Paired superfluidity in resonant atomic gases

Leo Radzihovsky

for details see: *Gurarie, L.R., Annals of Physics, 322, 2-119 (2007)*
Veillette, Sheehy, L.R., PRA 75, 043614 (2007)

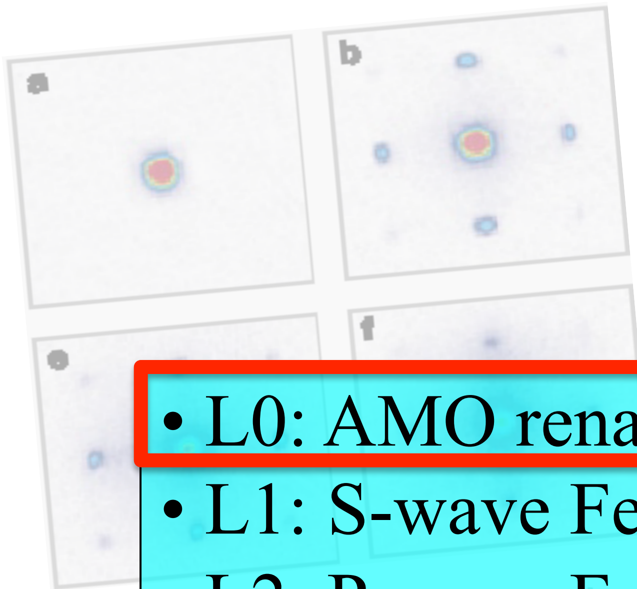
Sheehy, L.R., Annals of Physics, 322, 1790 (2007)
Nicolic, Sachdev, PRA 75, 033608 (2007)
Giorgini, et al., RMP, 80, 885 (2008)
Ketterle and Zwierlein, Varenna lectures (2006)

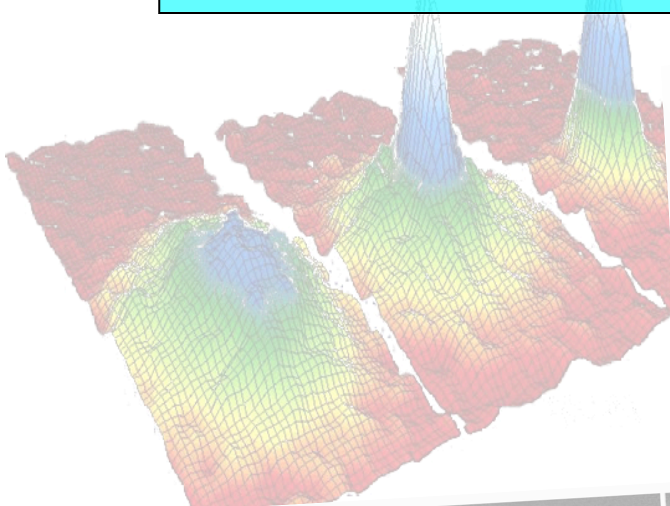
$: NSF

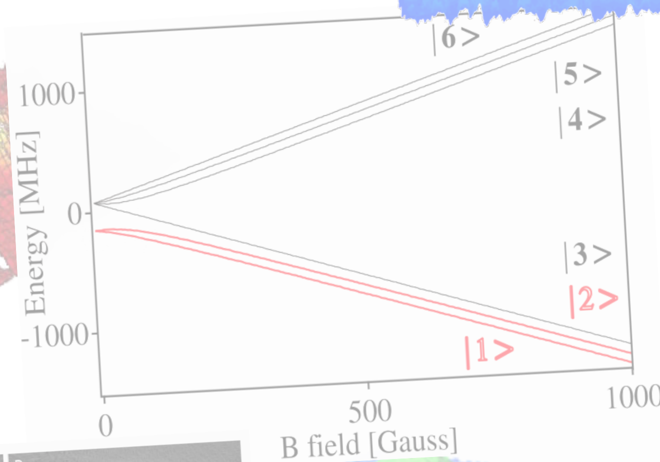
BSS2014, Boulder, CO, July 2014

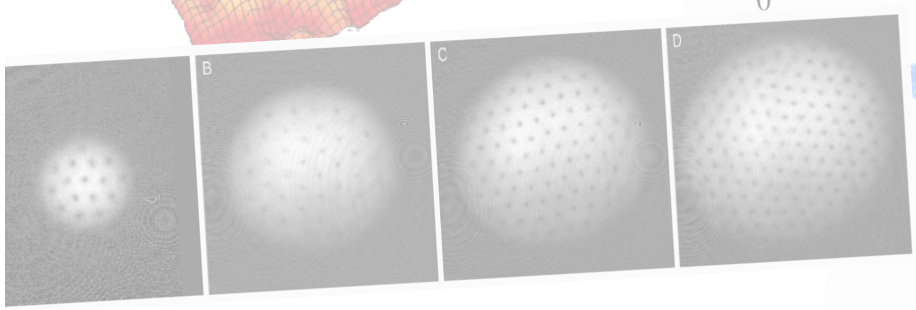
Course outline

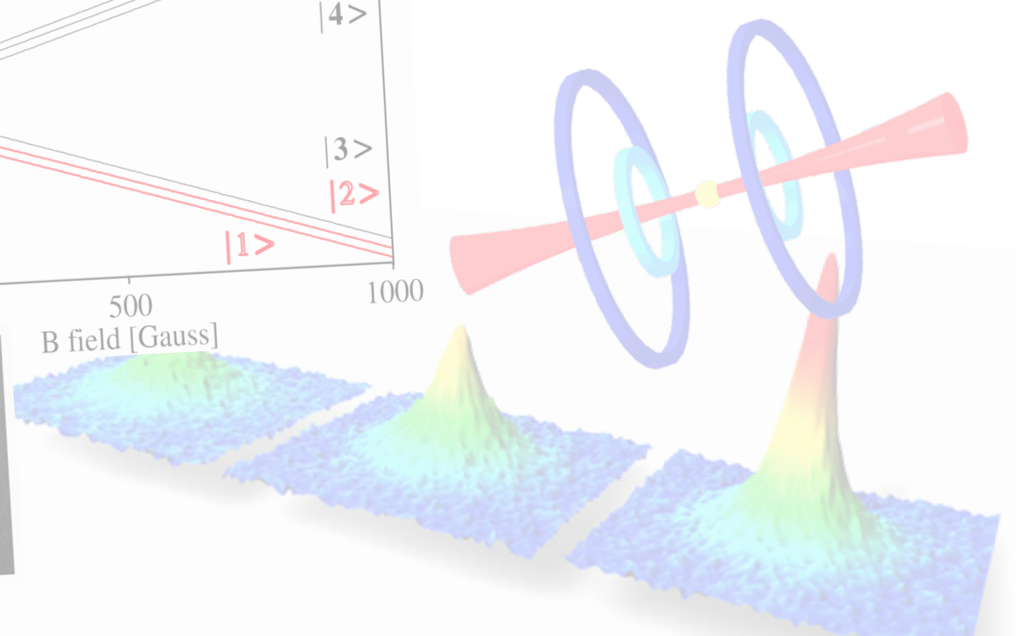
- L0: AMO renaissance and scattering theory overview
- L1: S-wave Feshbach resonant superfluidity
- L2: P-wave Feshbach resonant superfluidity



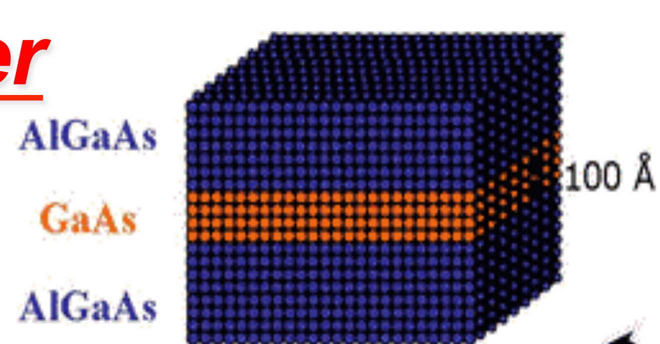


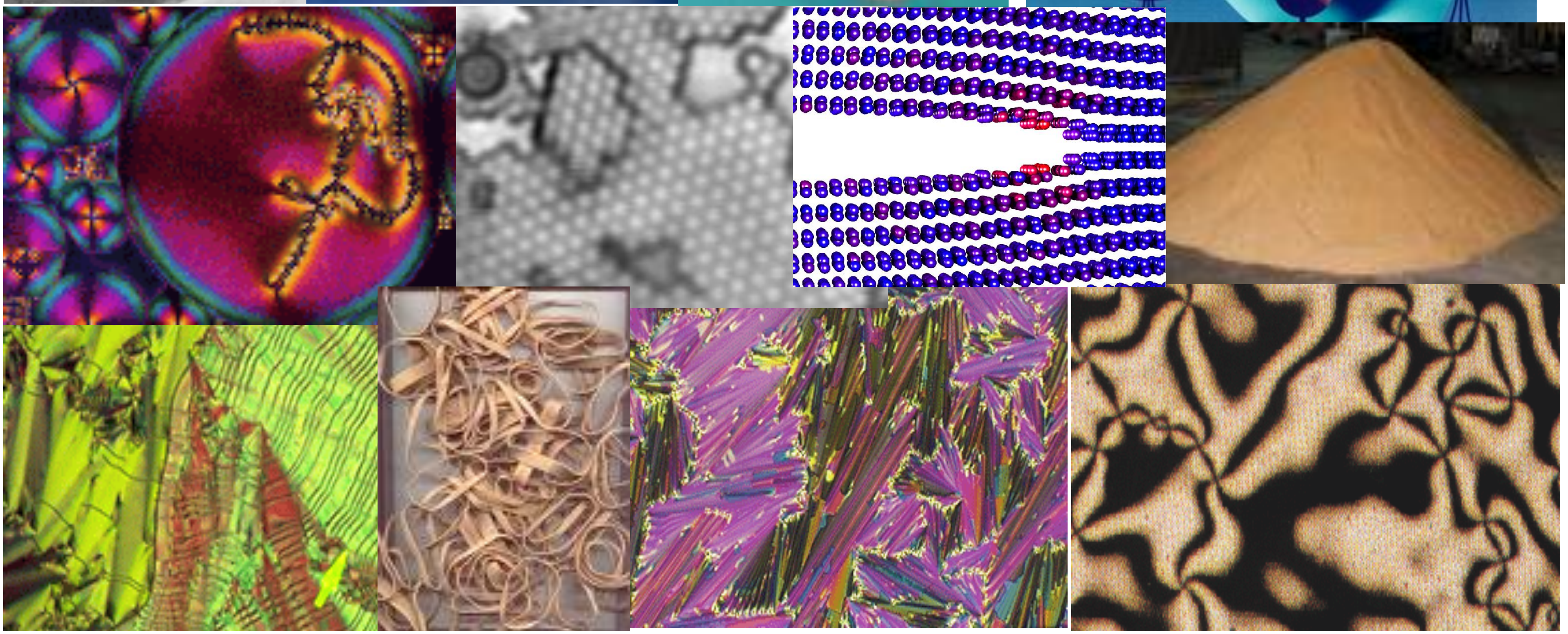






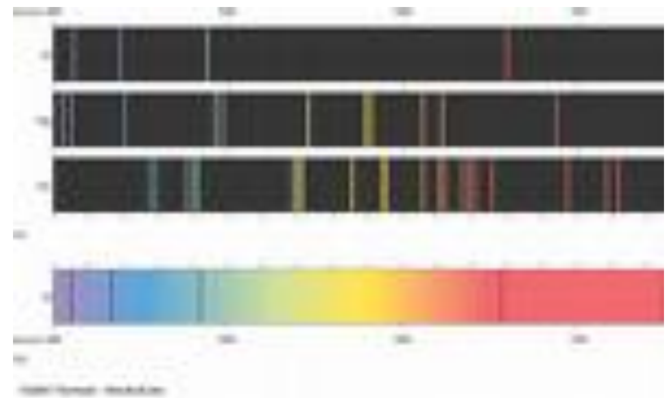
Condensed matter

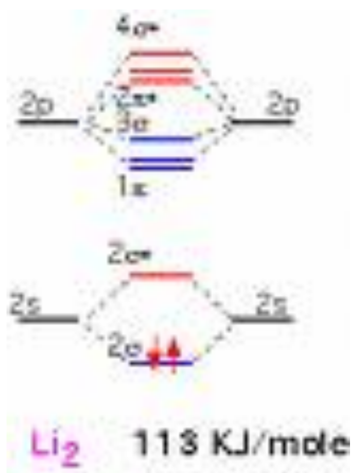


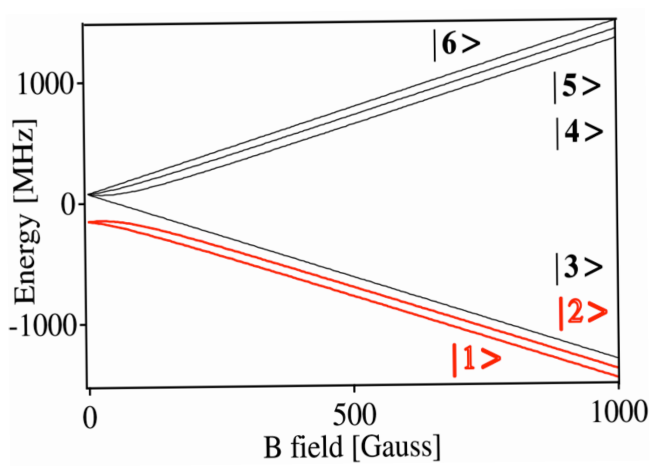


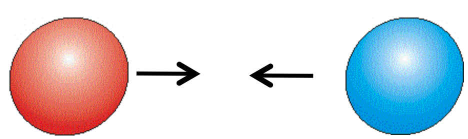
<u>***Atomic physics***</u> *(naïve view)*

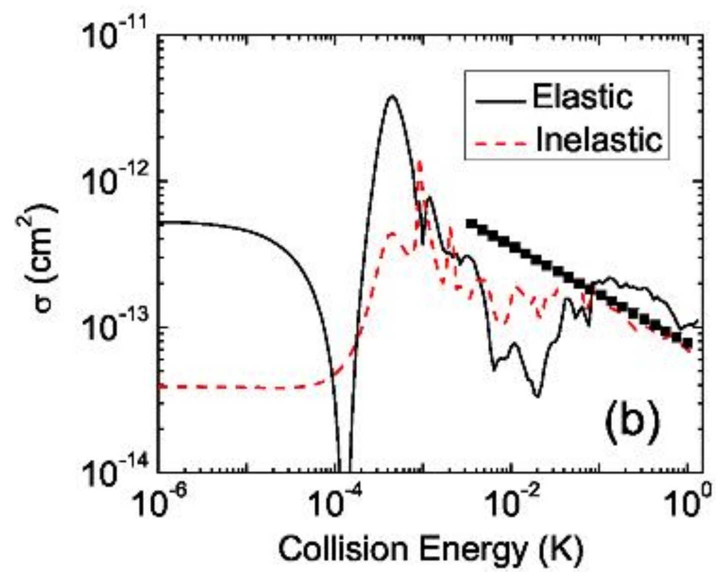
- atomic spectra
- collisions
- molecules
- laser-atom interaction







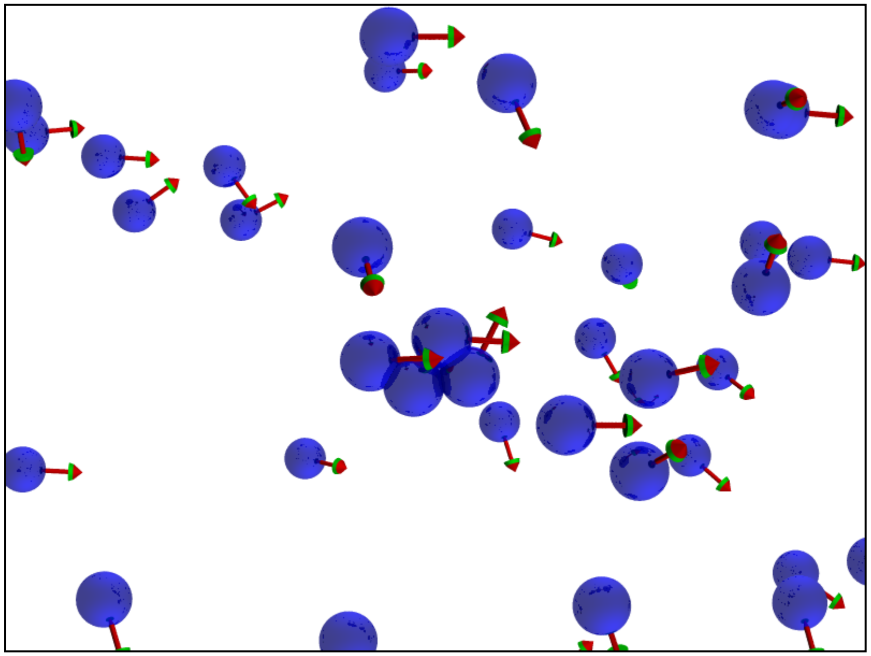




Dilute atomic gases

• density ~ 10^{12} cm^{-3} ⇔ d ~ 10^4 Å, mfp ~ 10 cm

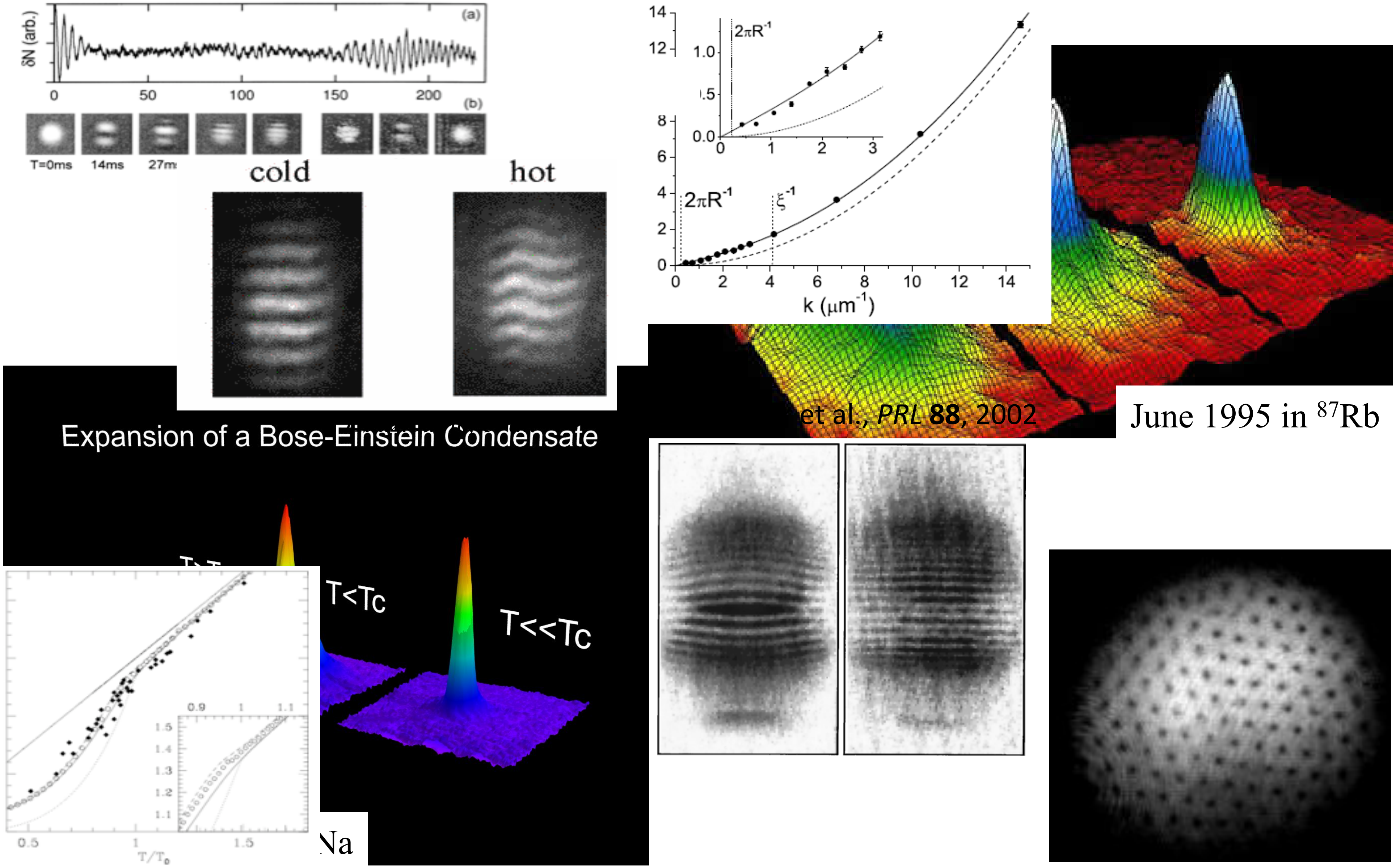
(cf. density$_{air}$ = 10^{19} cm^{-3} ⇔ d$_{air}$ ~ 10^2 Å)



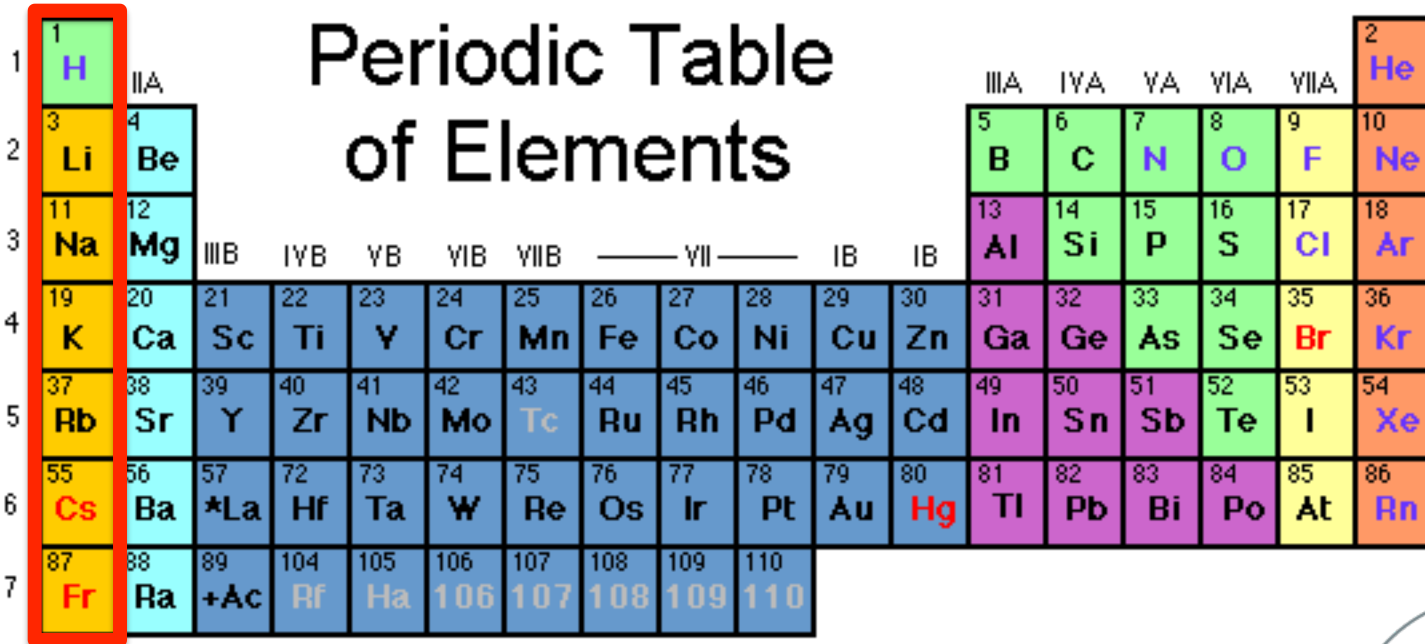
• classically: ⇒ (boring) IDEAL GAS

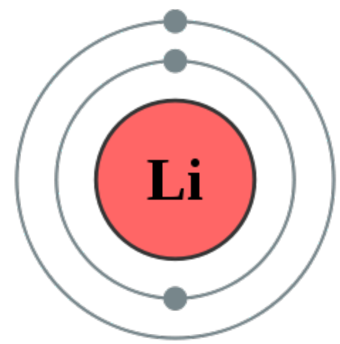
Revolution in AMO physics

- degenerate Bose and Fermi atomic gases

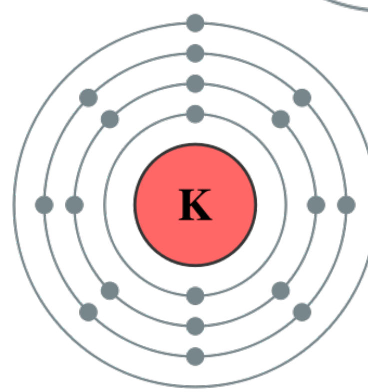


Alkali atoms



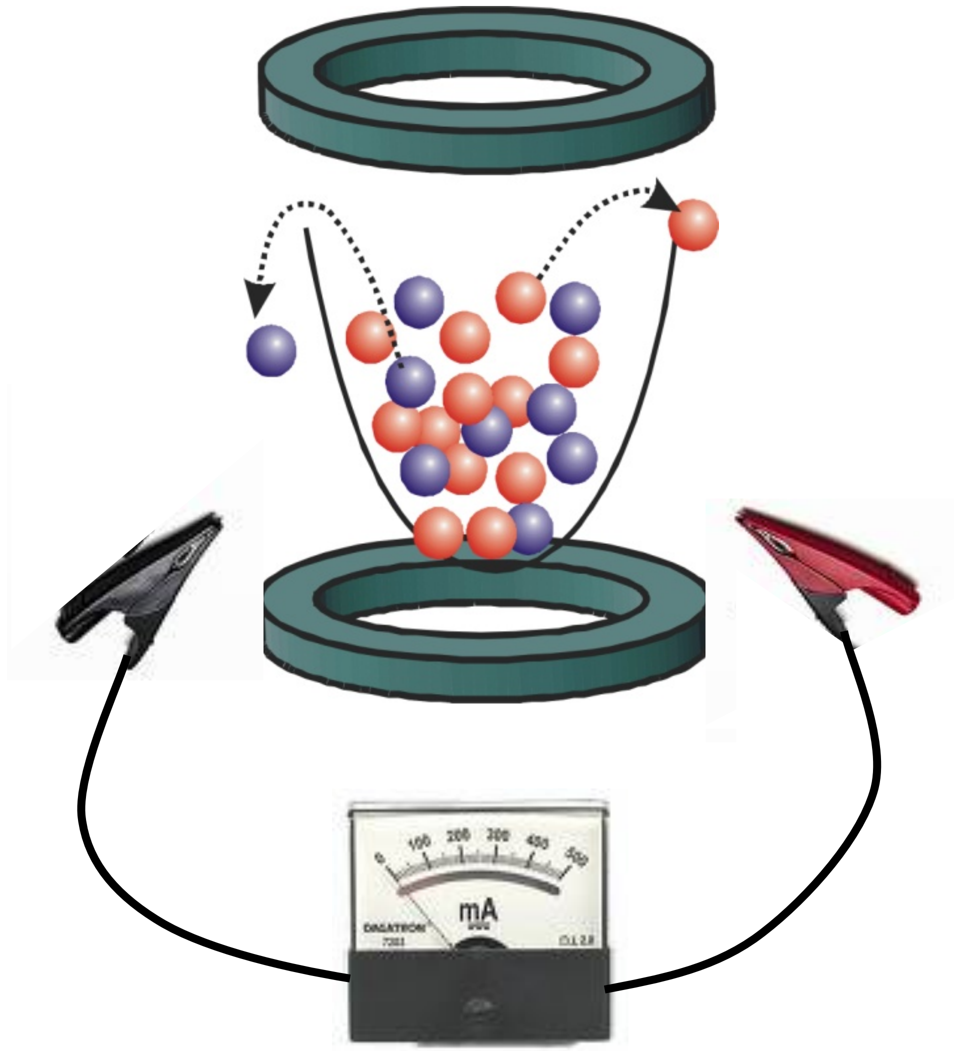


- Li 6: $2S_{1/2}$ $|n=2,l=0,s=1/2,s_z>|i=1,i_z>$



- K40: $4S_{1/2}$ $|n=4,l=0,s=1/2,s_z>|i=4,i_z>$

Condensed matter with cold atomic gases?

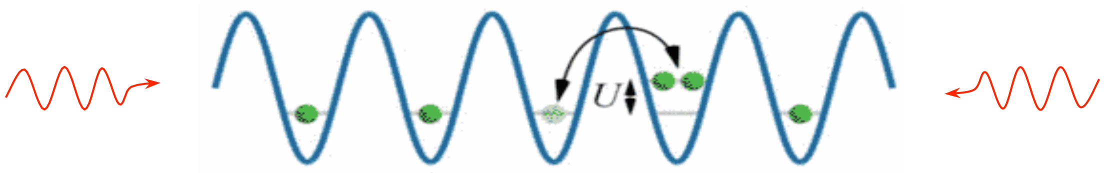


need strong interactions

Optical lattices

I. Bloch '98

- standing-wave of interfering laser beams *(cf optical tweezers)*

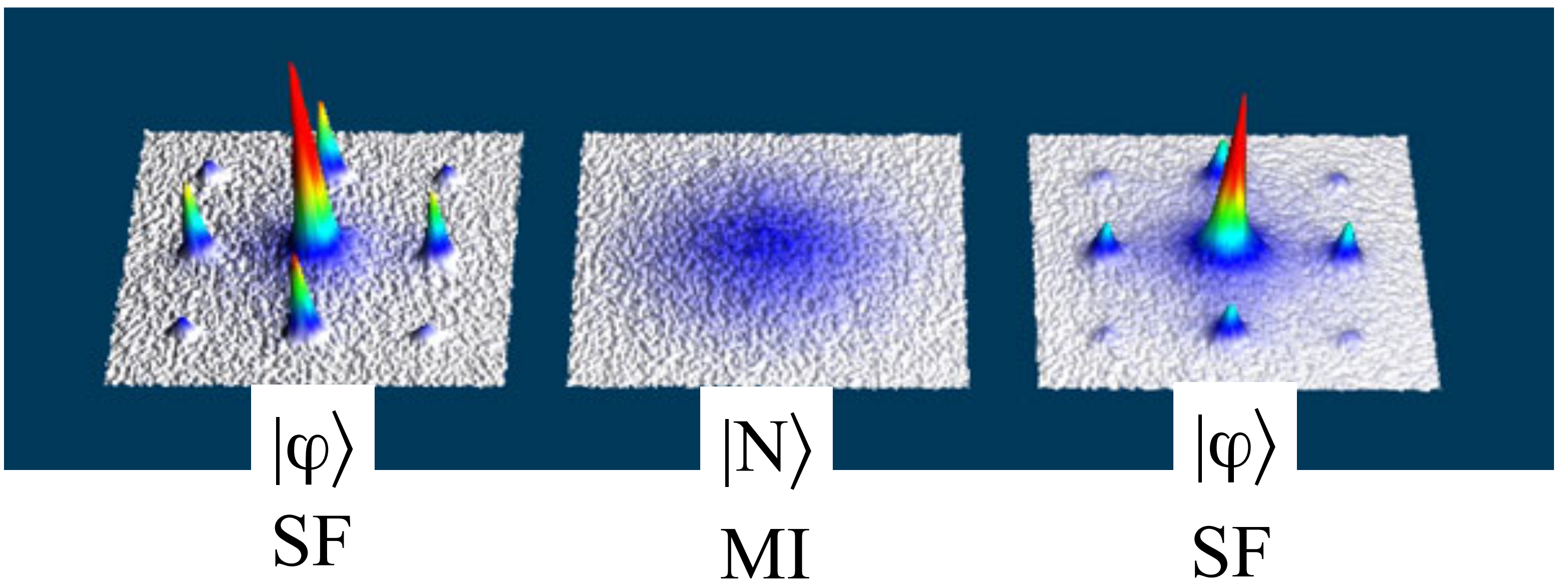


ac-Stark effect
(red-detuned, attractive)

$$V(r) = E_g - \frac{\frac{1}{2} d^2 I(r)}{E_{eg} - \omega_L}$$

- Superfluid-Insulator transition of bosons *(Doniach'81, Fisher, et al. '89)*

- realization in cold atoms *(M. Greiner, et al., '01, Jaksch, et a '98)*



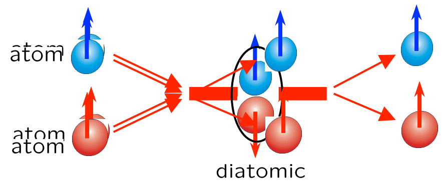
- …and much, much more

Feshbach resonance

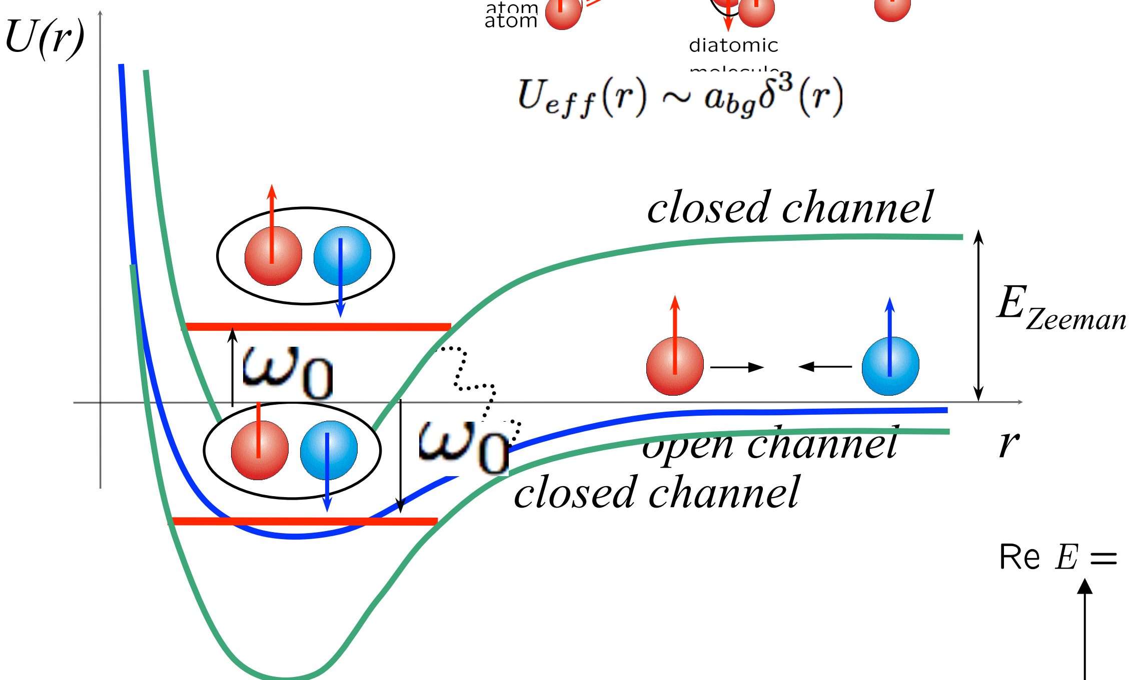
Ketterle, '98

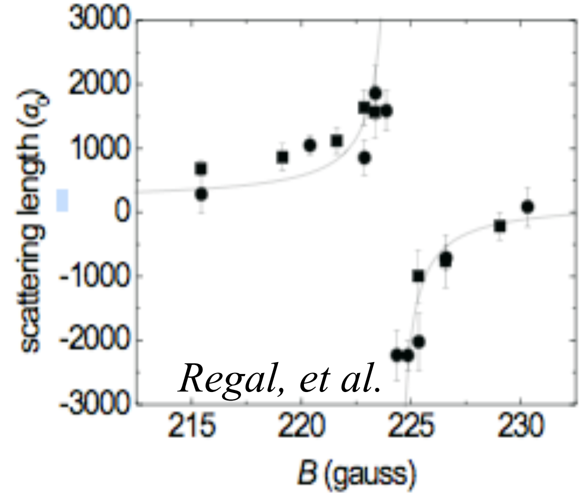
- tunability (strength and sign) of interactions (sudden and adiabatic) via Feshbach resonances

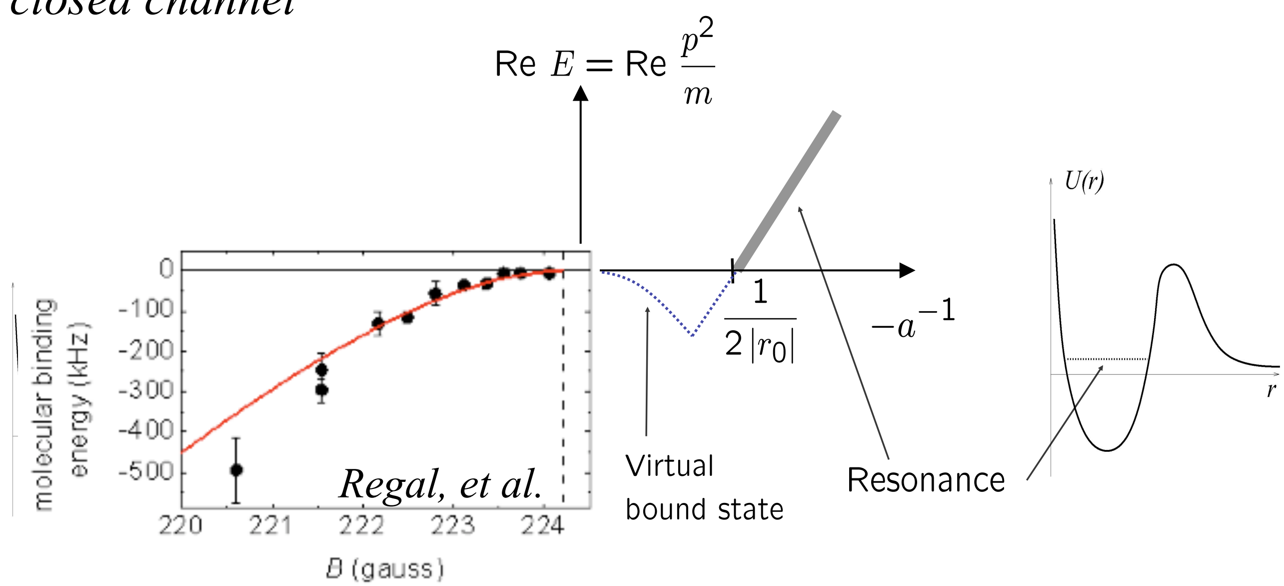
$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



$$U_{eff}(r) \sim a_{bg}\delta^3(r)$$





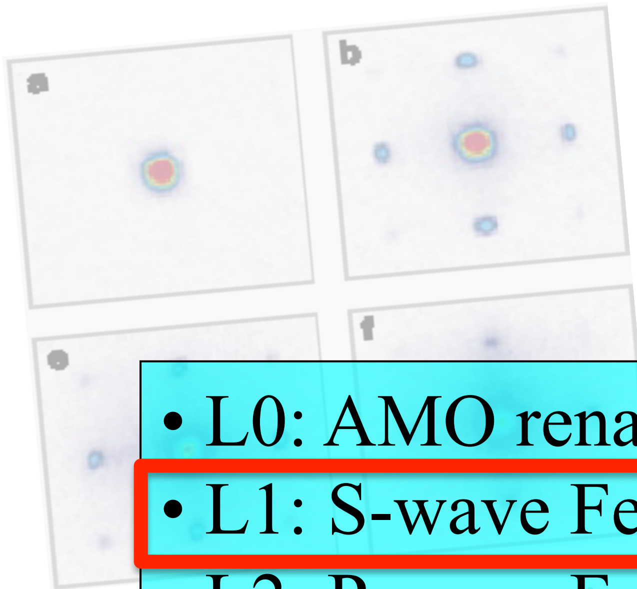


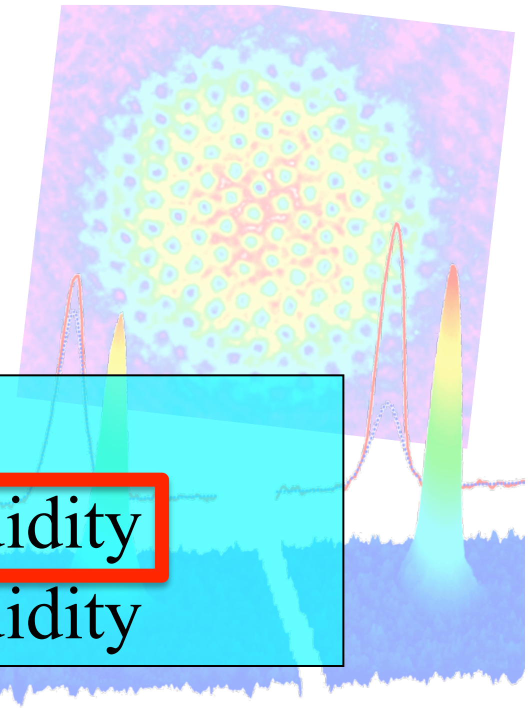
Feshbach resonances on youtube

"*Quantum decoupling transition in a one-dimensional* superfluid", Sheehy and Radzihovsky, PRL (2005)

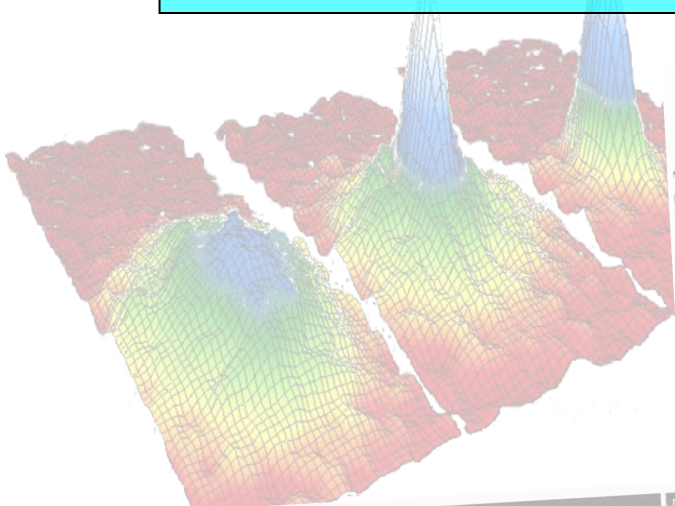
(song by Jonathan Mann, 2009)

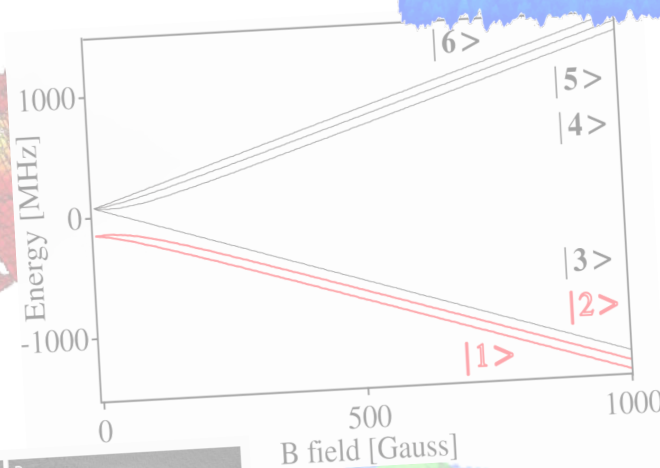
Course outline

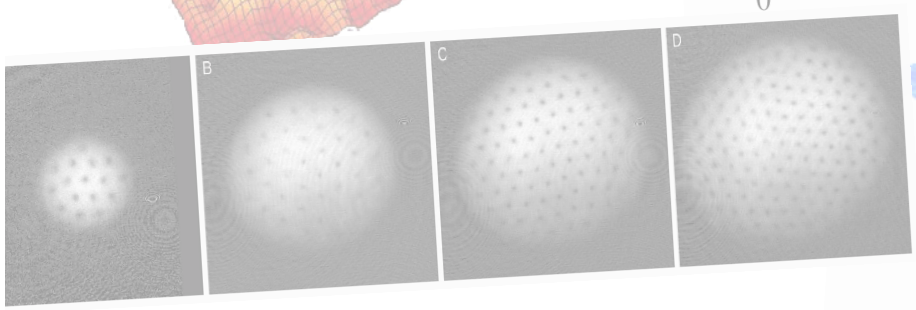


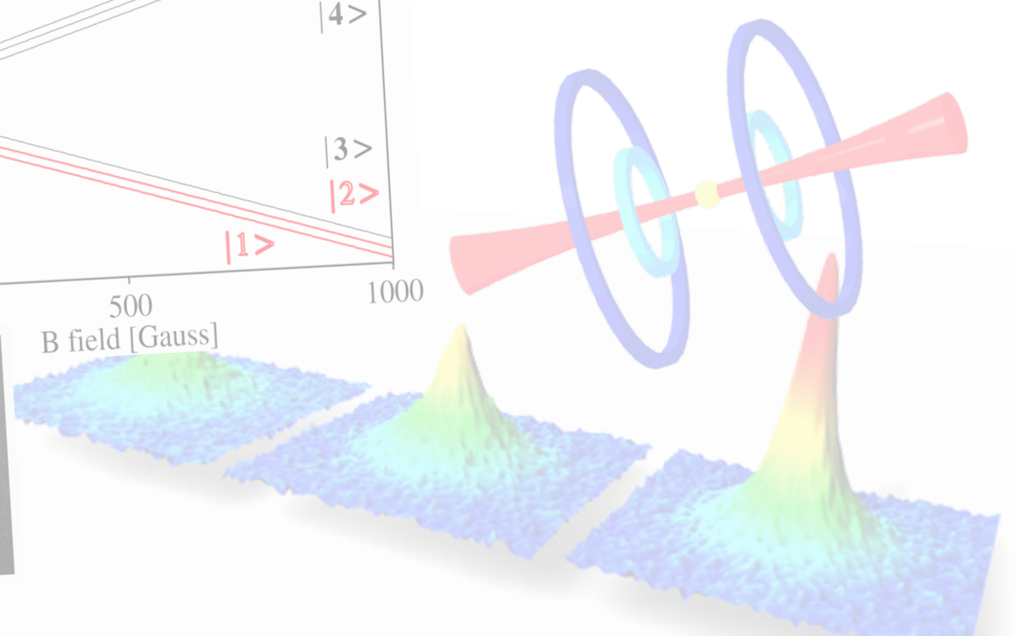


- L0: AMO renaissance overview
- L1: S-wave Feshbach resonant superfluidity
- L2: P-wave Feshbach resonant superfluidity



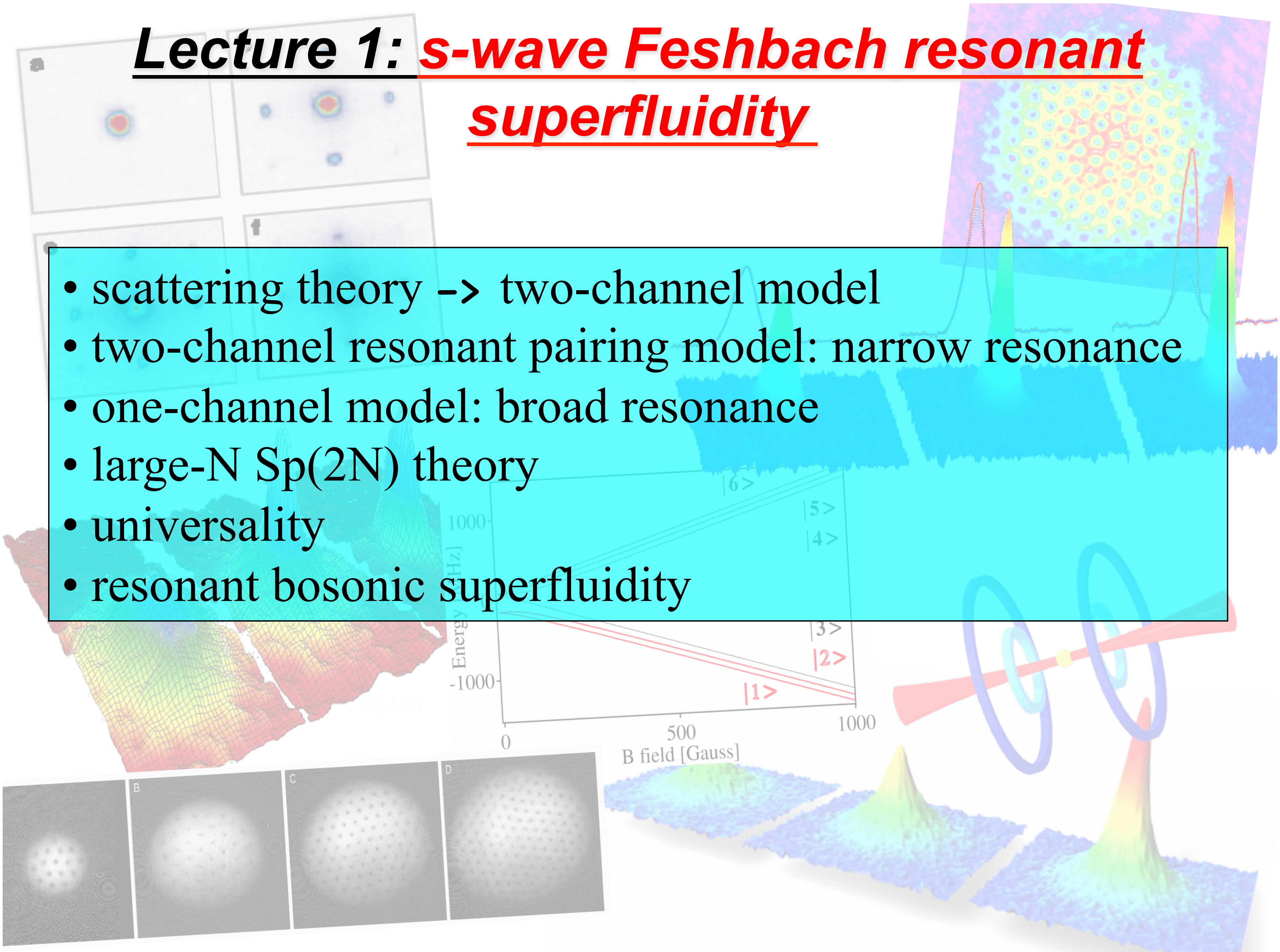




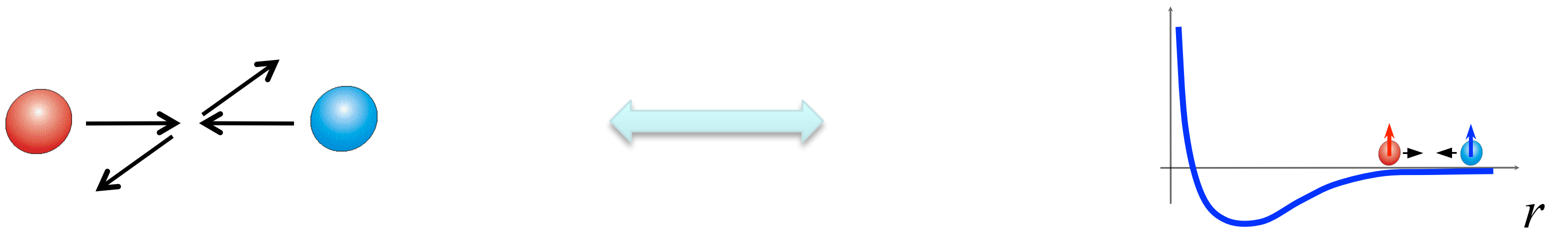


Lecture 1: ***s-wave Feshbach resonant superfluidity***

- scattering theory -> two-channel model
- two-channel resonant pairing model: narrow resonance
- one-channel model: broad resonance
- large-N Sp(2N) theory
- universality
- resonant bosonic superfluidity



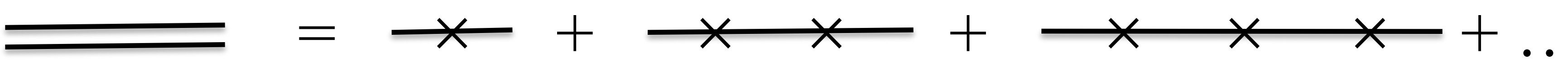
Review of scattering theory



- Schrodinger eqn via Greens function: $(H_0 + V)\psi = E\psi$

$$\psi = \psi_0 + \frac{1}{E - H_0} V\psi = \psi_0 + \frac{1}{E - H_0} T\psi_0$$

- T-matrix: $T = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V\ldots = V + V\frac{1}{E-H_0}T$



$$\psi = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{2m_r}{\hbar^2}\int_{k'} \frac{T(k',k)e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 - k'^2 + i\varepsilon} = e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{e^{ikr}}{r}\underbrace{\frac{-2m_r}{4\pi\hbar^2}T(\mathbf{k}',\mathbf{k})}$$

- Scattering amplitude: $f(\mathbf{k}',\mathbf{k}) = \sum_\ell (2\ell+1) f_\ell(k) P_\ell(\hat{\mathbf{k}}'\cdot\hat{\mathbf{k}})$

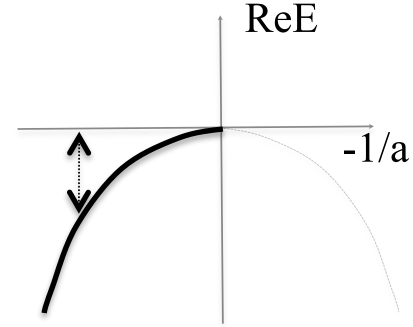
- Scattering matrix S and phase shift δ: $S_\ell = e^{i2\delta_\ell} = 2ikf_\ell + 1 = \frac{f_\ell}{f_\ell^*}$

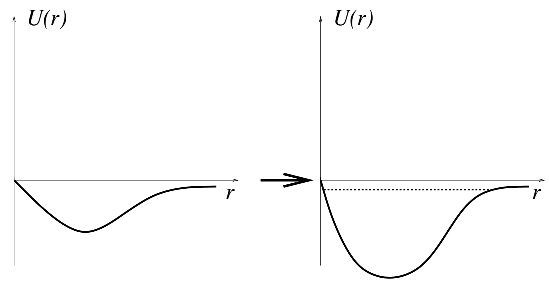
Low-E resonant scattering phenomenology

- unitarity: $$f_s = \frac{1}{F_s(k^2) - ik} \approx \frac{1}{-a_s^{-1} + \frac{r_0}{2}k^2 - ik} \qquad \left(f_\ell = \frac{1}{k^{-2\ell} F_\ell(k^2) - ik} \right)$$

$$-ka = \tan\delta_0 \qquad (F_\ell(k^2) = k^{2\ell+1}\cot\delta_\ell)$$

- low energy: $$k_{pole} = ia^{-1} \rightarrow E_{bound} = -\frac{\hbar^2}{2m_r a^2}$$

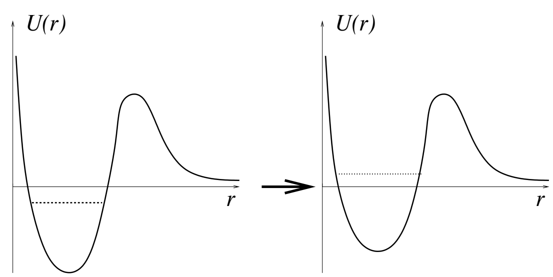


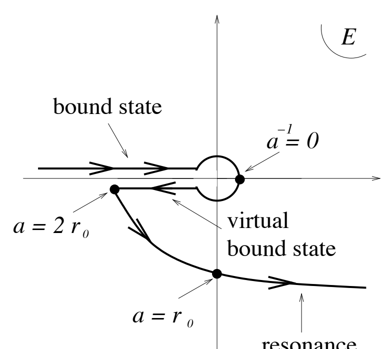


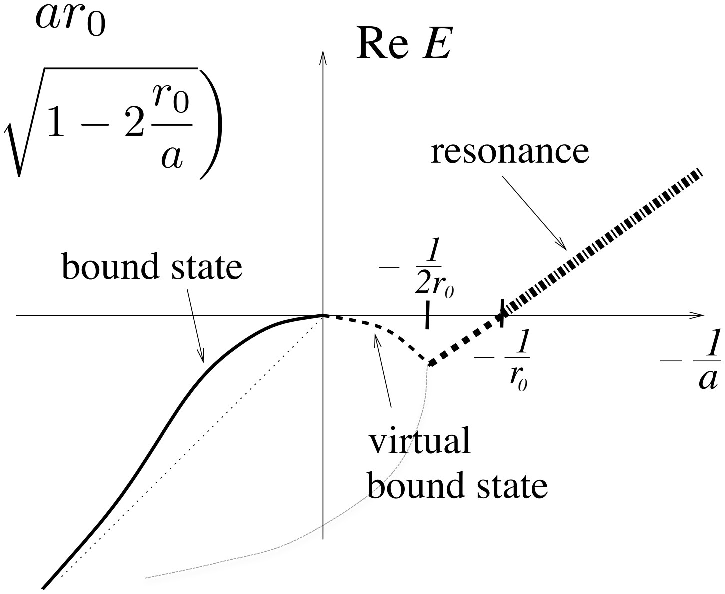
$$\psi \sim e^{ik_{\text{pole}}r - iE_{\text{pole}}t}$$

- intermediate energy ($r_0 < 0$): $$k^{\pm}_{\text{pole}} = \frac{i}{r_0} \pm \frac{\sqrt{2ar_0 - a^2}}{ar_0}$$

$$\rightarrow E_{\text{pole}} = \frac{1}{m_r r_0^2}\left(\frac{r_0}{a} - 1 + \sqrt{1 - 2\frac{r_0}{a}}\right)$$

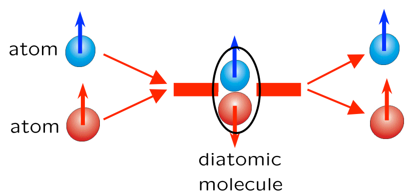


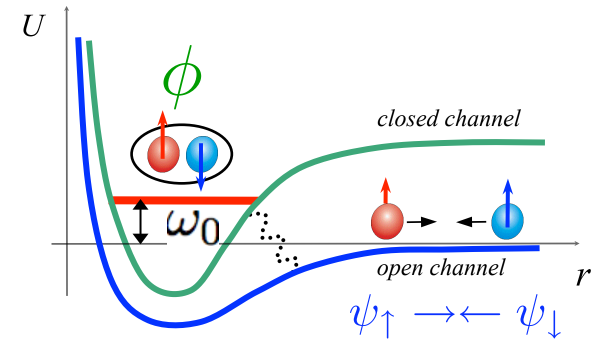




S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)





two-channel model:

("bare") detuning

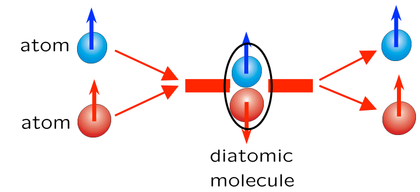
$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

atoms (open channel)

molecules (closed channel)

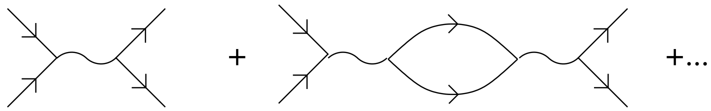
atom – molecules interconversion

S-wave FR scattering:details



$$\mathcal{S}_{2ch} = \overline{\psi}_\sigma(i\partial_t - \frac{p^2}{2m})\psi_\sigma + \overline{\phi}(i\partial_t - \frac{p^2}{4m})\phi + g\overline{\phi}\psi_\uparrow\psi_\downarrow + c.c.$$

$$T(\mathbf{k},\mathbf{k}') = gD_0g + gD_0g\Pi gD_0g + \cdots = \frac{g^2}{\omega - \frac{p^2}{4m} - \epsilon_0 - g^2\Pi}$$



$$\Pi(k) = \int_{\nu,\mathbf{q}} \frac{i}{(\omega - \nu - \frac{k_1^2}{2m} + i0)(\nu - \frac{k_2^2}{2m} + i0)}$$

$$= \int \frac{d^3q}{(2\pi)^3}\frac{1}{(\omega - \frac{p^2}{4m}) - \frac{q^2}{m} + i0} = -\frac{m\Lambda}{2\pi^2} - i\frac{m}{4\pi}k$$

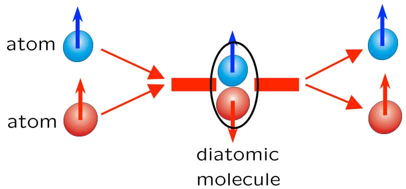
$$f_s(k) = \frac{1}{F(k^2) - ik}$$

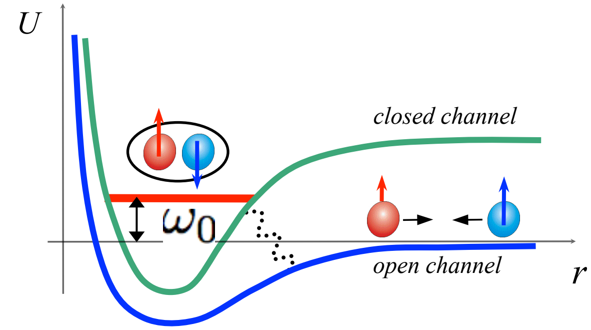
$$\longrightarrow f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2}k^2 - ik}, \text{ with } a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

$$= \frac{-\sqrt{\Gamma_0/m}}{E - \omega_0 + i\sqrt{\Gamma_0 E}} \qquad (\omega_0 = \epsilon_0 - g^2\Lambda m, \quad \Gamma_0 = g^4m^3)$$

S-wave Feshbach resonant scattering

- tunability (strength and sign) of interactions (sudden and adiabatic)

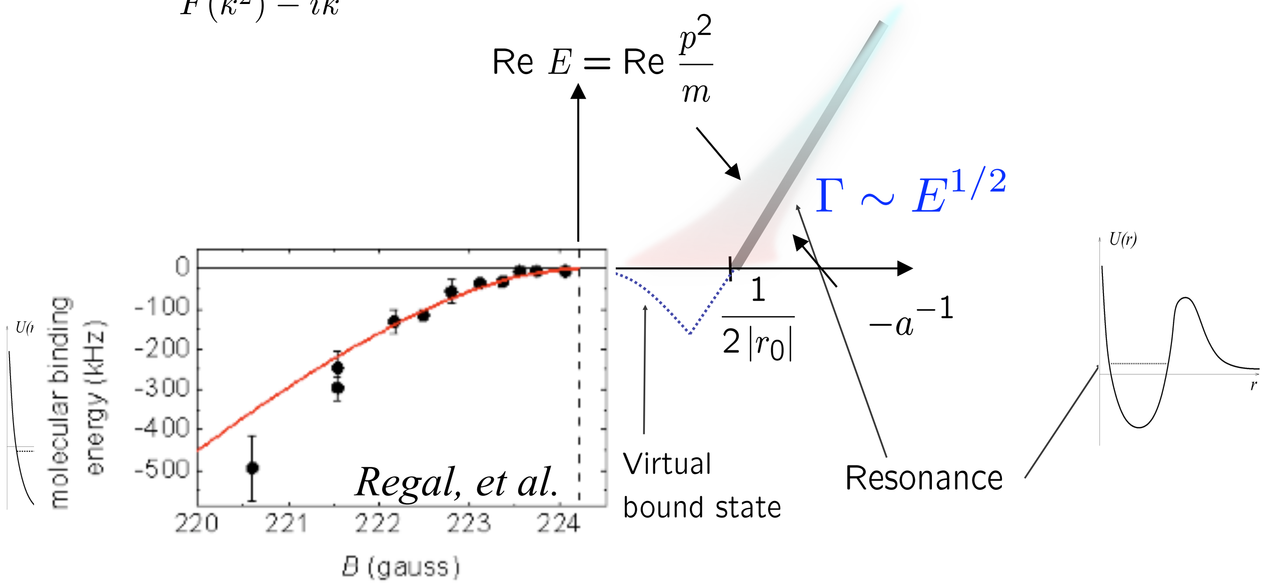


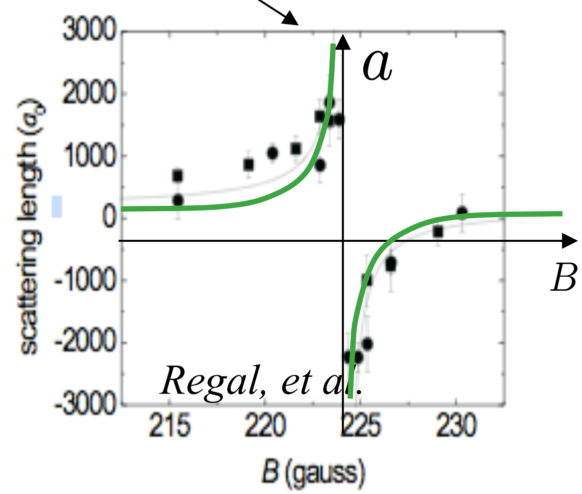


$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \frac{\hat{p}^2}{2m}\psi_\sigma + \phi^\dagger\Big(\frac{\hat{p}^2}{4m} + \epsilon_0\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

$$\longrightarrow\; f_s(k) = \frac{1}{-a^{-1} + \frac{r_0}{2}k^2 - ik}, \quad \text{with} \quad a \sim -\frac{g^2}{\omega_0} \sim a_{bg}\frac{\Delta B}{B_0 - B}, \quad r_0 \sim -\frac{1}{g^2}$$

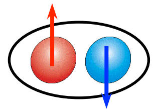
$$f_s(k) = \frac{1}{F(k^2) - ik}$$



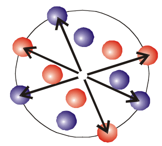


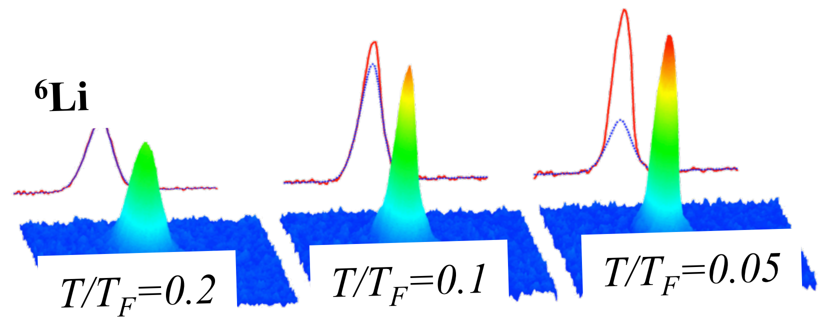
S-wave resonant fermionic superfluidity

• molecular BEC *(Regal, Jin '03)*



• BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*





• BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \Big(\frac{\hat{p}^2}{2m} - \mu\Big)\psi_\sigma + \phi^\dagger \Big(\frac{\hat{p}^2}{4m} + \epsilon_0 - 2\mu\Big)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$

$(\phi = B + \delta\phi)$

$$F(B,\mu) = -T\log[\mathrm{Tr}(e^{-H/T})]$$

$\longrightarrow$ gap equation: $\dfrac{\delta F}{\delta B} = 0,$

number equation: $-\dfrac{\delta F}{\delta \mu} = n = n_a + 2n_m$

S-wave resonant superfluidity

$$H = \sum_k \left[\left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k\sigma} a_{k\sigma} + \left(\frac{k^2}{4m} + \epsilon_0 - 2\mu\right) b^\dagger_k b_k - gB a^\dagger_{-k\downarrow} a^\dagger_{k\uparrow} \right]$$

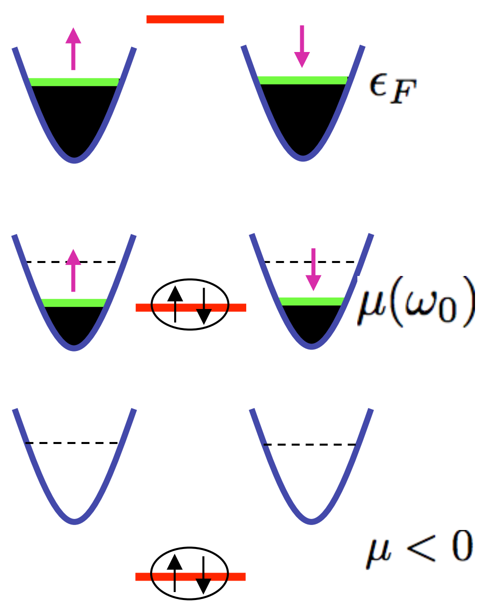
exactly solvable g -> 0^+ limit: *free Fermi- and molecular Bose gases in chemical equilbrium*

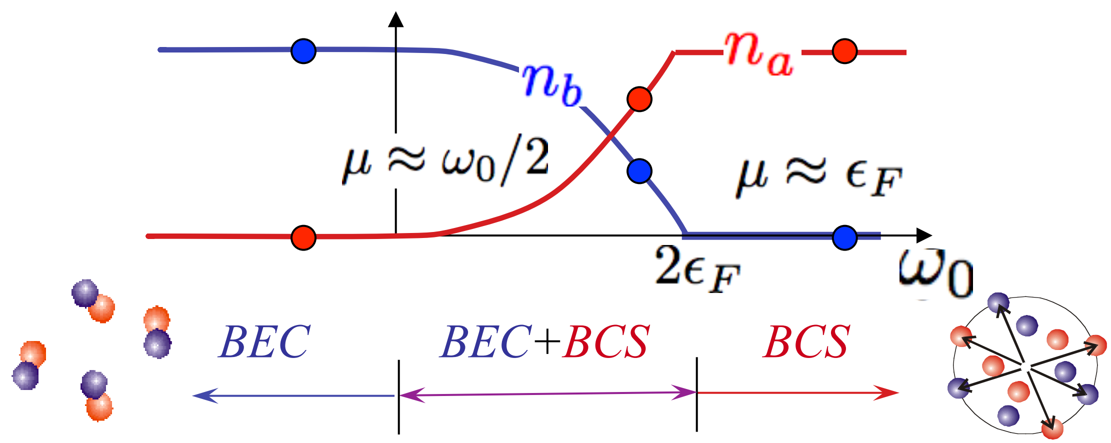
$$H^{g=0} = \sum_k \left(\frac{k^2}{2m} - \mu\right) n^a_k + (\epsilon_0 - 2\mu) n^m_0$$

T=0:

$$n_a = \frac{(2m)^{3/2}}{3\pi^2} \mu^{3/2}, \text{ for } \mu > 0 \quad \Longrightarrow n^m_0 = |B|^2 = \frac{n}{2}\left[1 - \left(\frac{\mu}{\epsilon_F}\right)^{3/2}\right]$$

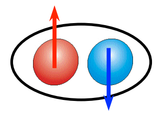
$$\begin{aligned} \mu(\epsilon_0) &= \epsilon_F, \text{ for } \epsilon_0 > 2\epsilon_F \to \text{no molecules} \\ &= \epsilon_0/2, \text{ for } 0 < \epsilon_0 < 2\epsilon_F \to \text{atomic Fermi sea \& molecular BEC} \\ &= \epsilon_0/2, \text{ for } \epsilon_0 < 0 \to \text{molecular BEC, no free atoms} \end{aligned}$$





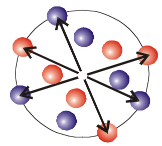
S-wave resonant fermionic superfluidity

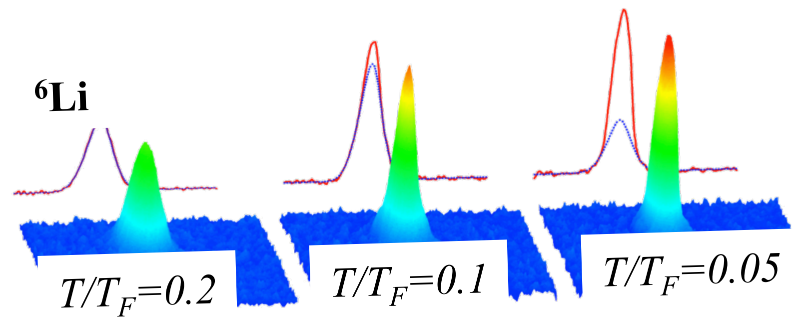
- molecular BEC *(Regal, Jin '03)*



K_2 , Li_2

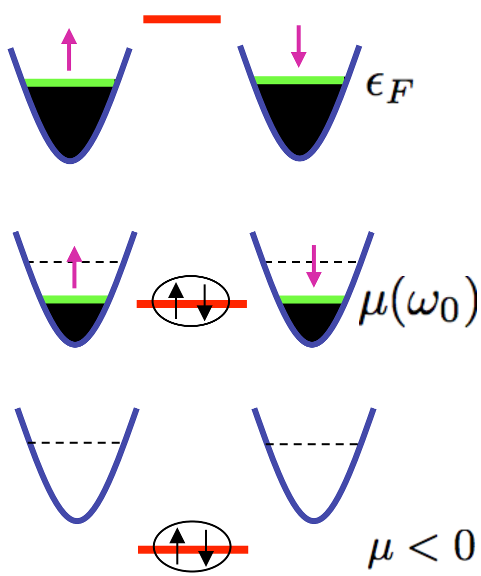
- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*

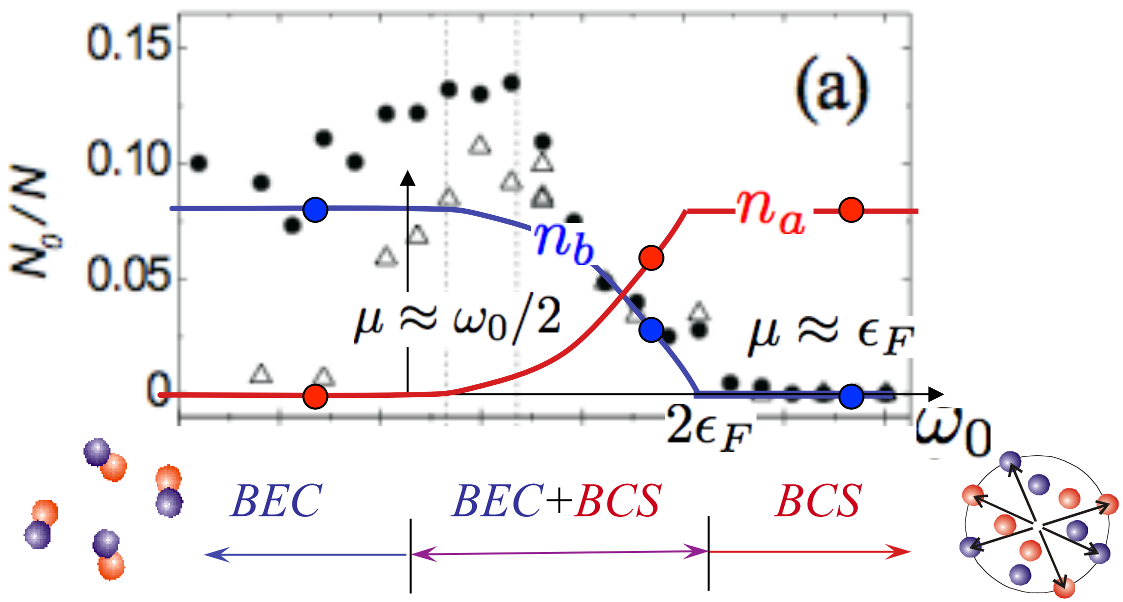




- BCS-BEC crossover:

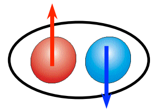
$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger \left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0 - 2\mu\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$





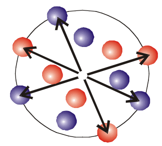
S-wave resonant fermionic superfluidity

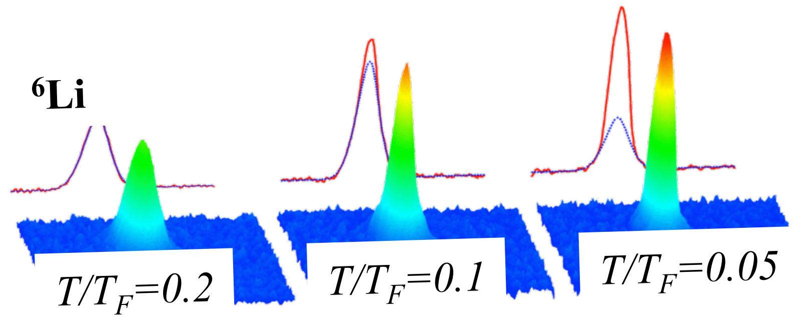
- molecular BEC *(Regal, Jin '03)*



K_2 , Li_2

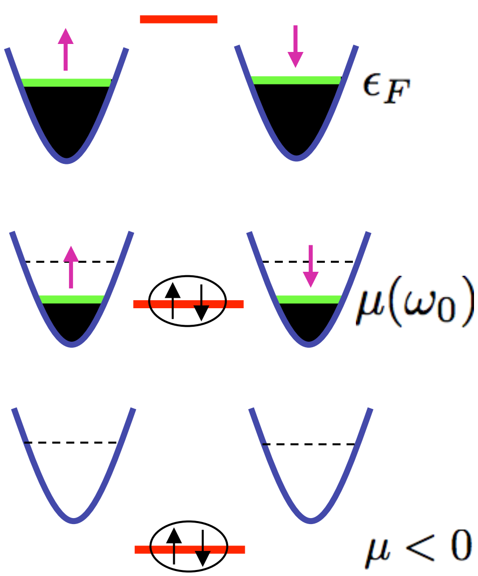
- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*

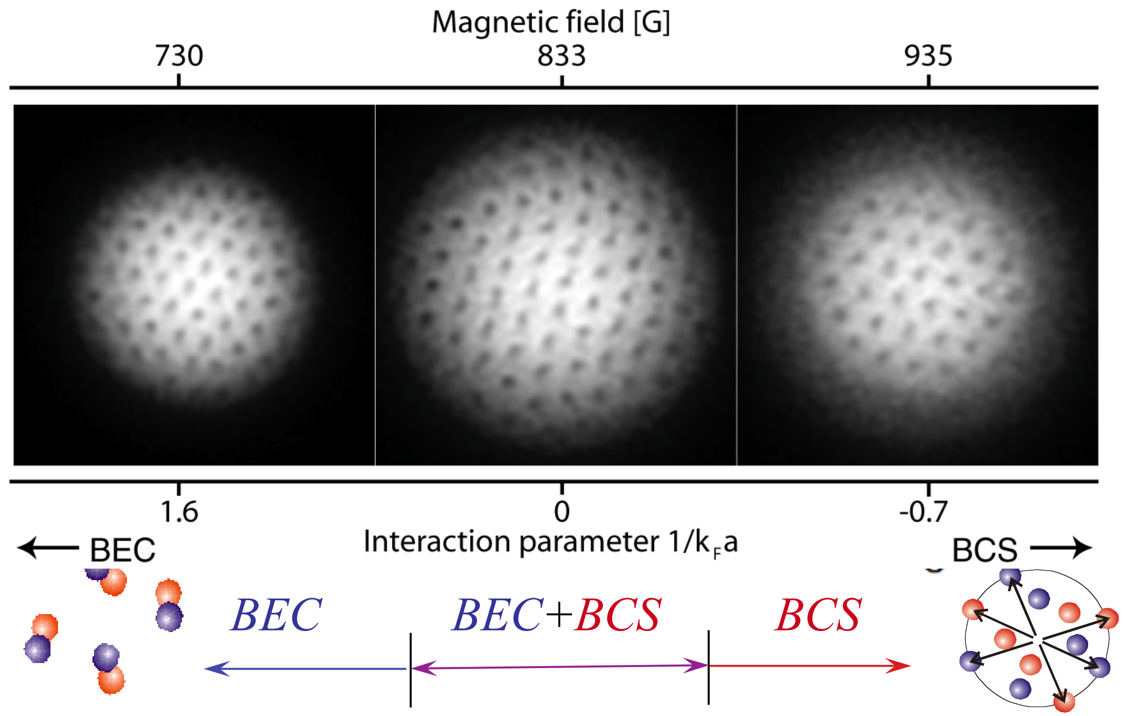




- BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m} - \mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m} + \epsilon_0 - 2\mu\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \text{h.c.}$$





S-wave resonant superfluidity: details

$$H = \sum_k \left[\left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k\sigma} a_{k\sigma} + \left(\frac{k^2}{4m} + \epsilon_0 - 2\mu\right) b^\dagger_k b_k - gB a^\dagger_{-k\downarrow} a^\dagger_{k\uparrow} + h.c. \right]$$

small g: *hybridized paired Fermi-sea (Cooper pairs) and molecular BEC*

no UV cutoff

T=0:

$$E_{gs}(B,\mu) = (\omega_0 - 2\mu)|B|^2 - \sum_k \left(E_k - \varepsilon_k - \frac{g^2}{2\epsilon_k}|B|^2\right)$$

$$= \frac{\omega_0 - 2\mu}{g^2}|\Delta|^2 - E^a_{\text{condense}}$$

$E_k n^a_k$

$$\begin{cases} -\Delta^2 \ln \frac{8e^{-3/2}\mu}{\Delta}, & \text{for } \mu > 0, \\ \alpha\Delta^2 + \beta\Delta^4, & \text{for } \mu < 0, \end{cases}$$

gap eqn: $$\frac{\omega_0 - 2\mu}{g^2} = \frac{1}{2}\int_k \left[\frac{1}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}} - \frac{1}{\epsilon_k}\right]$$

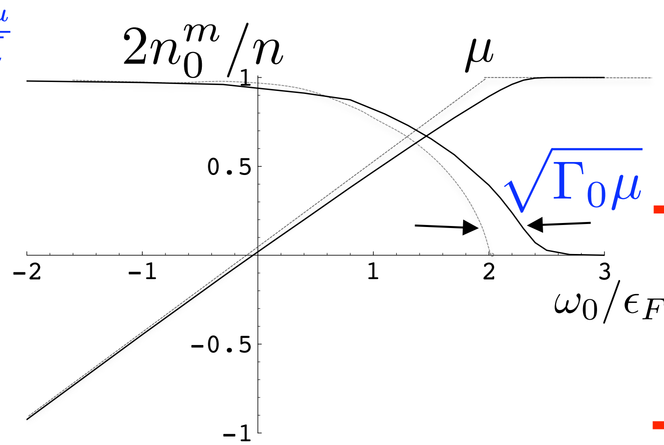
N eqn: $$n = \int_k \left[1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}}\right] + 2|B|^2$$

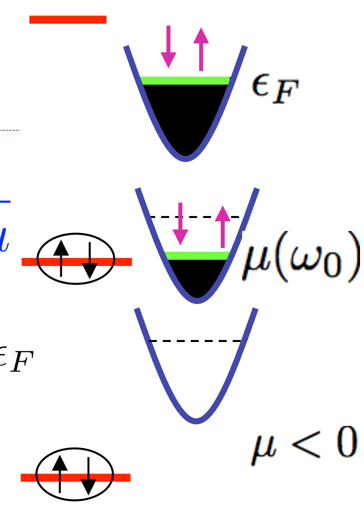
➡ μ > 0 BCS: $\Delta \approx gB = 8e^{-2}\mu e^{-c\frac{\omega_0 - 2\mu}{\sqrt{\Gamma_0 \mu}}}$

$n \approx c(m\mu)^{3/2} + 2B^2$

➡ μ < 0 BEC: $\omega_0 - 2\mu \approx \sqrt{\Gamma_0 \mu}$

$n \approx \left(\sqrt{\frac{\Gamma_0}{|\mu|}} + 2\right) B^2$





S-wave resonant SF: small parameter

$$\mathcal{H}_{2ch} = \psi^\dagger_\sigma\Big(\frac{\hat{p}^2}{2m} - \mu\Big)\psi_\sigma + \phi^\dagger\Big(\frac{\hat{p}^2}{4m} - 2\mu + \epsilon_0\Big)\phi - g\phi\psi^\dagger_\uparrow\psi^\dagger_\downarrow$$

dimensionless coupling:

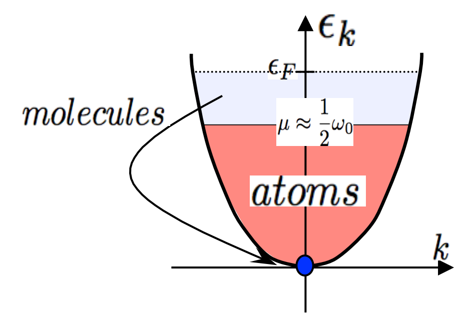
$$\gamma \sim \frac{g^2\nu(\epsilon_F)}{\epsilon_F} \sim \left(\frac{g\sqrt{n}}{\epsilon_F}\right)^2 \sim \frac{1}{k_F r_0} \sim \sqrt{\frac{\Gamma_0}{\epsilon_F}}$$

$$\gamma^{^{40}K}_{202G} \approx 5,\ \Delta B \sim 1G \sim 100\mu K$$

$$\gamma^{^{6}Li}_{544G} \approx 0.1,\ \Delta B \sim 0.1G \sim 10\mu K$$

$$\epsilon_F \sim 1\mu K$$

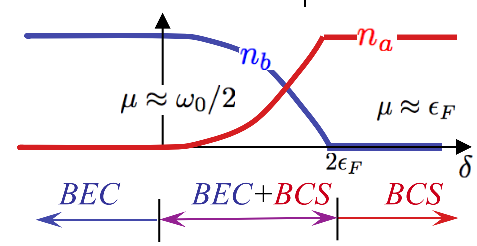
- **narrow** resonance $\gamma \ll 1 \ \rightarrow \ \mathrm{MFT}: \phi(x) = B$

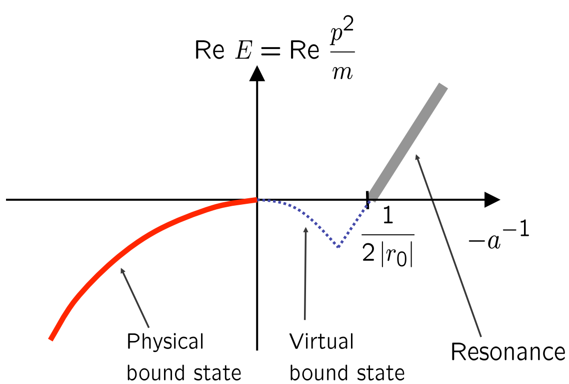


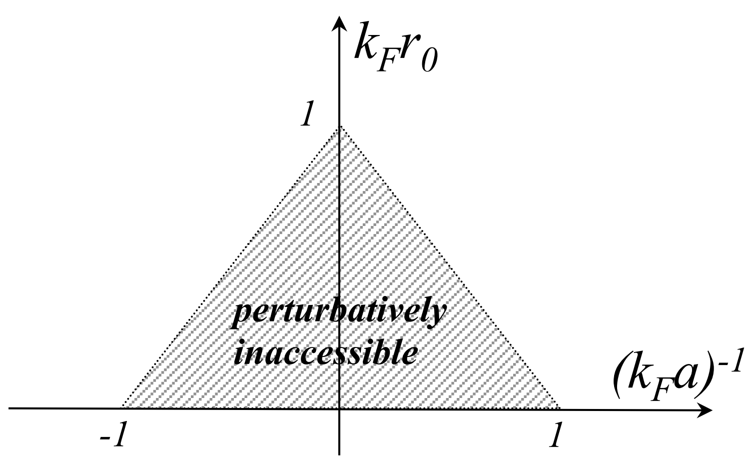
- **broad** resonance $\gamma \gg 1$

Strongly coupled ϕ and ψ

⇨ *MFT quantitatively uncontrolled*







$$\gamma \approx |T_{k_F}| n/\epsilon_F \approx \frac{1}{(k_F a)^{-1} - k_F r_0 + 1}$$

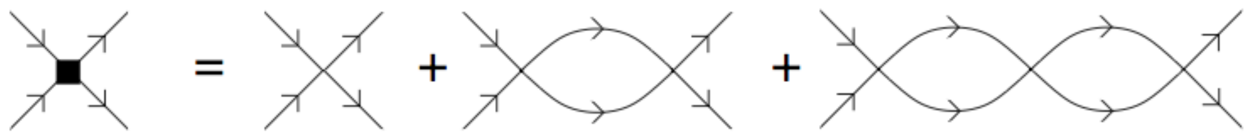
$\gamma >> 1$

Broad resonance scattering

$$\mathcal{H}_{2ch} \longrightarrow \mathcal{H}_{1ch} = \psi_\sigma^\dagger\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_\sigma + \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$$

- scattering T-matrix relates λ to a:

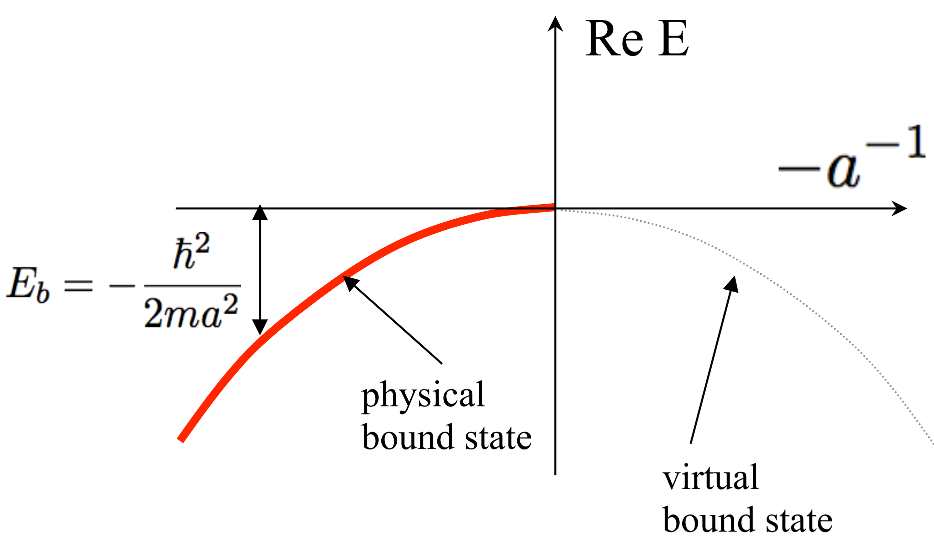
$T_{k\,k'} =$

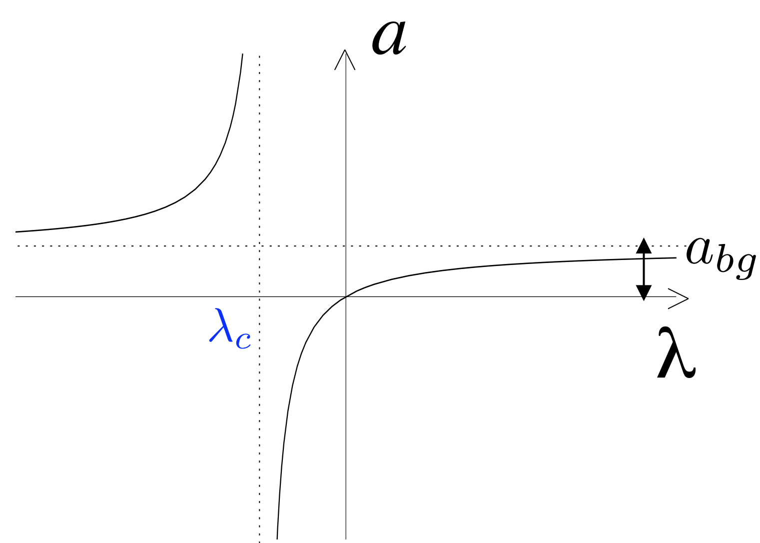


$+ \ldots = \dfrac{\lambda}{1 - \lambda\Pi}$

$$= -\frac{4\pi\hbar^2}{m} f_{kk'} \approx \frac{4\pi\hbar^2}{m}\frac{1}{a^{-1} + ik} \longrightarrow a = \frac{m}{4\pi\hbar^2}\frac{\lambda}{1 + \lambda/\lambda_c}$$

$(\lambda_c = \pi\hbar^2 d/m)$





$\gamma >> 1$ **Broad resonance superfluidity: Large N**

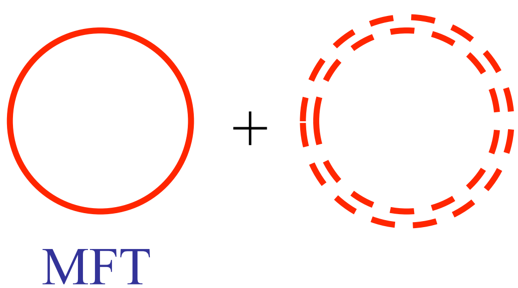
- no small parameter for $k_F a \sim n^{1/3} a >> 1$ → *introduce 1/N*

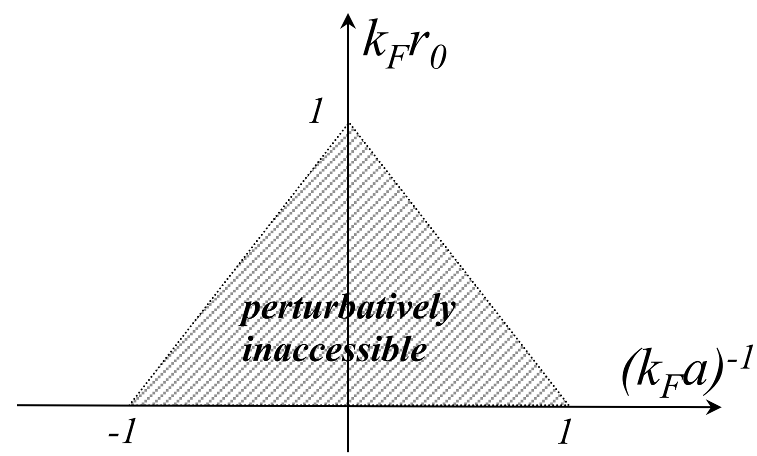
$$\mathcal{H}_{1ch} \xrightarrow{Sp(2N)} \mathcal{H}_N = \psi^\dagger_{\sigma\alpha}\left(\frac{p^2}{2m} - \mu_\sigma\right)\psi_{\sigma\alpha} + \frac{\lambda}{N}\psi^\dagger_{\uparrow\alpha}\psi^\dagger_{\downarrow\alpha}\psi_{\downarrow\beta}\psi_{\uparrow\beta}$$

$$S[\phi] = -\frac{N}{\lambda}\int_0^\beta d\tau d^3r|\phi|^2 - N\mathrm{Tr}\log\left[-G_\phi^{-1}\right]$$

$$G_\phi^{-1} = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m} + \mu_\uparrow & \phi_x \\ \phi_x^* & -\partial_\tau - \frac{\nabla^2}{2m} - \mu_\downarrow \end{pmatrix}$$

$$\begin{aligned} f &= -\frac{1}{\beta V}\log\int D\phi e^{-S[\phi]}, \\ &= Nf^{(0)} + f^{(1/N)} + \dots \end{aligned}$$





$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm}\int_k \log\left[1 + e^{-\beta(E_k + \sigma h)}\right]$$

Veillette, Sheehy, LR
Nikolic, Sachdev
also Nishida, Son
ε-expansion

$\gamma >> 1$ # Broad resonance superfluidity: N->∞

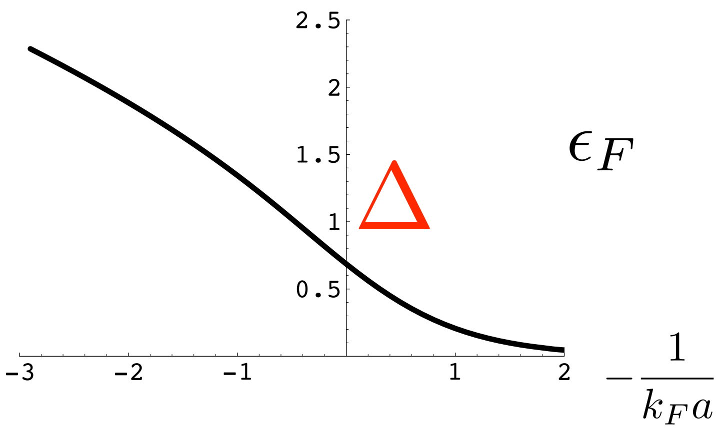
T=0:

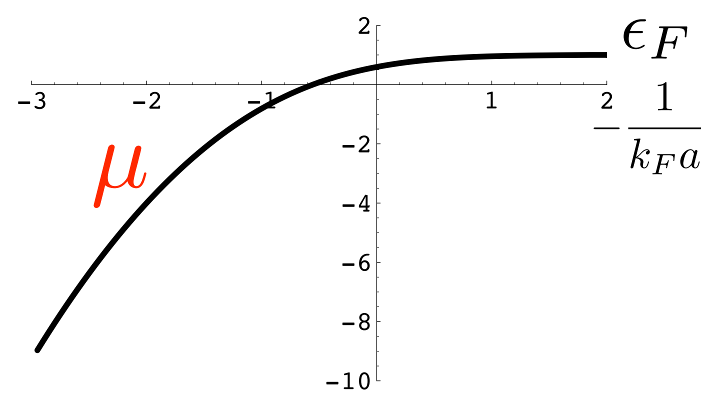
$$f^{(0)} = -\frac{|\Delta|^2}{\lambda} - \int_k (E_k - \xi_k) - \sum_{\sigma=\pm} \int_k \log\left[1 + e^{-\beta(E_k+\sigma h)}\right]$$

- $$E_{gs}(\Delta,\mu) = -\frac{m}{4\pi\hbar^2}\frac{|\Delta|^2}{a} - \sum_k (E_k - \varepsilon_k - \frac{1}{2\epsilon_k}|\Delta|^2)$$
- $$-\frac{m}{4\pi\hbar^2 a} = \frac{1}{2}\int_k \left[\frac{1}{\sqrt{(\epsilon_k-\mu)^2+\Delta^2}} - \frac{1}{\epsilon_k}\right]$$ $$n = \int_k \left[1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k-\mu)^2+\Delta^2}}\right]$$

for a < 0 (BCS): $\Delta \sim \mu e^{\frac{\pi}{2k_F a}\sqrt{\frac{\mu}{\epsilon_F}}}$ $\mu \approx \epsilon_F(1 - c\frac{\Delta^2}{\epsilon_F^2}\frac{1}{k_F|a|})$

for a > 0 (BEC): $\Delta \sim \epsilon_F\sqrt{\frac{1}{k_F a}}$ $\mu \approx -\frac{\hbar^2}{2ma^2}$



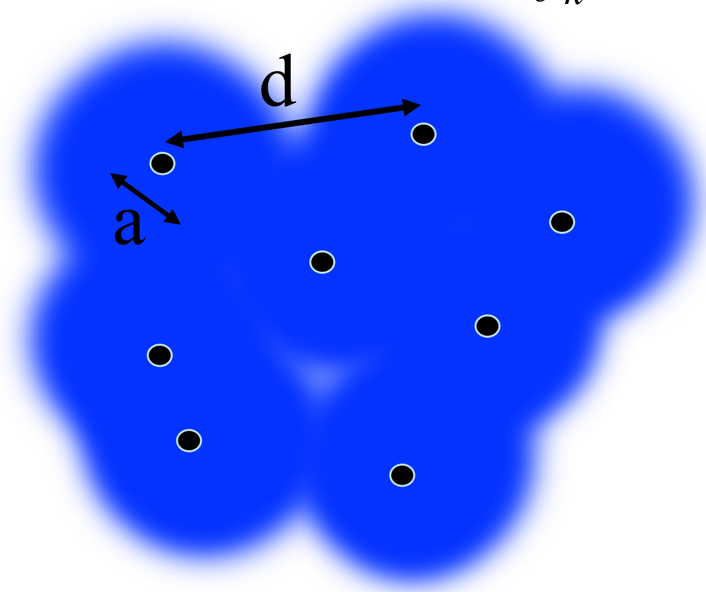


$\gamma >> 1,\ k_F a \to \infty$

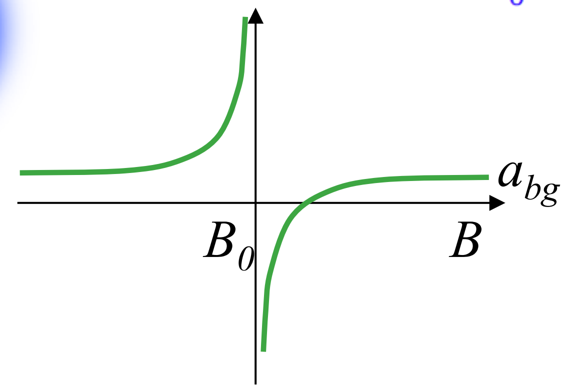
Universality at unitary point

T.L. Ho '04

$f_k = -1/(a^{-1} + i\,k) \to i/k$, -> k_F is the only scale



$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



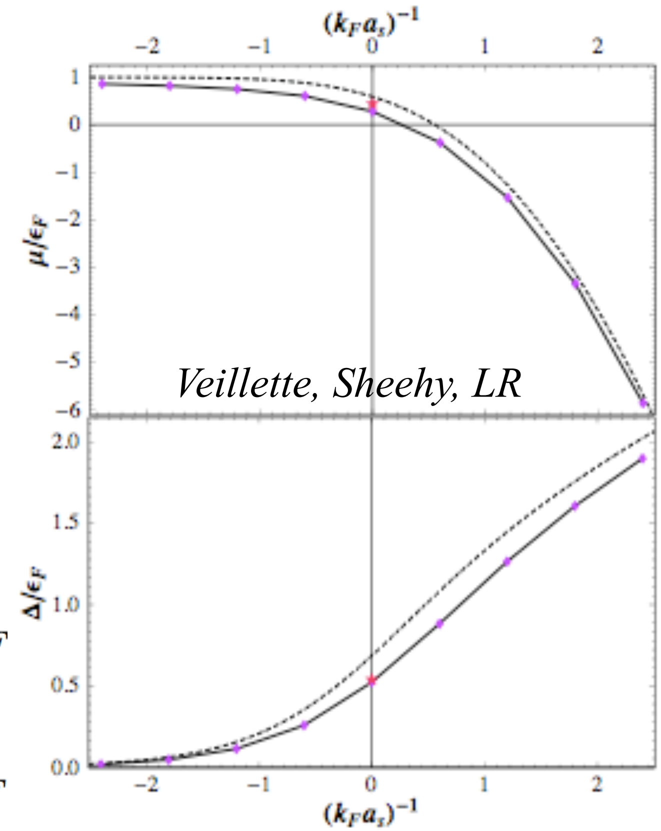
check in $N \to \infty$ (BCS) limit:

$$\frac{m}{2\pi\hbar^2 a} \to 0 = \int_k \left(\frac{1}{E_k} - \frac{1}{\epsilon_k}\right)$$

$$0 = \ln(\Delta/\alpha\epsilon_F)$$

$$f(T,n) = n\epsilon_F \hat{f}(k_B T/\epsilon_F)$$

$$\begin{aligned}
\epsilon &= \xi \frac{3}{5}\epsilon_F \\
\mu &= \xi\epsilon_F \\
\Delta &= \alpha\epsilon_F \\
\Delta_{exc} &= \alpha_{exc}\epsilon_F \\
k_B T_c &= \gamma\epsilon_F \\
B &= \xi \frac{2}{3} n\epsilon_F
\end{aligned}$$



1/N theory $\xi = 0.5906 - 0.312/N + \cdots$

Exp with ^{40}K $\xi = 0.46^{+0.05}_{-0.12}$

Summary s-wave

- Revolution in AMO physics
- Feshbach resonances as a road to strong interactions
- s-wave Feshbach resonant scattering
- s-wave Feshbach resonant paired superfluidity
  - *2-channel model (narrow resonance) – small coupling*
  - *1-channel model (broad resonance) – large N theory*
  - *universality at unitary point*

Questions of interest

- What are the big fundamental questions?

- Specific questions of current experimental interest:
  - *Unitary Fermi gas (universality)*
  - *Feshbach resonant superfluidity in an optical lattice*
  - *Resonant Bose gas (beyond Beliaev)*
  - *Stability to 3-body collisions and other inelastic processes*
  - *Cooling and thermalization*
  - *Experimental probes (development and understanding)*
  - *Phases realizations (e.g., FFLO, p-wave SF, magnetism, …)*
  - *Nonequilibrium quantum dynamics*
  - …

Supplementary material

- Feshbach resonances classic references

- Hyperfine structure of alkali

- Probes of Feshbach resonances

- Experimental realizations

- AMO experimental probes

Feshbach resonances

from C. Greene

- O. K. Rice, JCP 1, 375 (1933) - basic treatment of how a bound state autoionizes into a degenerate continuum
- U. Fano, Nuovo Cimento 12, 156 (1935) – shows that quantum interference has opposite signs above and below the resonance, leading to asymmetric line profiles analogous to anomalous dispersion
- G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936) – Basic formula developed for symmetric resonance profile when only the "bound part" of the reaction dominates
- H. Feshbach, Ann. Phys. 5, 357 (1958) and 19, 287 (1962) – developed general projection operator formalism that cleanly separates "bound" and "continuum" subspaces and systematically treats their interaction
- U. Fano, Phys. Rev. 124, 1866 (1961) – more elegant reformulation of his 1935 theory of asymmetric line profiles from discrete-continuum interactions
- P. Anderson, Phys. Rev. 124, 41 (1961) – model of localized impurity state in a continuous band

Feshbach resonances

Feshbach resonances in neutron-sulfur scattering, from Blatt&Weisskopf, 1950s

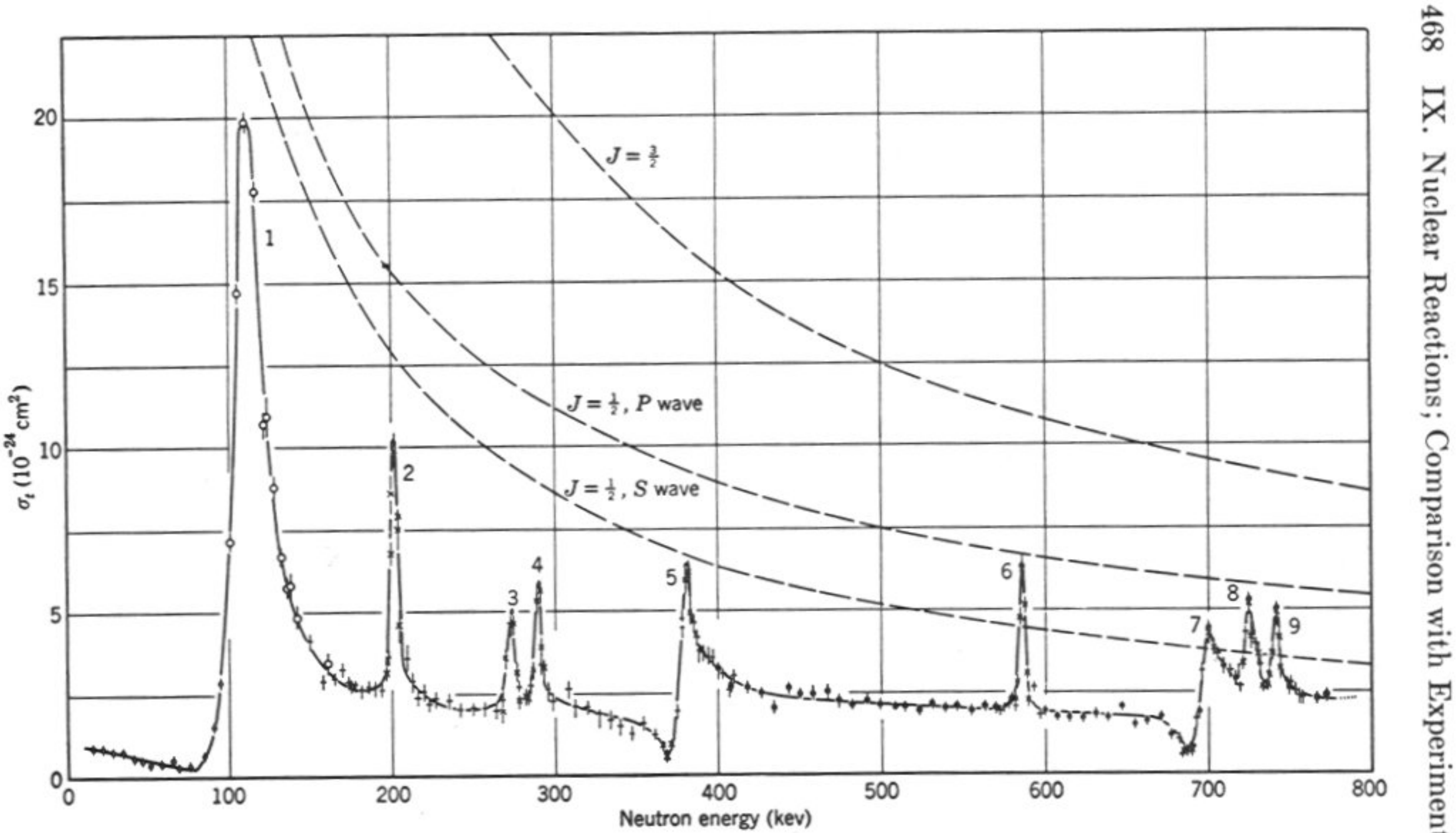


FIG. 2.2. Total neutron cross section for sulfur; experimental data taken from Adair (49) and Peterson (50).

Hyperfine interaction in a B field

$$H_{HF} = \alpha_{HF} \vec{I} \cdot \vec{S} - (g_I \mu_N \vec{I} + g_S \mu_B \vec{S}) \cdot \vec{B}$$

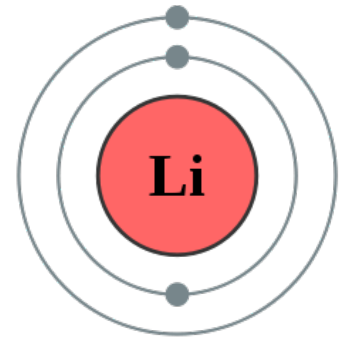
- Li 6: $2S_{1/2}$ $\quad |n=2, l=0, s = 1/2, s_z>|i=1, i_z>$

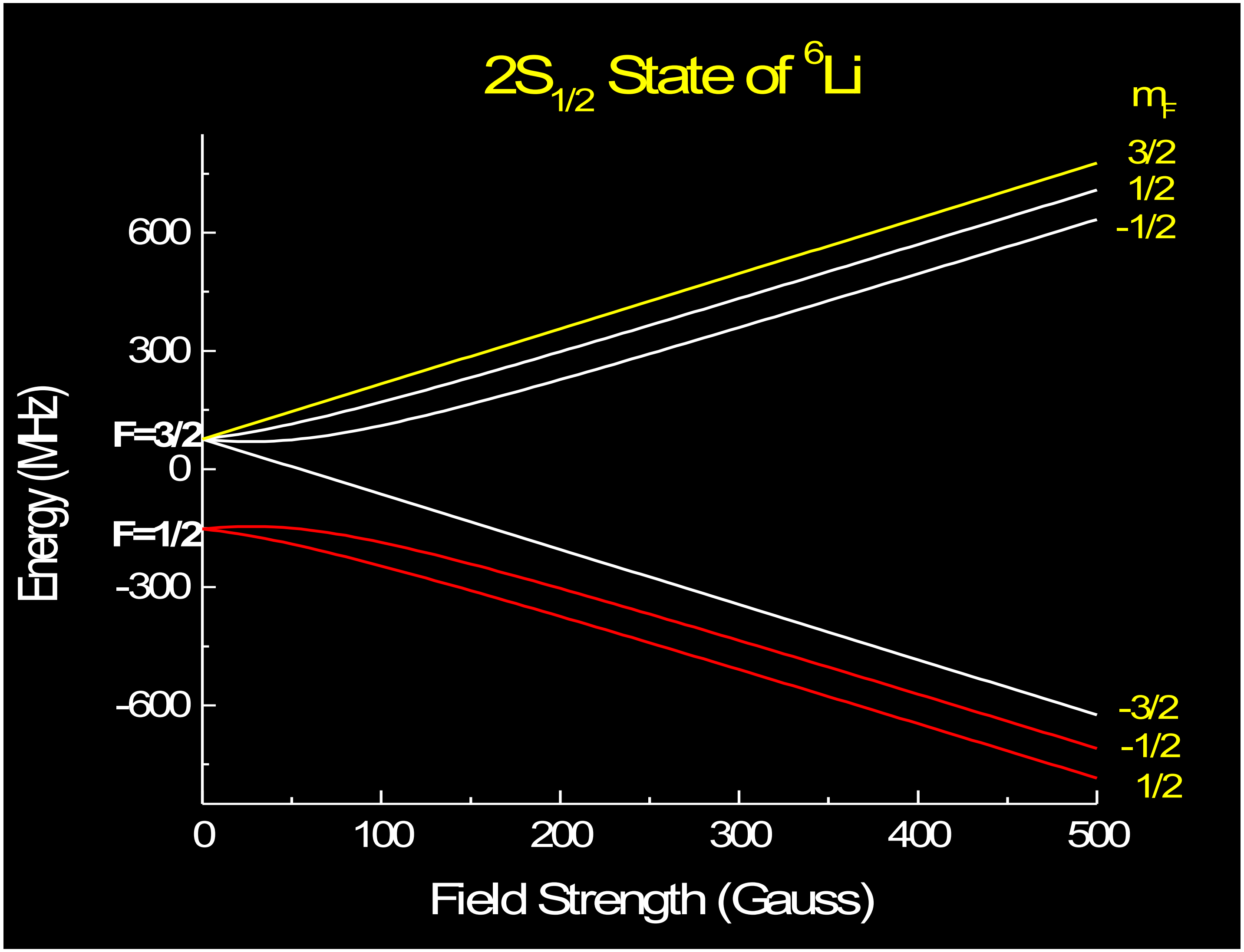
$\Longrightarrow \frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$ $\quad |n=2, l=0, f, m_f>$ $\quad$ *(B=0)*

- K40: $4S_{1/2}$ $\quad |n=4, l=0, s=1/2, s_z>|i=4, i_z>$

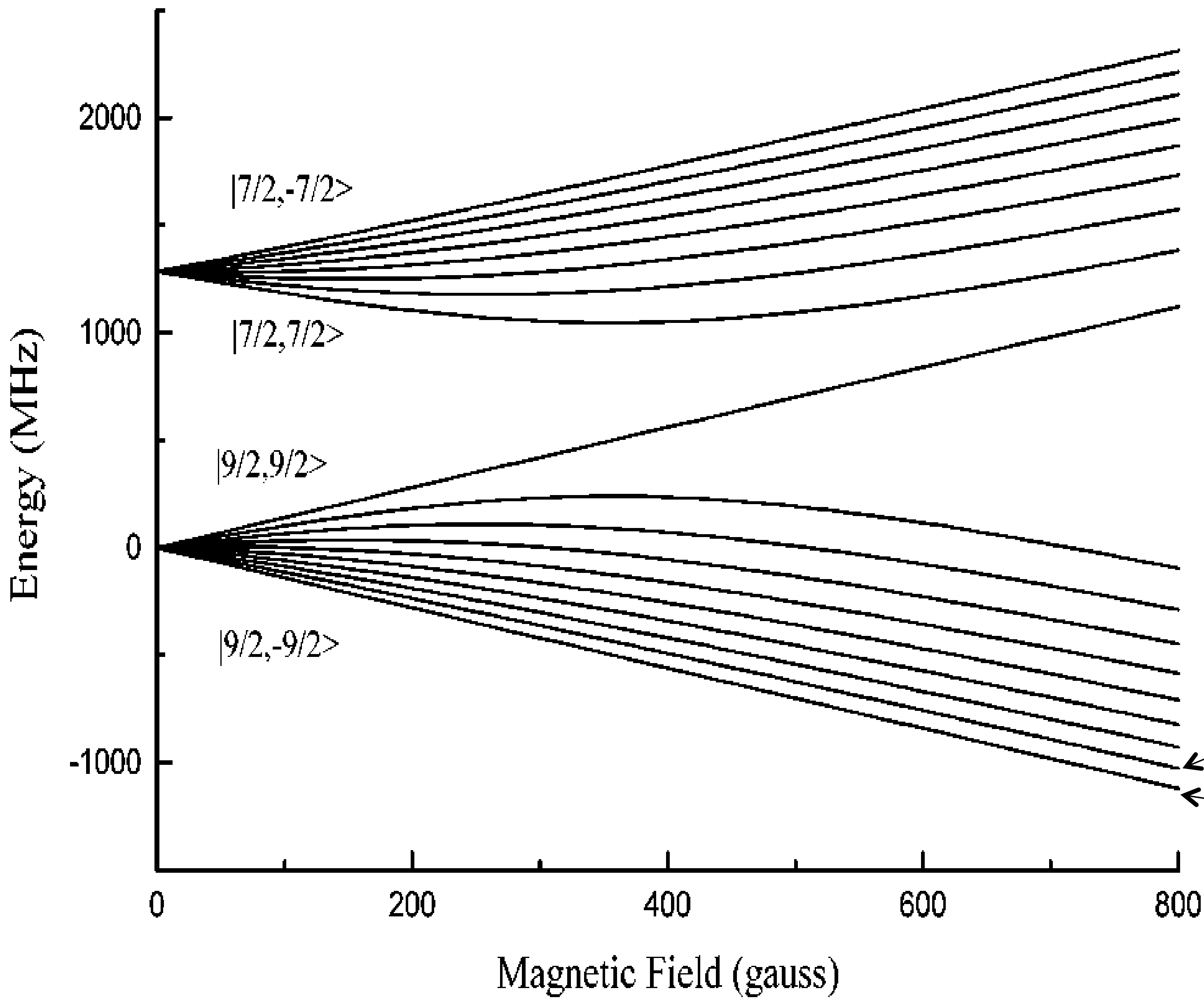
$\Longrightarrow \frac{1}{2} \otimes 4 = \frac{9}{2} \oplus \frac{7}{2}$ $\quad |n=4, l=0, f, m_f>$ $\quad$ *(B=0)*

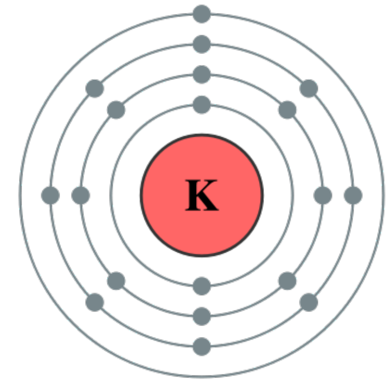
Hyperfine states of Li6

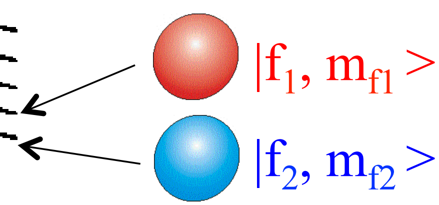




Hyperfine states of K40

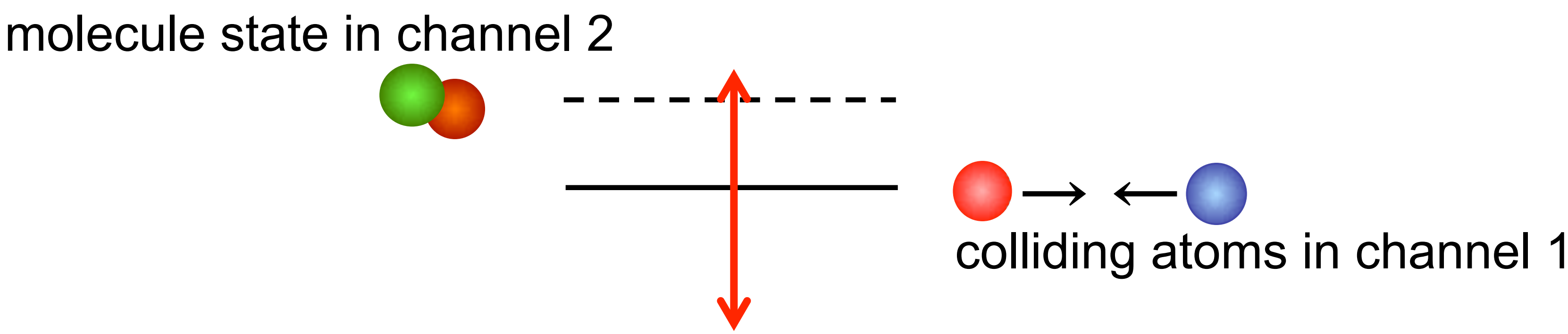






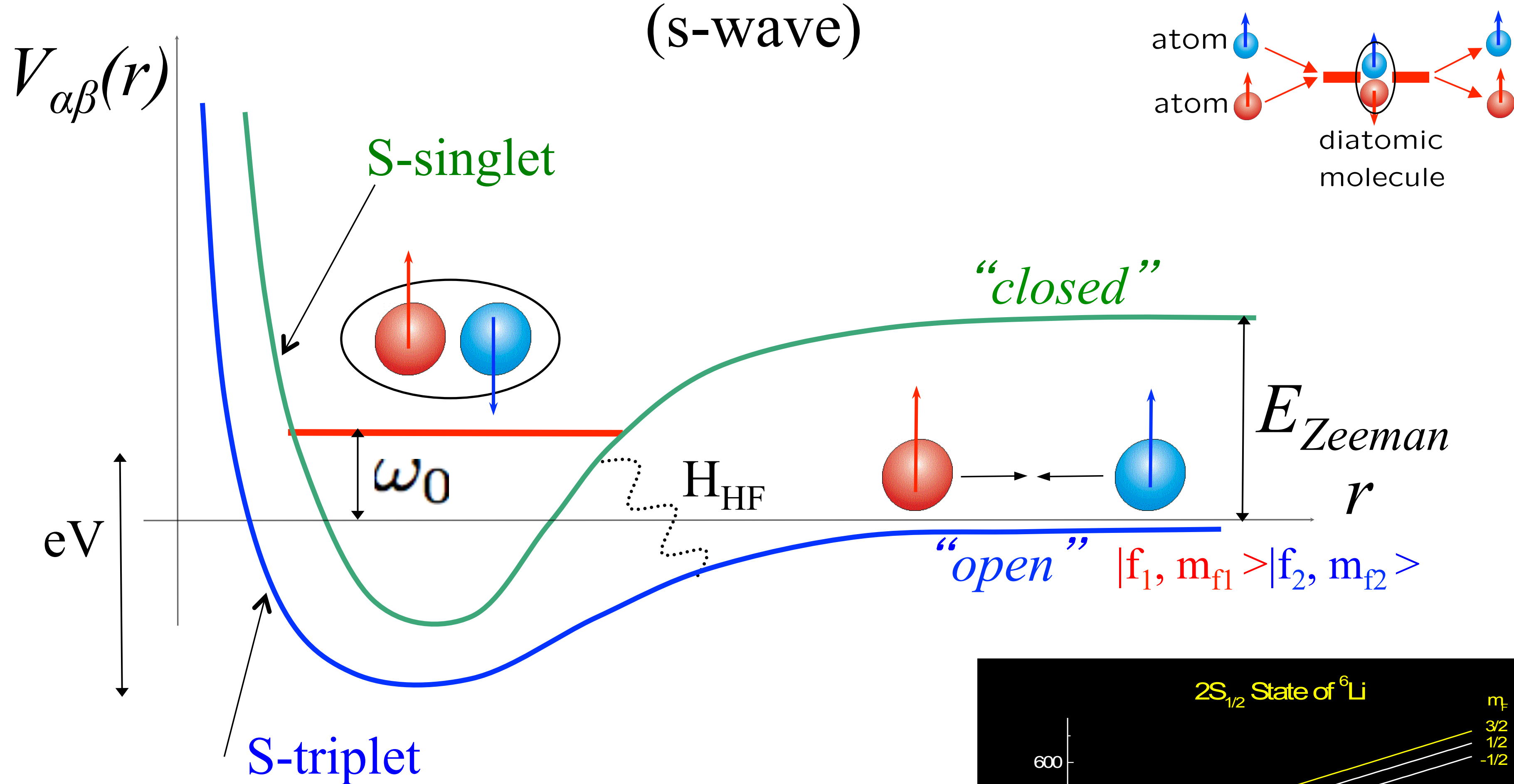
Atomic Feshbach resonances

A magnetic-field tunable atomic scattering resonance



Channels are coupled by the hyperfine interaction

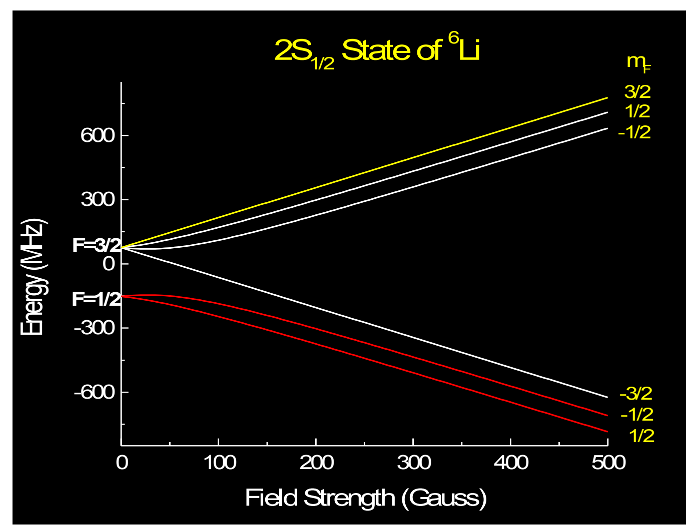
Atomic Feshbach resonances



microscopics: project quantum chemistry short-scale calculation of $V^{s/t}(r)$ onto hyperfine states at long scales

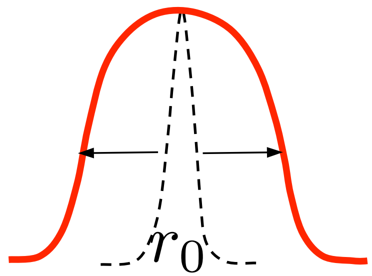
-> diagonalize 36 X 36 (e.g., for Li6)

$$V_{\alpha\beta}(r) = \langle \alpha_1 \alpha_2 | \hat{V}_s + \hat{V}_t | \beta_1 \beta_2 \rangle$$



Detect Feshbach resonances in alkali atoms

- cloud size:

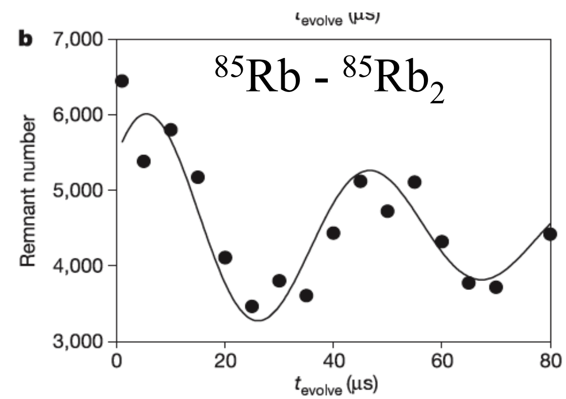


$$R(N) \sim (a r_0^4)^{1/5} N^{1/5}$$

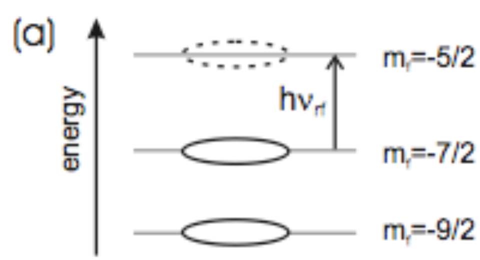
- atom loss via enhanced three-body decay rate: $\Gamma_3 \sim \frac{\hbar^2}{m} a^4 n^2$

- bound state Rabi oscillations (Ramsey fringes):

$$E_{\text{bound}} = -\frac{\hbar^2}{ma^2}$$



- RF spectroscopy resonance interaction shifts:



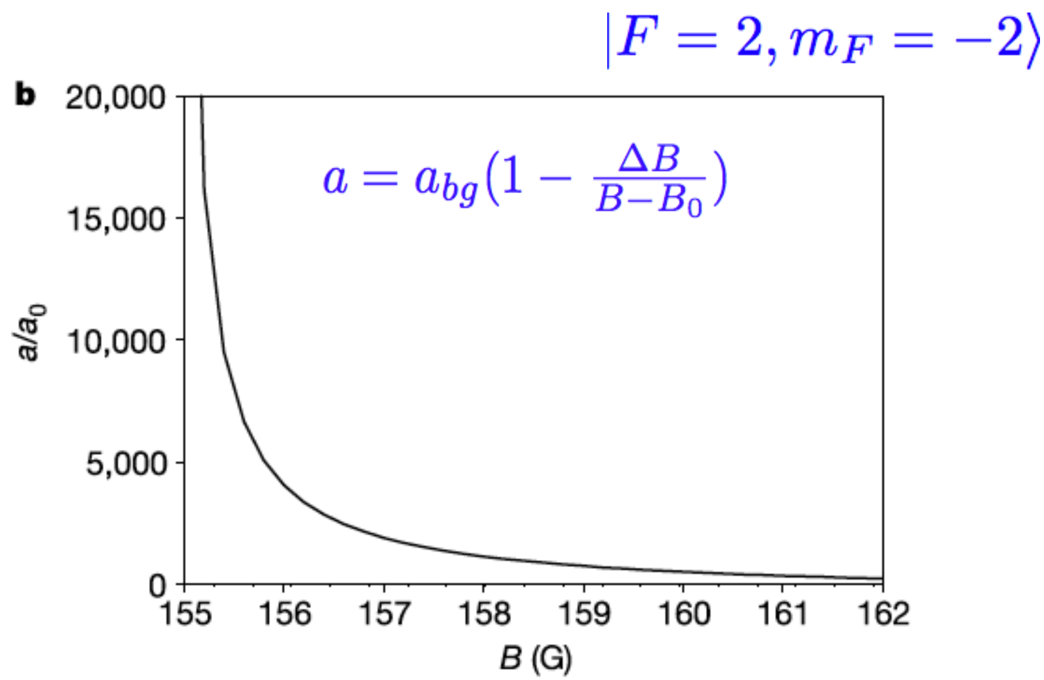
Rb85-Rb85 Feshbach resonance

Atom–molecule coherence in a Bose–Einstein condensate

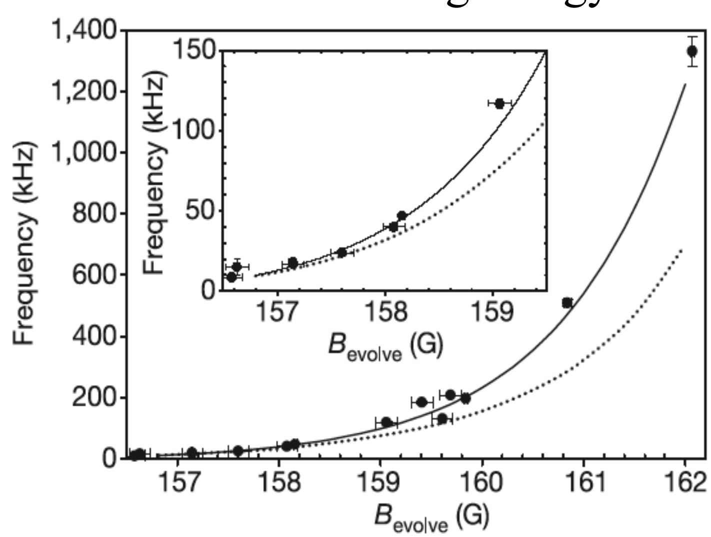
Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

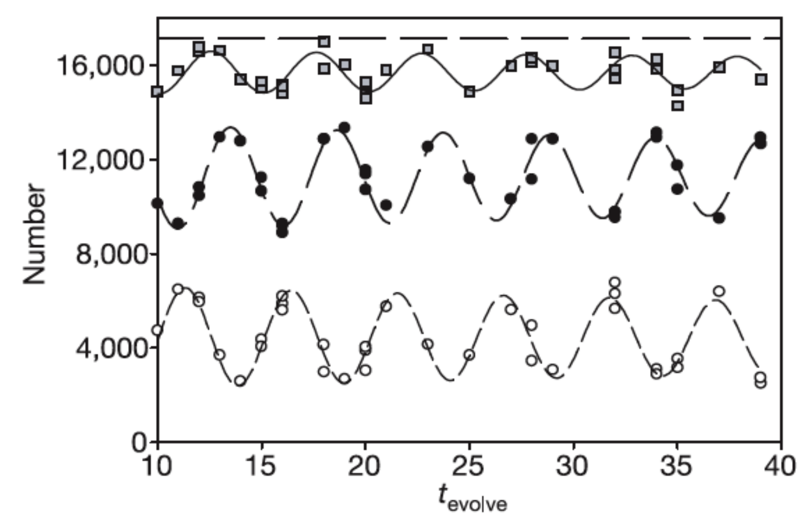
NATURE | VOL 417 | 30 MAY 2002



molecular binding energy

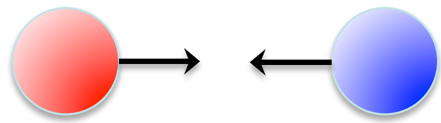


Rabi oscillations

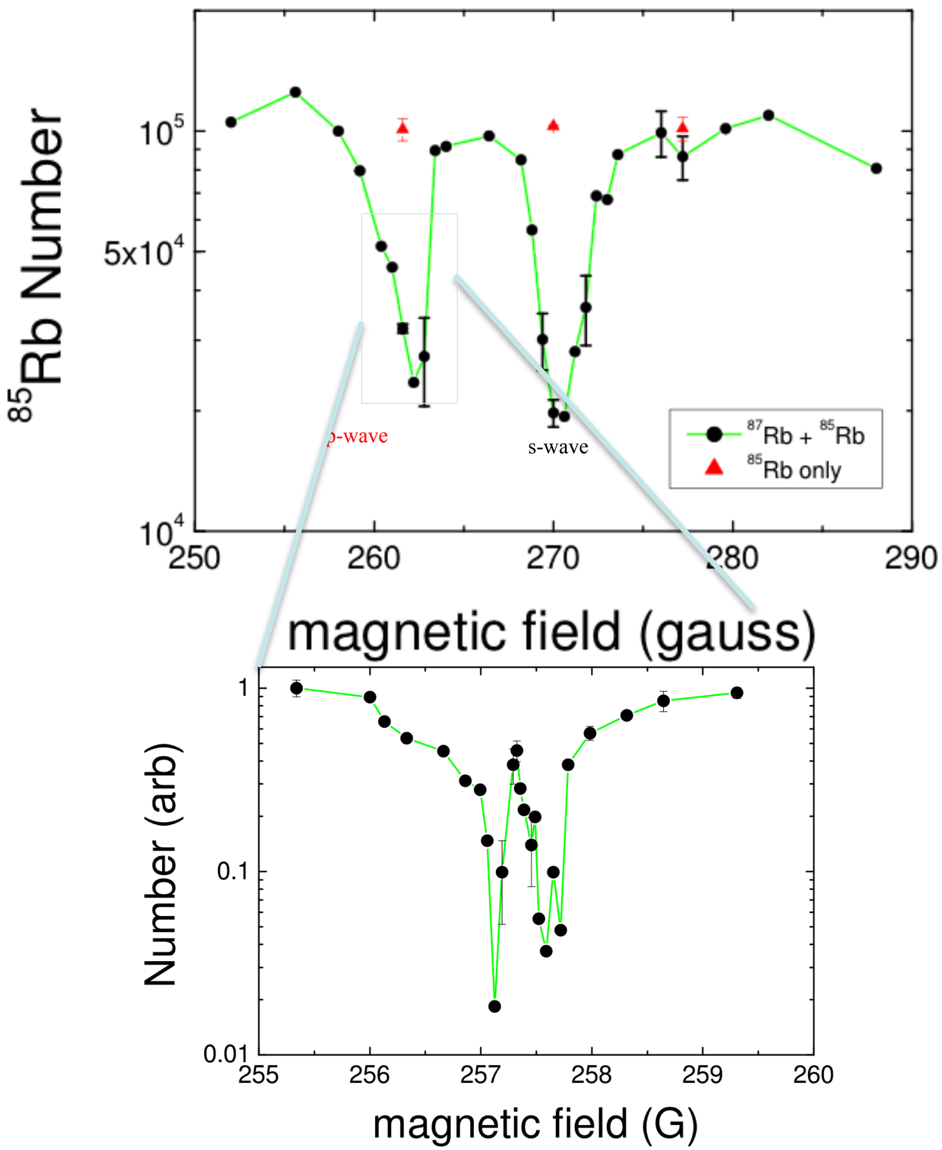


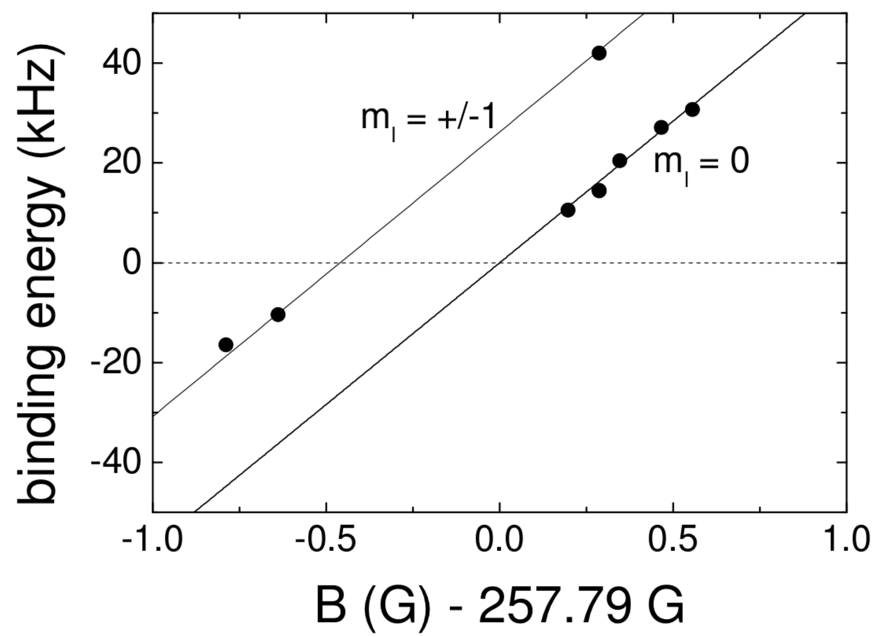
Rb85-Rb87 s- and p-wave Feshbach resonance

^{85}Rb : F=2, m_F= - 2



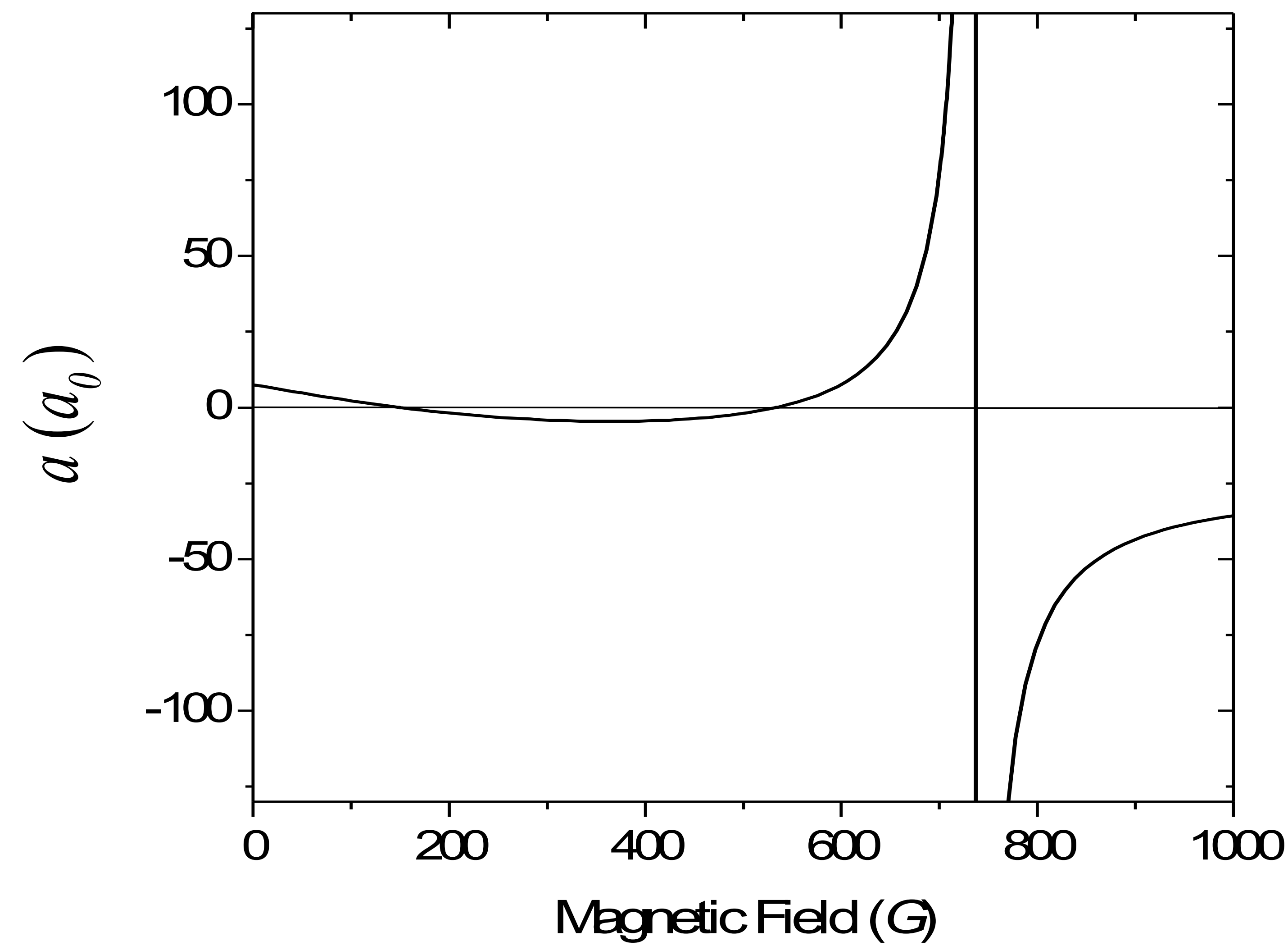
^{87}Rb : F=1, m_F= - 1



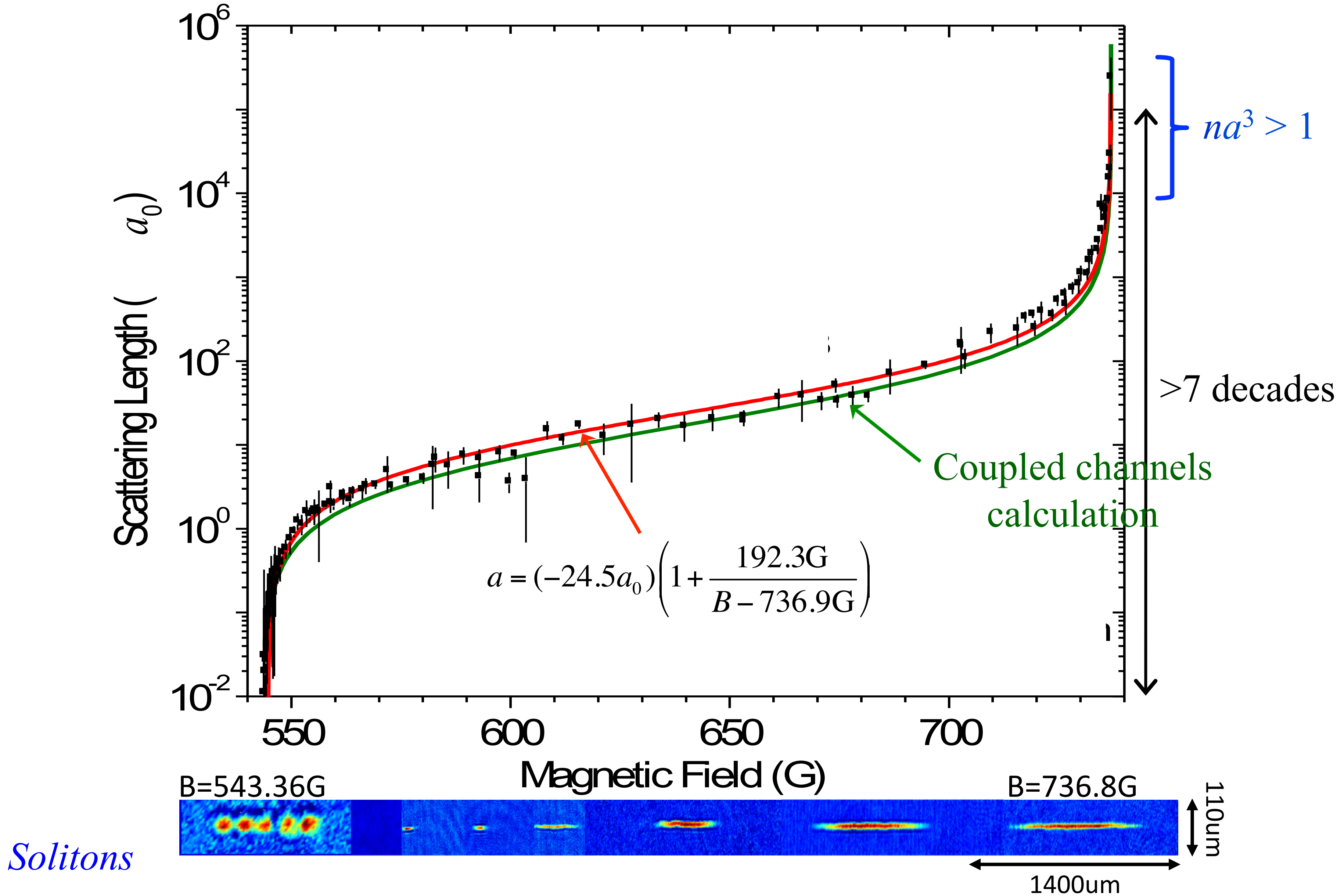


Papp's Thesis
Papp, Pino, Wieman

Li7-Li7 s-wave Feshbach resonance

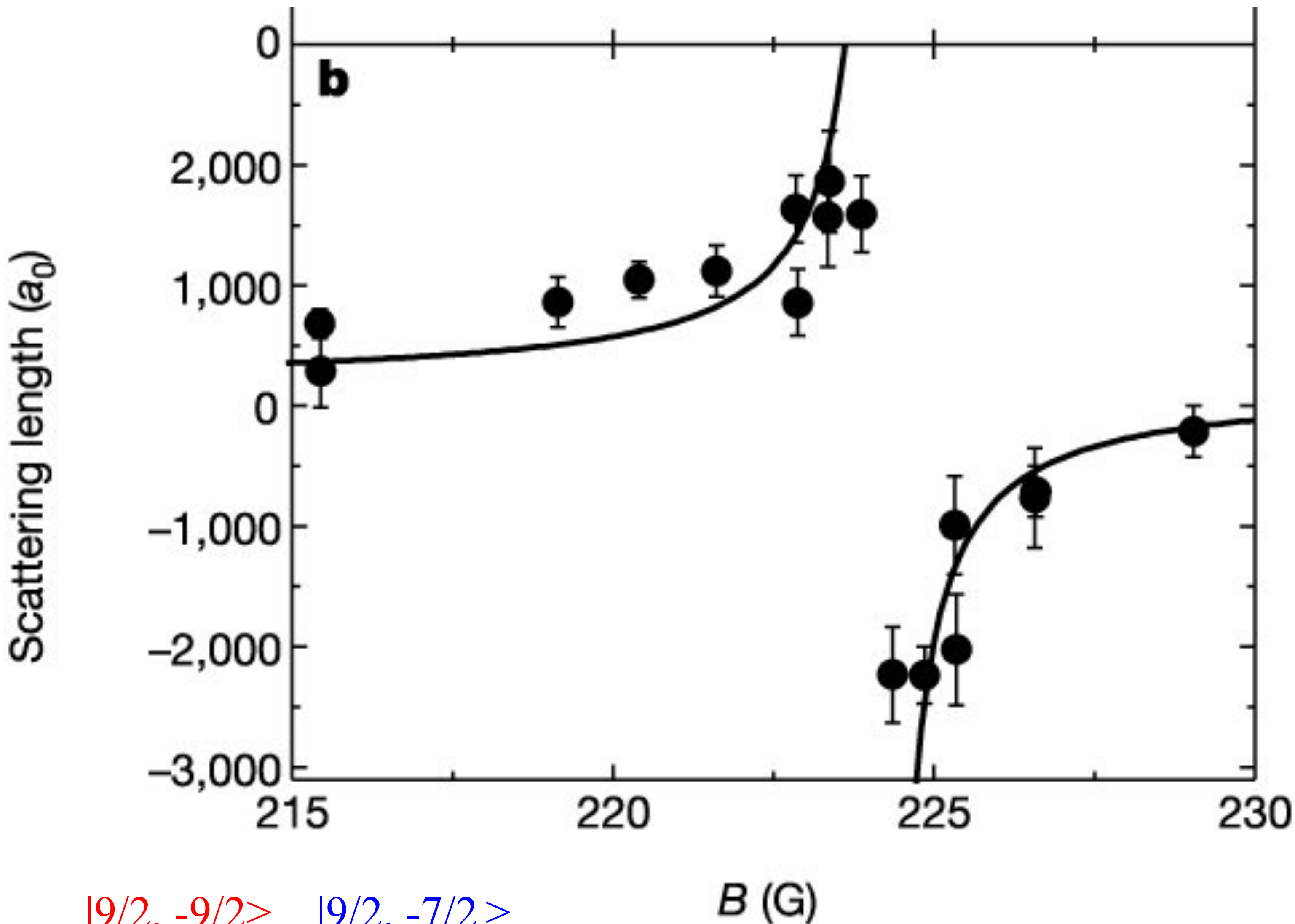


Li7-Li7 s-wave Feshbach resonance

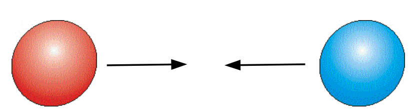


Pollack, Dries, Junker, Chen, Corcovilos, Hulet, PRL **102**, 090402 (2009)

K40-K40 s-wave Feshbach resonance

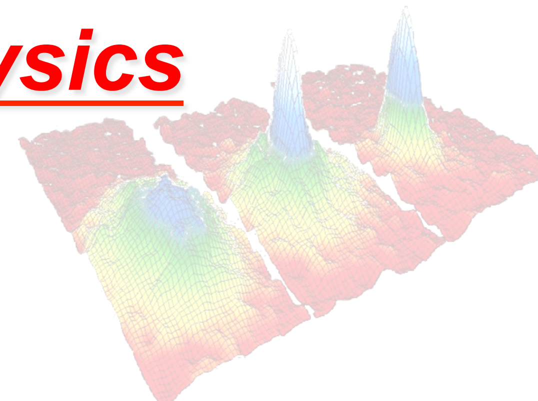


|9/2, -9/2> |9/2, -7/2 >



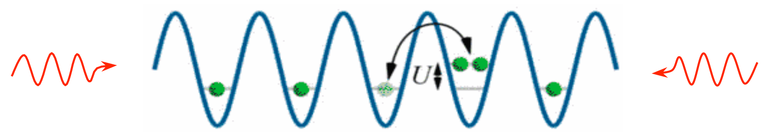
Regal, Jin 2003

Revolution in AMO physics

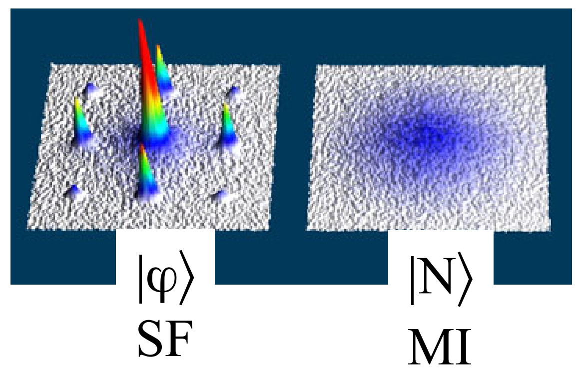


• degenerate Bose and Fermi atomic gases

• optical lattices

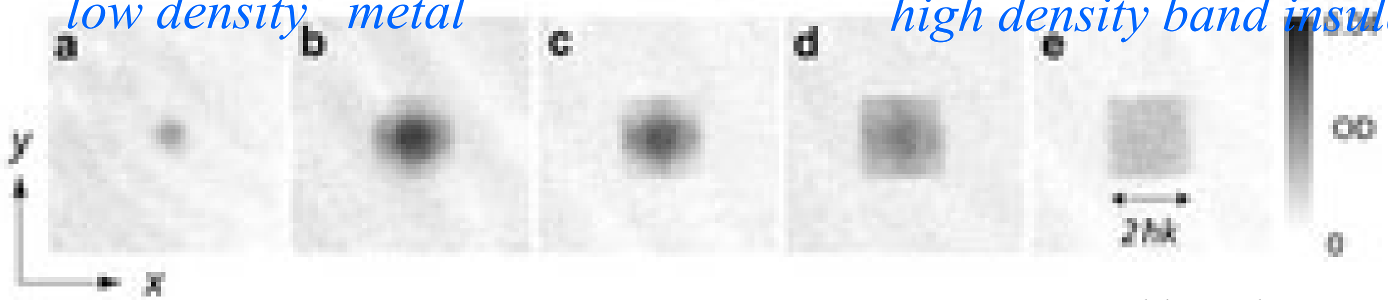


ac-Stark effect
(red-detuned, attractive)



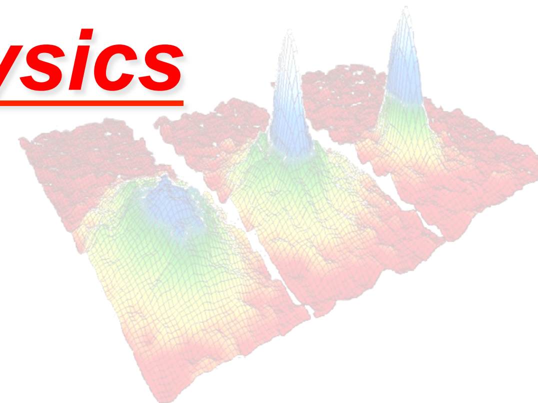
Greiner, et al.

low density "metal" *high density band insulator*



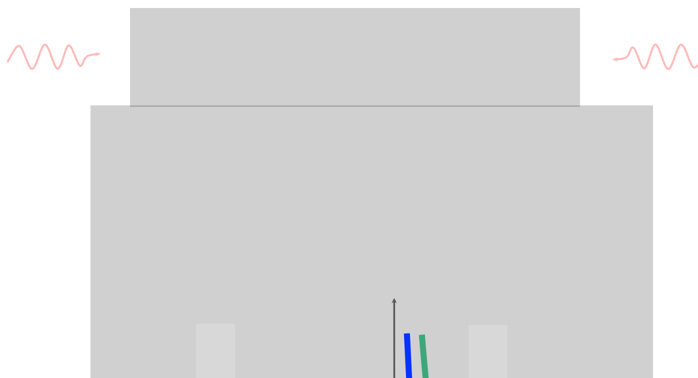
Kohl, Esslinger, et al. '05

Revolution in AMO physics

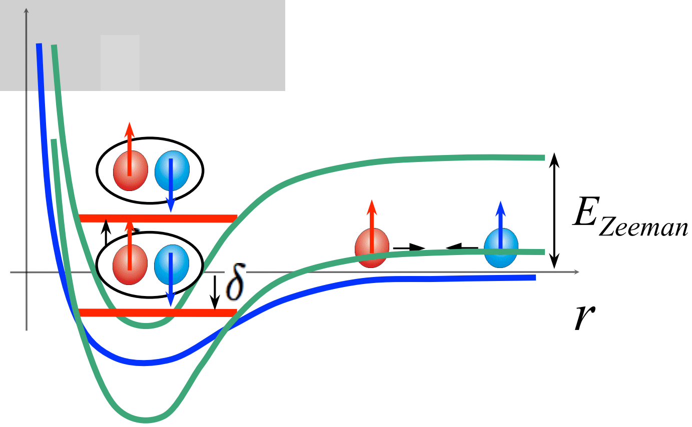


- degenerate Bose and Fermi atomic gases

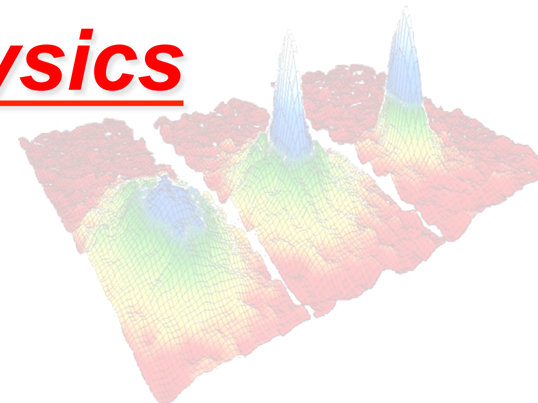
- optical lattices



- Feshbach resonance

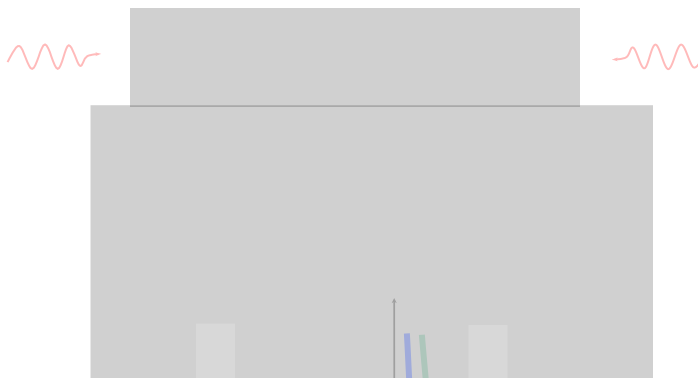


Revolution in AMO physics



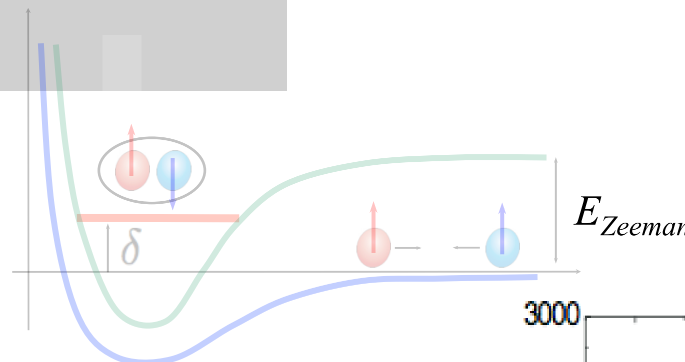
• degenerate Bose and Fermi atomic gases

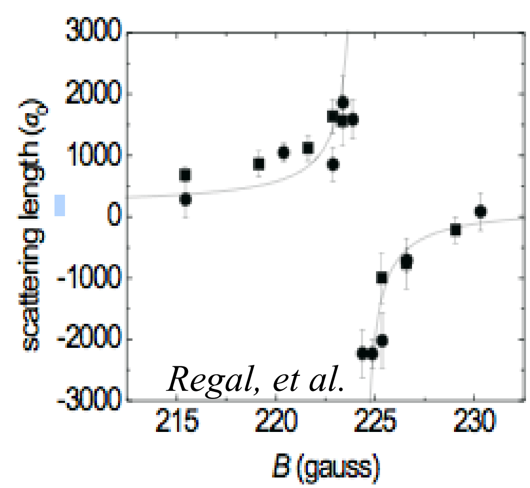
• optical lattices



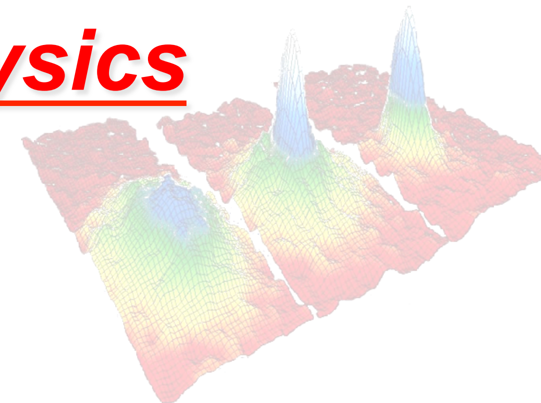
• Feshbach resonance

§ *weak to strong interactions*



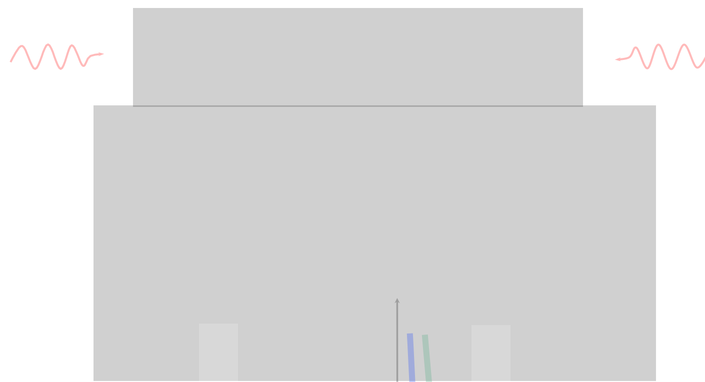


Revolution in AMO physics



- degenerate Bose and Fermi atomic gases

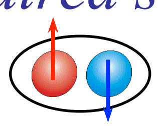
- optical lattices

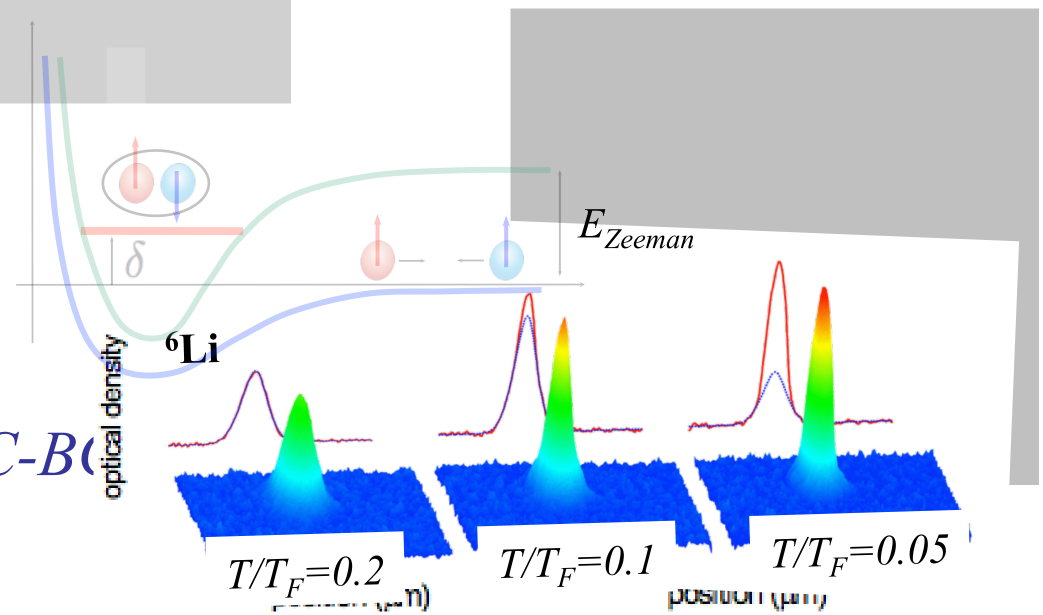


- Feshbach resonance

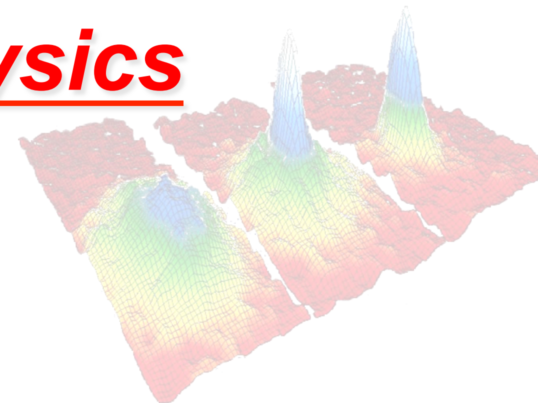
  § *weak to strong interactions*

  § *paired superfluidity and BEC-B*



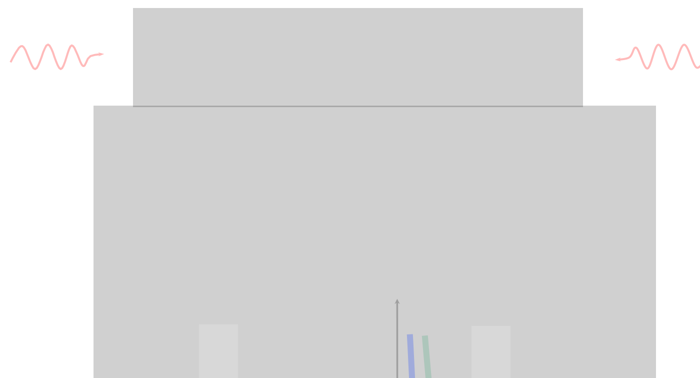


Revolution in AMO physics



- degenerate Bose and Fermi atomic gases

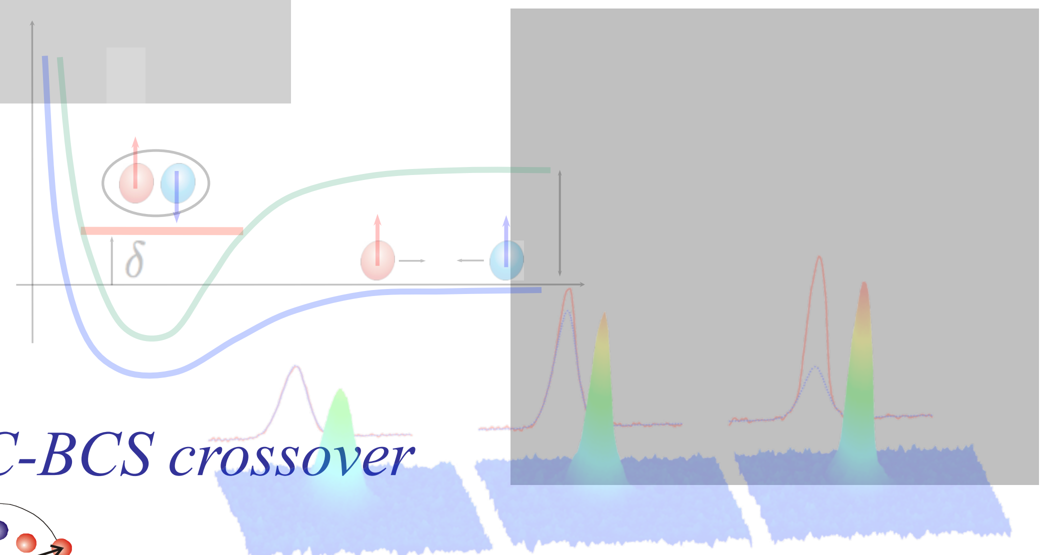
- optical lattices



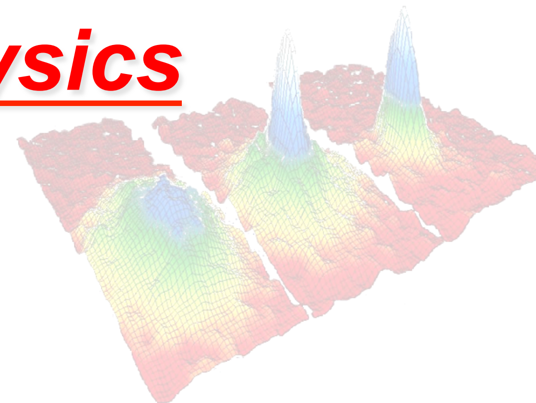
- **Feshbach resonance**

  § *weak to strong interactions*

  § *paired superfluidity and BEC-BCS crossover*

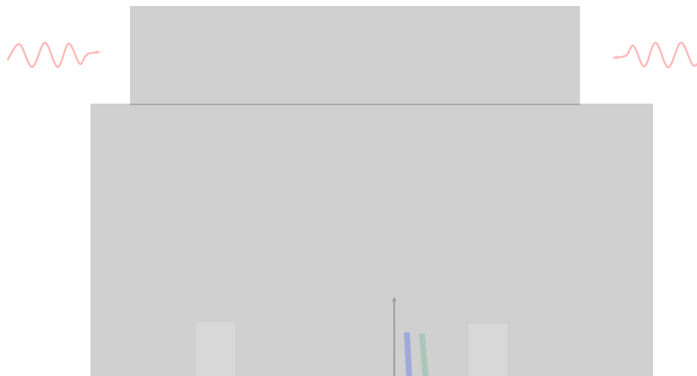


Revolution in AMO physics

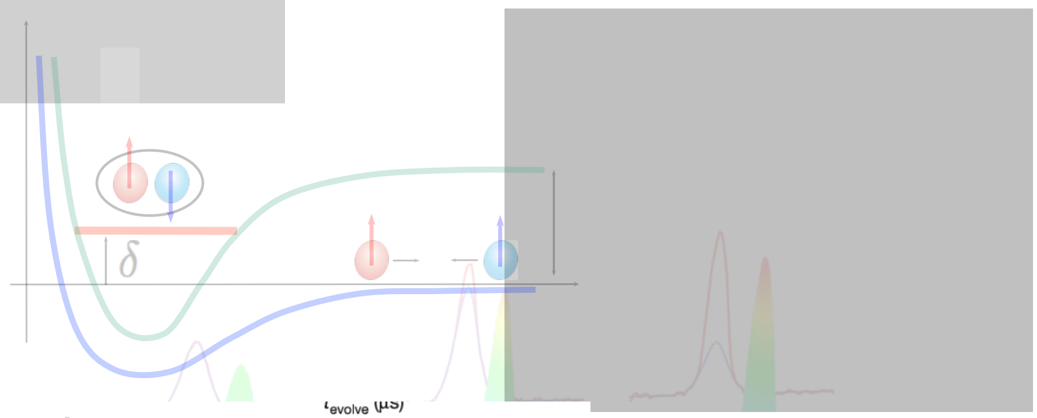


• degenerate Bose and Fermi atomic gases

• optical lattices



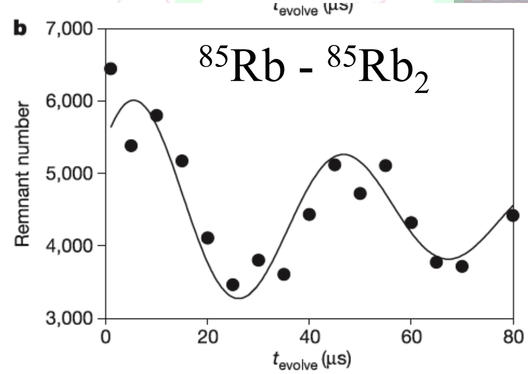
• Feshbach resonance

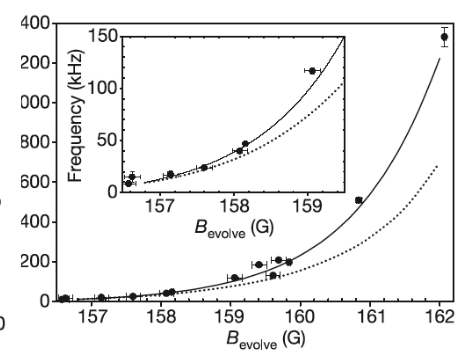


§ *weak to strong interactions*

§ *paired superfluidity and BEC-BCS cr*

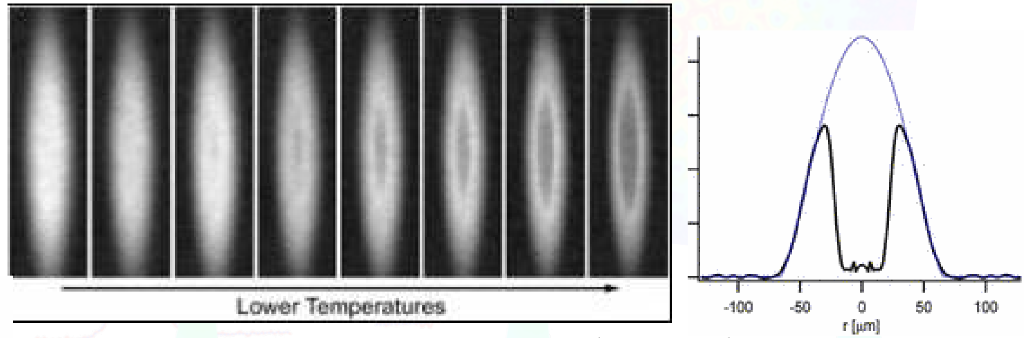
§ *quantum nonequilibrium CMP*



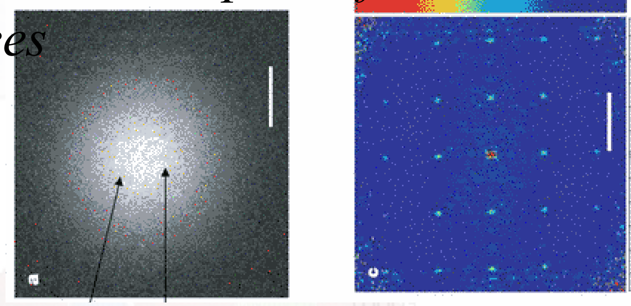


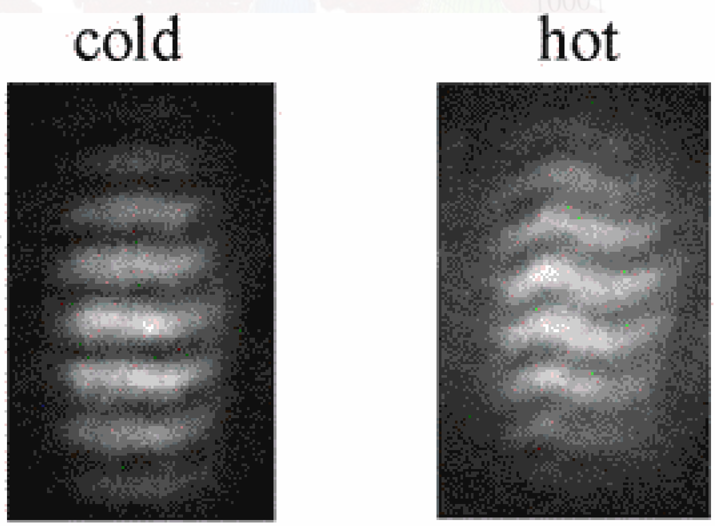
Variety of experimental probes

- Time-of-flight density imaging
  - § *momentum distribution function*
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *interference → phase fluctuations*
  - § *vortices*



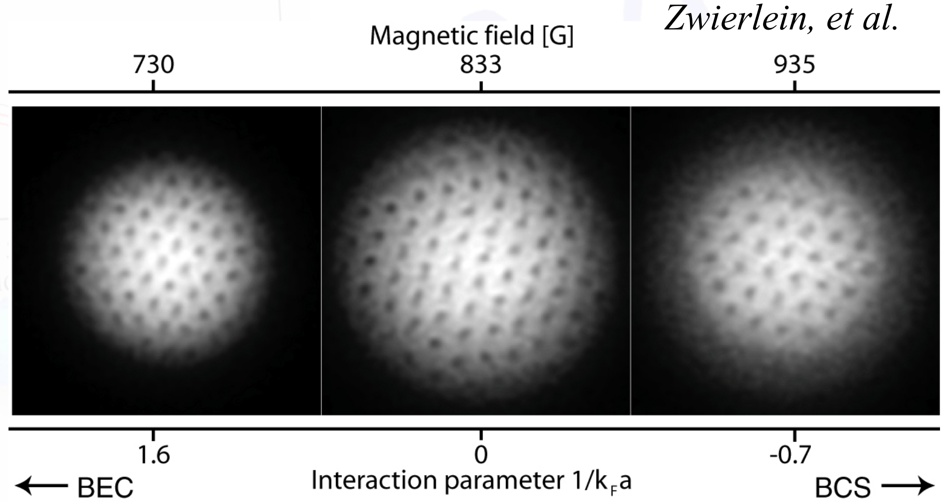
Zwierlein, et al.





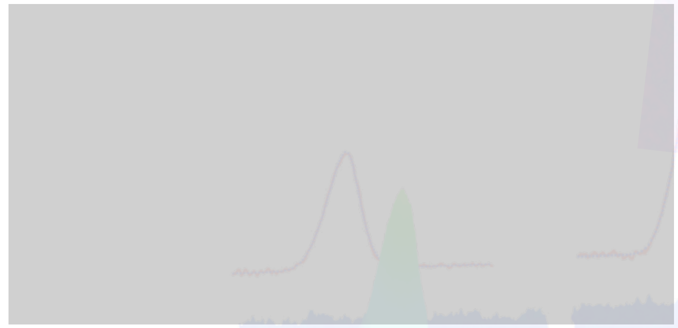
Hadzibabic, et al.

Zwierlein, et al.



Variety of experimental probes

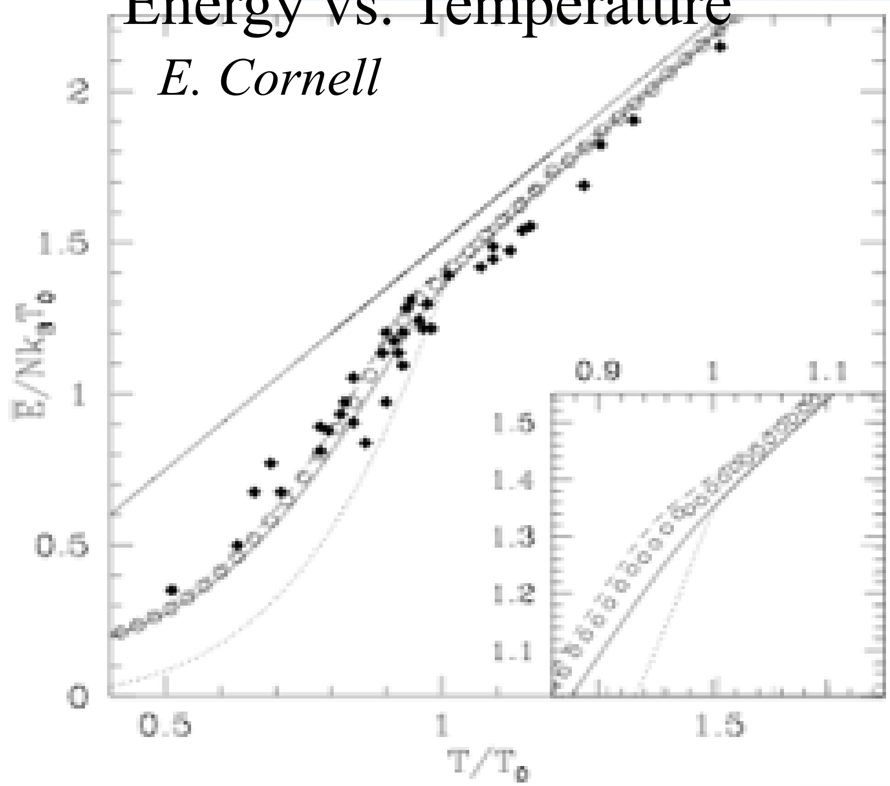
- Time-of-flight density imaging
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *quantum phase fluctuations*



- Thermodynamics

Energy vs. Temperature

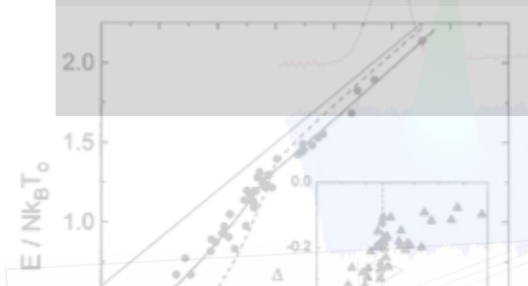
E. Cornell



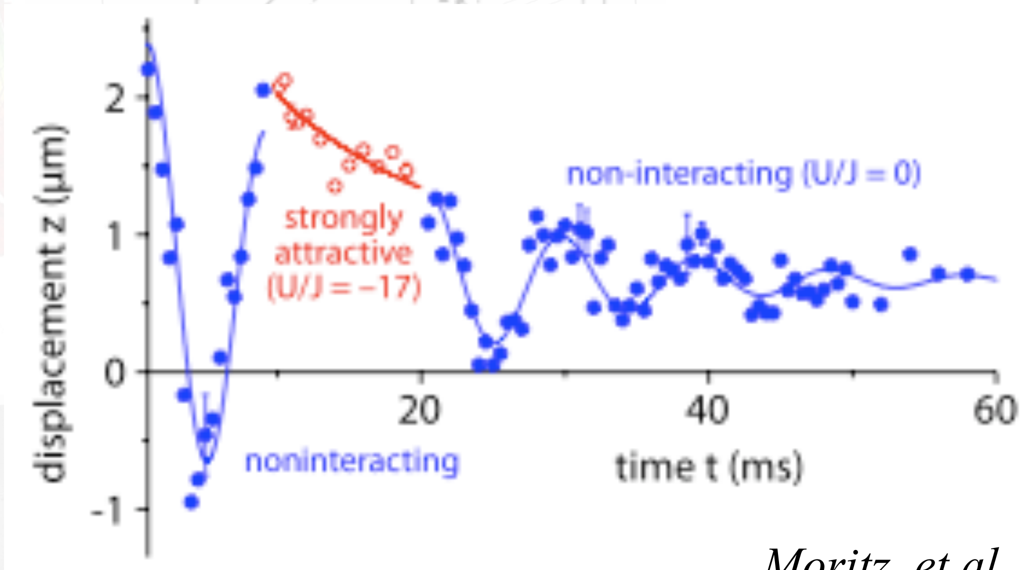
Variety of experimental probes

- Time-of-flight density imaging
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *quantum phase fluctuations*

- Thermodynamics



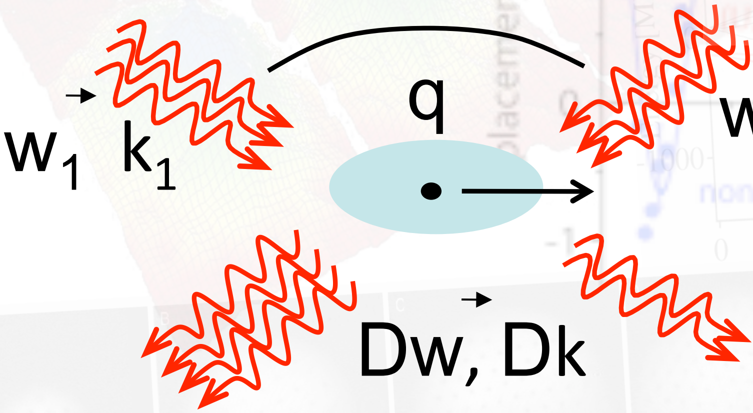
- Transport



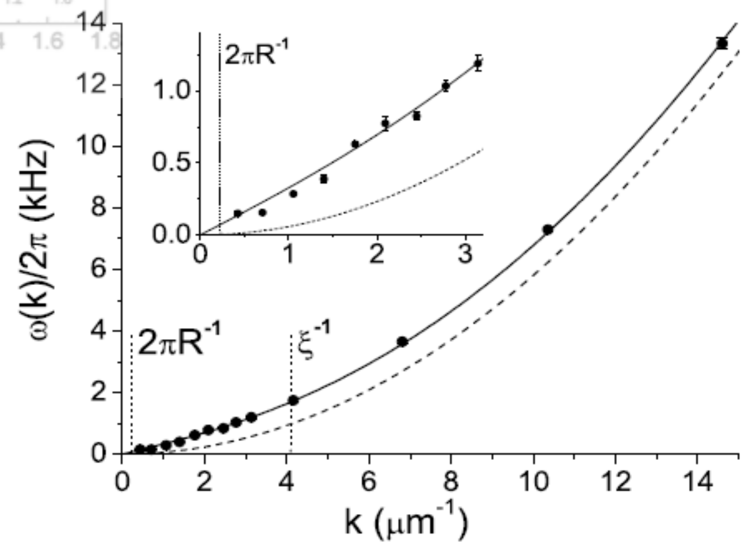
Moritz, et al.

Variety of experimental probes

- Time-of-flight density imaging
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *quantum phase fluctuations*
- Thermodynamics
- Transport
- Bragg spectroscopy: *collective excitations*



$Dk = 2\ k \sin q/2$



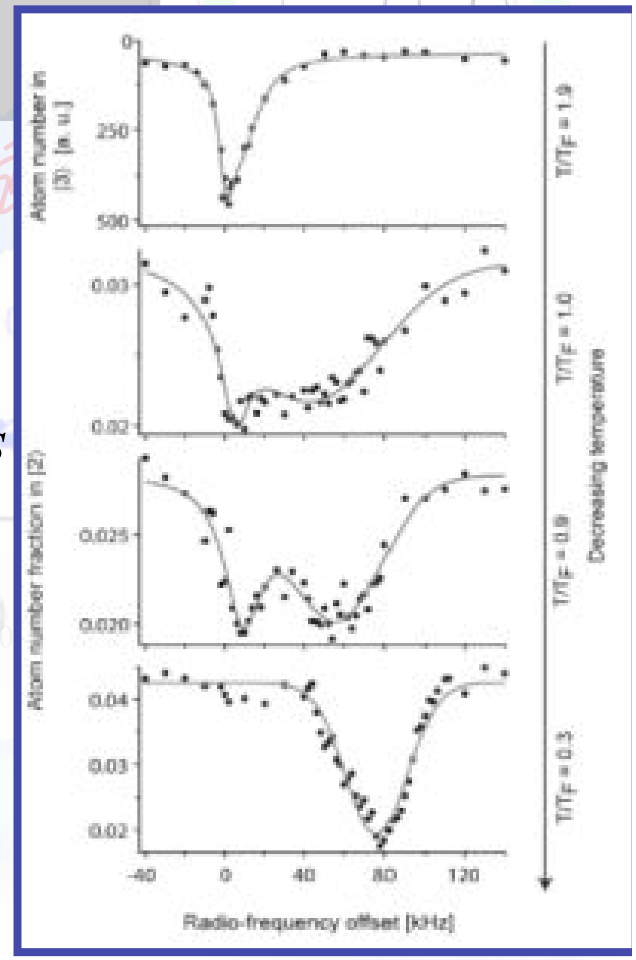
Steinhauer et al., *PRL* **88**, 2002

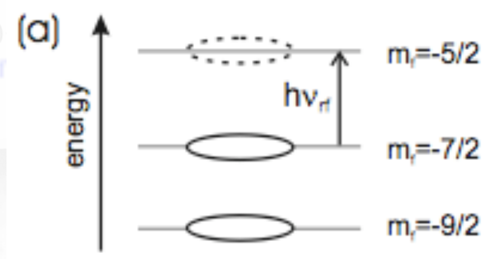
Variety of experimental probes

- Time-of-flight density imaging
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *quantum phase fluctuations*
- Thermodynamics
- Transport
- Bragg spectroscopy: *collective excitations*

- RF spectroscopy: *single atom excitations*

Schunck, et al.





Regal, Jin '03

Variety of experimental probes

- Time-of-flight density imaging
  - § *scattering length*
  - § *temperature*
  - § *noise → pairing correlations*
  - § *quantum phase fluctuations*
- Thermodynamics
- Transport
- Bragg spectroscopy: *collective excitations*
- RF spectroscopy: *single atom excitations*
- k-resolved photoemission

Stewart, et al