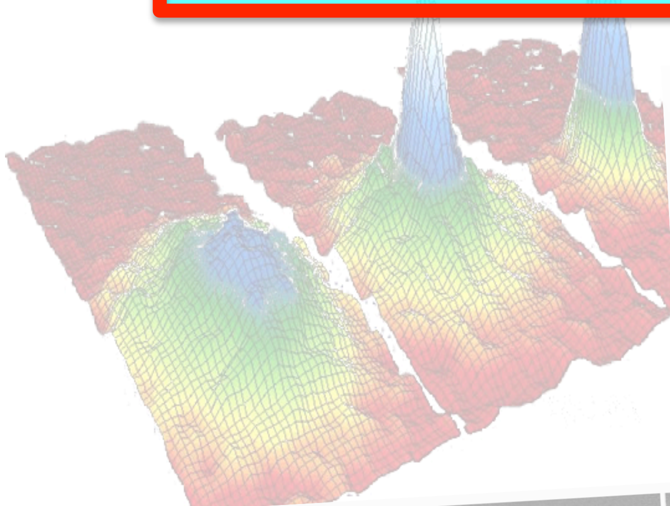
BSS2014, Boulder, CO, July 2014

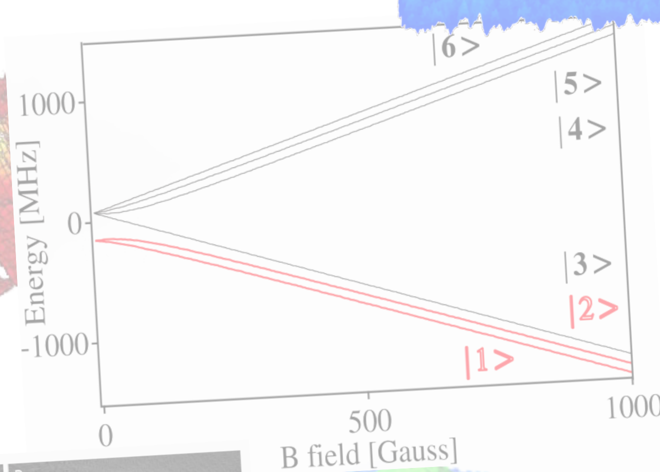
Course outline

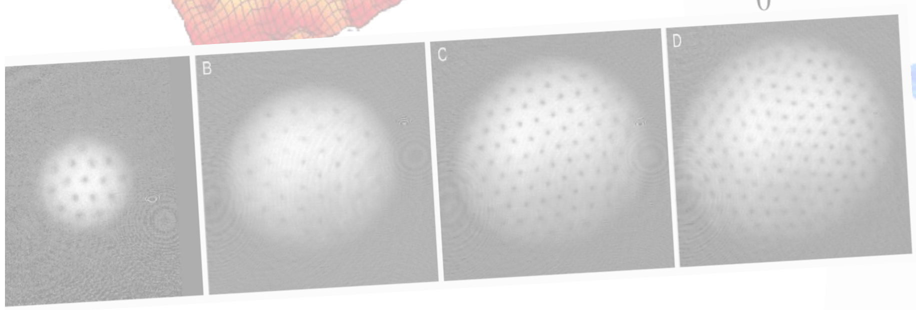


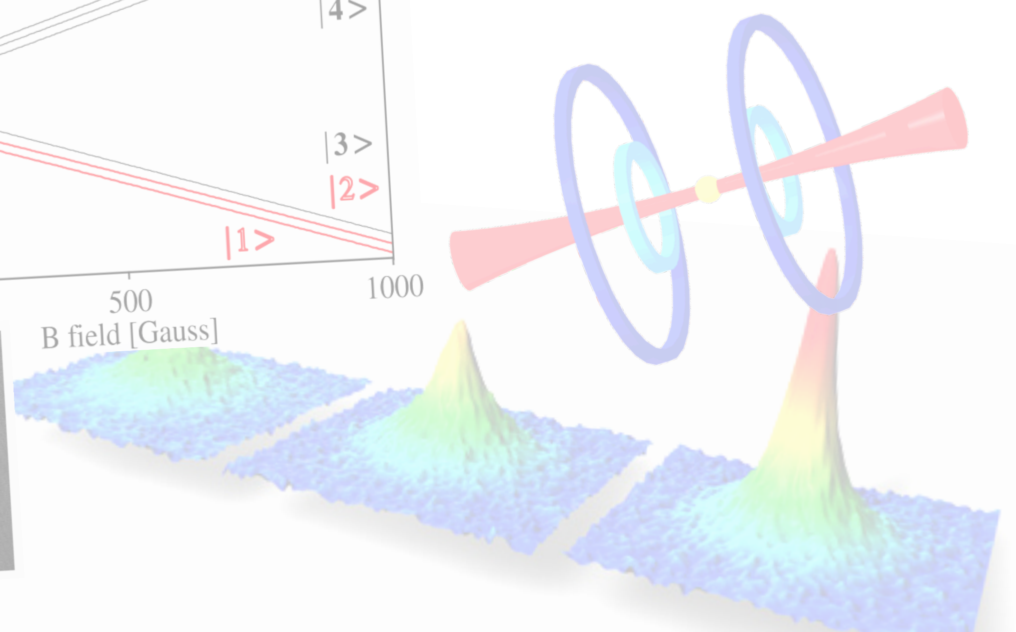


- L0: AMO renaissance and scattering theory overview
- L1: S-wave Feshbach resonant superfluidity
- L2: P-wave Feshbach resonant superfluidity



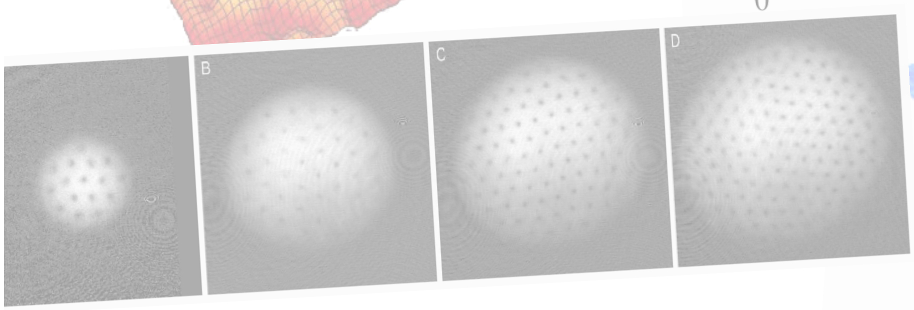


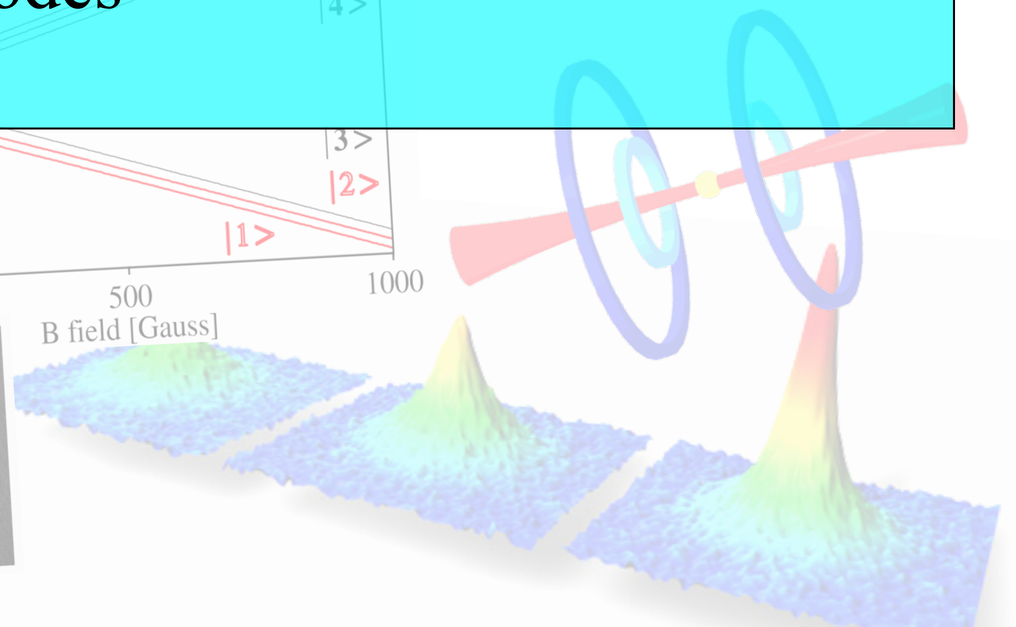




Lecture 2: p-wave Feshbach resonant superfluidity

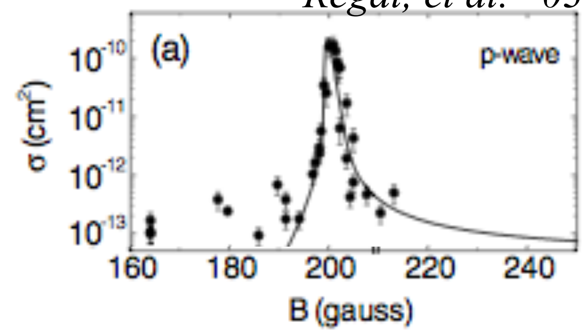
- motivation and experiments
- review of p-wave scattering theory
- two-channel model
- phases and transitions
- vortices and Majorana modes
- experimental prospects





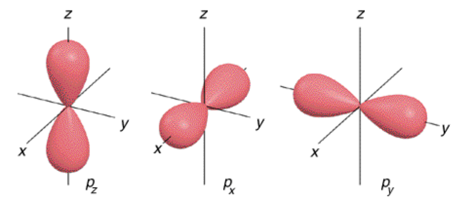
Finite angular momentum superfluidity

Regal, et al. '03

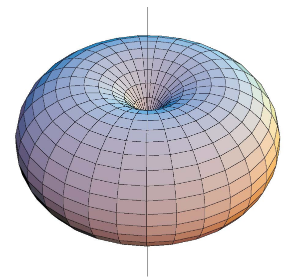


Motivation:

- *p-wave Feshbach resonances exist*
- *examples of ^{3}He and high-Tc superconductors*
- *multiple superfluids phases*

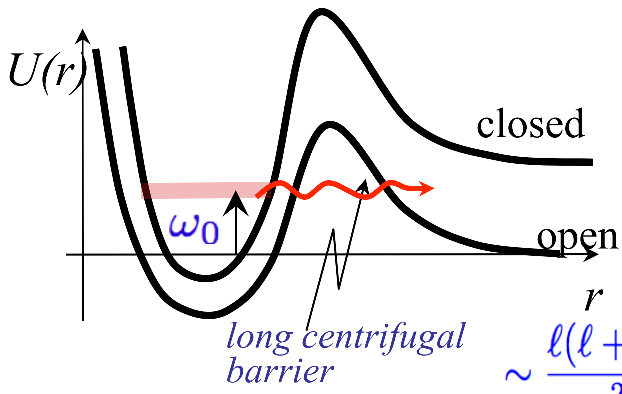


- *anisotropic gap with gapless excitations*

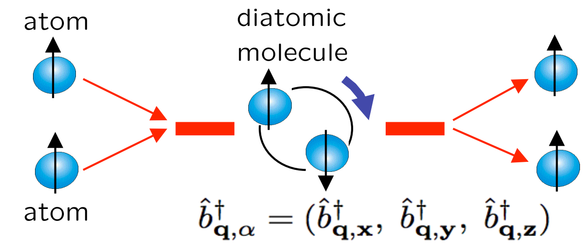


- *conventional (thermal and quantum) and topological phase transitions with detuning*
- *non-Abelian vortex excitations ⇒ topological QC?*

P-wave Feshbach resonant scattering



naturally ***narrow!***



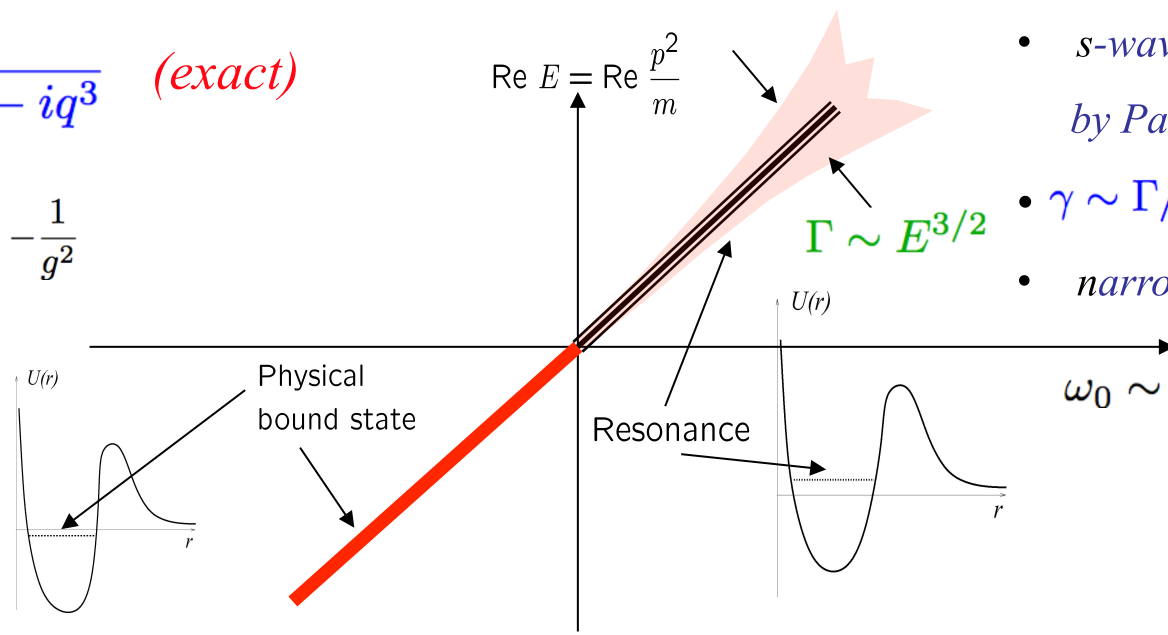
$\sim \frac{\ell(\ell+1)}{r^2}$ ➡ escape (molecular life) time $\tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}$, for $E \to 0$

$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger$$

$$f_p = \frac{q^2}{-v^{-1} + \frac{q_0}{2}q^2 - iq^3}$$ *(exact)*

with $v^{-1} \sim -\frac{g^2}{\omega_0}$, $q_0 \sim -\frac{1}{g^2}$

$$f_p(q) = \frac{q^2}{F(q^2) - iq^3}$$

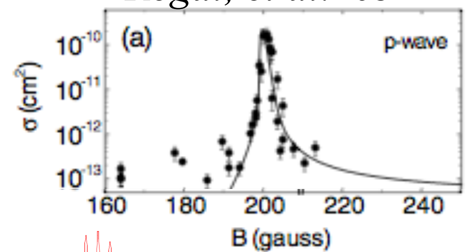


$\omega_0 \sim B - B_0$

- *s-wave suppressed by Pauli principle*
- $\gamma \sim \Gamma/E \sim E^{\frac{1}{2}} \ll 1$
- *narrows with* ϵ_F, n

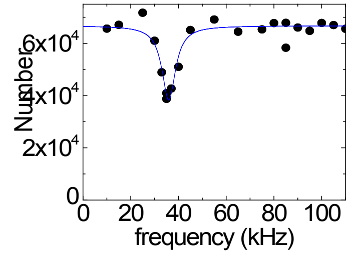
Experimental hopes for p-wave superfluidity

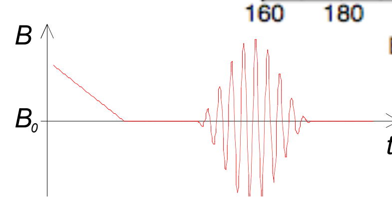
Regal, et al. '03



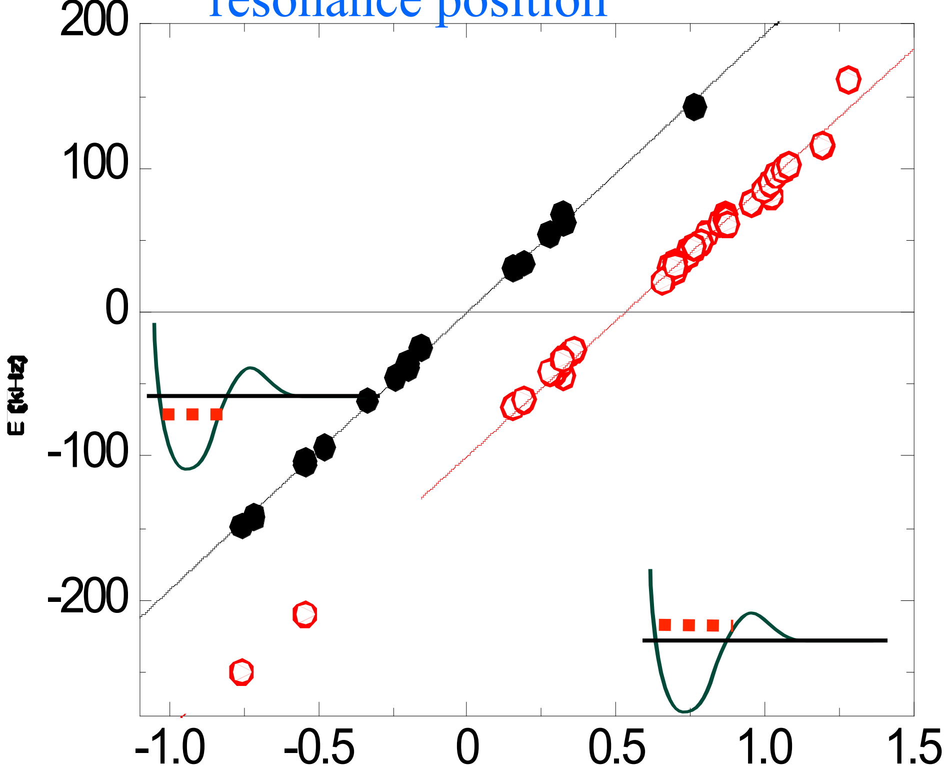
- p-wave Feshbach resonance in ^{40}K, ^{6}Li
- making p-wave molecules:

resonant disappearance of atoms with oscillating B(t)

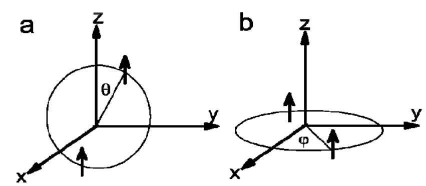




resonance position



Gaebler, et al. '07



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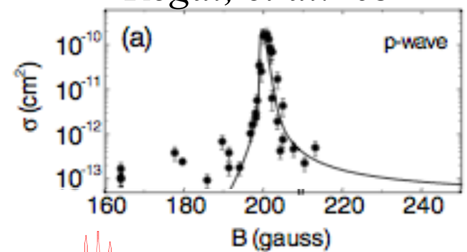
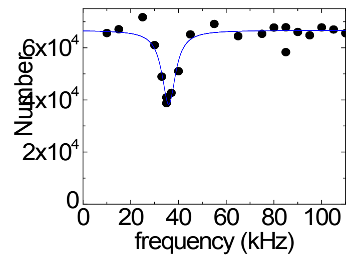
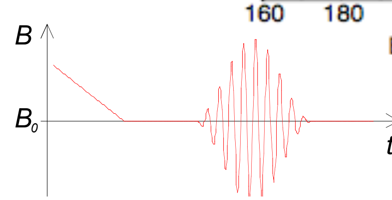
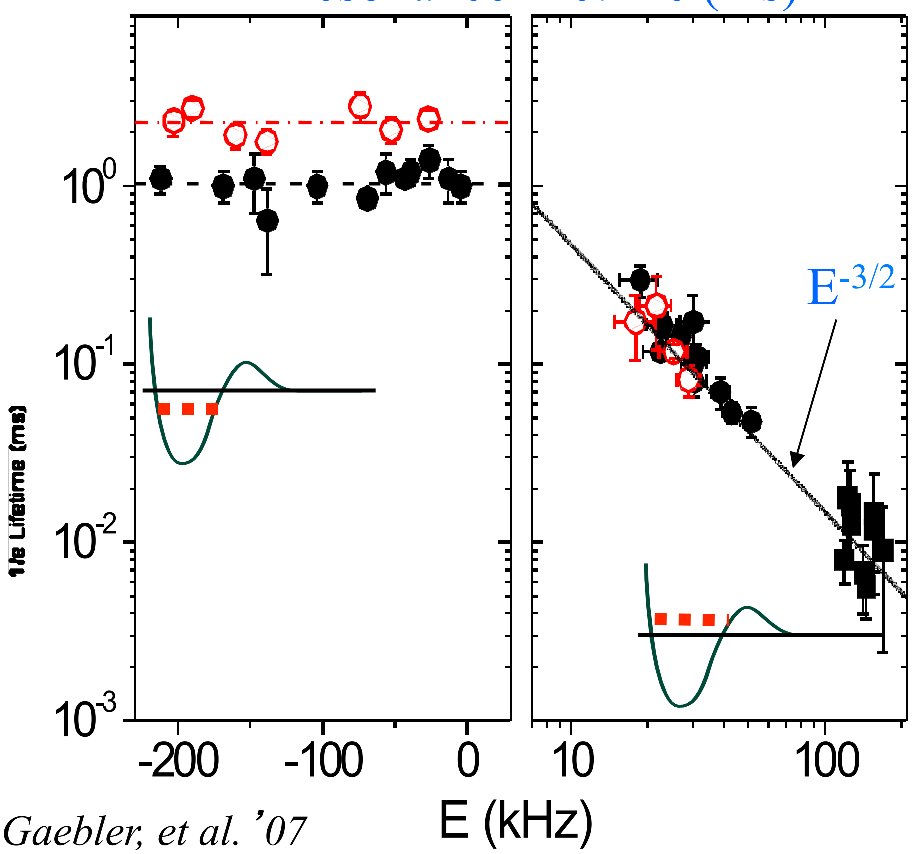


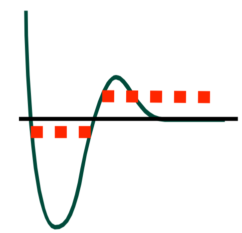




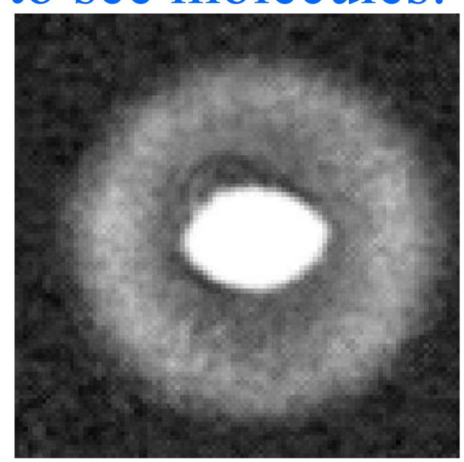
resonance lifetime (ms)



Gaebler, et al. '07



to see molecules:



look for energetic atoms

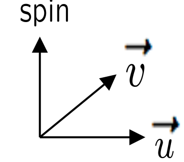
P-wave resonant superfluidity

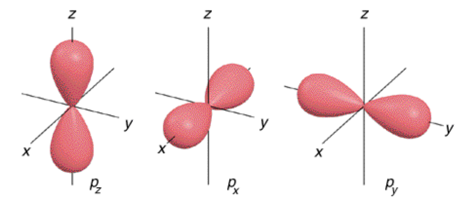
$$\mathcal{H}_{2ch} = \psi^\dagger \frac{\hat{p}^2}{2m}\psi + \vec{\phi}^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\vec{\phi} - ig\vec{\phi}\cdot\psi^\dagger\nabla\psi^\dagger$$

dimensionless coupling: $\gamma \sim \left(\frac{g\sqrt{n}k_F}{\epsilon_F}\right)^2 \sim g^2\epsilon_F^{1/2} \sim \frac{n^{1/3}}{q_0}$

- ***narrow*** resonance $\gamma \ll 1 \;\rightarrow\; \mathrm{MFT}: \vec{\phi}(x) = \vec{B}$

- ***complex vector*** order parameter:





$$\vec{B} = \vec{u} + i\,\vec{v} \;\Longleftrightarrow\; \psi_0 = B_z,\; \psi_\pm = \pm(B_x \pm iB_y)$$

$$\vec{B}\cdot\vec{k} = \sum_{m=0,\pm k}\psi_m Y_{1,m}(\hat{k})k$$

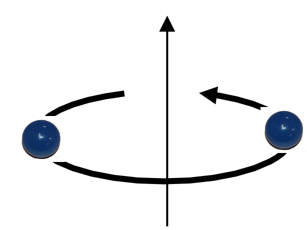
- sample states:

$v = 0 \;\Longleftrightarrow\; |m = 0\rangle$ along $\vec{u}$

$(k_x \quad \beta - state\ in\ ^3He)$

$u = v \;\Longleftrightarrow\; |m = 1\rangle$ along $\vec{u}\times\vec{v}$

$(k_x + ik_y$ *"axial" Anderson – Morel state in* $^3He)$



$L_z = \pm 1, 0$

Mean-field theory $(\gamma \sim g^2 \epsilon_F^{1/2} \ll 1)$

$$H_p = \sum_{\mathbf{k}} \frac{k^2}{2m} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{q},\alpha} (\frac{q^2}{4m} + \epsilon_{0\alpha}) b^\dagger_{\mathbf{q},\alpha} b_{\mathbf{q},\alpha} + \sum_{\mathbf{k}} [\Delta_{\mathbf{k}}(\vec{B}) a^\dagger_{\mathbf{k}} a^\dagger_{-\mathbf{k}} + h.c.]$$

- *superfluid ground state:*

$$\text{molecular BEC } |\vec{B}\rangle \; + \; \text{Cooper pairing } |\mathrm{BCS}_{\vec{\mathrm{B}}}\rangle = \Pi_k (u_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{-\mathbf{k}} a^\dagger_{\mathbf{k}})|0\rangle$$

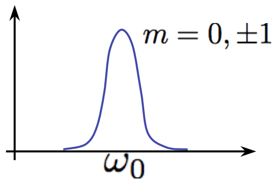
(closed) *(open)*

- *excitation spectrum:* $H_{ex} = \sum_{\mathbf{k}} E^{(a)}_{\mathbf{k}} \alpha^\dagger_{\mathbf{k}} \alpha_{\mathbf{k}} + \sum_{\mathbf{k},\alpha} E^{(m)}_{\mathbf{k},\alpha} \beta^\dagger_{\mathbf{k},\alpha} \beta_{\mathbf{k},\alpha}$

$$E^{(a)}_{\mathbf{k}} = \sqrt{\varepsilon^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}, \qquad E^{(m)}_{\mathbf{k},\alpha} = \sqrt{\epsilon^2_{\mathbf{k}} + \mu_\alpha \epsilon_{\mathbf{k}}} \qquad \textit{with gap}: \; \Delta_k = 2g|\vec{B} \cdot \vec{k}|$$

- $\vec{B}, \; n_b, \; , n_a, \; \mu$ *determined by :*

  - Ø energy minimization (gap equation) $\rightarrow \dfrac{\partial E(\vec{B})}{\partial B_\alpha} = 0$
  - Ø atom number equation $\rightarrow 2n_b + n_a = n$



Isotropic resonance at T=0 $(\omega_\alpha = \omega_0)$

$$E = (u^2+v^2)\left[\omega_0 - 2\mu + a_1 \ln\{a_0(u+v)\}\right] + a_1 \frac{u^3+v^3}{u+v} + a_2\left[\left(u^2+v^2\right)^2 + \frac{1}{2}\left(u^2-v^2\right)^2\right]$$

BCS

$\text{BEC}: \; (\vec{B}^* \cdot \vec{B})^2 + \frac{1}{2}|\vec{B}\cdot\vec{B}|^2$

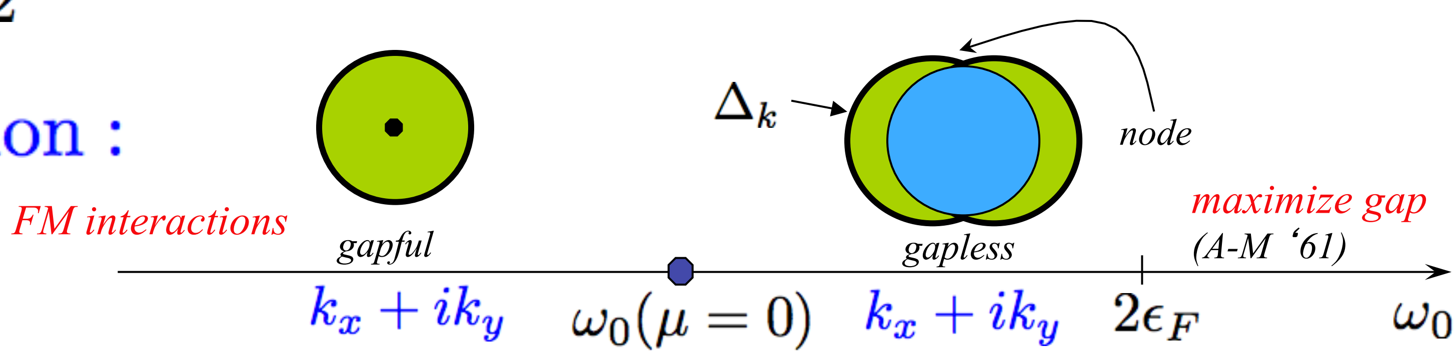
- BCS $(\omega_0 \gg 2\epsilon_F)$:
  - Ø $\mu \approx \epsilon_F + \mathcal{O}(\gamma)$
  - Ø $\dfrac{E_{k_x+ik_y}}{E_{k_x}} = \dfrac{1}{2}e > 1$

  ⇨ $(Anderson - Morel\ A_1\ phase)$

  $u = v \sim e^{-(\omega_0 - 2\epsilon_F)/\gamma\epsilon_F} \Longrightarrow k_x + ik_y \; (m=1)$

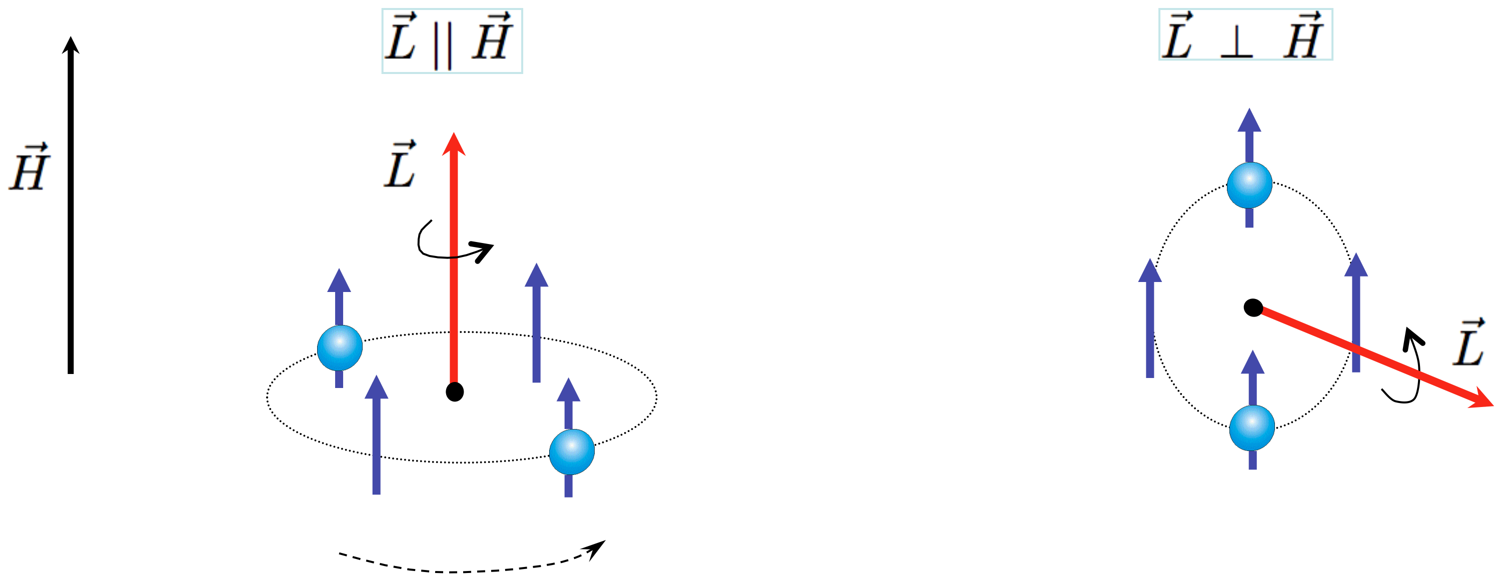
- BEC $(\omega \ll 2\epsilon_F)$:
  - Ø $\mu \approx \dfrac{1}{2}\omega_0 + \mathcal{O}(\gamma)$ ⇨ $u = v \approx \sqrt{n - n(\omega_0/2)} \Longrightarrow k_x + ik_y \; (m=1)$

- transition :

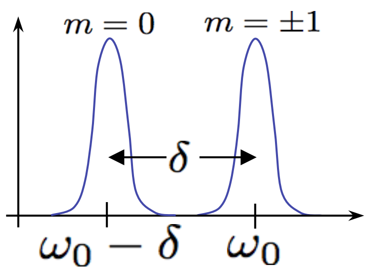


Dipolar-interaction FR splitting

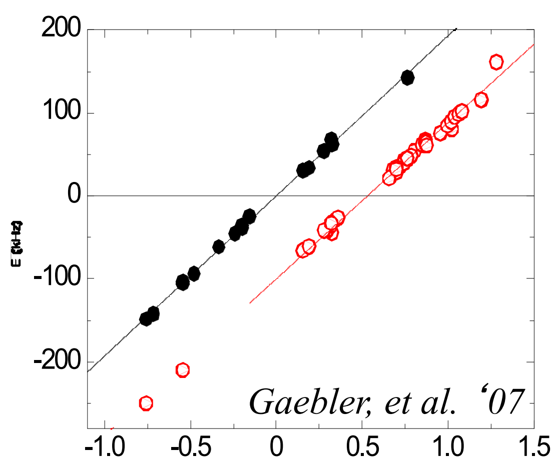
Ticknor, et al '03



$$H_{\text{molecule}} = \omega_{||} b^{\dagger}_{||} b_{||} + \omega_{\perp} \vec{b}^{\dagger}_{\perp} \cdot \vec{b}_{\perp} \qquad \omega_{||} < \omega_{\perp}$$



Ticknor, Regal, et al. '03

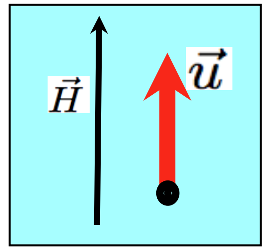


$$E = \sum_{\alpha} (u_{\alpha}^2 + v_{\alpha}^2)[\omega_{\alpha} - 2\mu + \ldots$$

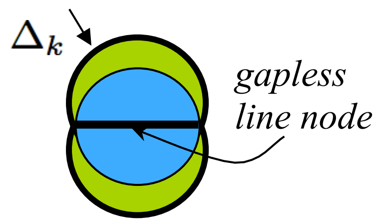
P-wave superfluid phases

$$E=\sum_\alpha (u_\alpha^2+v_\alpha^2)\left[\omega_{0\alpha}-2\mu+a_1\ln\{a_0(u+v)\}\right]+a_1\frac{u^3+v^3}{u+v}+a_2\left[\left(u^2+v^2\right)^2+\frac{1}{2}\left(u^2-v^2\right)^2\right]$$

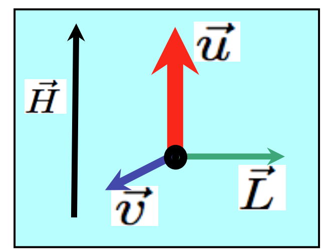
- k_x - state: *β-phase of ^3He* $(m_{||}=0)$



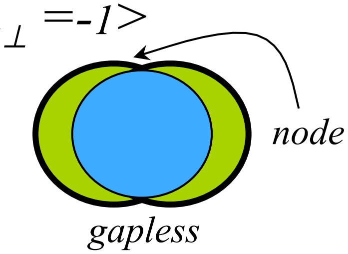
  - $u \approx u_0 e^{\delta},\ \ v=0$
  - equatorial node line for $\mu > 0 \ \Rightarrow\ C \sim T^{\alpha}$, …
  - fully gapped for $\mu < 0 \ \Rightarrow C \sim e^{-|B-B_0|^2/T}$, …
  - spontaneously broken symmetries: *U(1)*



- $k_x + i\,\sigma\, k_y$- state: *"deformed" A_1 -phase of ^3He* $(m_\perp=1)$



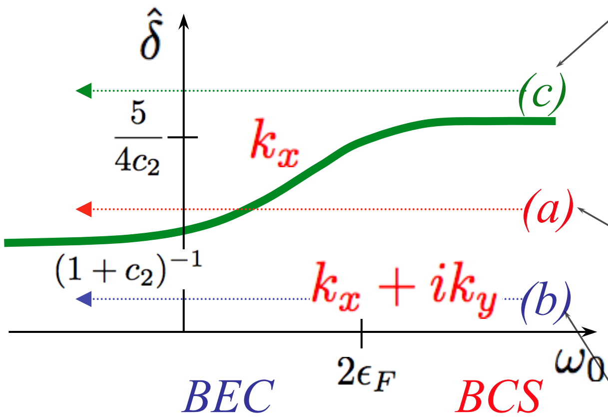
  - $u \approx u_0\,(1+\delta)\, e^{\delta/2},\ v \approx u_0\,(1-\delta)\, e^{\delta/2} \ \Rightarrow\ |m_\perp=1\rangle + \delta\,|m_\perp=-1\rangle$
  - polar point nodes for $\mu > 0 \Rightarrow C \sim T^{\alpha}$, …
  - fully gapped for $\mu < 0 \ \Rightarrow C \sim e^{-|B-B_0|^2/T}$, …
  - spontaneously broken symmetries: *U(1), O(2), T*

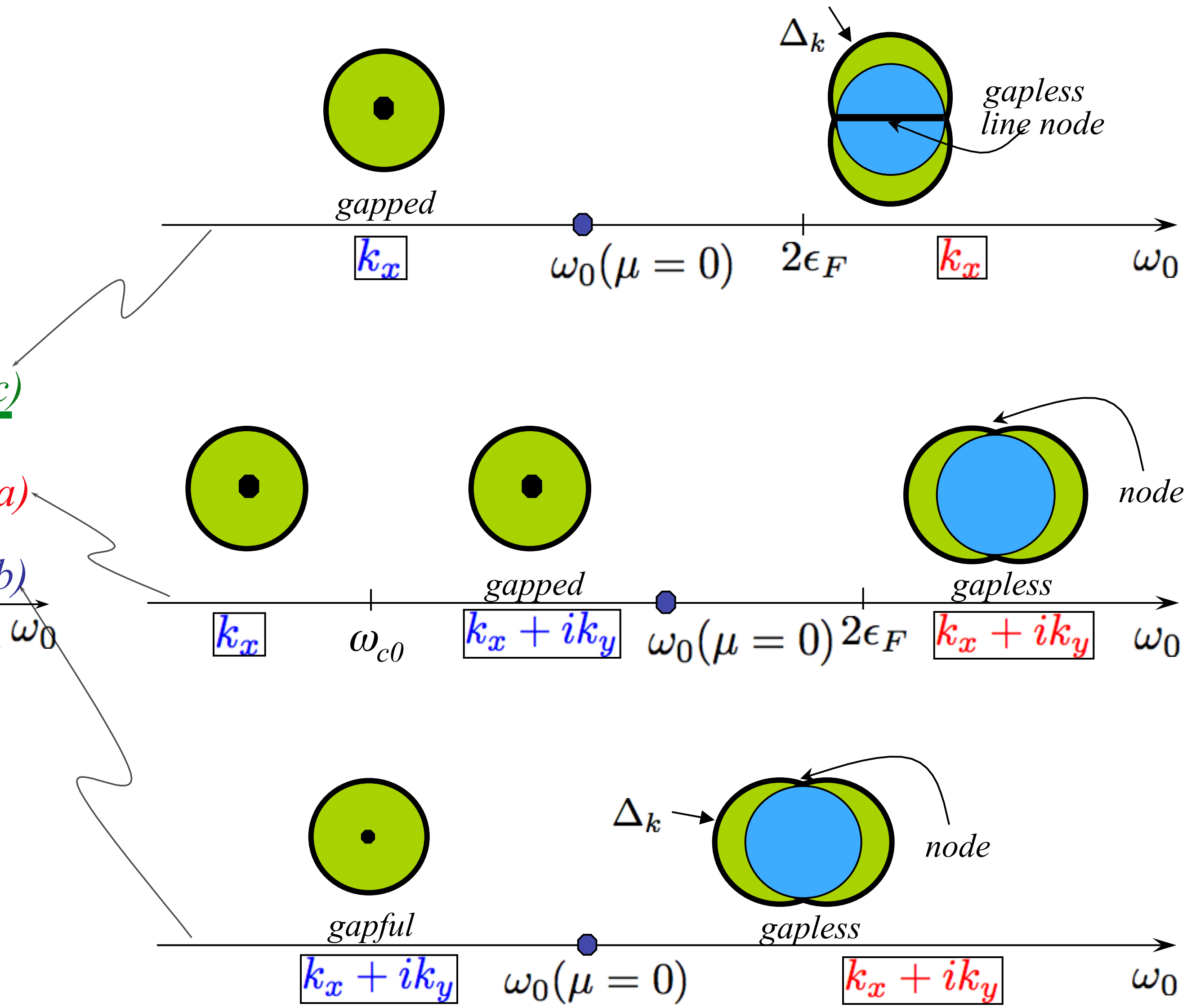


T=0 phase diagrams

also see Cheng, Yip '05

$$E = \sum_\alpha (u_\alpha^2 + v_\alpha^2)\left[\omega_{0\alpha} - 2\mu + a_1 \ln\{a_0 (u+v)\}\right] + a_1 \frac{u^3+v^3}{u+v} + a_2 \left[(u^2+v^2)^2 + \frac{1}{2}(u^2-v^2)^2 \right]$$



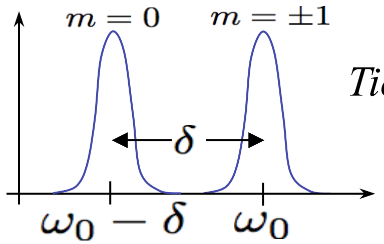


$$c_2 = \frac{m^2}{3\pi^2}\int_0^\infty dk g_k^2 \sim m^2 g_\Lambda^2 \Lambda \sim \frac{\Gamma_\Lambda}{E_\Lambda}$$

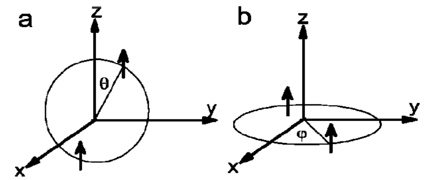
Anisotropic p-wave superfluidity

Gurarie, L.R., Andreev '05
Cheng and Yip '05

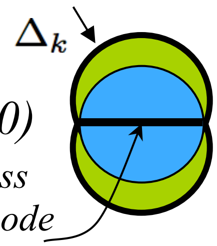
- *resonance splits:*

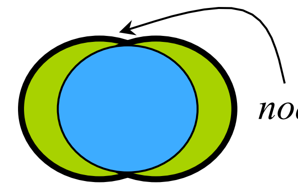


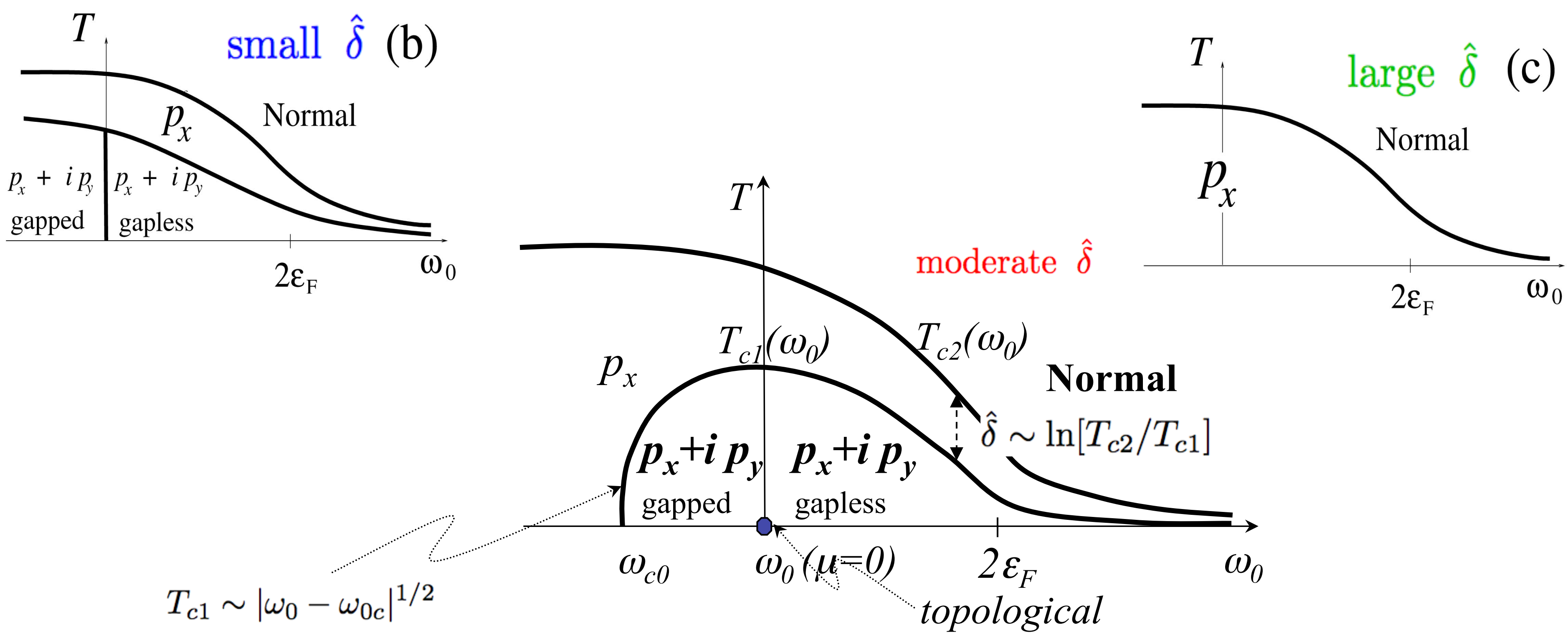
Ticknor, Regal, et al.



- *two competing states:* p_x $(m_z=0)$ and $p_x+i\,p_y$ $(m_z=\pm1)$

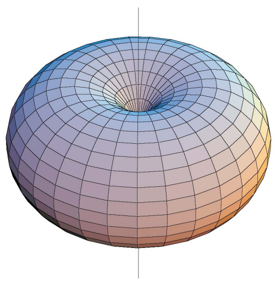


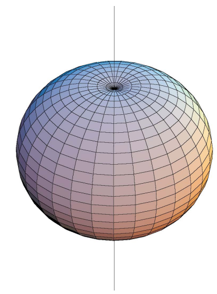




Gapless ⟷ gapped superfluid transitions

$p_x + i\,p_y$ $\xrightarrow{\mu \text{ changes sign}}$ $p_x + i\,p_y$





$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,(p_x^2 + p_y^2)\,|B|^2}$$

G. E. Volovik, JETP Lett. 80, 343 (2004)

p_x $\xrightarrow{\mu \text{ changes sign}}$ p_x

$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + g^2\,p_x^2\,|B|^2}$$

$$E_p = \sqrt{\left(\frac{p^2}{2m} + |\mu|\right)^2 + g^2\,p_x^2\,|B|^2}$$

$p_x + i\,p_y$ superfluid in 2D

- Pfaffian (Moore-Read) state from FQH $|p_x + ip_{y\,BCS}\rangle = \prod_p \left[u_p + v_p a^\dagger_{-p} a^\dagger_p\right] |0\rangle$

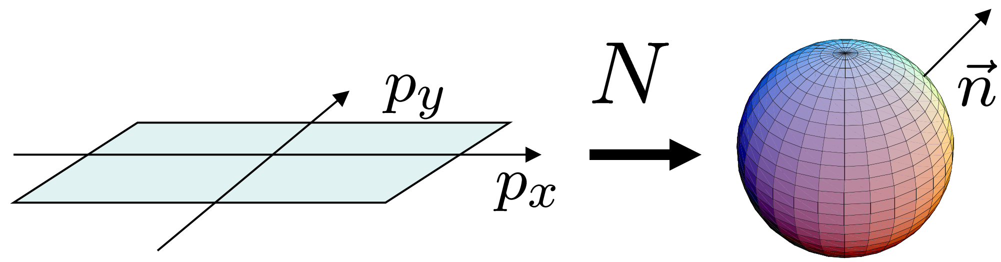
$$\Psi(z_1, z_2, \ldots, z_{2N}) = \sum_P (-1)^P \frac{1}{z_{P_1} - z_{P_2}} \frac{1}{z_{P_3} - z_{P_4}} \cdots \frac{1}{z_{P_{N-1}} - z_{P_N}}$$

- topological classification in terms of u_p and v_p

Anderson's pseudospin

$$\begin{cases} n_x + in_y = 2v^* u \\ n_z = |v|^2 - |u|^2 \end{cases}$$

$$\vec{n} = \frac{1}{\sqrt{\left(\frac{p^2}{2m_a} - \mu\right)^2 + 4B^2\left(p_x^2 + p_y^2\right)}} \begin{pmatrix} 2gBp_x \\ -2gBp_y \\ \frac{p^2}{2m_a} - \mu \end{pmatrix}$$



Explicit calculations show that

N=0 if $\mu < 0$

N=1 if $\mu > 0$

$$N = \frac{1}{8\pi} \int d^2p \,\left[\vec{n} \cdot \partial_\alpha \vec{n} \times \partial_\beta \vec{n}\; \epsilon_{\alpha\beta}\right]$$

topological invariant

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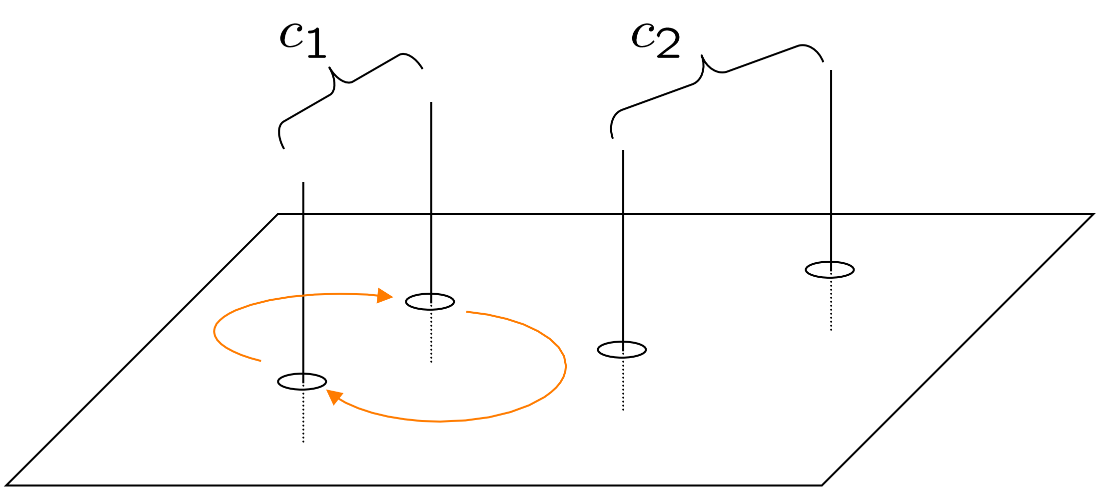
- topological classification in terms of u_p and v_p
- gapped (N=1, BCS) ⇒ gapped (N=0, BEC) superfluid transition at $\mu=0$

  Read and Green, PRB 61, 10267 (2000)

- vortex excitations with non-Abelian statistics *Ivanov, PRL (2001)*



one fermion (2 states –either empty or occupied fermion) per two vortices

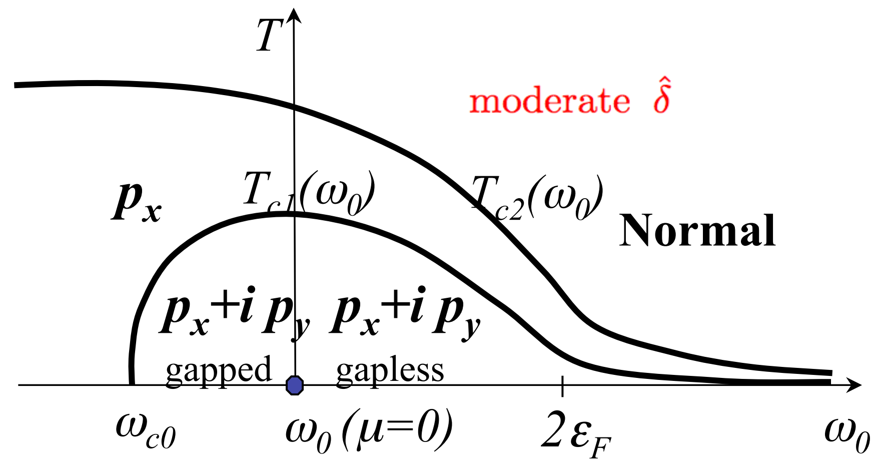
$2^{\frac{n}{2}}$ states per n vortices

- suggested to be used as qubits for quantum computers

  Kitaev, Ann. Phys. 303, 2 (2003)

Summary of p-wave superfluidity

- mapped out T, $\omega_0 \propto B$, δ phase diagram for p-wave Feshbach resonant Fermi gas
  - *p_x and $p_x + i\,p_y$ superfluids*
  - *thermal, quantum and topological SF => SF transitions*



- quantitatively accurate description for small $\gamma=\Gamma/\varepsilon_F$ (low n)

- realization of topological states, majorana zero modes, and non-Abelian statistics of Pfaffian (Moore-Read) state

- p-wave Feshbach molecules observed in K^{40}

- …BUT
  - *short (msec) molecular lifetime (see Levinson, et al, PRL 2007)*
  - *what about Li^6 ?*
  - *need better quantitative understanding of stability*